Language: 英語/English

KEK 物理セミナー Physics Seminar

7月14日 (月) July 14 (Mon)

16:00 ~ 17:00

3号館 セミナーホール
Seminar Hall in 3rd Building

Approaches To Quark Confinement

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(Institute For Advanced Study)

- Abstract:
  Although QCD has been with us for a long time, confinement of quarks is still not fully understood. In this talk, I will survey some old and new approaches to quark confinement, centering on electric-magnetic duality, relations to string theory, and the large N limit.

Past presentations are archived at our home page, http://seminar.kek.jp/physics/ .

Organizers: T. Komatsubara and I. Adachi

2003.6.18

(1)

I AM SURE THE PROBLEM OF QUARK CONFWEMENT IS FAMILIAR TO MANY ...

HADRONS SEEM TO BE MADE OF QUARKS

uud proton

udd neutron

BUT WE DON'T SEE INDIVIDUAL QUARKS ...

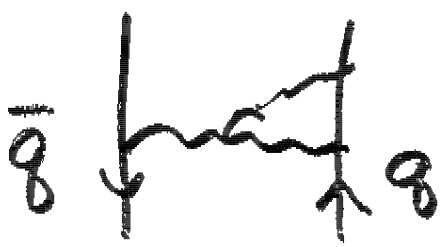
WE INFER THE EXISTENCE (2) OF QUARKS FROM

* HADRON QUANTUM NUMBERS
* DEEP INELASTIC SCATTERING
* e^+e^- ANNIHILATION
* AND MORE GENERALLY FROM ALL THE SUCCESSES OF QCD

IN QCD WE START WITH A DESCRIPTION

$$\mathcal{L} = \frac{1}{4g^2}\int d^4x \left(\mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \bar{q}(iD\!\!\!/ - m)q \right)$$

THAT LOOKS LIKE IT WILL LEAD TO COULOMB-LIKE FORCES



$$V(r) = \frac{g^2}{r} \; ??$$

ASYMPTOTIC FREEDOM IMPLIES THAT THIS ISN'T THE WHOLE STORY ...

THE "COUPLING CONSTANT" ④

g^2 "RUNS" AND IS REALLY A FUNCTION OF r:

$$V(r) = \frac{g^2(r)}{r}$$

WHERE $g^2(r) \sim \frac{\text{const}}{\ln 1/r}$ FOR $r \to 0$

$g^2(r) \sim$?? FOR $r \to \infty$

FROM THIS POINT OF VIEW, QUARK CONFINEMENT ROUGHLY

HAS TO DO WITH THE BEHAVIOR OF $g^2(r)$ FOR $r \to \infty$, WHERE IT IS HARD TO CALCULATE AS PERTURBATION THEORY IS NO GOOD.

ROUGHLY, CONFINEMENT

$\Longleftrightarrow$

$$\frac{g^2(r)}{r} \to \infty$$

FOR $r \to \infty$

(6)

WE ARE CONVINCED THAT QUARKS ARE CONFINED BECAUSE OF A MIXTURE OF PARTICLE EXPERIMENTS AND COMPUTER SIMULATIONS ... BUT THAT DOESN'T MEAN WE REALLY UNDERSTAND IT.

THE PROBLEM OF CONFINEMENT IS TO GIVE SOME SORT OF THEORETICAL EXPLANATION OF CONFINEMENT, STARTING FROM QCD ... AND A COMPUTATION OF THE PROPERTIES OF THE COLOR-SINGLET HADRONS THAT FORM AS A RESULT OF CONFINEMENT.

THE PROBLEM IS NOT REALLY SOLVED, SO ALL WE HAVE - AS IN MY TITLE - ARE "APPROACHES", i.e. METHODS THAT GIVE SOME INSIGHT BUT ARE NOT DEFINITIVE, EITHER BECAUSE THE APPROXIMATIONS ARE NOT RELIABLE OR BECAUSE THE MODELS THAT ARE SOLVED ARE NOT REALLY QCD.

IN THE FIRST PART OF THIS TALK, I WILL EXPLAIN SOME INSIGHTS FROM THE 1970s

* ANALOGY WITH SUPERCONDUCTIVITY; CONJECTURED ROLE OF ELECTRIC-MAGNETIC DUALITY
* STRONGLY COUPLED LATTICE GAUGE THEORY
* $1/N$ EXPANSION

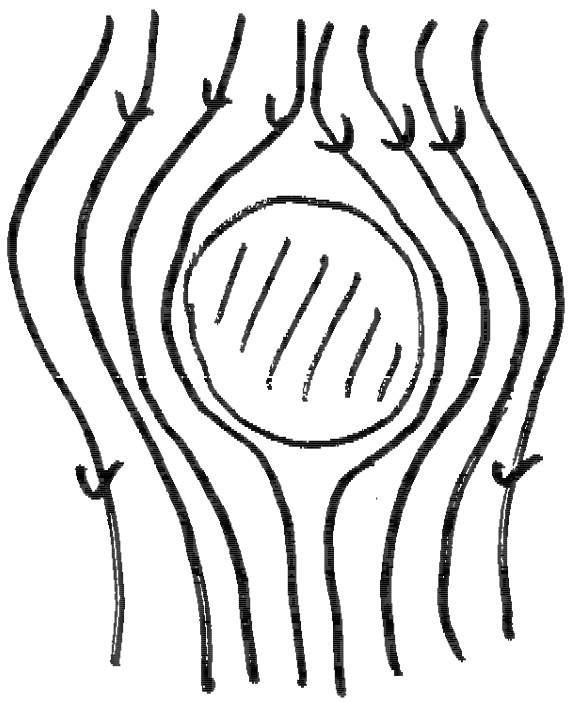
THEN I WILL CONCLUDE BY EXPLAINING NEW INSIGHTS OF THE 1990's, IN WHICH MODELS OF CONFINEMENT HAVE BEEN DEDUCED, RESPECTIVELY, FROM

* SUPERSYMMETRY AND DUALITY
* BRANES AND M-THEORY
* GAUGE-GRAVITY DUALITY

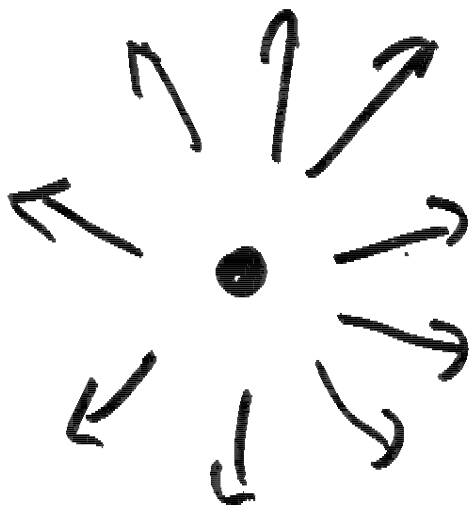
(1) ONE IMPORTANT INSIGHT OF THE '70's WAS THE ANALOGY BETEEN QUARK CONFINEMENT AND THE MEISSNER EFFECT

Mandelstam, 't Hooft, Polyakov Nambu

A SUPERCONDUCTOR EXPELS MAGNETIC FLUX

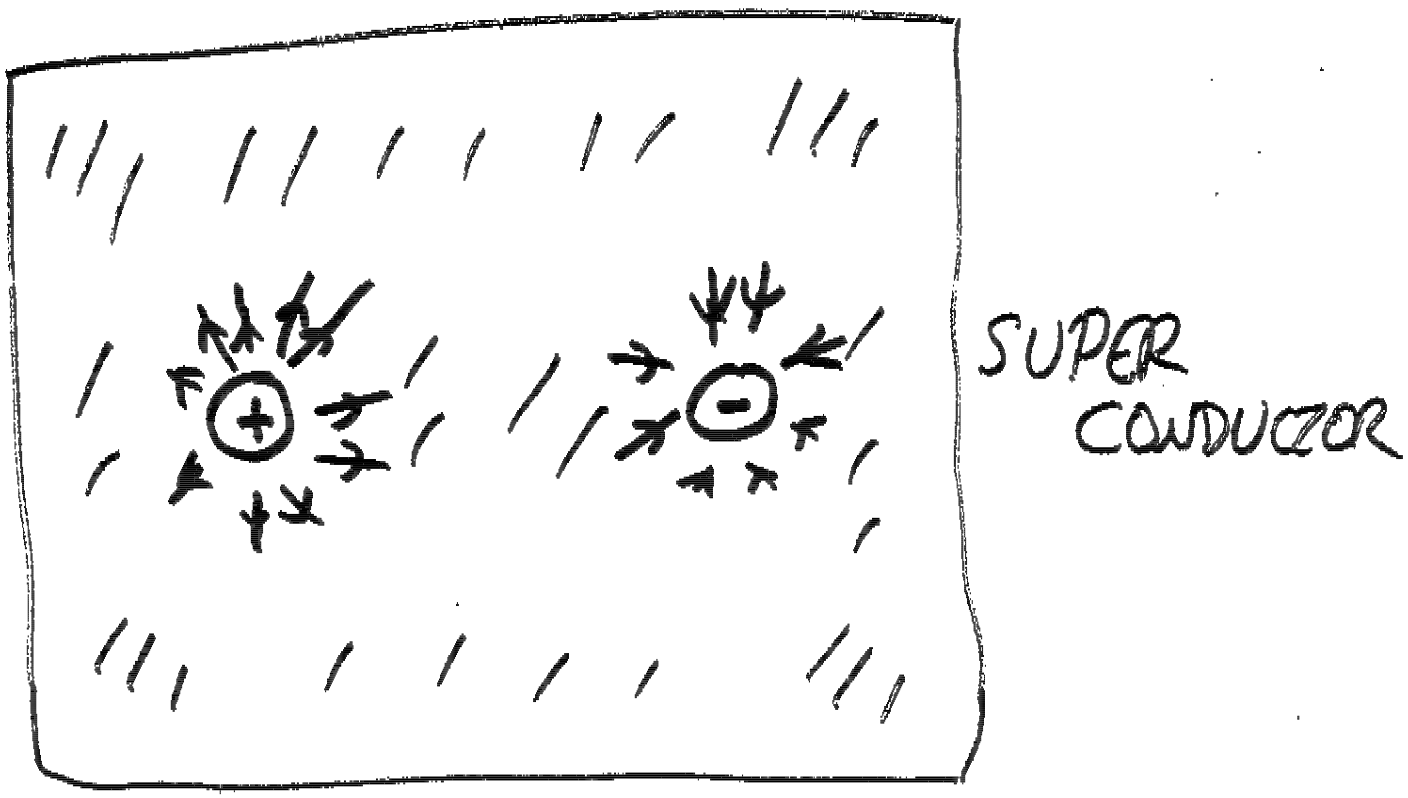


NOW SUPPOSE MAGNETIC MONOPOLES EXIST



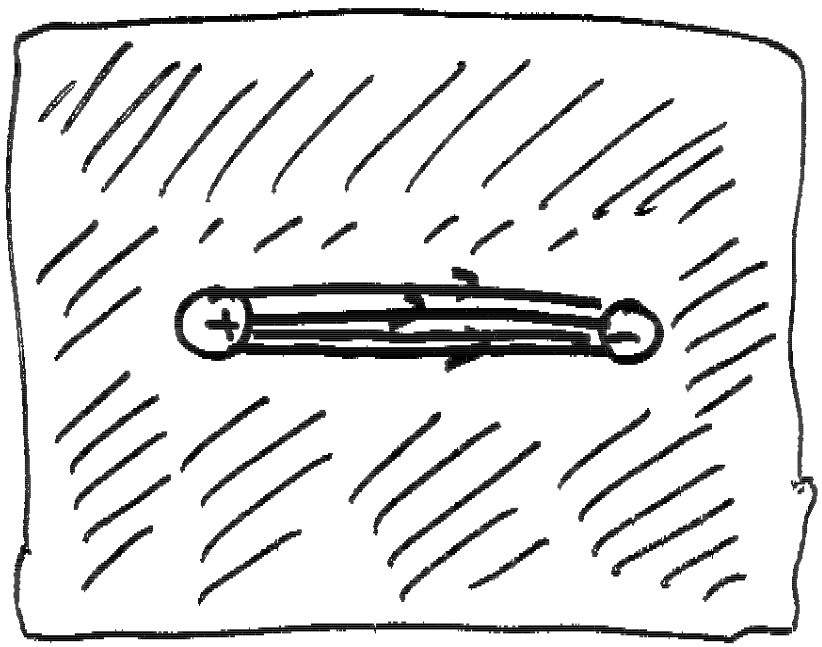
A MONOPOLE IS A SOURCE OF $4\pi g$ UNITS OF FLUX

INJECT A MONOPOLE-ANTIMONOPOLE PAIR INTO A SUPERCONDUCTOR AND WE HAVE A PROBLEM:



THE + AND - MAGNETIC CHARGES ARE SOURCES AND SINKS OF MAGNETIC FLUX, WHICH HOWEVER IS NOT ALLOWED IN A SUPERCONDUCTOR

ENERGETICALLY, THE BEST SOLUTION IS THAT A TUBE OF NORMAL MATERIAL FORMS BETWEEN THE MONOPOLE AND ANTIMONOPOLE, WITH THE MAGNETIC FLUX TRAPPED IN IT:



"ABRIKOSOV-GORKOV FLUX TUBE"

(15)

IF μ = ENERGY PER UNIT LENGTH OF AN ABRIKOSOV-GORKOV FLUX TUBE, THEN THE POTENTIAL ENERGY $V(r)$ FOR A MONOPOLE-ANTIMONOPOLE PAIR SEPARATED A DISTANCE r IS

$$V(r) = \mu r + \text{CONSTANT}$$

FOR LARGE r

THUS, MONOPOLES ARE CONFINED IN THIS SITUATION BECAUSE OF A PHYSICAL OBJECT, A STRING OR FLUX TUBE, THAT CONNECTS A MONOPOLE- ANTIMONOPOLE PAIR

ALL EXPLANATIONS OF CONFINEMENT INVOLVE SUCH A STRING...

THE PROPOSAL, THEN, IS THAT CONFINEMENT IS AN ELECTRIC ANALOG, FOR NONABELIAN GAUGE FIELDS, OF THE MEISSNER EFFECT:

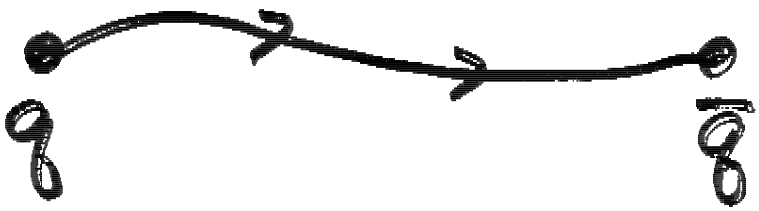
MONOPOLE	QUARK
U(1) MAGNETIC FIELD	COLOR ELECTRIC FIELD
ABRIKOSOV-GORKOV FLUX TUBE	"QCD ELECTRIC FLUX TUBE"

THIS ANALOGY DOESN'T EXPLAIN WHAT A QCD FLUX TUBE IS, OR WHY THEY EXIST, OR HOW TO CALCULATE ANYTHING, BUT IT DOES SUGGEST THAT THESE ARE THE KEY QUESTIONS, AND IT MAKES A PREDICTION: THE $q-\bar{q}$ POTENTIAL IS LINEAR FOR LARGE r

μ = "STRING TENSION"

$$V(r) = \mu r + \text{CONSTANT}$$

SINCE IT ARISES FROM THE FORMATION OF A QCD FLUX TUBE OR "STRING" CONNECTING QUARKS:



THE LINEAR POTENTIAL AGREES WELL WITH COMPUTER SIMULATIONS, AND THE STRING PICTURE HAS SOME SUPPORT IN HADRON PHENOMENOLOGY

(19.5)

INDEED, THIS PICTURE

OF A MESON

ACCOUNTS FOR THE PARTIAL

SUCCESSES OF THE

"DUAL RESONANCE MODEL"

OR STRING THEORY OF HADRONS

THAT PREDED QCD

TO DEMONSTRATE THE EXISTENCE OF QCD FLUX TUBES, WE NEED A MECHANISM THAT TRAPS THE "COLOR ELECTRIC FLUX" CREATED BY QUARKS LIKE THE MEISSNER EFFECT -OR SUPERCONDUCTING HIGGS MECHANISM- TRAPS ORDINARY MAGNETIC FLUX.

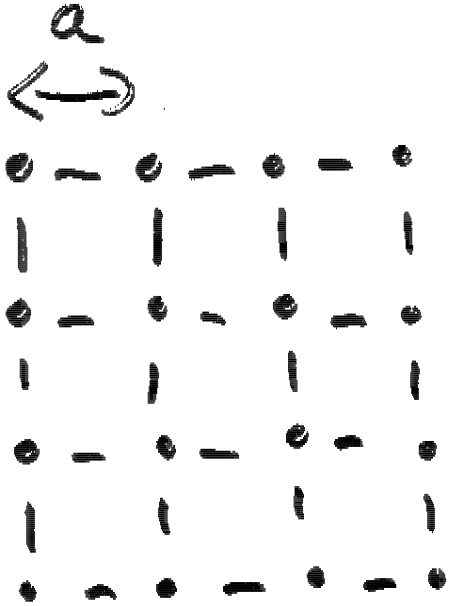
THIS IS TROUBLESOME, BECAUSE IN QCD IT IS HARD TO EVEN DEFINE THE COLOR ELECTRIC FLUX THAT SHOULD BE TRAPPED!

(20.5)

(2) WE STILL DON'T KNOW HOW TO DEMONSTRATE THE EXISTENCE OF QCD FLUX TUBES, BUT IN THE MID-1970's A SIMPLER MODEL WAS FOUND IN WHICH THEY CAN BE DEMONSTRATED:

STRONGLY COUPLED LATTICE GAUGE THEORY

WILSON, POLYAKOV



THIS IS A LATTICE VERSION OF QCD, IN WHICH QUARKS ARE ALLOWED ONLY ON LATTICE SITES AND ELECTRIC FIELDS ONLY ON THE LINKS BETWEEN THEM; IN A HIGHLY NONRELATIVISTIC LIMIT IN WHICH THE STRING TENSION μ IS SO LARGE THAT $\sqrt{\mu}a \gg 1$

(i.e. THERE IS A LARGE COST IN ENERGY TO SEPARATE A q-$\bar{q}$ PAIR EVEN ONE LATTICE SITE)

FLUX TUBES AND CONFINEMENT CAN BE SEEN BY HAND.

CONFINEMENT CAN ALSO BE STUDIED IN SUPERCOMPUTER SIMULATIONS IN THE OPPOSITE LIMIT $\sqrt{\sigma}a \ll 1$, WHERE ANALYTIC UNDERSTANDING FAILS. THIS IS THE LIMIT WHERE THE LATTICE THEORY AGREES WITH QCD.

(22,2)

MAXWELL'S EQUATIONS IN VACUUM

$$\nabla \times B = \frac{\partial E}{\partial t}, \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot E = \nabla \cdot B = 0$$

HAVE A SYMMETRY

$$E \to B, \quad B \to -E$$

SUCH A SYMMETRY, EXCHANGING ELECTRIC AND MAGNETIC FIELDS

...

22.4

WOULD MAD THE MEISSNER EFFECT TO QUARK CONFINEMENT.

BUT IN NONABELIAN GAUGE THEORY, THERE ARE NO CLASSICAL SYMMETRIES EXCHANGING MAGNETIC AND ELECTRIC FIELDS

{IN RELATIVISTIC PHYSICS

MEISSNER EFFECT

$\Leftrightarrow$ HIGGS MECHANISM}

'THE OTHER DEVELOPMENT OF (83)

THIS PERIOD WAS 't HOOFT'S INSIGHT

OF WHERE ONE SHOULD LOOK TO FIND

A PRECISE DESCRIPTION OF QCD

IN TERMS OF STRINGS; HE

CONSIDERED QCD WITH N COLORS

RATHER THAN 3, AND SHOWED

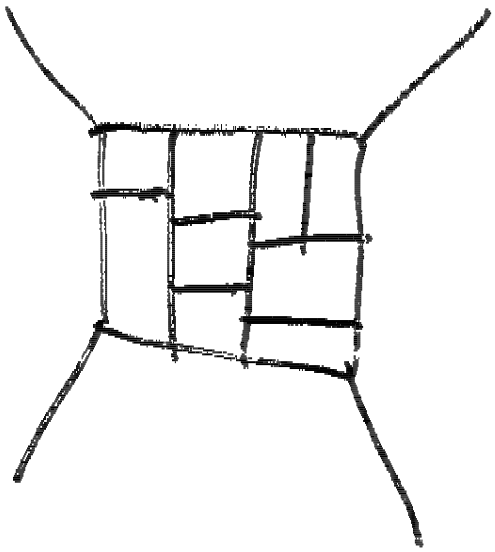
THAT FOR LARGE N THE

DOMINANT FEYNMAN DIAGRAMS

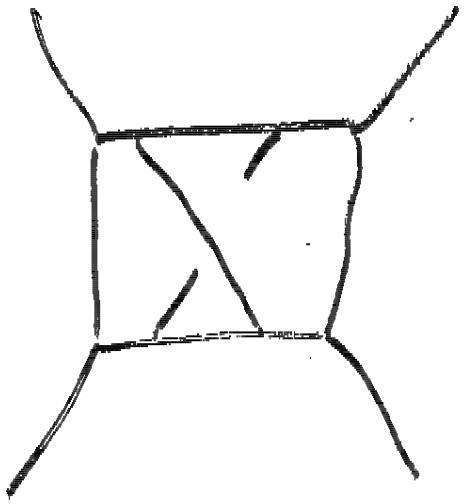
ARE "PLANAR" DIAGRAMS, WHICH

CAN BE DRAWN ON THE PLANE WITH

NO LINES CROSSING:

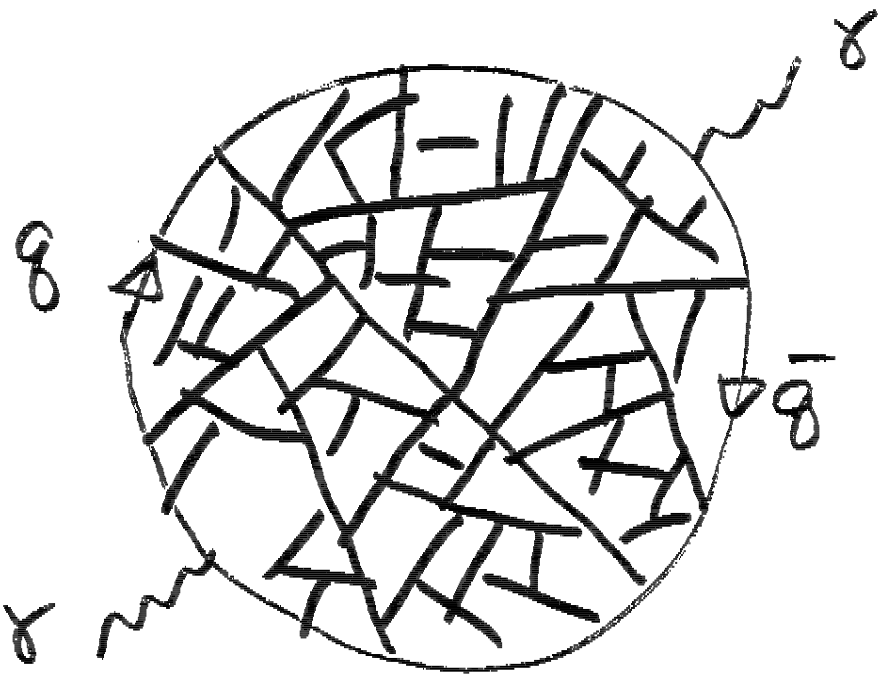


BUT NOT

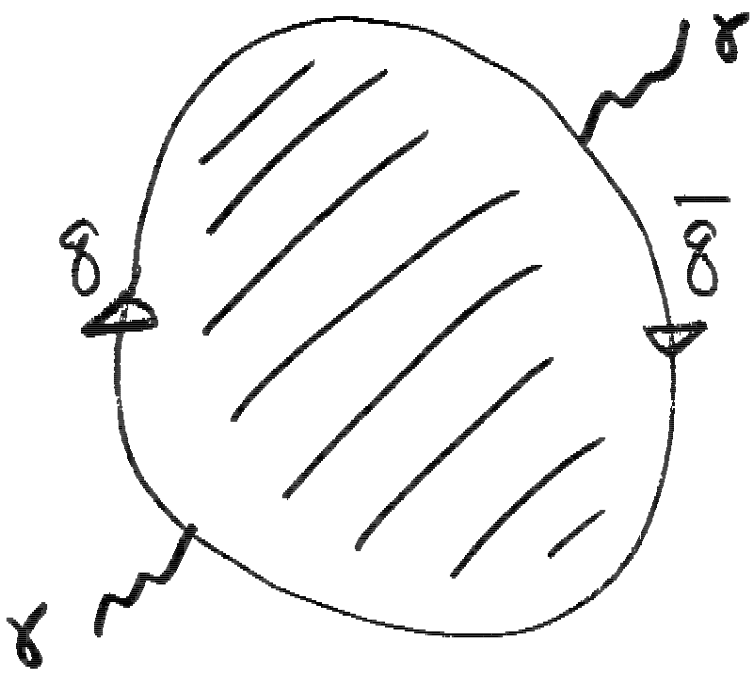


'T HOOFT THEN PROPOSED THAT SOME UNSPECIFIED NON PERTURBATIVE PROCESS FILLS IN THE "HOLES" IN THE FEYNMAN DIAGRAM TO MAKE A CONTINUOUS TWO-DIMENSIONAL SURFACE THE "STRING WORLD-SHEET"

HERE IS A QUARK LOOP IN PERTURBATION THEORY:



FILL IN THE HOLES:



"STRING WORLDSHEET"

SO: CONJECTURE

QCD IS EQUIVALENT TO A STRING THEORY IN WHICH THE STRING COUPLING CONSTANT IS $1/N$

BUT STRING THEORY DESCRIBES GRAVITY SO PROVING 't HOOFT'S CONJECTURE WOULD MEAN ESTABLISHING A GAUGE/GRAVITY RELATION.

OF THE FRAMEWORKS I AM MENTIONING, ONLY THE $1/N$ EXPANSION LOOKS LIKE IT COULD GIVE A METHOD, NOT JUST TO PROVE CONFINEMENT, BUT TO COMPUTE THE PROPERTIES OF THE HADRONS

ZWEIG'S RULE SUGGESTS IT IS A GOOD APPROXIMATION (NOTE $1/N = 1/3$, $e = .303$ $e^2/4\pi = 1/137$)

BUT TANGIBLE PROGRESS WAS OUT OF REACH IN THE 1970's AND 1980's

NOW I WILL TELL YOU ABOUT SOME "NEW" APPROACHES TO QUARK CONFINEMENT, WHERE - IN THE 1990's - NEW QCD-LIKE MODELS WERE FOUND IN WHICH QUARK CONFINEMENT CAN BE UNDERSTOOD AND DEMONSTRATED ... OFTEN IN THE PROCESS MAKING MORE CONCRETE SOME OF THE OLD IDEAS. FOR BREVITY, I'LL JUST GIVE TWO EXAMPLES:

(1) SUPERCONDUCTIVITY ARISES WHEN AN ELECTRICALLY CHARGED COOPER PAIR "CONDENSES" OR IN PARTICLE PHYSICS LANGUAGE GETS A VACUUM EXPECTATION VALUE (VEV) SO QUARK CONFINEMENT SHOULD ARISE IF A "MAGNETICALLY CHARGED FIELD," WHATEVER THAT IS, GETS A VEV.

SO 't HOOFT PROPOSED (ca. 1980)

START WITH A GAUGE

THEORY WITH HIGGS FIELDS

$$V(\vec{\phi}) = \lambda(\vec{\phi}^2 - a^2)^2$$

THAT HAS MAGNETIC MONOPOLES,

MAKE a^2 SMALLER UNTIL THE

MONOPOLE BECOMES MASSLESS

AND THEN "CONDENSES".... AND

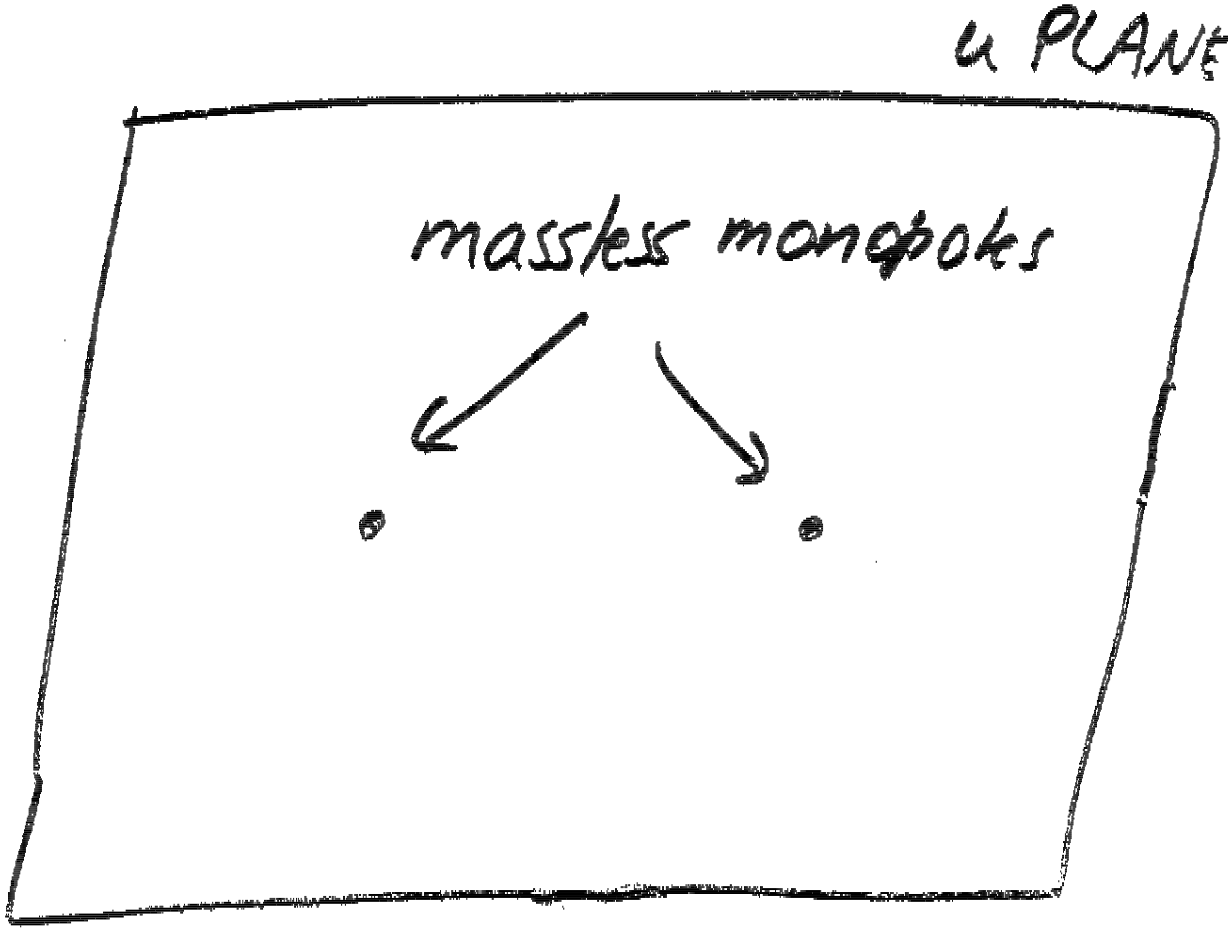
PRESTO CONFINEMENT

THE ONLY TROUBLE WAS THAT ONE COULDN'T REALLY CALCULATE IF THIS IS RIGHT...

BUT SEIBERG AND I (1994) FOUND A SUPERSYMMETRIC THEORY WITH HIGGS FIELDS AND MONOPOLES IS WHICH EVERYTHING CAN BE COMPUTED AND WORKS OUT AS 't HOOFT HAD PREDICTED

31

THE COMPUTATIONS USED HOLOMORPHY TO DETERMINE WHAT HAPPENS FOR STRONG COUPLING, GIVEN KNOWLEDGE OF WEAK COUPLING



IN ORDER TO DESCRIBE

A PHASE OF GAUGE THEORY

IN WHICH MONOPOLES "CONDENSE"

ONE CANNOT USE THE

"ORIGINAL" GAUGE FIELD ...

ONE NEEDS TO MAKE AN

"ELECTRIC-MAGNETIC DUALITY

TRANSFORMATION" ....

I MENTIONED BEFORE

THAT ELECTROMAGNETISM HAS

A SYMMETRY

$$E \rightarrow B$$

$$B \rightarrow -E$$

AND CLASSICAL YANG-MILLS

THEORY DOES NOT....

(34)

BUT IT WAS CONJECTURED IN THE 1970's (MONTONEN-OLIVE) THAT QUANTUM YANG-MILLS THEORIES CAN HAVE SUCH SYMMETRIES

$$E \longleftrightarrow B$$

$$\text{QUARKS} \longleftrightarrow \text{MONOPOLES}$$

"HIGGS MECHANISM" OF WEAK INTERACTIONS $\longleftrightarrow$ QUARK CONFINEMENT

AND THIS HAS BEEN PUT ON A SOLID FOOTWG IN THE 1980's ...

AND QUANTUM DUALITY

TRANSFORMATIONS THAT $\frac{gk}{e}$ $\frac{\text{monopole}}{\frac{2\pi\hbar c}{e}}$

EXCHANGE

WEAK AND STRONG

COUPLING

ELECTRIC AND MAGNETIC

FIELDS

HAVE BEEN USED IN MANY

CASES TO PROBE

CONFINEMENT

(2) WHAT I'VE EXPLAINED SO FAR IS BASICALLY THAT THE VISION "CONFINEMENT FROM SYMMETRY BETWEEN ELECTRICITY AND MAGNETISM" HAS BEEN ESTABLISHED IN SOME EXAMPLES.

MORE RECENTLY, THE VISION "LARGE N GAUGE THEORY EQUALS STRING THEORY" HAS ALSO BEEN REALIZED, IN SOME EXAMPLES.

IT ALL DEPENDS ON THE DISCOVERY OF D-BRANES (Polchinski, Green)

A D-BRANE IS A STRANGE OBJECT IN STRING THEORY

- ITS POSITION IS DESCRIBED BY NONCOMMUTING MATRICES

NOT JUST $[p, x] = -i\hbar$

BUT $[x_i, x_j] \neq 0$

MORE EXACTLY, IN A SYSTEM OF N D-BRANES, THE COMPONENTS $X_1, X_2 \ldots$ OF D-BRANE POSITION ARE MEASURED BY [illegible] $N \times N$ MATRICES; FOR EXAMPLE

$$X_1 = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{pmatrix}$$

$$X_2 = \begin{pmatrix} b_1 & & & \\ & b_2 & & \\ & & \ddots & \\ & & & b_N \end{pmatrix}$$

IF THE X_i COMMUTE - AS IN THAT EXAMPLE - THEIR EIGENVALUES ARE THE D-BRANE POSITIONS. IN THAT EXAMPLE THE N D-BRANES ARE LOCATED AT

$$(X_1, X_2) = \begin{matrix} (a_1, b_1) \\ (a_2, b_2) \\ \vdots \\ (a_N, b_N) \end{matrix} \Bigg\} \begin{matrix} \text{COORDINATES} \\ \text{OF THE} \\ N \\ \text{D-BRANES} \end{matrix}$$

ASSUMING OUR D-BRANE IS A PARTICLE, THERE IS A HAMILTONIAN DESCRIBING ITS MOTION, ROUGHLY

$$H = \frac{1}{2m} \sum_{i=1,2} \mathrm{Tr}\left(\left(-i\hbar \frac{\partial}{\partial X_i} \right)^2 \right)$$

$$+ \lambda \sum_{i<j} \mathrm{Tr}\left(\left(-i[X_i, X_j] \right)^2 \right)$$

THE FIRST TERM IS

THE KINETIC ENERGY; FOR

$N=1$ -- A SINGLE D-BRANE...

IT IS

$$\sum_i \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x_i}\right)^2$$

AS IN QUANTUM MECHANICS

TEXTBOOKS.

THE SECOND TERM IS A POTENTIAL

ENERGY THAT TENDS TO MAKE

THE MATRICES COMMUTE

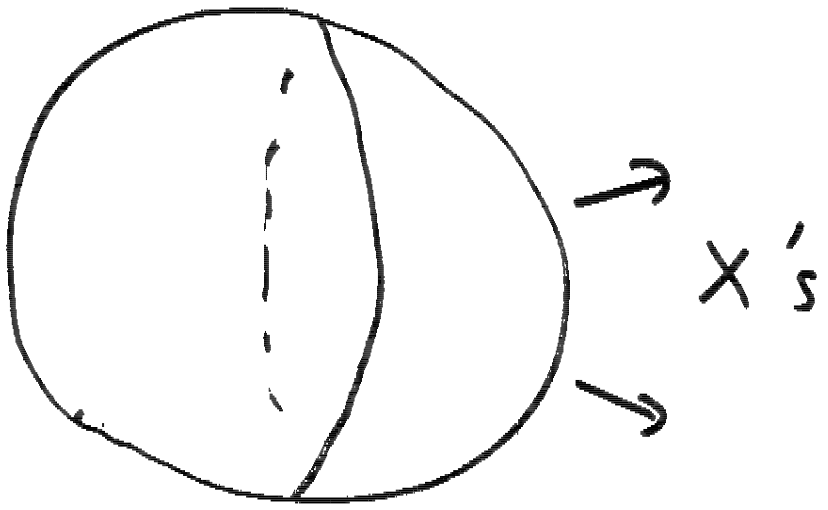
FOR COMMUTING MATRICES

$$X_1 = \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_N \end{pmatrix} \quad X_2 = \begin{pmatrix} b_1 & & \\ & \ddots & \\ & & b_N \end{pmatrix}$$

WITH $|\vec{a} - \vec{b}|$ LARGE

THIS REDUCES TO QUANTUM MECHANICS OF N IDENTICAL PARTICES.

NOW, A D-BRANE INSTEAD OF BEING A PARTICLE, CAN BE AN EXTENDED OBJECT LIKE A SOAP BUBBLE



IN THIS CASE, THE "NORMAL" OSCILLATIONS OF THE D-BRANE ARE DESCRIBED BY NONCOMMUTING MATRICES X_i

TO MAKE A MODEL OF QUARK CONFINEMENT IN FOUR DIMENSIONS, WE WANT THE CASE THAT THE D-BRANE IS A 3+1 - DIMENSIONAL "MEMBRANE" IN 10 DIMENSIONS.

THERE ARE 6 X_i DESCRIBING D-BRANE MOTION, AND - THE KEY POINT - THE U(N) SYMMETRY OF THE X_i IS A GAUGE SYMMETRY

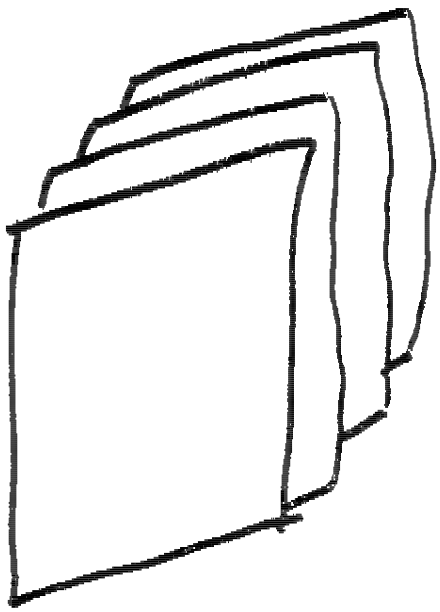
SO THE LAGRANGIAN DESCRIBING MOTION OF N SUCH D-BRANES IS ROUGHLY

$$\mathcal{L} = \frac{1}{4g_s}\int d^4x \Big[\mathrm{Tr}\Big(F_{\mu\nu}F^{\mu\nu} + (D_m X_i)^2 + \sum_{i<j} [X_i, X_j]^2 \Big) \Big]$$

g_s IS THE STRING COUPLING CONSTANT

NOW A D-BRANE IS A PHYSICAL OBJECT IN THE CONTEXT OF A STRING THEORY OF QUANTUM GRAVITY SO IT HAS A GRAVITATIONAL FIELD.

IF YOU PLACE N D-BRANES IN AN



OTHERWISE FLAT SPACE THE GRAVITATIONAL DISTORTION IS

$\sim g_s N$

THIS IS NOT TOO

INTERESTING IF $g_s N << 1$

THE GOOD CASE TURNS OUT TO

BE

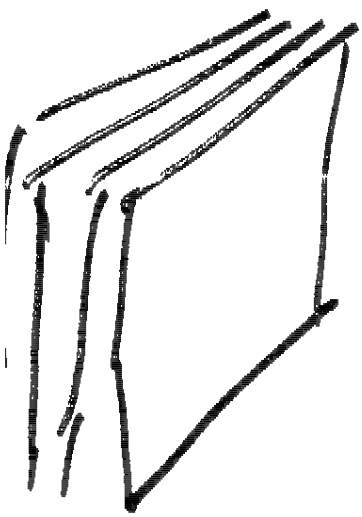
$g_s << 1$ SO WE CAN
UNDERSTAND
THE MODEL

$g_s N$ FIXED SO THE
GRAVITY OF THE D-BRANES
IS INTERESTING
and ~~therefore~~

$N >> 1$

THE LIMIT g_s SMALL, N LARGE, $g_s N$ FIXED IS EXACTLY THE LIMIT IN WHICH 't HOOFT CONJECTURED (1974) THAT GAUGE THEORY WOULD TURN INTO STRING THEORY, AND AS FIRST ARGUED BY MALDACENA (1997) THAT IS WHAT HAPPENS HERE

TWO THINGS ARE EQUIVALENT



GAUGE THEORY OF N PARALLEL D-BRANES

PHYSICS IN CURVED SPACETIME CREATED BY THE D-BRANES

ONE CAN UNCOVER A "ROSETTA STONE" MAPPING PROBLEMS IN GAUGE THEORY TO PROBLEMS IN GRAVITY

(50)

HERE IS A PORTION OF THE ROSETTA STONE

GAUGE:

DO GAUGE THEORY AT HIGH TEMPERATURE

DECIDE IF QUARKS ARE CONFINED

GRAVITY:

COUNT THE QUANTUM STATES OF A BLACK HOLE

FIND THE TOPOLOGY OF A EUCLIDEAN BLACK HOLE

SO WE CAN UNDERSTAND QUARK CONFINEMENT IN CERTAIN GAUGE THEORIES WITH CERTAIN SETS OF MATTER FIELDS, BUT NOT YET IN QCD.

WE ONLY HAVE APPROACHES TO QUARK CONFINEMENT.