Language: 日本語/Japanese

KEK 物理セミナー Physics Seminar

6月4日 (水) June 4 (Wed)

15:00 ~ 16:00

4号館 345号室 Room 345 in the 4th Building

ハイゼンベルク不確定性原理再考
Reformulation of the Heisenberg uncertainty principle

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- Abstract:
ハイゼンベルクの不確定性原理は，一般にはその意味を深く吟味しないまま、量子論の基礎的原理として漠然と受け入れられている。本講演では、不確定性の物理的意味とその適用範囲を一般的な測定理論の観点から再検討し，ハイゼンベルクの不確定性関係が成立するためにはどのような物理的前提が必要であるか，またその前提なしに一般的状況でも成立する不確定性関係とはどのようなものかを議論する。

* 参考: http://www.math.is.tohoku.ac.jp/int/lab/ms3-j.html

http://www.math.is.tohoku.ac.jp/~ozawa/

M. Ozawa, Phys. Rev. A 67, 042105 (2003)

M. Ozawa, quant-ph/0210044

ハイゼンベルク不確定性原理再考

2003年6月4日・KEK 物理セミナー

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講演要旨

- ハイゼンベルクの不確定性原理の物理的意味
- ハイゼンベルクの不確定性原理の形式的表現
- 問題点：物理的意味と形式的表現の不一致
- 任意の測定に関する形式的表現
- 任意の測定に対して成立する新しい関係式
- 従来の関係式の適用範囲
- 応用：重力波の検出限界，量子計算素子精度の限界
- 展望

不確定性原理発見の歴史的背景

１９２５： ハイゼンベルク行列力学

１９２６： シュレーディンガー方程式

１９２６： 波動関数の物理的意味に関する論争

１９２６： ボルン確率解釈

１９２７： ハイゼンベルク不確定性原理

- シュレーディンガーの波動関数の実在的解釈を批判
- 正準交換関係の物理的意味
- アインシュタインの影響「観測の可能性はその理論自身が決める」

ハイゼンベルクの論文

ハイゼンベルク，量子論的運動学および力学の直感的内容について，**Z. Phys. 43, 172**（１９２７）.

1. ガンマ線顕微鏡の思考実験
2. 不確定性原理は交換関係 $QP - PQ = i\hbar$ の直接の数学的帰結である
3. ガウス型波動関数の位置空間での広がりと運動量空間での広がりの関係

ケナード **(Kennard)**（１９２７）：
標準偏差の導入，任意の波動関数に一般化

ロバートソン（**Robertson**）（１９２９）：
任意の物理量に一般化

I.3 THE PHYSICAL CONTENT OF QUANTUM KINEMATICS AND MECHANICS

WERNER HEISENBERG

First we define the terms *velocity*, *energy*, etc. (for example, for an electron) which remain valid in quantum mechanics. It is shown that canonically conjugate quantities can be determined simultaneously only with a characteristic indeterminacy (§1). This indeterminacy is the real basis for the occurrence of statistical relations in quantum mechanics. Its mathematical formulation is given by the Dirac-Jordan theory (§2). Starting from the basic principles thus obtained, we show how microscopic processes can be understood by way of quantum mechanics (§3). To illustrate the theory, a few special *gedankenexperiments* are discussed (§4).

When one wants to be clear about what is to be understood by the words "position of the object," for example of the electron (relative to a given frame of reference), then one must specify definite experiments with whose help one plans to measure the "position of the electron"; otherwise this word has no meaning. There is no shortage of such experiments, which in principle even allow one to determine the "position of the electron" with arbitrary accuracy. For example, let one illuminate the electron and observe it under a microscope. Then the highest attainable accuracy in the measurement of position is governed by the wavelength of the light. However, in principle one can build, say, a γ-ray microscope and with it carry out the determination of position with as much accuracy as one wants. In this measurement there is an important feature, the Compton effect. Every observation of scattered light coming from the electron presupposes a photoelectric effect (in the eye, on the photographic plate, in the photocell) and can therefore also be so interpreted that a light quantum hits the electron, is reflected or scattered, and then, once again bent by the lens of the microscope, produces the photoeffect. At the instant when position is determined—therefore, at the moment when the photon is scattered by the electron—the electron undergoes a discontinuous change in momentum. This change is the greater the smaller the wavelength of the light employed—that is, the more exact the determination of the position. At the instant at which the position of the electron is known, its momentum therefore can be known up to magnitudes which correspond to that discontinuous change. Thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely. In this circumstance we see a direct physical interpretation of the equation $\mathbf{pq} - \mathbf{qp} = -i\hbar$. Let q_1 be the precision with which the value q is known (q_1 is, say, the mean error of q), therefore here the wavelength of the light. Let p_1 be the precision with which the value p is determinable; that is, here, the discontinuous change of p in the Compton effect. Then, according to the elementary laws of the Compton effect p_1 and q_1 stand in the relation

$$p_1 q_1 \sim h. \tag{1}$$

That this relation (1) is a straightforward mathematical consequence of the rule $\mathbf{pq} - \mathbf{qp} = -i\hbar$ will be shown below. Here we can note that equation (1) is a precise expression for the facts which one earlier sought to describe by the division of phase space into cells of magnitude h. For the determination of the position of the electron one can also do other experiments—for example, collision experiments. A precise measurement of the position demands collisions with very fast particles, because for slow electrons the diffraction phenomena—which, according to Einstein, are consequences of de Broglie waves (as, for example, in the Ramsauer effect)—prevent a sharp specification of location. In a precise measurement of position the momentum of the electron again changes discontinuously. An elementary estimate of the precision using the formulas for de Broglie waves leads once more to relation (1).

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unknown (for example, electrodynamics), equation (1) makes a demand which may be useful for the discovery of the new laws. For quantum mechanics equation (1) can be derived from the Dirac-Jordan formulation by a slight generalization. If, for any definite state variable η we determine the position q of the electron as q' with an uncertainty q_1, then we can express this fact by a probability amplitude $S(\eta, q)$ which differs appreciably from zero only in a region of spread q_1 near q'. For example, one can write

$$S(\eta, q) \text{ proportional to } \exp[-(q - q')^2/2q_1^2 - (ip'/\hbar)(q - q')], \tag{3a}$$

with therefore

$$S\bar{S}' \text{ proportional to } \exp[-(q - q')^2/q_1^2]. \tag{3b}$$

Then for the probability amplitude for any given value of p we have

$$S(\eta, p) = \int S(\eta, q)S(q, p)\,dq. \tag{4}$$

For $S(q, p)$, according to Jordan, we can write

$$S(q, p) = \exp(ipq/\hbar). \tag{5}$$

Then, according to (4), $S(\eta, p)$ differs appreciably from zero only for values of p for which $(p - p')q_1/\hbar$ is not significantly greater than 1. Specifically, employing (3), we find $S(\eta, p)$ is proportional to

$$\int \exp[i(p - p')q/\hbar - (q - q')^2/2q_1^2]\,dq;$$

that is, proportional to

$$\exp[-(p - p')^2/2p_1^2 + iq'(p - p')/\hbar];$$

and thus

$$S\bar{S} \text{ is proportional to } \exp[-(p - p')^2/p_1^2],$$

where

$$p_1q_1 = \hbar. \tag{6}$$

The assumption (3) for $S(\eta, q)$ corresponds therefore to the experimental fact that the value p' is measured for p and the value q' for q [with the limit (6) on the precision].

From the purely mathematical point of view it is characteristic of the Dirac-

ermöglichen, als es der Gleichung (1) entspricht, so wäre die Quantenmechanik unmöglich. Diese Ungenauigkeit, die durch Gleichung (1) festgelegt ist, schafft also erst Raum für die Gültigkeit der Beziehungen, die in den quantenmechanischen Vertauschungsrelationen

$$pq - qp = \frac{h}{2\pi i}$$

ihren prägnanten Ausdruck finden; sie ermöglicht diese Gleichung, ohne daß der physikalische Sinn der Größen p und q geändert werden mußte.

Für diejenigen physikalischen Phänomene, deren quantentheoretische Formulierung noch unbekannt ist (z. B. die Elektrodynamik), bedeutet Gleichung (1) eine Forderung, die zum Auffinden der neuen Gesetze nützlich sein mag. Für die Quantenmechanik läßt sich Gleichung (1) durch eine geringfügige Verallgemeinerung aus der Dirac-Jordanschen Formulierung herleiten. Wenn wir für den bestimmten Wert η irgend eines Parameters den Ort q des Elektrons zu q' bestimmen mit einer Genauigkeit q_1, so können wir dieses Faktum durch eine Wahrscheinlichkeitsamplitude $S(\eta, q)$ zum Ausdruck bringen, die nur in einem Gebiet der ungefähren Größe q_1 um q' von Null merklich verschieden ist. Insbesondere kann man z. B. setzen

$$S(\eta, q) \text{ prop } e^{-\frac{(q-q')^2}{2q_1^2} - \frac{2\pi i}{h}p'(q-q')}, \text{ also } S\bar{S} \text{ prop } e^{-\frac{(q-q')^2}{q_1^2}}. \quad (3)$$

Dann gilt für die zu p gehörige Wahrscheinlichkeitsamplitude

$$S(\eta, p) = \int S(\eta, q) S(q, p)\, dq. \quad (4)$$

Für $S(q, p)$ kann nach Jordan gesetzt werden

$$S(q, p) = e^{\frac{2\pi i p q}{h}}. \quad (5)$$

Dann wird nach (4) $S(\eta, p)$ nur für Werte von p, für welche $\frac{2\pi(p-p')q_1}{h}$ nicht wesentlich größer als 1 ist, merklich von Null verschieden sein. Insbesondere gilt im Falle (3):

$$S(\eta, p) \text{ prop} \int e^{\frac{2\pi i(p-p')q}{h} - \frac{(q'-q)^2}{2q_1^2}} dq,$$

d. h.

$$S(\eta, p) \text{ prop } e^{-\frac{(p-p')^2}{2p_1^2} + \frac{2\pi i}{h}q'(p-p')}, \text{ also } S\bar{S} \text{ prop } e^{-\frac{(p-p')^2}{p_1^2}},$$

wo

$$p_1 q_1 = \frac{h}{2\pi}. \quad (6)$$

This excerpt from Heisenberg's original paper shows his derivation of the uncertainty relation for momentum and position.

粒子の位置(x 座標)を測定するために，$+x$ 方向に波長 λ (運動量 h/λ)の光子を入射させ，散乱光を顕微鏡で観測する(図 1-b)．そのためには，図の O 点で散乱された光子は角度 2α の範囲(図の OA と OB の間)に入らなければならない．その OA と OB の間の散乱光の波長が λ' から λ'' までで，反跳を受ける粒子の運動量の x 成分が p'_x から p''_x までの範囲の値をとるとすれば，観測以前の粒子の運動量を p_x として，x 方向の運動量保存則は

$$\frac{h}{\lambda}+p_x=\frac{h}{\lambda'}\sin\alpha+p'_x=-\frac{h}{\lambda''}\sin\alpha+p''_x.$$

これより，測定後の粒子の運動量は

$$\Delta p_x=|p'_x-p''_x|=\frac{h}{\lambda'}\sin\alpha+\frac{h}{\lambda''}\sin\alpha\fallingdotseq 2\frac{h}{\lambda}\sin\alpha$$

の範囲で不確定である．

他方，こうして得られる粒子の x 座標の不確定性 Δx は，顕微鏡の分解能 $\lambda/2\varepsilon=\lambda/2\sin\alpha$ に等しいと考えられるので，

$\Delta x\,\Delta p_x=h.$

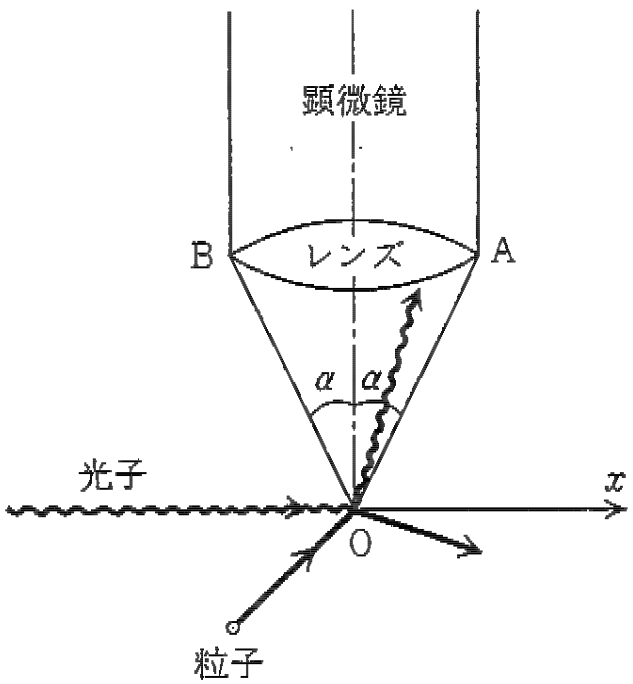


図 1-b

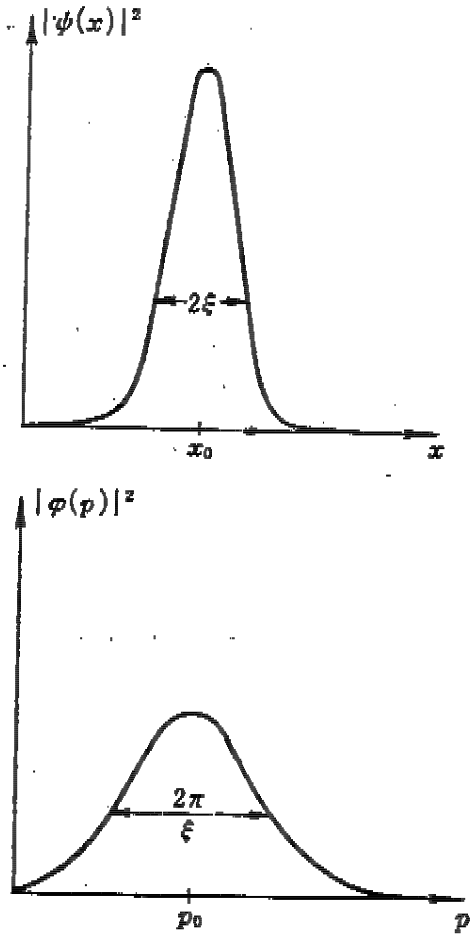
ガウス波束（1に規格化されていない）

$$\psi(x) = \exp\left[\frac{i}{\hbar} p_0 x - \frac{(x-x_0)^2}{2\xi^2}\right]$$

は点 x_0 のまわりの幅 ξ の領域を占める．これに対応する運動量空間の波

$$\varphi(p) = \frac{\xi}{\sqrt{\hbar}} \exp\left[\frac{i}{\hbar} x_0(p_0-p) - \frac{1}{2}\frac{\xi^2}{\hbar^2}(p-p_0)^2\right]$$

はこの空間のなかで点 p_0 のまわりの幅 $\hbar/\xi$ の領域を占める．ξ を小さくすると，Δx は小さくなるが，それにつれて Δp は大きくなり，それらの積は $\hbar$ 程度に保たれる．



(a)

Box 2.4: The Heisenberg uncertainty principle

Perhaps the best known result of quantum mechanics is the *Heisenberg uncertainty principle*. Suppose A and B are two Hermitian operators, and $|\psi\rangle$ is a quantum state. Suppose $\langle\psi|AB|\psi\rangle = x + iy$, where x and y are real. Note that $\langle\psi|[A,B]|\psi\rangle = 2iy$ and $\langle\psi|\{A,B\}|\psi\rangle = 2x$. This implies that

$$|\langle\psi|[A,B]|\psi\rangle|^2 + |\langle\psi|\{A,B\}|\psi\rangle|^2 = 4\,|\langle\psi|AB|\psi\rangle|^2 . \tag{2.105}$$

By the Cauchy–Schwarz inequality

$$|\langle\psi|AB|\psi\rangle|^2 \leq \langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle, \tag{2.106}$$

which combined with Equation (2.105) and dropping a non-negative term gives

$$|\langle\psi|[A,B]|\psi\rangle|^2 \leq 4\langle\psi|A^2|\psi\rangle\langle\psi|B^2|\psi\rangle. \tag{2.107}$$

Suppose C and D are two observables. Substituting $A = C - \langle C\rangle$ and $B = D - \langle D\rangle$ into the last equation, we obtain Heisenberg's uncertainty principle as it is usually stated:

$$\Delta(C)\Delta(D) \geq \frac{|\langle\psi|[C,D]|\psi\rangle|}{2} . \tag{2.108}$$

You should be wary of a common misconception about the uncertainty principle, that measuring an observable C to some 'accuracy' $\Delta(C)$ causes the value of D to be 'disturbed' by an amount $\Delta(D)$ in such a way that some sort of inequality similar to (2.108) is satisfied. While it is true that measurements in quantum mechanics cause disturbance to the system being measured, this is most emphatically *not* the content of the uncertainty principle.

The correct interpretation of the uncertainty principle is that if we prepare a large number of quantum systems in identical states, $|\psi\rangle$, and then perform measurements of C on some of those systems, and of D in others, then the standard deviation $\Delta(C)$ of the C results times the standard deviation $\Delta(D)$ of the results for D will satisfy the inequality (2.108).

不確定性原理に関する混乱

A. ガンマ線顕微鏡における分解能と運動量の擾乱の関係：

（分解能）×（運動量の擾乱）$= h$

B. 位置測定の精度と運動量の擾乱の関係：

（位置の測定精度）×（運動量の擾乱）$\geq \frac{\hbar}{2}$

$\varepsilon(A) \times \eta(B) \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$

C. 位置と運動量の標準偏差の関係：

（位置の標準偏差）×（運動量の標準偏差）$\geq \frac{\hbar}{2}$

$\sigma(A) \times \sigma(B) \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$

C は **Schwarz** の不等式から証明可能．では，**B** は **C** から証明可能か？

ハイゼンベルクは **A** を例示して，**B** を主張したが，実際に証明されたのは，**C** である．

測定精度と擾乱の不確定性関係の問題点

1.「$\eta(P) = 0$ ならば $\varepsilon(Q) = \infty$」という関係は疑問. 何も相互作用をかけなくても初期状態の不確定性程度の精度で測定は出来る.

2.「$\varepsilon(Q) = 0$ ならば $\eta(P) = \infty$」という関係も疑問. 有限のエネルギーで正確な測定が可能.

測定の数学的表現

量子力学において，

状態＝密度作用素（または，状態ベクトル）

物理量＝自己共役作用素

と数学的に表現される．それでは，測定は何に対応するだろうか？

- 測定結果の確率分布
- 測定による状態変化

の数学的特徴付けが必要．

測定結果の確率分布

物理量⇒自己共役作用素⇒スペクトル分解⇒射影作用素

$$\mathbf{Pr}\{\mathbf{x} = x\} = \langle \psi, \boldsymbol{E}^{\mathbf{A}}(x)\psi \rangle$$

射影作用素を正値作用素に一般化することができる.

$$\mathbf{Pr}\{\mathbf{x} = x\} = \langle \psi, \boldsymbol{F}(x)\psi \rangle$$

$\{\boldsymbol{F}(x) |\ x \in \mathbf{R}\}$ が確率作用素値測度(POVM) であるとは,

(i) $\mathbf{0} \leq \boldsymbol{F}(x) \leq \boldsymbol{I}$

(ii) $\sum_{\mathbf{x}} \boldsymbol{F}(x) = \boldsymbol{I}$

測定結果の確率分布は, 一般に確率作用素値測度(POVM) で表現されるが, POVM では測定に伴う状態変化が表現されない.

測定結果の確率分布と測定値に依存する状態変化を共に表現する数学的対象は次のように表される.

オペレーション＝完全正写像

状態変化の一般的表現

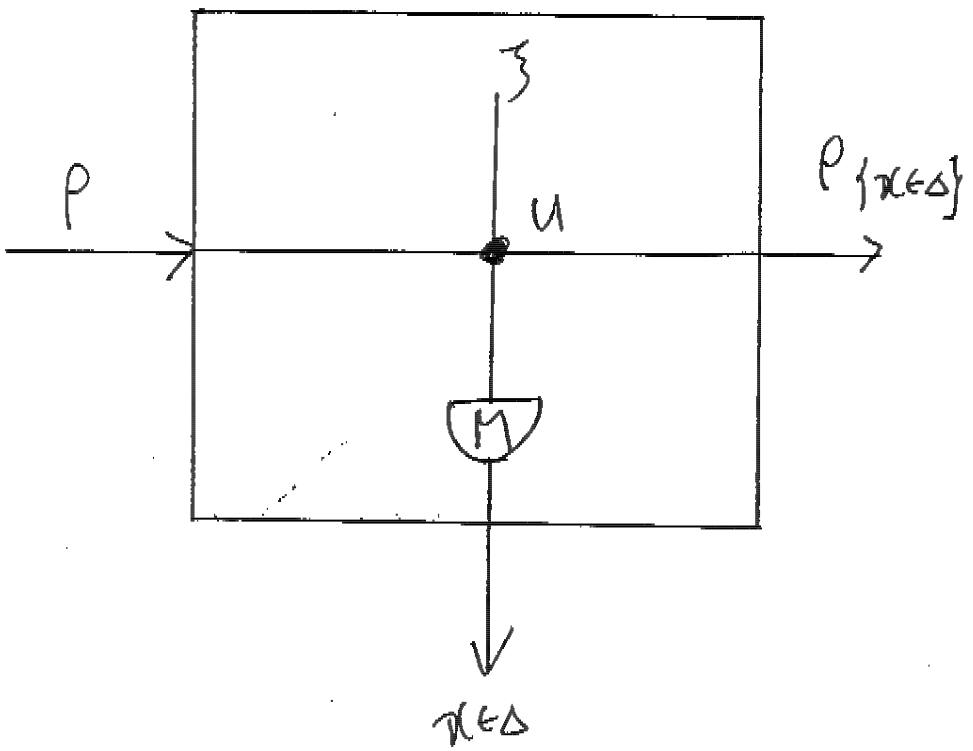
形式的定義：

T：$\boldsymbol{\rho \mapsto T\rho}$，密度作用素 ρ を正値作用素 $T\rho$ に変換する写像（正写像）で，$0 \leq \mathrm{Tr}[T\rho] \leq 1$ を満たし，外部の系との合成系にも拡大できるもの（完全性写像）．

$\mathrm{T} \otimes \boldsymbol{id} : \boldsymbol{\rho \otimes \rho^0 \mapsto T\rho \otimes \rho^0}$

物理的解釈：

確率 $\mathrm{Tr}[T\rho]$ で，状態 ρ は状態 $T\rho/\mathrm{Tr}[\rho]$ に変化する．


ξ
ρ
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ρ{x∈Δ}
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測定＝正規化された完全正写像値測度

$\{\mathcal{I}(x)|\ x \in \mathbf{R}\}$：正規化され完全正写像値測度

(インストルメント，**CP-instrument**)

形式的定義：

(i) 各 $x \in \mathbf{R}$ に対して，$\mathcal{I}(x)$ はオペレーション．

(ii) $\mathrm{Tr}[\sum_x \mathcal{I}(x)\rho] = \mathrm{Tr}[\rho]$.

物理的解釈：

(i) 測定値の確率分布：$\mathbf{Pr}\{\mathbf{x} = x\} = \mathrm{Tr}[\mathcal{I}(x)\rho]$.

(ii) 測定による状態変化：

$$\rho_{\{x=x\}} = \frac{\mathcal{I}(x)\rho}{\mathrm{Tr}[\mathcal{I}(x)\rho]}.$$

間接測定のモデル

$(\mathcal{K}, \xi, U, M)$：間接測定モデル

(i) $\mathcal{K}$＝プローブ系の状態空間

(ii) ξ＝プローブ系の初期状態を表す$\mathcal{K}$の単位ベクトル

(iii) U＝プローブ系と対象との相互作用を表す$\mathcal{H} \otimes \mathcal{K}$上のユニタリ作用素

(iv) M＝相互作用後に実際に測定されるプローブ系の物理量を表す $\mathcal{K}$ 上の自己共役作用素

モデル$(\mathcal{K}, \xi, U, M)$のインストルメント：

$$\mathcal{I}(x)\rho = \mathrm{Tr}_{\mathcal{K}}\{[I \otimes E^M(x)]U(\rho \otimes |\xi\rangle\langle\xi|)U^\dagger\}$$

定理：測定モデルの統計的同値類と正規化された完全正写像値測度は一対一に対応する．[JMP25(1984)79]

つまり，抽象的なインストルメント $\{\mathcal{I}(x)|\ x \in \mathbf{R}\}$ が与えられると，必ず，間接測定モデル $(\mathcal{K}, \xi, U, M)$ が構成できて，このモデルから計算された測定値の確率分布と状態変化は，インストルメントで抽象的に与えられたものと一致する．

したがって，インストルメント $\{\mathcal{I}(x)|\ x \in \mathbf{R}\}$ で記述される測定概念が，最も一般的かつ近似的実現可能な測定の全体を表す．

逆に，どんな測定も間接測定モデルで記述される測定と統計的に同等であるので，測定のモデルとしては間接測定モデルだけ考えれば十分である．

Quantum measuring processes of continuous observables

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The purpose of this paper is to provide a basis of theory of measurements of continuous observables. We generalize von Neumann's description of measuring processes of discrete quantum observables in terms of interaction between the measured system and the apparatus to continuous observables, and show how every such measuring process determines the state change caused by the measurement. We establish a one-to-one correspondence between completely positive instruments in the sense of Davies and Lewis and the state changes determined by the measuring processes. We also prove that there are no weakly repeatable completely positive instruments of nondiscrete observables in the standard formulation of quantum mechanics, so that there are no measuring processes of nondiscrete observables whose state changes satisfy the repeatability hypothesis. A proof of the Wigner–Araki–Yanase theorem on the nonexistence of repeatable measurements of observables not commuting conserved quantities is given in our framework. We also discuss the implication of these results for the recent results due to Srinivas and due to Mercer on measurements of continuous observables.

PACS numbers: 03.65.Bz, 02.50. + s

1. INTRODUCTION

In the last decade, some attempts were developed to construct a satisfactory theory of the quantum mechanical measurement of an observable with continuous spectrum.[1–9] However, we have found no satisfactory solution of the fundamental problem to determine the state changes caused by measurements of continuous observables. In spite of these difficulties in continuous spectrum, the theory for discrete spectrum has a conventionally accepted solution since the pioneering work of von Neumann.[10]

Let $A = \Sigma_i \lambda_i P_i$ be an observable with simple discrete spectrum $\lambda_1, \lambda_2, \cdots$. Then von Neumann[10] showed the following:

(1) By the repeatability hypothesis, the state change $\rho \rightarrow \rho'$ caused by the measurement of A is determined as $\rho' = \Sigma_i P_i \rho P_i$.

(2) The above state change $\rho \rightarrow \rho'$ is compatible with the Hamiltonian formalism in the description of the measuring process in terms of the time evolution of the composite system of the observed system and the measuring apparatus.

In the present paper, we shall show the following:

(1) The description of measuring processes has a satisfactory generalization to continuous observables.

(2) Every measuring process determines a state change caused by the measurement.

(3) There are no measuring processes of a nondiscrete observable whose state changes satisfy the repeatability hypothesis.

In order to clarify the present situation, we shall review some developments on the problem so far. In the early stage, Umegaki and Nakamura[11] showed that the state change $\rho \rightarrow \rho' = \Sigma_i P_i \rho P_i$ is just an example of Umegaki's noncommutative conditional expectations[12] onto the von Neumann algebra generated by A, and they conjectured that the state change caused by the measurement of a continuous observable would also be such a noncommutative conditional expectation. However, it is shown by Areveson[13] that such conditional expectations do not exist for continuous observables. In view of these results, Davies and Lewis[1] established the mathematical concept of instruments which enables us to treat statistical correlations of outcomes of successive measurements, and formulate the repeatability hypothesis for continuous observables. They conjectured the nonexistence of repeatable instruments for continuous observables and proposed the more flexible approach to measurements of continuous observables abandoning repeatability hypothesis. Recently, Srinivas[8] generalized the concept of instruments and showed the existence of such generalized instruments for continuous observables which satisfy the repeatability hypothesis. He proposed a generalized collapse postulate which determines such repeatable generalized instruments to describe the state changes caused by measurements of continuous observables. More recently, Mercer[9] considered a wider class of state transformations than conditional expectations and proposed the state change should be described by such a transformation with the locality introduced by him. It is a remarkable fact that these attempts are concerned only with the first half of von Neumann's work cited above. An operator theoretical analysis on von Neumann's second result was done by Kraus.[14] He established the complete positivity of state changes caused by the general measuring processes, but his result is concerned only with the yes–no measurements.

In this paper, we shall show that the state changes determined by measuring processes naturally correspond to completely positive instruments and vice versa. We prove Davies and Lewis's conjecture for completely positive instruments, i.e., completely positive instruments cannot be weakly repeatable unless the corresponding observable is discrete. These results show that Srinivas's generalized col-

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間接測定モデルの例：von Neumann のモデル

測定対象 S：位置 $\hat{x}$, 運動量 $\hat{p}_x$

装置 A： 位置 $\hat{y}$, 運動量 $\hat{p}_y$

プローブ物理量$=\hat{y}$

測定相互作用：

$$H_{\mathbf{SA}} = K\hat{x}\hat{p}_y$$

結合定数：K, 十分に大

測定時間$=(t, t+\Delta t)$, $K\Delta t = 1$

$$U = e^{-i\hat{x}\hat{p}_y/\hbar}$$

対象の初期状態： $\rho(t) = |\psi\rangle\langle\psi|$

プローブの初期状態：$\sigma = |\xi\rangle\langle\xi|$

$$\Psi = U(\psi \otimes \xi)$$

とおくと，測定直後の合成系の状態は

$$\rho(t + \Delta t)_{\mathbf{S+A}} = |\Psi\rangle\langle\Psi|$$

となる．**Schrödinger** 方程式を解くことにより，

$$\Psi(x, y) = \psi(x)\xi(y - x)$$

がえられる．測定値の確率分布はこの状態におけるプローブ物理量 $\hat{y}$ の確率分布であるから，

$$\Pr\{\mathbf{x} = y\} = \int_{\mathbf{R}} |\psi(x)|^2 |\xi(y - x)|^2 dx$$

である．畳み込みの性質から，これは関数 $|\xi(y)|^2$ が δ 関数に近付くにしたがって，**Born** の統計公式で予言される位置 $\hat{x}$ の確率密度関数 $|\psi(y)|^2$ に近付く．

測定精度の定義

状態 ψ において物理量 A を測定するために実際は物理量 B を測定したとする．$A-B$ の二乗平均平方根をこの測定の二乗平均平方根誤差と定義し，$\varepsilon(A)$ と書く．

$$\varepsilon(A) = \langle\psi|(A-B)^2|\psi\rangle^{1/2} = \|A\psi - B\psi\|.$$

測定モデル$(\mathcal{K}, \xi, U, M)$においては，状態 ψ において物理量 A を測定するために，状態 $\psi\otimes\xi$ において物理量 $M' = U^\dagger(I\otimes M)U$ を測定するので，$A\otimes I - M'$ の二乗平均平方根をこの測定モデルの二乗平均平方根誤差と定義し，$\varepsilon(A)$ と書く．

$$\varepsilon(A) = \|(A\otimes I)(\psi\otimes\xi) - M'(\psi\otimes\xi)\|.$$

定理：これはモデルによらず，統計的同値類に対して一定の値に定まる．

擾乱の定義

測定モデル$(\mathcal{K}, \xi, U, M)$において，対象の任意の物理量 B に対して，$B \otimes I - U^\dagger (B \otimes I) U$ の二乗平均平方根をこの測定による B の二乗平均平方根変化と定義し，$\eta(B)$ と書く．

$$\begin{aligned} \eta(B) &= \| (B \otimes I)\psi \otimes \xi - U^\dagger (B \otimes I) U (\psi \otimes \xi) \| \\ &= \| [U, B \otimes I](\psi \otimes \xi) \|. \end{aligned}$$

von Neumann の測定の精度と擾乱

Heisenberg の運動方程式を$t < t+\tau < t+\Delta t$ で解く：

$$\begin{aligned}
\hat{x}(t+\tau) &= \hat{x}(t), \\
\hat{y}(t+\tau) &= K\tau\hat{x}(t) + \hat{y}(t), \\
\hat{p}_x(t+\tau) &= \hat{p}_x(t) - K\tau\hat{p}_y(t), \\
\hat{p_y}(t+\tau) &= \hat{p}_y(t).
\end{aligned}$$

$\tau = \Delta t = 1/K$では：

$$\begin{aligned}
\hat{x}(t+\Delta t) &= \hat{x}(t), \\
\hat{y}(t+\Delta t) &= \hat{x}(t) + \hat{y}(t), \\
\hat{p}_x(t+\Delta t) &= \hat{p}_x(t) - \hat{p}_y(t), \\
\hat{p_y}(t+\Delta t) &= \hat{p}_y(t).
\end{aligned}$$

誤差作用素と擾乱作用素：

$$N(\hat{x}) = \hat{y}(t+\Delta t) - \hat{x}(t) = \hat{y}(t),$$

$$D(\hat{p}_x) = \hat{p}_x(t+\Delta t) - \hat{p}_x(t) = -\hat{p}_y(t).$$

位置測定の誤差と運動量の擾乱：

$$\varepsilon(\hat{x})^2 = \langle \hat{y}(t)^2 \rangle,$$

$$\eta(\hat{p}_x)^2 = \langle \hat{p}_y(t)^2 \rangle.$$

測定の時刻のプローブの位置と運動量の標準偏差を$\sigma(\hat{y})$ 及び $\sigma(\hat{p}_y)$ とすると：

$$\sigma(\hat{x})^2 = \langle \hat{y}(t)^2 \rangle - \langle \hat{y}(t) \rangle^2 \leq \varepsilon(\hat{x})^2,$$

$$\sigma(\hat{p}_x)^2 = \langle \hat{p}_y(t)^2 \rangle - \langle \hat{p}_y(t) \rangle^2 \leq \eta(\hat{p}_x)^2.$$

Robertson の不確定性関係から,

$$\varepsilon(\hat{x})\eta(\hat{p}_x) \geq \sigma(\hat{y})\sigma(\hat{p}_y) \geq \frac{\hbar}{2}.$$

したがって, **von Neumann** のモデルでは, 測定精度と擾乱に関する不確定性原理が成立する.

不確定性原理を打ち破るモデル

被測定系 S ： 位置 $\hat{x}$, 運動量 $\hat{p}_x$

装置 A ：位置 $\hat{y}$, 運動量 $\hat{p}_y$

プローブ物理量$=\hat{y}$

測定相互作用：

$$H_{\mathbf{SA}} = \frac{K\pi}{3\sqrt{3}}\{2(\hat{x}\hat{p}_y - \hat{p}_x\hat{y}) + (\hat{x}\hat{p}_x - \hat{y}\hat{p}_y)\} \qquad (1)$$

結合定数： K は十分に大

測定時間$=$ $(t, t+\Delta t)$, $K\Delta t = 1$

$$U = \exp\frac{-i\pi}{3\sqrt{3}\hbar}\{2(\hat{x}\hat{p}_y - \hat{p}_x\hat{y}) + (\hat{x}\hat{p}_x - \hat{y}\hat{p}_y)\} \qquad (2)$$

対象の初期状態： $\rho(t) = |\psi\rangle\langle\psi|$

プローブの初期状態： $\sigma = |\xi\rangle\langle\xi|$

測定直後の合成系の状態を

$$\rho(t+\Delta t)_{\mathbf{S+A}} = |\Psi\rangle\langle\Psi|$$

とする．**Schrödinger** 方程式を解くことにより，

$$\Psi(x,y) = \psi(y)\xi(y-x) \tag{3}$$

となる．したがって，測定値の確率密度は

$$\Pr\{\mathbf{x}(t) = y\} = \int_{\mathbf{R}} |\psi(y)|^2 |\xi(y-x)|^2 dx = |\psi(y)|^2 \tag{4}$$

である．これは，**Born** の統計公式で予言される位置 $\hat{x}$ の確率密度関数 $|\psi(y)|^2$ に一致するので，この測定は対象の位置の正確な測定である．

Heisenberg の運動方程式を$t < t+\tau < t+\Delta t$ で解く：

$$\hat{x}(t+\tau) = \frac{2}{\sqrt{3}}\hat{x}(t)\sin\frac{(1+K\tau)\pi}{3} + \frac{-2}{\sqrt{3}}\hat{y}(t)\sin\frac{K\tau\pi}{3},$$

$$\hat{y}(t+\tau) = \frac{2}{\sqrt{3}}\hat{x}(t)\sin\frac{K\tau\pi}{3} + \frac{2}{\sqrt{3}}\hat{y}(t)\sin\frac{(1-K\tau)\pi}{3},$$

$$\hat{p}_x(t+\tau) = \frac{2}{\sqrt{3}}\hat{p}_x(t)\sin\frac{(1-K\tau)\pi}{3} + \frac{-2}{\sqrt{3}}\hat{p}_y(t)\sin\frac{K\tau\pi}{3},$$

$$\hat{p}_y(t+\tau) = \frac{2}{\sqrt{3}}\hat{p}_x(t)\sin\frac{K\tau\pi}{3} + \frac{2}{\sqrt{3}}\hat{p}_y(t)\sin\frac{(1+K\tau)\pi}{3}.$$

$\tau = \Delta t = 1/K$では：

$$\begin{aligned}
\hat{x}(t+\Delta t) &= \hat{x}(t) - \hat{y}(t), \\
\hat{y}(t+\Delta t) &= \hat{x}(t), \\
\hat{p}_x(t+\Delta t) &= -\hat{p}_y(t), \\
\hat{p}_y(t+\Delta t) &= \hat{p}_x(t) + \hat{p}_y(t).
\end{aligned}$$

誤差作用素と擾乱作用素：

$$\begin{aligned}
N(\hat{x}) &= \hat{y}(t+\Delta t) - \hat{x}(t) = 0, \\
D(\hat{p}_x) &= \hat{p}_x(t+\Delta t) - \hat{p}_x(t) = -\hat{p}_y(t) - \hat{p}_x(t).
\end{aligned}$$

位置測定の誤差と運動量の擾乱：

$$\begin{aligned}
\varepsilon(\hat{x}) &= 0, \\
\eta(\hat{p}_x)^2 &= \langle [\hat{p}_x(t) + \hat{p}_y(t)]^2 \rangle \\
&= \sigma(\hat{p}_x)^2 + \sigma(\hat{p}_y)^2 + [\langle \hat{p}_x(t) \rangle + \langle \hat{p}_y(t) \rangle]^2.
\end{aligned}$$

したがって，

$$\varepsilon(\hat{x})\eta(\hat{p}_x) = 0. \tag{5}$$

よって，このモデルはハイゼンベルクの関係を満たさない．

正しい不確定性関係

定理. 任意の測定モデル (ξ, U, M) に対して,

$$\varepsilon(A)\eta(B) + \varepsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2}|\langle[A, B]\rangle|$$

が成立する. $\varepsilon(A)$ と $\eta(B)$ が ψ によらずに一定ならば,

$$\varepsilon(A)\eta(B) \geq \frac{1}{2}|\langle[A, B]\rangle|$$

が成立する. **[MO, PRA (in press), quant-ph/0207121]**

証明

$[M(\Delta t), P(\Delta t)] = 0.$

$M(\Delta t) = Q(0) + N(Q)$

$P(\Delta t) = P(0) + D(P)$

$[Q, P] = i\hbar$

より,

$$[N(Q), D(P)] + [N(Q), P(0)] + [Q(0), D(P)] = -i\hbar.$$

絶対値を取って，三角不等式より,

$$|\langle [N(Q), D(P)] \rangle| + |\langle [N(Q), P(0)] \rangle| + |\langle [Q(0), D(P)] \rangle| \geq \hbar.$$

Robertsonの不確定性関係から,

$$\begin{aligned}
\varepsilon(Q)\eta(P) &\geq \sigma[N(Q)]\sigma[D(P)] \geq \frac{1}{2}|\langle[N(Q), D(P)]\rangle|, \\
\varepsilon(Q)\sigma(P) &\geq \sigma[N(Q)]\sigma[P(0)] \geq \frac{1}{2}|\langle[N(Q), D(P)]\rangle|, \\
\sigma(Q)\eta(P) &\geq \sigma[Q(0)]\sigma[D(P)] \geq \frac{1}{2}|\langle[Q(0), D(P)]\rangle|.
\end{aligned}$$

以上を代入すればよい.

Heisenbeg 不確定性原理の破れ

1．（I型の破れ）入力状態によらずに $\eta(B) = 0$ となる場合：

$$\varepsilon(A)\sigma(B) \geq \tfrac{1}{2}|\langle[A, B]\rangle|$$

2.（II型の破れ）入力状態によらずに $\varepsilon(A) = 0$ となる場合：

$$\sigma(A)\eta(B) \geq \tfrac{1}{2}|\langle[A, B]\rangle|$$

[MO, quant-ph/0207121, quant-ph/0210044]

3. Heisenberg 不確定性原理を打破する測定相互作用

$$\hat{H} = \hat{x}\hat{p}_y \qquad \Rightarrow \quad \text{成立}$$

$$\hat{H} = \hat{x}\hat{p}_y - \hat{p}_x\hat{y} + \tfrac{1}{2}(\hat{x}\hat{p}_x - \hat{y}\hat{p}_y) \qquad \Rightarrow \quad \text{不成立}$$

[MO, PLA 299 (2002) 1]

Wigner-Araki-Yanase の定理

加法的保存量と非可換な物理量の射影測定は実現不可能である．測定精度をよくするには測定器系に含まれる保存量を大きくする必要がある．装置が巨視的なら保存量の分散を無限大として，実用精度が実現される．**[Wigner, Z. Phys. 133 (1952), 101; H. Araki and M. M. Yanase PR120 (1960), 622]**

数学的定式化： A, L_1：対象の物理量，L_2：プローブの物理量

(i) $[U, L_1 \otimes I + I \otimes L_2] = 0$

(ii) $U|A = a_j\rangle \otimes \xi = |A = a_j, M = m_j\rangle$

ならば，

$$[A, L_1] = 0$$

Yanase のスピン測定の精度

$A = S_z$, $L_1 = S_x$, $L_2 =$ プローブの角運動量のx成分 とする.

(i) $[U, L_1 \otimes I + I \otimes L_2] = 0$

(ii) (**Yanase** の条件) $[M, L_2] = 0$

ならば,

$$\text{測定の誤り確率} \geq \frac{\hbar^2}{16\langle L_2^2 \rangle}$$

が得られる. **[M. M. Yanase, PR123 (1961) 666]**

この場合,

測定の二乗平均誤差$=\hbar^2 \times$測定の誤り確率

だから,

$$\varepsilon(S_z)^2 \geq \frac{\hbar^4}{16\langle L_2^2 \rangle}$$

不確定性原理とWigner-Araki-Yanase の定理の矛盾点

基本原理：測定対象と測定器の境目は任意であり，対象＋プローブを新たに測定対象と見なして測定の記述をすることができる．

すると，保存則とYanase の条件をみたす測定は，$L_1 + L_2$ を乱さない A の測定と見なすことができる．

したがって，この測定では，

$$\eta(L_1 \otimes I + I \otimes L_2) = 0$$

不確定性原理から，

$$\varepsilon(A) = \infty$$

これは，**Yanase** の結果

$$\varepsilon(S_z)^2 \geq \frac{\hbar^4}{16\langle L_2^2 \rangle}$$

および $\langle L_2^2 \rangle$ を大きくすれば，幾らでも正確な測定が可能であるという **Wigner-Araki-Yanase** の説に矛盾する.

I型の破れと Yanase の限界

定理 [MO, PRL 88 (2002) 0504302]：

$[U, L_1 + L_2] = 0$ かつ $[M, L_2] = 0$ ならば,

$$\varepsilon(A)^2 \geq \frac{|\langle [A, L_1] \rangle|^2}{4\sigma(L_1)^2 + 4\sigma(L_2)^2}$$

証明： $\eta(L_1 + L_2) = 0$ より，I型の破れから,

$$\varepsilon(A)\sigma(L_1 + L_2) \geq \frac{1}{2}|\langle [A, L_1 + L_2] \rangle|$$

ここで, $[A, L_2] = 0$

$\sigma(L_1 + L_2)^2 = \sigma(L_1)^2 + \sigma(L_2)^2$

だから,

$$\varepsilon(A)^2 \geq \frac{|\langle [A, L_1] \rangle|^2}{4\sigma(L_1)^2 + 4\sigma(L_2)^2}$$

証明終わり.

$|\langle [A, L_1] \rangle| = |\langle [S_z, S_x] \rangle| = \hbar|\langle S_y \rangle| \leq \frac{\hbar^2}{2}$ より,

$$\max_{\psi} \varepsilon(A)^2 \geq \frac{\hbar^4}{4\hbar^2 + 16\sigma(L_2)^2}$$

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Conservation Laws, Uncertainty Relations, and Quantum Limits of Measurements

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The uncertainty relation between the noise operator and the conserved quantity leads to a bound on the accuracy of general measurements. The bound extends the assertion by Wigner, Araki, and Yanase that conservation laws limit the accuracy of "repeatable," or "nondisturbing," measurements to general measurements, and improves the one previously obtained by Yanase for spin measurements. The bound represents an obstacle to making a small quantum computer.

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In 1952, Wigner [1] found that conservation laws put a limit on measurements of quantum mechanical observables. In 1960, Araki and Yanase [2] proved the following assertion known as the Wigner-Araki-Yanase (WAY) theorem: *Observables which do not commute with bounded additive conserved quantities have no "exact" measurements* [3]. Subsequently, Yanase [4] found a bound for the accuracy of measurements of the x component of spin in terms of the "size" of the apparatus, where the size is characterized by the mean square of the z component of the angular momentum [5]. Yanase [4] and Wigner [6] concluded from this result that in order to increase the accuracy of spin measurement one has to use a very large measuring apparatus.

In the WAY theorem, for a measurement to be "exact" the following two conditions are required to be satisfied: (i) the Born statistical formula (BSF) and (ii) the repeatability hypothesis (RH), asserting that, if an observable is measured twice in succession in a system, then we obtain the same value each time. Yanase's bound does not assume the RH. Instead, a condition, to be referred to as Yanase's condition, is assumed that the probe observable, the observable in the apparatus to be measured after the measuring interaction, commutes with the conserved quantity, to ensure the measurability of the probe observable [4]. Elaborating the suggestions given by Stein and Shimony [3], Ohira and Pearle [7] constructed a simple measuring interaction that satisfies the conservation law and the BSF, assuming the precise probe measurement, but does not satisfy the RH. Based on their model, Ohira and Pearle claimed that it is possible to have an accurate measurement of the spin component regardless of the size of the apparatus, if the RH is abandoned. However, their model does not satisfy Yanase's condition, so that the problem remains as to the measurability of the probe observable.

Yanase's argument, however, assumes a large (but of variable size) measuring apparatus having the continuous angular momentum from the beginning for technical reasons and concludes that accurate measurement requires a very large apparatus. To avoid a circular argument, a rigorous derivation without such an assumption is still demanded. Moreover, Wigner [6] pointed out the necessity for generalizing the bound to general quantum systems other than spin 1/2 systems, as well as including all additive conservation laws.

In order to accomplish the suggested generalization, a new approach to the problem is proposed in this Letter based on the uncertainty relation between the conserved quantity and the noise operator, defined as the difference between the post-measurement probe and the measured quantity. We obtain a bound for the mean-square error of general measuring interactions imposed by any additive conservation laws without assuming the RH. This bound also clarifies the trade-off between the size and the commutativity of the noise operator with the conserved quantity, unifying the suggestion by WAY and others and the one suggested by Ohira and Pearle. For spin measurements, this bound with Yanase's condition leads to a tight bound for the error probability of spin measurement, which improves Yanase's bound.

Let $\mathbf{A}(\mathbf{x})$ be a measuring apparatus with macroscopic output variable $\mathbf{x}$ to measure, possibly with some error, an observable A of the *object* $\mathbf{S}$, a quantum system represented by a Hilbert space $\mathcal{H}$. The measuring interaction turns on at time t, the *time of measurement*, and turns off at time $t + \Delta t$ between the object $\mathbf{S}$ and the *probe* $\mathbf{P}$, a part of the apparatus that interacts with the object, represented by a Hilbert space $\mathcal{K}$. Denote by U the unitary operator on $\mathcal{H} \otimes \mathcal{K}$ representing the time evolution of $\mathbf{S} + \mathbf{P}$ in the time interval $(t, t + \Delta t)$.

At the time of measurement, the object is supposed to be in an unknown (vector) state ψ and the probe is supposed to be prepared in a known (vector) state ξ; all state vectors are assumed to be normalized unless stated otherwise. Thus, the composite system $\mathbf{S} + \mathbf{P}$ is in the state $\psi \otimes \xi$ at time t. Just after the measuring interaction, the probe is subjected to a local interaction with the subsequent stages of the apparatus. The last process is assumed to measure an observable M, called the *probe observable*, of the probe with arbitrary precision, and the outcome is recorded as the value of the macroscopic outcome variable $\mathbf{x}$.

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反復測定に関する標準量子限界(SQL)と量子非破壊(QND)測定

QND 物理量＝時間発展で変化しない物理量

非 QND 物理量の反復測定には，SQL が存在する．

自由質点の反復測定に関する標準量子限界 (SQL)

- **Heisenberg 不確定性原理 $\Rightarrow$ SQL:**

  自由質点 m の位置 x を時間間隔 τ で反復測定すると，2 回目の測定値を

  $$\sqrt{\frac{\hbar\tau}{m}}$$

  より小さい不確定性で予測することはできない．

- **Yuen** はこの正当性を疑い，**SQL** を打破する測定法として，**contractive state measurement** を提唱した．**[H. Yuen, PRL 51 (1983) 719]**

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Contractive States and the Standard Quantum Limit for Monitoring Free-Mass Positions

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The familiar minimum-uncertainty wave packets for masses are generalized in analogy with the two-photon coherent states of the radiation field. The free evolution of a subclass of these states, the contractive states, leads to a narrowing of the position uncertainty in contrast with the usual minimum-uncertainty wave packets. As a consequence the standard quantum limit for monitoring the positions of a free mass can be breached. Further implications on quantum nondemolition measurements are discussed.

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There has been considerable recent interest in ascertaining and achieving the fundamental quantum limits on signal processing and precision measurements, in particular for applications to optical communications[1-3] and gravitational-wave detection.[4-6] A major result of this work is that one can beat the so-called standard quantum limit for amplitude measurements on harmonic oscillators. However, for the gravitational-wave interferometer[9] it is usually supposed[7,8] that the resolution is limited by the "standard quantum limit" (SQL) for measuring the positions of a free mass.[4-5] In this paper it is shown that the latter SQL is also *not* generally valid; it can be breached by a specific quantum measurement without special preparation of the free-mass quantum state. Toward this end I will describe a class of generalized minimum-uncertainty wave packets for masses, to be called twisted coherent states, which are also of interest in their own right. The breakdown of the SQL for free-mass position measurements demonstrates the fact that back actions from a conjugate observable do *not* necessarily, at least in accordance with the principle of quantum mechanics, limit the accuracy of subsequent measurements on an observable.

The evolution of a free mass is given by $X(t) = X(0) + P(0)\,t/m$, so that the position fluctuation at time t is

$$\langle \Delta X^2(t)\rangle = \langle \Delta X^2(0)\rangle + \langle \Delta P^2(0)\rangle\, t^2/m^2 + \langle \Delta X(0)\Delta P(0) + \Delta P(0)\Delta X(0)\rangle\, t/m. \quad (1)$$

In the previous derivation[4-5] of the general SQL for monitoring free-mass positions, it is implicitly assumed that the $t = 0$ state of the mass (or the state after measurement) is such that the last term in (1) either vanishes or is positive. Under this assumption the uncertainty principle can be applied to minimize (1) at any time t with the resulting SQL

$$\langle \Delta X^2(t)\rangle_{\mathrm{SQL}} = \hbar t/m. \quad (2)$$

On the other hand, it is clear that $\langle \Delta X^2(t_0)\rangle = 0$ if the initial state is an eigenstate of the self-adjoint operator $X(0) + P(0)\,t_0/m$. Thus, the last term in (1) can surely be negative and the SQL is not generally valid. However, $\langle \Delta X^2(t)\rangle = 0$ implies $\langle \Delta P^2(t)\rangle = \infty$ so that $\langle P^2(0)/2m\rangle = \langle P^2(t)/2m\rangle = \infty$, i.e., an infinite average energy is needed to produce such a state.[3] A more realistic description can be developed as follows.

For an oscillator of mass m and frequency ω, the twisted or two-photon coherent states (TCS)[1,3]

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Beating the quantum limits (cont'd)

Heisenberg's Uncertainty Principle is for many an irksome constraint on the freedom to make measurements accurately. Can the constraint be overturned?

Is it possible to evade the restrictions of Heisenberg's uncertainty principle by the clever design of measuring equipment? Plainly, the answer to a question such as this would be of great practical importance in, say, the information processing industry. The trend towards ever-smaller processing devices, for example, will ultimately be limited by quantum mechanics, or by the fidelity with which a device put into a specific state so as to represent a bit of information will persist in that state despite unavoidable interactions with external systems.

Historically, the search for stratagems to beat the quantum limit has, in practice, been driven by those looking for gravitational waves, and who are naturally anxious to beat the quantum limit in their design of the masses which, supposedly, would be set vibrating by interaction with gravitational waves. Inevitably, the argument centres around the question of precisely what is meant by the process of making a quantum measurement.

As is customary in these connections, the vigorous arguments there have been over the past several years have entailed the splitting of several recondite hairs. Until recently, it has seemed that those who hold that the "standard quantum limit" (SQL) is inescapable were in the ascendant. But now Masanao Ozawa of Nagoya University has put a cat among the pigeons by specifying a quantum system in which, he says, it is possible to do better than SQL (*Phys. Rev. Lett.* 54, 2465; 1988)

The conventional view has been put well by Carlton Caves of the California Institute of Technology (*Phys. Rev. Lett.* 54, 2465; 1985) for the case of what must be the simplest quantum measurement of all, that of the position of a free particle at two instants separated in time by an interval τ. The strength of Caves's argument stems in part from his ready acceptance of an earlier assertion by Horace P. Yuen (*Phys. Rev. Lett.* 51, 719; 1983) that there is indeed a flaw in the standard textbook derivation of SQL.

That argument is as follows. Suppose the measurement of the position of the particle at time t_0 is uncertain by an amount Δx, so that the corresponding uncertainty of the momentum (p) is that given by the Heisenberg relation as Δp, which must be $\geq \hbar/2\Delta x$. The uncertainty of the position of the particle after an interval τ is then a function of two parts — the uncertainty of its position at the beginning of the interval and the extent to which the position of the particle is further spread by the spread of the possible momentum (or velocity) after the first measurement. Combining the two components as statistical variances, the textbooks show that $(\Delta x)^2$ after an interval τ is $\geq \hbar\tau/m$, where m is the mass of the particle.

The flaw in this argument, pointed out by Yuen and now, apparently, generally accepted, is that there is more to say about the evolution of the quantum system during the interval τ. In terms of the operators representing position and momentum, $x(t) = x(t_0) + p(t_0)t/m$. In suitable circumstances, the outcome at the end of the interval is a correlation term which may be negative and which, when subtracted from the variances due to the initial uncertainties, will help to beat the quantum limit. Everything depends on the state in which the initial measurement leaves the particle.

Caves's counterargument, reminiscent of the things that people were saying at Copenhagen in the 1920s, is that the old textbook argument is deficient in its neglect of the measuring equipment. Specifically, he defines the imperfect "resolution" of the equipment as a quantity σ whose square must be added to the variance of the position of the free particle to yield the uncertainty of the second measurement. Whether or not there are states of the particle, called "contractive states" by Yuen, the SQL is a simple consequence of Heisenberg's Uncertainty Principle in Caves's view. One interesting caveat in the argument is the assumption that the coupling between the measuring apparatus and the system observed must be linear. In 1985, Caves was guessing that those seeking violations of SQL should see what non-linear couplings have to offer.

What Ozawa has now done is to pull apart Cave's definition of "resolution", which he says may mean either the uncertainty of the result of a measurement, the uncertainty in the position of the particle after the measurement and the uncertainty in the measuring apparatus immediately before it. He defines instead the "precision" of the measurement, which is the uncertainty that arises in the measurement of the position of a particle known to be in an eigenstate of the position operator (and whose momentum is thus entirely uncertain) and the "resolution", which is the difference between the result of a measurement and the position of the free particle just after it. The question is whether the precision can ever be less than the resolution, as thus defined.

Naturally, Ozawa's conclusion is that such a state of affairs is indeed attainable. The crux of his argument is the construction of a solvable model to represent the interaction between the measuring equipment and the free particle whose position is to be measured, which has the virtue that (regarded as a quantum mechanical hamiltonian) it can be solved exactly. In reality, what he has done is to show that, in Yuen's language, contractive states exist and have exactly the properties predicted for them. On the face of things, it seems, the SQL can be beaten by this means to any desired extent.

Against the background of past arguments, it seems unlikely that this argument, neat though it is, will turn the conventional position on the SQL. One of the virtues of Ozawa's case is that the quantities arising in his calculations are indeed precisely defined and are related directly to quantities that can be measured. On the other hand, there are difficulties about the assumptions made about the way that noise associated with the measuring system would be allowed for as well as in the formality with which the yardstick itself is defined. Yet Ozawa's calculation will undoubtedly lift the spirits of those involved with the design of gravitational wave detectors; it will be interesting to see where this leads.

But, as the past half century has shown, this is not a field in which arguments come neatly to generally accepted conclusions. On this occasion, there may be continuing arguments about Ozawa's assumption that the coupling between the free particle and the measuring equipment is so strong that the unperturbed motions of the particle and the yardstick can be neglected. It is also far from obvious how the particular example on which the conclusion rests can be turned into a realistic measuring equipment that would allow those who design equipment to exploit this recipe for beating SQL. But none of this will damp enthusiasm for overturning what often seems an intolerable constraint on the freedom to design accurate measuring equipment.

John Maddox

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PHYSICAL REVIEW

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Defense of the Standard Quantum Limit for Free-Mass Position

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(Received 6 April 1984)

Measurements of the position x of a free mass m are thought to be governed by the standard quantum limit (SQL): In two successive measurements of x spaced a time τ apart, the result of the second measurement cannot be predicted with uncertainty smaller than $(\hbar\tau/m)^{1/2}$. Yuen has suggested that there might be ways to beat the SQL. Here I give an improved formulation of the SQL, and I argue for, but do not prove, its validity.

PACS numbers: 03.65.Bz, 06.20.Dk

Conventional wisdom[1,2] holds that in two successive measurements of the position x of a free mass m, the result of the second measurement cannot be predicted with uncertainty smaller than $(\hbar\tau/m)^{1/2}$, where τ is the time between measurements. This limit is called the *standard quantum limit* (SQL) *for monitoring the position of a free mass.*

The standard "textbook" argument for the SQL runs as follows. Suppose that the first measurement of x at $t=0$ leaves the free mass with position uncertainty $\Delta x(0)$. This first measurement disturbs the momentum p and leaves a momentum uncertainty $\Delta p(0) \geq \hbar/2\Delta x(0)$. By the time τ of the second measurement the variance of x (squared uncertainty) increases to

$$(\Delta x)^2(\tau) = (\Delta x)^2(0) + [(\Delta p)^2(0)/m^2]\tau^2 \geq 2\Delta x(0)\Delta p(0)\tau/m \geq \hbar\tau/m. \tag{1}$$

The standard argument views the SQL as a straightforward consequence of the position-momentum uncertainty principle $\Delta x(0)\Delta p(0) \geq \frac{1}{2}\hbar$.

Yuen[3] has pointed out a serious flaw in the standard argument. Between the two measurements the free mass undergoes unitary evolution. In the Heisenberg picture the position operator $\hat{x}$ evolves as

$$\hat{x}(t) = \hat{x}(0) + \hat{p}(0)t/m. \tag{2}$$

Thus the variance of x at time τ is given not by Eq. (1), but by

$$(\Delta x)^2(\tau) = (\Delta x)^2(0) + \frac{(\Delta p)^2(0)}{m^2}\tau^2 + \frac{\langle \hat{x}(0)\hat{p}(0) + \hat{p}(0)\hat{x}(0)\rangle - 2\langle \hat{x}(0)\rangle\langle \hat{p}(0)\rangle}{m}\tau. \tag{3}$$

The standard argument assumes implicitly that the last term in Eq. (3) is zero or positive. Yuen's point[3] is that some measurements of x leave the free mass in a state for which this term is negative. He calls such states *contractive states* because the variance of x decreases with time, at least for a while. As a result, the uncertainty $\Delta x(\tau)$ can be smaller than the SQL. Yuen[3,4] concludes that there are measurements of x that beat the SQL. My conclusion is different: The flaw lies in the standard argument, not in the SQL. In this Letter I give a new, heuristic argument for the SQL, formulate an improved statement of the SQL, and analyze a measurement model that supports the heuristic argument.

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On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle*

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The monitoring of a quantum-mechanical harmonic oscillator on which a classical force acts is important in a variety of high-precision experiments, such as the attempt to detect gravitational radiation. This paper reviews the standard techniques for monitoring the oscillator, and introduces a new technique which, in principle, can determine the details of the force with arbitrary accuracy, despite the quantum properties of the oscillator. The standard method for monitoring the oscillator is the "amplitude-and-phase" method (position or momentum transducer with output fed through a narrow-band amplifier). The accuracy obtainable by this method is limited by the uncertainty principle ("standard quantum limit"). To do better requires a measurement of the type which Braginsky has called "quantum nondemolition." A well known quantum nondemolition technique is "quantum counting," which can detect an arbitrarily weak classical force, but which cannot provide good accuracy in determining its precise time dependence. This paper considers extensively a new type of quantum nondemolition measurement—a "back-action-evading" measurement of the real part X_1 (or the imaginary part X_2) of the oscillator's complex amplitude. In principle X_1 can be measured "arbitrarily quickly and arbitrarily accurately," and a sequence of such measurements can lead to an arbitrarily accurate monitoring of the classical force. The authors describe explicit *Gedanken* experiments which demonstrate that X_1 can be measured arbitrarily quickly and arbitrarily accurately. In these experiments the measuring apparatus must be coupled to both the position (position transducer) and the momentum (momentum transducer) of the oscillator, and both couplings must be modulated sinusoidally. For a given measurement time the strength of the coupling determines the accuracy of the measurement; for arbitrarily strong coupling the measurement can be arbitrarily accurate. The "momentum transducer" is constructed by combining a "velocity transducer" with a "negative capacitor" or "negative spring." The modulated couplings are provided by an external, classical generator, which can be realized as a harmonic oscillator excited in an arbitrarily energetic, coherent state. One can avoid the use of two transducers by making "stroboscopic measurements" of X_1, in which one measures position (or momentum) at half-cycle intervals. Alternatively, one can make "continuous single-transducer" measurements of X_1 by modulating appropriately the output of a single transducer (position or momentum), and then filtering the output to pick out the information about X_1 and reject information about X_2. Continuous single-transducer measurements are useful in the case of weak coupling. In this case long measurement times are required to achieve good accuracy, and continuous single-transducer measurements are almost as good as perfectly coupled two-transducer measurements. Finally, the authors develop a theory of quantum nondemolition measurement for arbitrary systems. This paper (Paper I) concentrates on issues of principle; a sequel (Paper II) will consider issues of practice.

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- **contractive state measurement** を実現する測定相互作用を発見. **[MO, PRL 60 (1988) 385]**
- これにより, **SQL** が正しくないことが示された.

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Measurement Breaking the Standard Quantum Limit for Free-Mass Position

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(Received 2 July 1987)

An explicit interaction-Hamiltonian realization of a measurement of the free-mass position with the following properties is given: (1) The probability distribution of the readouts is exactly the same as the free-mass position distribution just before the measurement. (2) The measurement leaves the free mass in a contractive state just after the measurement. It is shown that this measurement breaks the standard quantum limit for the free-mass position in the sense sharpened by the recent controversy.

PACS numbers: 03.65.Bz, 04.80.+z

For monitoring the position of a free mass such as the gravitational-wave interferometer,[1] it is usually supposed[2,3] that the predictability of the results is limited by the so-called standard quantum limit (SQL). In the recent controversy,[4-8] started with Yuen's proposal[4] of a measurement which beats the SQL, the meaning of the SQL has been much clarified and yet no one has given a general proof nor a counterexample for the SQL. Recently, Ni[9] succeeded in constructing a repeated-measurement scheme to monitor the free-mass position to an arbitrary accuracy. However, it is open whether this scheme beats the SQL in the sense sharpened by the recent controversy. In particular, the following problem remains open: Can we realize a high-precision measurement which leaves the free mass in a contractive state?

In the present paper, I shall give a model of measurement of a free-mass position which breaks the SQL in its most serious formulation. An explicit form of the system-meter interaction Hamiltonian will be given and it will be shown that if the meter is prepared in an appropriate contractive state[4] then the measurement leaves the free mass in a contractive state and the uncertainty of the prediction for the next identical measurement decreases in a given duration to a desired extent. Thus Yuen's original proposal[4] is fully realized. This result will open a new way to an arbitrarily accurate nonquantum-nondemolition monitoring for gravitational wave detection and other related fields such as optical communications.

The precise formulation of the SQL is given by Caves[8] as follows: Let a free mass m undergo unitary evolution during the time τ between two measurements of its position x, made with identical measuring apparatus; the result of the second measurement cannot be predicted with uncertainty smaller than $(\hbar\tau/m)^{1/2}$ in average over all the first readout values. Caves[8] showed that the SQL holds for a specific model of a position measurement due to von Neumann[10] and he also gave the following heuristic argument for the validity of the SQL. His point is the notion of the imperfect resolution σ of one's measuring apparatus. His argument runs as follows: *The first assumption* is that the variance of the measurement of x is the sum of σ^2 and the variance of x at the time of the measurement; this is the case when the measuring apparatus is coupled linearly to x. *The second assumption* is that just after the first measurement, the free mass has position uncertainty $\Delta x(0) \leq \sigma$. Under these conditions, he derived the SQL from the uncertainty relation $\Delta x(0)\Delta x(\tau) \geq \hbar\tau/2m$.

However, his definition of the resolution of a measurement is ambiguous. In fact, he used three different definitions in his paper: (1) the uncertainty in the result, (2) the position uncertainty after the measurement, and (3) the uncertainty of the meter before the measurement. These three notions are essentially different, although they are the same for von Neumann's model. I

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量子計算実現に関する量子限界

現行のパラダイム：

1．スピン成分で1量子ビットを表現

2．1 量子ビットの回転と2量子ビットのCNOTを物理相互作用で実現

3．実現の精度（誤り確率）が $10^{-5} - 10^{-6}$ なら，環境からのデコーヒーレンスによる誤りを訂正をしながら幾らでも大きな回路を組むことが出来る

これは合理的かつ絶対的な指針か？

保存法則による量子状態制御の限界

[MO, PRL 89 (2002), 057902]

数学的ユニタリ変換を物理相互作用で実現させるためには，外部の制御系との相互作用が必要．保存量が存在すると，保存量に関するエンタングルメントが生じ，ユニタリ性と矛盾する．

Wigner-Araki-Yanase の定理の定量化：

$[U, L_1 + L_2] = 0$ ならば，

$$\varepsilon(A)\sigma(L_1') + \eta(A)\sigma(L_2') \geq \frac{1}{2}|\langle [A, L_1] \rangle|$$

定理： **CNOT**（及び，**Toffoli** ゲート，**Fredkin** ゲート）のユニタリ性と角運動量保存則は矛盾する．n 量子ビットのアンシラを利用しても角運動量を保存するユニタリで実現される**CNOT**の誤り確率は $1/4(n+2)^2$（及び，$1/4(n+3)^2$）以上である．

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Conservative Quantum Computing

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(Received 8 October 2001; published 16 July 2002)

The Wigner-Araki-Yanase theorem shows that conservation laws limit the accuracy of measurement. Here, we generalize the argument to show that conservation laws limit the accuracy of quantum logic operations. A rigorous lower bound is obtained of the error probability of any physical realization of the controlled-NOT gate under the constraint that the computational basis is represented by a component of spin, and that physical implementations obey the angular momentum conservation law. The lower bound is shown to be inversely proportional to the number of ancilla qubits or the strength of the external control field.

DOI: 10.1103/PhysRevLett.89.057902 PACS numbers: 03.67.Lx, 03.65.Ta

Since the discovery of Shor's algorithm [1], physical realization of quantum computers is one of the major topics in physics. One of the formidable obstacles to the realization of quantum computers is the decoherence induced by the environment. The theory of quantum error correction and the theory of fault-tolerant quantum computing have been developed to overcome this difficulty [2,3]. One of the main achievements of this field is the threshold theorem: Provided the noise in individual quantum gates is below a certain threshold it is possible to efficiently perform arbitrarily large quantum computing. However, the threshold is rather demanding and the problem turns to whether there is any fundamental limit for implementing quantum gates. Recently, Lloyd [4] and Ng [5] have discussed how fundamental constants provide limits on speed and memory of quantum computers. Here, I will propose another approach based on conservation laws.

If we consider the ultimate performance of computing allowed by the laws of physics, elementary quantum gates should be isolated and small, so that the corresponding unitary operators should satisfy fundamental symmetries, or conservation laws. From this point of view, it is likely that the degree of conflict with a conservation law depends on the nature of its logic to be performed and that the imperfection can be reduced by increasing the size of implementation. However, no serious investigation has ever taken place. In this letter we model qubits as spin-1/2 objects and investigate the quantum limit induced by the angular momentum conservation law. We show that, although the SWAP gate has no conflict with the conservation law, the controlled-NOT gate, which is one of the universal quantum logic gates, cannot be implemented by any 2-qubit rotationally invariant unitary operation within error probability 1/16. Thus, to obtain more accuracy, we need to blow up the unitary operation to an ancilla system. Then, the size of an implementation of the quantum gate is defined as the total number of qubits in the computational basis and the ancilla. It is shown that any physically realizable unitary operator with size less than n qubits cannot implement the controlled-NOT gate within the error probability $1/4n^2$. An analogous limit for bosonic ancillae will also be obtained by defining the size of the ancilla as 2 times the square root of the average number of photons, and thus the lower bound is inversely proportional to the average number of photons. It is also shown that, in any set of universal gates, for any size limit s there is at least one gate which cannot be implemented within the error probability $1/ks^2$ for some constant k. Thus, we cannot circumvent this limitation by a clever choice of the set of universal gates.

Let U_{CN} be a controlled-NOT gate on a 2-qubit system $\mathbf{C} + \mathbf{T}$. Let X_i, Y_i, and Z_i be the Pauli operators of qubit $\mathbf{C}$ for $i = 1$ or qubit $\mathbf{T}$ for $i = 2$ defined by $X_i = |1\rangle\langle 0| + |0\rangle\langle 1|$, $Y_i = i|1\rangle\langle 0| - i|0\rangle\langle 1|$, and $Z_i = |0\rangle\langle 0| - |1\rangle\langle 1|$ with the computational basis $\{|0\rangle, |1\rangle\}$. On the computational basis, U_{CN} acts as $U_{\mathrm{CN}}|a, b\rangle = |a, b \oplus a\rangle$ for $a, b = 0, 1$, where $\oplus$ denotes the addition modulo 2. Thus, in particular, we have

$$U_{\mathrm{CN}}|a, 0\rangle = |a, a\rangle \tag{1}$$

for $a = 0, 1$. The above relation shows that the unitary operator U_{CN} serves as an interaction between the "object" $\mathbf{C}$ and the "probe" $\mathbf{T}$ for a measurement of Z_1 satisfying the projection postulate. Thus, by the Wigner-Araki-Yanase theorem [6,7], if there are additive conserved quantities not commuting with Z_1, the unitary operator U_{CN} cannot be implemented. To be precise, let L_1 and L_2 be a pair of observables of $\mathbf{C}$ and $\mathbf{T}$, respectively, such that

$$[Z_1, L_1] \neq 0. \tag{2}$$

Then, the controlled-NOT gate U_{CN} cannot satisfy the conservation law [8]

$$[U_{\mathrm{CN}}, L_1 + L_2] = 0. \tag{3}$$

A simple proof runs as follows. Assume that Eq. (3) holds. If $a \neq b$, we have

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— 本来2量子ビットのユニタリを実現するのに100量子ビットのユニタリを実現する必要がある.

参考：新しい符号化に基づく量子計算（encoded universality）

D. Bacon et al., Phys. Rev. Lett. 85, 1758 (2000).

D. P. Divincenzo et al., Nature (London) 408, 339 (2000).

— 新しい不確定性原理から導かれる量子計算の誤差を免れているか？

アンシラがボゾンの場合は次のようになる.

定理：平均光子数 N の外場をアンシラとして利用する場合に，角運動量を保存するユニタリで実現されるCNOT（及び，Toffoli ゲート，Fredkin ゲート）の誤り確率は $1/16N$ 以上である.

参考： **S. J. van Enk and H. J. Kimble, e-print quant-ph/010788; J. Gea-Banacloche, Phys. Rev. A 65, 022308 (2002).**

— 特定の相互作用のモデルにもとづいて誤差確率が外場の強度に反比例することを導いている．本研究では，保存則を満たすという仮定だけから同じ結論を導いた.

新しい不確定性原理を検証する実験の提案

新しい不確定性原理を検証する実験の提案

The noiseless quadrature transducer can be implemented, in principle, as follows. Consider two degenerate modes a, the signal, and b, the probe, with frequency ω and orthogonal polarization, which undergo successive parametric interactions in the following five steps; see Refs. [24–28] for similar implementations of BAE amplifiers.

(i) The two polarization modes undergo a mixing interaction using a polarization rotator which rotates the angle of polarization by θ. The operation of the polarization rotator is represented by the mixing operator

$$T(\theta)=\exp[\theta(ab^{\dagger}-a^{\dagger}b)]. \tag{25}$$

(ii) The mixture of the signal and the probe fields will propagate along each of the ordinary and orthogonal extraordinary axis of a KTP crystal pumped by a pulsed intense classical field. This interaction is a nondegenerate parametric amplifier described by the two-mode squeeze operator

$$S(r)=\exp[r(ab-a^{\dagger}b^{\dagger})], \tag{26}$$

where r corresponds to the squeezing parameter.

(iii) After the amplification step, the fields pass through a second polarization rotator with the mixing angle 2θ so that the operation is represented by $T(2\theta)=T(\theta)^2$.

(iv) The mixture of the signal and the probe fields undergoes the second parametric amplification described by the two-mode squeeze operator $S(-r)$.

(v) After the second amplification step, the field passes through a third polarization rotator with the mixing angle θ so that the operation is represented by $T(\theta)$.

Thus, this process defines the measuring interaction operator

$$U(r,\theta)=T(\theta)S(-r)T(\theta)T(\theta)S(r)T(\theta). \tag{27}$$

We shall determine the parameters θ and r for $U(r,\theta)$ to realize a noiseless quadrature transducer, Eq. (22). Suppose that θ and r satisfies the relation $\sin 2\theta=\tanh r$. In this case, it is well known [24–28] that the unitary operator $U_-=T(\theta)S(-r)T(\theta)$ realizes the BAE quadrature amplifier with $G=2\sinh r$, i.e.,

Thus, if $\sinh r=1/2$, i.e.,

$$r=\ln\frac{1+\sqrt{5}}{2} \tag{31a}$$

$$\theta=\frac{1}{2}\sin^{-1}\frac{1}{\sqrt{5}}, \tag{31b}$$

the resulting unitary operator $U=U(r,\theta)$ realizes the noiseless quadrature transducer, Eq. (22).

Thus, if r and θ are chosen as Eq. (31), the parametric process steps (i)–(v) realize the noiseless quadrature transducer. Therefore, this process followed immediately by the homodyne detection of the X_b component implements a nearly nondisturbing and noiseless measurement of the X_a quadrature component that disturbs the conjugate observable Y_a much less than the quantum limit set by Heisenberg's lower bound for the noise-disturbance product, $\epsilon(X_a)\eta(Y_a)\geqslant 1/4$.

まとめ

1．正しい不確定性関係:

$$\varepsilon(A)\eta(B)+\varepsilon(A)\sigma(B)+\sigma(A)\eta(B)\geq\tfrac{1}{2}|\langle[A,B]\rangle|$$

2. Heisenberg の不確定性関係：$\varepsilon(A)$ と $\eta(B)$ が入力状態によらずに一定ならば,

$$\varepsilon(A)\eta(B)\geq\tfrac{1}{2}|\langle[A,B]\rangle|$$

3．I型の破れ：$\eta(B)=0$ ならば,

$$\varepsilon(A)\sigma(B)\geq\tfrac{1}{2}|\langle[A,B]\rangle|$$

4． II型の破れ：$\varepsilon(A)=0$ ならば,

$$\sigma(A)\eta(B)\geq\tfrac{1}{2}|\langle[A,B]\rangle|$$

5．Yanase の限界の一般化：$[U,L_1+L_2]=0$ かつ $[M,L_2]=0$ ならば,

$$\varepsilon(A)^2\geq\frac{|\langle[A,L_1]\rangle|^2}{4\sigma(L_1)^2+4\sigma(L_2)^2}$$

6．Wigner-Araki-Yanase の定理の定量化

：$[U, L_1 + L_2] = 0$ ならば，

$$\varepsilon(A)\sigma(L_1') + \eta(A)\sigma(L_2') \geq \frac{1}{2}|\langle [A, L_1] \rangle|$$

6．単一スピン量子ビット量子計算の限界：n 量子ビットのアンシラを利用しても角運動量を保存するユニタリで実現されるCNOTの誤り確率は $1/4(n+2)^2$ 以上である．

展望：ハイゼンベルク不確定性原理の解明と量子情報理論への応用

1. 新しい不確定性原理の理論的基礎づけ：任意の完全正写像値測度が満たす不等式として新しい不確定性原理を再定式化

2. 一般の同時測定に関する不確定性原理の導出：任意の確率作用素測度に関する不等式として新しい不確定性原理を再定式化

3. 新しい不確定性原理の検証実験

4. 新しい不確定性原理の量子計算の精度への応用：符号化された新しい計算基底に基づく計算素子の計算精度の研究

5. 新しい不確定性原理のエンタングルメントへの応用：測定精度とエンタングルメントの相互関係の研究

6. 新しい不確定性原理の量子暗号への応用：新しい不

確定性原理にもとづいて量子暗号の安全性を解明

7. 新しい不確定性原理とno-cloning 定理の相互関係：no-cloning 定理を新しい不確定性原理の特殊ケースとして演繹

8. 連続測定における不確定性原理の研究：連続測定のモデルに対するより限定された不確定性原理の導出

9. 新しい不確定性原理と量子情報量への応用：新しい不確定性原理から情報量の限界を導出