# CP-violating Asymmetry in $B \to \pi\pi\pi$, $KK\pi$ Decays

( Phenomenological Application of Chiral-Heavy Quark Effective Theory)

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## Motivation

1) For $B \to 3$ body decays, there is no well developed Method.

   - ⊙ chiral ⊕ heavy quark symmetry
   - ⊙ PQCD approach [ Chen and Li ]
     - ↳ $B \to f_0 \pi \to \pi\pi\pi$

⇒ Challenging to find a viable dynamical description.

2) Large direct CP-violation should occur in $B^{\pm} \to h\pi^{\pm}$ ($h = \pi^+\pi^-, K^+K^-$) has energy corresponding to the resonance $\chi_{c0}$.

Contents ;

1) Non-Resonant Background of $B \to 3\pi$
2) Resonant Decay for $B \to \chi_{c0}\pi \to 3\pi$
3) Partial width CP-Asymmetry

# Present Experimental limits :

## First measurements by Belle collaboration

$$Br(B^+ \to K^+\pi^-\pi^+) = (55.6 \pm 5.8 \pm 7.7) \times 10^{-6}$$

$$Br(B^+ \to K^+K^-K^+) = (35.3 \pm 3.7 \pm 4.5) \times 10^{-6}$$

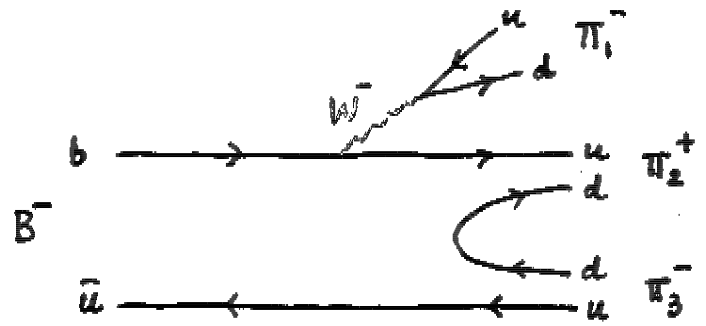
$$B^+ \to (\pi^+\pi^-\pi^+)_{NR} < 4.1 \times 10^{-5}$$

$$\to (K^+\pi^-\pi^+)_{NR} < 2.8 \times 10^{-5}$$

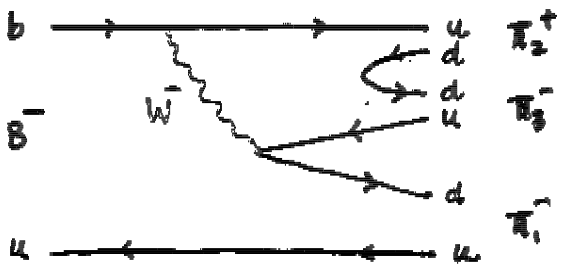
$$\to (K^+K^-\pi^+)_{NR} < 7.5 \times 10^{-5}$$

$$\to (K^+K^-K^+)_{NR} < 3.8 \times 10^{-5}$$

# Non-Resonant Background of $B^{\pm} \to 3\pi$.




(a)




(b)

- The relevant effective Hamiltonian is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \left[ C_1 O_1 + C_2 O_2 \right]$$

$$O_1 = \bar{u} \gamma_\mu (1-\gamma_5) b \cdot \bar{d} \gamma^\mu (1-\gamma_5) u$$

$$O_2 = \bar{d} \gamma_\mu (1-\gamma_5) b \cdot \bar{u} \gamma^\mu (1-\gamma_5) u$$

- Under the Factorization approximation,

$$A(\bar{B} \to 3\pi)_{Fact} = \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \langle \pi_1^- \pi_2^+ \pi_3^- | C_1 O_1 + C_2 O_2 | \bar{B} \rangle_{Fact}$$

$$= \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^* \Big\{ a_1 \langle \pi_2^+ \pi_3^- | (\bar{u} b)_{V-A} | \bar{B} \rangle \langle \pi_1^- | (\bar{d} u)_{V-A} | 0 \rangle$$

$$+ a_2 \langle \pi_1^- | (\bar{d} b)_{V-A} | \bar{B} \rangle \langle \pi_2^+ \pi_3^- | (\bar{u} u)_{V-A} | 0 \rangle$$

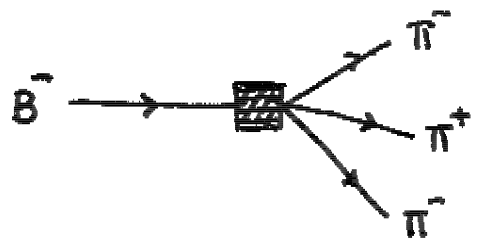
$$+ (\pi_1^- \longleftrightarrow \pi_3^-) \Big\} \otimes \frac{1}{2} \qquad \cdots\cdots (1)$$

$$a_1 = C_1 + \frac{C_2}{N_c} \quad ; \quad a_2 = C_2 + \frac{C_1}{N_c}$$

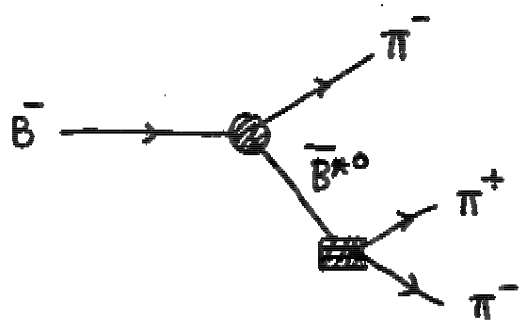
- The $a_2$ part : can be easily calculated using the current matrix elements of the form factors.

- The $a_1$ part : including $B^*$-pole contributions

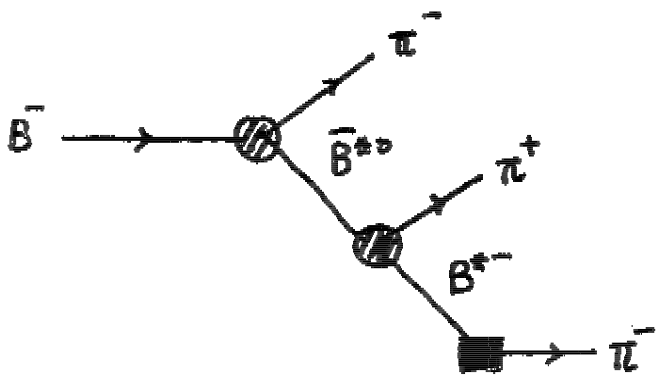
  ↳ HQET ⊕ $\chi$-PT. (chPT)



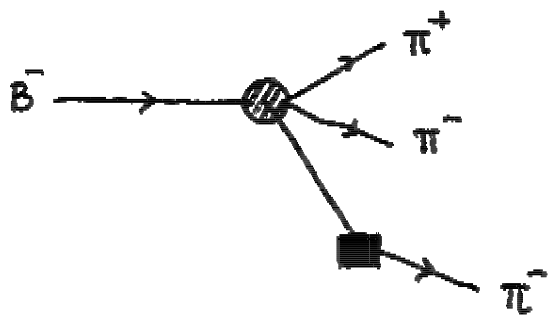

(2-a)



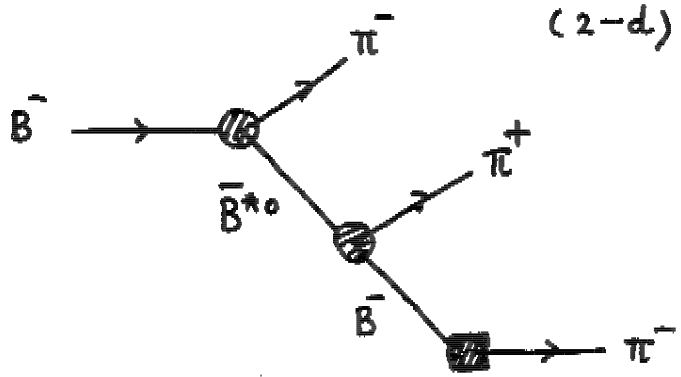

(2-b)




(2-c)




(2-d)




(2-e)

⊗ : Strong vertex

▨ : Weak vertex

# $a_2$ - part Calculation

$$\langle \pi_1^-(P_1) | (\bar{d}b)_{V-A} | B^-(P_B) \rangle = (P_B + P_1)_\mu F_1^{B\pi}(q_0^2) + (P_B - P_1)_\mu \frac{(m_B^2 - m_\pi^2)}{q_0^2} [F_0^{B\pi}(q_0^2) - F_1^{B\pi}(q_0^2)]$$

$$\langle \pi_2^+(P_2)\, \pi_3^-(P_3) | (\bar{u}u)_{V-A} | 0 \rangle = \langle \pi_2^+(P_2) | \bar{u}\gamma_\mu u | \pi_3^+(-P_3) \rangle = (P_1 - P_3)_\mu F_1^{\pi\pi}(q_0^2)$$

with $q_0 = P_B - P_1 = P_2 + P_3$

➡

$$A(B^- \to \pi^+\pi^-\pi^+) \overset{a_2\text{-part}}{=\!=} \frac{G_F}{\sqrt{2}} m_B^2 V_{ub} V_{ud}^* \frac{a_2}{m_B^2} \otimes \left\{ (2s_3 + s_1 - m_B^2 - 3m_\pi^2) F_1^{B\pi}(s_1) F_1^{\pi\pi}(s_2) + (s_1 \leftrightarrow s_3) \right\}$$

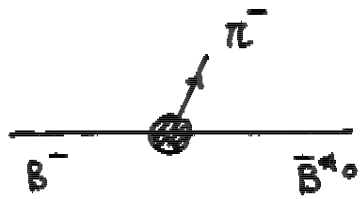
- $F_1^{B\pi}(s_1) = \dfrac{F_1^{B\pi}(0)}{1 - s_1/m_1^2}$ ( $F_1^{B\pi}(0) = 0.53$ ; $m_1 = 5.32$ GeV )

- $F_1^{\pi\pi}(s_1) = \dfrac{(1 - s_1/m_\sigma^2) - i\,\Gamma_\sigma/m_\sigma}{(1 - s_1/m_\sigma^2)^2 + \Gamma_\sigma^2/m_\sigma^2}$ ( $\Gamma_\sigma = 0.2$ GeV, $m_\sigma = m_{\pi\pi} = 0.7$ GeV )

# $a_1$ - part calculation

⊙ by using both Heavy Quark Effective Theory (HQET) and Chiral perturbation theory (ChPT)

With combining chiral and heavy quark symmetry, the relevant Lagrangian for the hadronic interaction of $B \to B^* \pi$ is given




$$\mathcal{L}_{int} = i g \langle H_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba} \bar{H}_a \rangle + \cdots$$

$$H_a = \frac{(1+\not{v})}{2} [ P^*_{a\mu} \gamma^\mu - P_a \gamma_5 ]$$

$$\bar{H}_a = \gamma_0 H_a^+ \gamma_0$$

} Heavy meson ($Q\bar{q}_a$)

and $\mathcal{A}^\mu_{ba} = \frac{1}{2} ( \xi^+ \partial^\mu \xi - \xi \partial^\mu \xi^+ )_{ba}$ : Light mesons

$v$ : heavy meson velocity

$a = 1, 2, 3$ for $u, d$ and $s$ respectively

$P_a = (B^-, \bar{B}^0, \bar{B}_s)$

$P^*_a = (B^{-*}, \bar{B}^{0*}, \bar{B}^*_s)$

⊙ *Pseudoscalar light mesons :*

$$\xi = \exp\left[ i \frac{M}{f_\pi} \right] \quad ; \quad f_\pi = 132 \text{ MeV}$$

with $$M = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta^0 & , & \pi^+ & , & K^+ \\ \pi^- & , & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & , & K^0 \\ K^- & , & \bar{K}^0 & , & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

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⊙ Lagrangian of heavy meson effective theory :
upto $O(p^2)$ in the chiral expansion [Grinstein]
$O(1/M_Q^2)$ in $1/M_Q$ expansion in Heavy quark mass.

$$\mathcal{L} = -\mathrm{Tr}\left[ \bar{H}_a(v) \; i v\cdot D_{ba} \, H_b(v) \right]$$

$$+ \left( g + \frac{g_1}{M} \right) \mathrm{Tr}\left[ \bar{H}_a(v) H_b(v) \not{A}_{ba} \gamma_5 \right]$$

$$+ \frac{g_2}{M} \mathrm{Tr}\left[ \bar{H}_a(v) \not{A}_{ba} \gamma_5 H_b(v) \right]$$

$$+ \frac{\lambda_2}{M} \mathrm{Tr}\left[ \bar{H}_a(v) \sigma^{\mu\nu} H_a(v) \sigma_{\mu\nu} \right]$$

$$= -\mathrm{Tr}\left[ \bar{H}_a(v) \; i v\cdot D_{ba} \, H_b(v) \right] + \tilde{g}_{\bar{H}H} \, \mathrm{Tr}\left[ \bar{H}_a(v) H_b(v) \not{A}_{ba} \gamma_5 \right] \quad \text{---- (2)}$$

$$\tilde{g}_{\bar{H}H} = \begin{cases} \tilde{g}_{B^*} = g + \frac{1}{M}(g_1 + g_2) & \text{for } B^*B^*\pi \text{ coupling} \\ \tilde{g}_B = g + \frac{1}{M}(g_1 - g_2) & \text{for } BB^*\pi \text{ coupling} \end{cases}$$

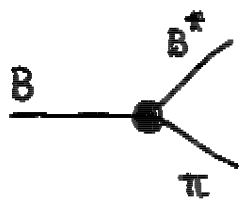
⊙ The chiral representation of the left-handed current $\bar{q}_{aL}\Gamma b$ proceeds similarly with $\Gamma = \gamma^\lambda(1-\gamma_5)$ :

$$J_a^\lambda = \frac{\lambda\alpha}{2}\left(1+\frac{\beta_1}{M}\right) Tr\left[\Gamma H_b(v)\bar{\xi}^+_{ba}\right]$$

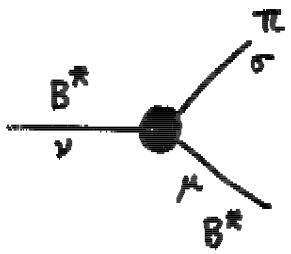
$$+ \frac{\lambda\alpha}{2}\frac{\beta_2}{M} Tr\left[\gamma^\mu\Gamma\gamma_\mu H_b(v)\bar{\xi}^+_{ba}\right]$$

$$+ \frac{\lambda\alpha}{4M} Tr\left[\left[\not{P}, \gamma_\mu\right] i D^\mu_{cb} H_c(v)\bar{\xi}^+_{ba}\right] \quad \cdots (3)$$

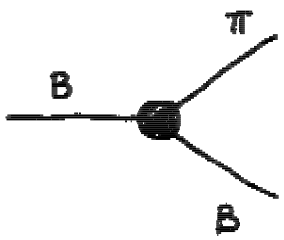
⊙ From Lagrangian $Eq^{(2)}$ :

ⓐ 


$$g_{BB^*\pi} = -2\tilde{g}_B\frac{1}{f_\pi}(P_\pi)_\mu$$

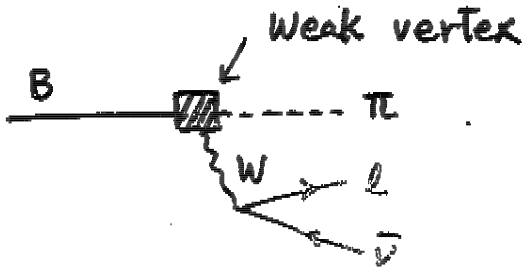
ⓑ 


$$g_{B^*B^*\pi} = 2i\tilde{g}_{B^*}\frac{1}{f_\pi}\varepsilon^{\mu\alpha\nu\sigma}v_\alpha(P_\pi)_\lambda$$

ⓒ 


$$g_{BB\pi} = 0$$

⊙ $B \to \pi \ell \nu$ form factors :




$B\pi W$ vertex :

$$\langle \pi | V_\mu | B \rangle = \lim_{P_\pi \to 0} \langle 0 | [ Q_\pi , V_\mu ] | B \rangle$$

$$= \langle 0 | A_\mu | B \rangle$$

$$= \langle 0 | J_a^\lambda | B \rangle$$

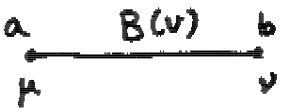
Since $\xi^\dagger_{ba} = \exp\left[-i \frac{m^+}{f_\pi}\right]$

$$= 1 + \underbrace{\frac{-i}{f_\pi} (m^+)_{ba}}_{\text{1-pion state}} + \frac{1}{2!}\left(\frac{-i}{f_\pi}\right)^2 \underbrace{(m^+)_{bc}(m^+)_{ca}}_{\text{2-pion state}} + \cdots$$

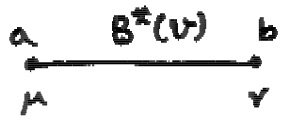
$$\therefore \langle \pi | V_\mu | B \rangle = -\frac{i}{f_\pi} \langle 0 | A_\mu | B \rangle$$

$$= -\frac{i}{f_\pi} \left[ -i f_B (P_B)_\mu \right] = -\frac{f_B}{f_\pi} (P_B)_\mu$$

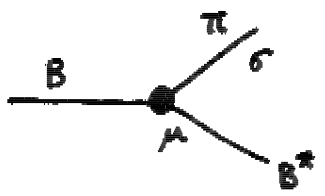
# $B \to \pi\pi\ell\nu$



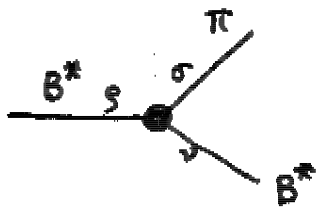

$$i\,\delta_{ab}\,\frac{1}{2(v\cdot k)}$$



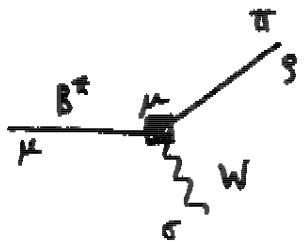

$$-i\,\delta_{ab}\,\frac{(g_{\mu\nu}-v_\mu v_\nu)}{2(v\cdot k+\Delta)}$$




$$g_{BB^*\pi} = -2\tilde{g}_B\,\frac{(p_\pi)_\mu}{f_\pi}$$




$$g_{B^*B^*\pi} = 2i\,\tilde{g}_{B^*}\,\frac{1}{f_\pi}\,\frac{1}{m_{B^*}}\,\varepsilon^{\rho\sigma\nu\alpha}(p_{B^*})_\alpha(p_\pi)_\sigma$$

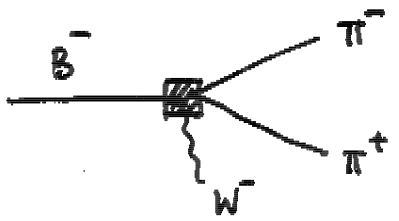



$$\frac{f_{B^*}}{f_\pi}\,(g^\mu_\sigma v_\rho - g^\mu_\rho v_\sigma)(p_{B^*})^\rho$$

$$i\,f_{B^*}(g^\sigma_\lambda v_\rho - g^\sigma_\rho v_\lambda)(p_{B^*})^\rho$$

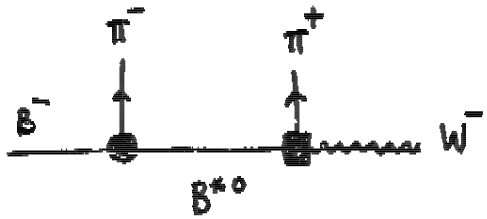
# Possible diagrams for $B \to \pi\pi l\nu$

(Fig-1)

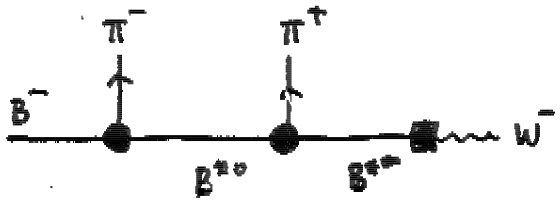



$$\langle \pi\pi | J_a^{\lambda}(\omega) | B \rangle = \frac{\lambda}{2} \frac{f_B}{f_\pi^2} (P_B)_\lambda$$

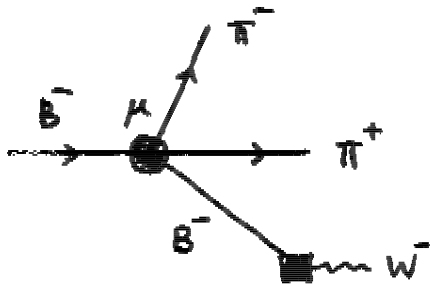
(Fig-2)



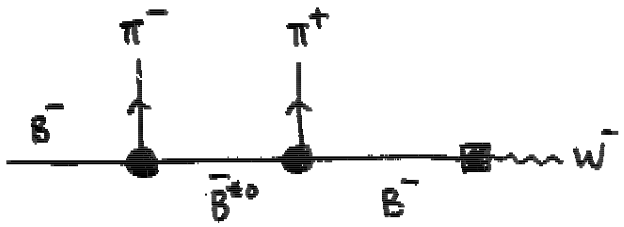

(Fig-3)




(Fig-4)




(Fig 5)



⊙ The hadronic matrix element $B \to \pi\pi$ can be written in terms of four form factors $\omega^{\pm}$, $r$, and $h$ :

$$\langle \pi^-(P_\pi^-)\, \pi^+(P_\pi^+) \mid \bar{u}\, \gamma_\mu (1-\gamma_5)\, b \mid B^-(P_B) \rangle$$
$$= i\,\omega_+ P_\mu + i\,\omega_- Q_\mu + i\,r\,(P_B - P)_\mu + h\,\varepsilon_{\mu\alpha\beta\gamma} P_B^\alpha P^\beta Q^\gamma$$

where $P_\mu = P_\pi^+ + P_\pi^-$

$Q_\mu = P_\pi^+ - P_\pi^-$

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In our case, We don't use form-factor Model :

(Fig 1) $\Rightarrow$ $\dfrac{i}{2}\dfrac{f_B}{f_\pi^2}\left[ (P_B - P)_\mu + P_\mu \right]$

(Fig 2) $\Rightarrow$ $-i\dfrac{f_{B^*}}{f_\pi^2}\tilde{g}_{B^*}\dfrac{1}{(v\cdot P_\pi^- + \Delta)}\Big[ \Big\{ \frac{1}{2}(m_B - v\cdot P_\pi^-) - v\cdot P_\pi^- \Big\} P_\mu$
$- \frac{1}{2}(m_B - v\cdot P_\pi^-)\, Q_\mu$
$- (v\cdot P_\pi^-)\,(P_B - P)_\mu \Big]$

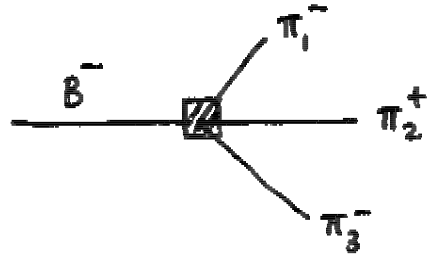
(Fig 3) $\Rightarrow$ $\dfrac{1}{2}\dfrac{f_{B^*}}{f_\pi^2}\tilde{g}_{B^*}^2\dfrac{1}{(v\cdot P_\pi^- + \Delta)}\dfrac{1}{[v\cdot(P_\pi^- + P_\pi^+) + \Delta]}\varepsilon_{\mu\alpha\beta\gamma} P_B^\alpha P^\beta Q^\gamma$

(Fig 4) $\Rightarrow$ $-i\dfrac{f_B}{2f_\pi^2}\dfrac{v\cdot(P_\pi^+ - P_\pi^-)}{v\cdot(P_\pi^- + P_\pi^+)}\left[ (P_B - P)_\mu + P_\mu \right]$

(Fig 5) $\Rightarrow$ $+i\dfrac{f_B}{f_\pi^2}\hat{g}_{B^*}^2\dfrac{1}{(v\cdot P_\pi^- + \Delta)}\dfrac{1}{v\cdot(P_\pi^- + P_\pi^+)}\left[ (P_\pi^- \cdot P_\pi^+) - (v\cdot P_\pi^-)(v\cdot P_\pi^+) \right]$
$\otimes \left[ (P_B - P)_\mu + P_\mu \right]$

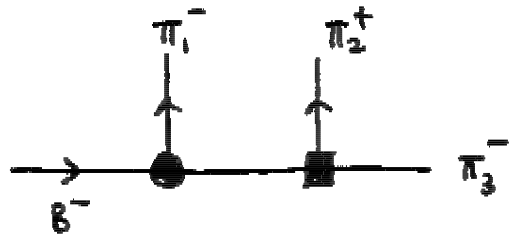
Dorminant contributions on $B \to 3\pi$ decays:

(Fig 1) $\Rightarrow$




$$\text{Amp 1} = -\frac{1}{8}\frac{f_B}{f_\pi}\left[2(m_B^2+m_\pi^2) - s_1 - s_3\right]$$

(Fig 2) $\Rightarrow$




$$\text{Amp 2} = -\frac{1}{2}\,\tilde{g}_{B^*}\left(\frac{f_{B^*}}{f_\pi}\right) m_B m_{B^*} \otimes \left[\frac{1}{(s_1 - m_{B^*}^2)}\left\{(s_2 - 2m_\pi^2) - \frac{1}{2m_{B^*}^2}(m_B^2 - m_\pi^2 - s_1)s_1\right\} + (s_1 \leftrightarrow s_3)\right]$$

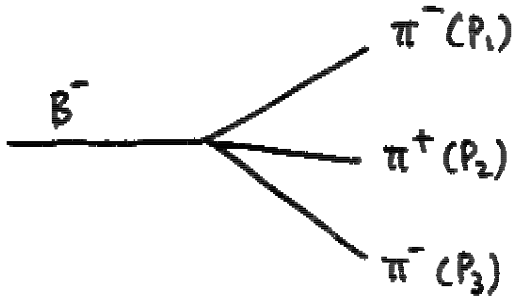
(Fig 3) $\Rightarrow$ "zero"

(Fig 4) and (Fig 5) : negligible !

* We found that (Fig 1) interfer distructively with (Fig 2).

$\Rightarrow$ provide smaller branching ratio.

# Kinematics and input parameters :




$$s_1 = (P_2+P_3)^2 = (P_B - P_1)^2$$

$$s_2 = (P_3+P_1)^2 = (P_B - P_2)^2$$

$$s_3 = (P_1+P_2)^2 = (P_B - P_3)^2$$

$$s_1 + s_2 + s_3 = m_B^2 + 3m_\pi^2$$

$$\boxed{d\Gamma = \frac{1}{256\pi^3}\frac{1}{m_B^3}\left|\langle 3\pi|T|B\rangle\right|^2 ds_1 ds_2}$$

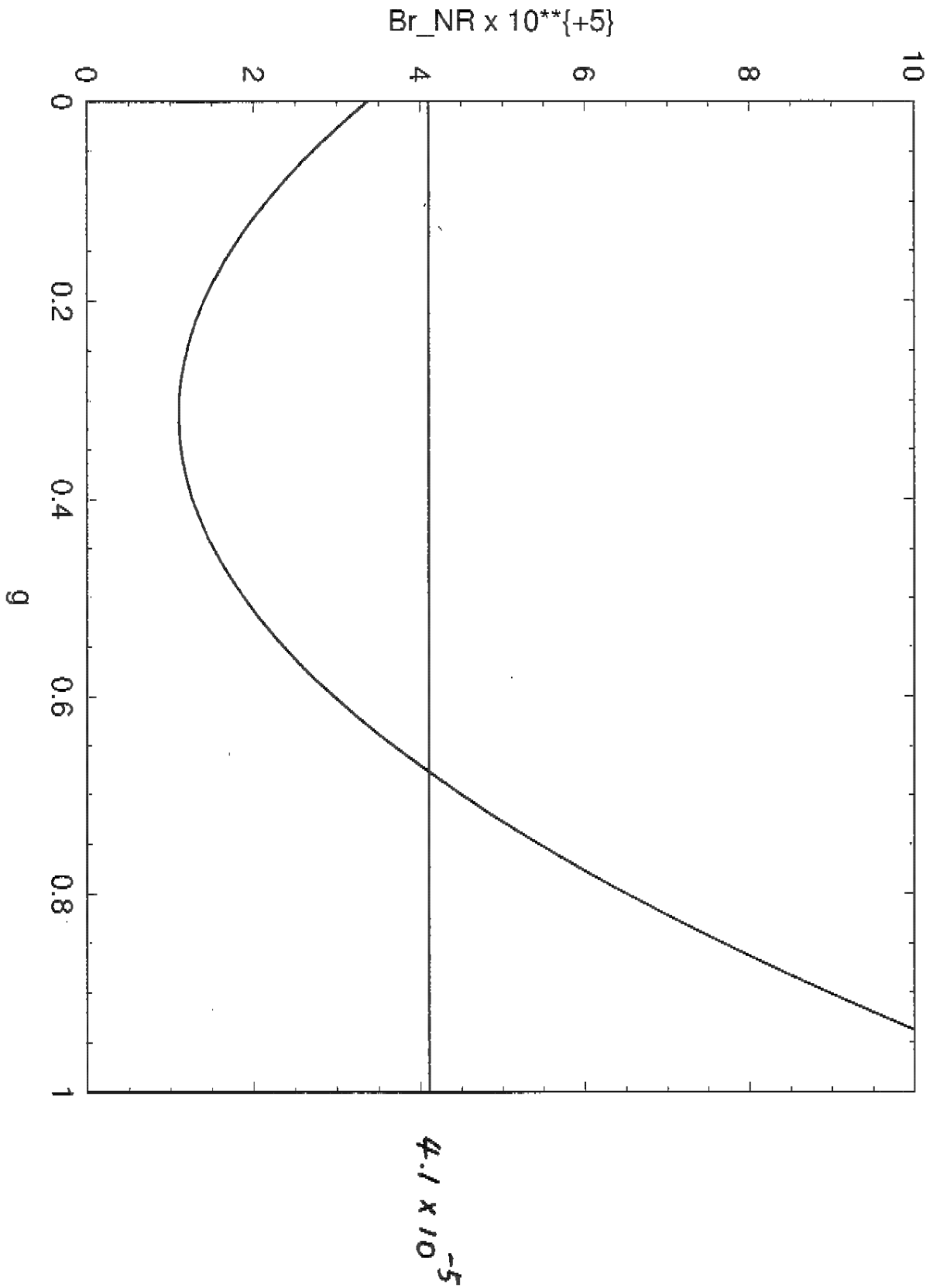
$$4m_\pi^2 \le s_1 \le (m_B - m_\pi)^2$$

$$s_-(s_1) \le s_2 \le s_+(s_1)$$

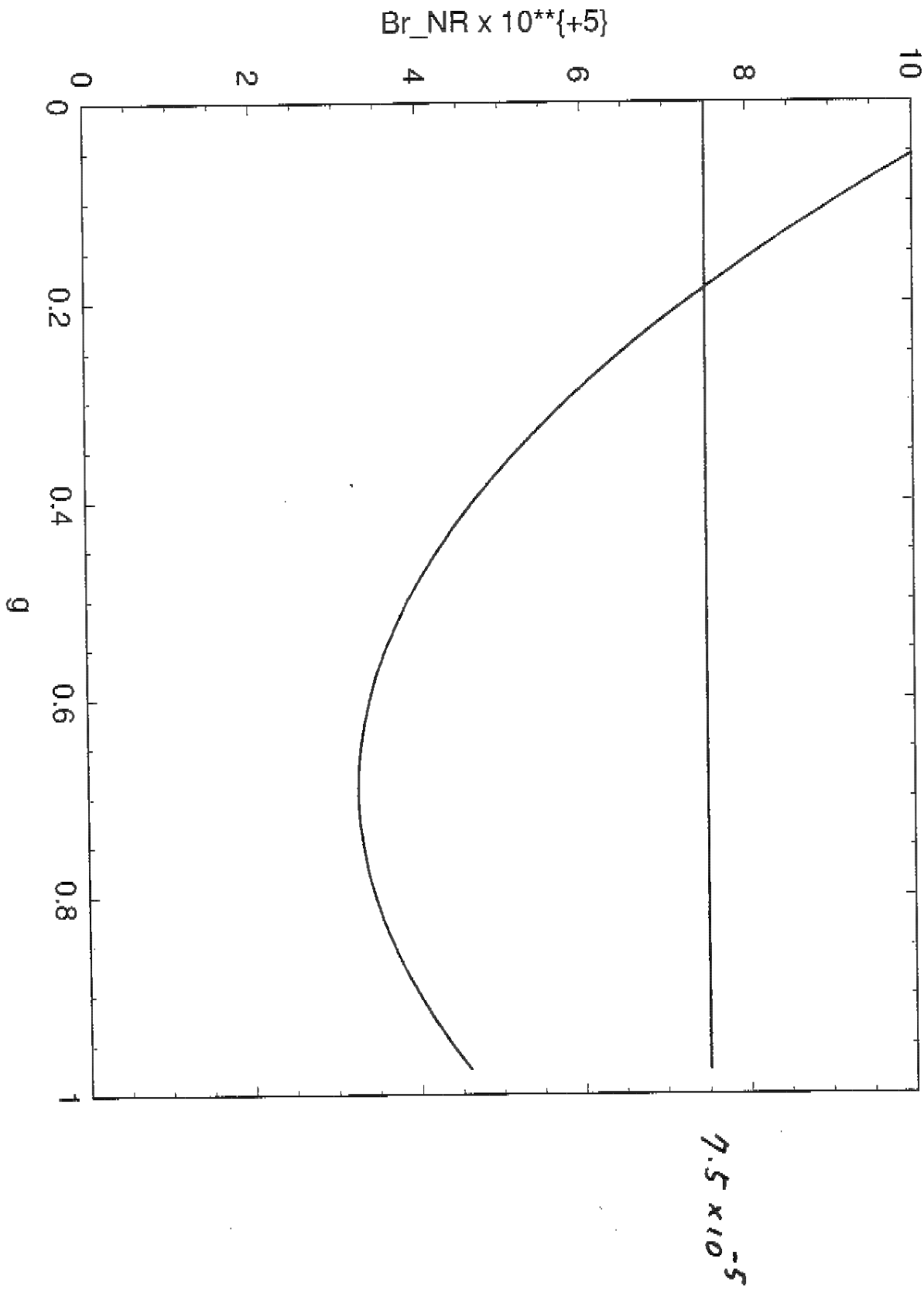
$$s_\pm(s) = \frac{1}{2}(m_B^2 + 3m_\pi^2 - s) \pm \frac{1}{2}\sqrt{\frac{(s-4m_\pi^2)[(m_B+m_\pi)^2-s][(m_B-m_\pi)^2-}{s}}$$

$m_B = 5.2787$ GeV $\quad \Delta_B = m_{B^*} - m_B = 46$ MeV

$m_{B^*} = 5.3248$ GeV $\quad f_B = 200$ MeV




$B^{+} \to \pi^{+}\pi^{+}\pi^{-}$



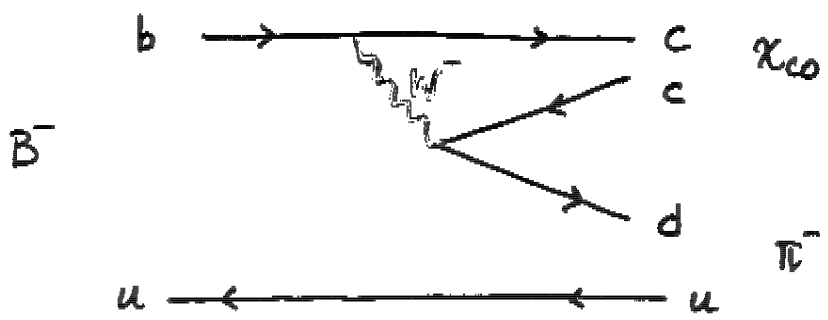

$B^{+} \to K^{+} K^{-} \pi^{+}$

$7.5 \times 10^{-5}$

# Resonant Decay for $B^- \to \chi_{c0}\pi^- \to 3\pi$

For $B^\pm \to \chi_{c0}(3415)\pi^\pm$ :

1) Color-suppressed decay mode




2) Since $\chi_{c0}$ is scalar partle ($0^{++}$)

Factorized term vanishes.

$$A(B^- \to \chi_{c0}\pi^-) = \frac{G_F}{\sqrt{2}} V_{bc}V_{cd}^* \left\{ a_2(\mu) \langle \pi^- | \bar{d}\gamma_\mu b | B \rangle \cdot \langle \chi_{c0} | \bar{c}\gamma^\mu c | 0 \rangle \right.$$

$$\left. + \underbrace{2C_1(\mu)}_{\text{big contribution!}} \underbrace{\langle \pi^- \chi_{c0} | (\bar{c}\Gamma T^a b)(\bar{s}\Gamma T_a c) | B \rangle}_{\text{Nonfactorizable term.}} \right\}$$

3) PQCD method can predict Br. ratios of $B \to \chi_{c0}\pi$, $\chi_{c0}K$.

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## (B) $B \to \chi_{c0} K$:

- Since $\chi_{c0}$ is scalar particle ($O^{++}$),

$$< \chi_{c0}(P)|\bar{c}_\alpha(y) c_\beta(0)|0 >= \frac{i}{\sqrt{2N_c}} \int_0^1 dx\ e^{ixP\cdot y} \left[ (\ \not{P}\ )_{\alpha\beta}\ \Phi^v_{\chi_{c0}}(x) + m_{c0}\ I\!I_{\alpha\beta}\ \Phi^s_{\chi_{c0}}(x) \right]$$

- Defined wave-functions as follows :

$$\begin{aligned} \Phi^s(x) &= \frac{f_{\chi_{c0}}}{2\sqrt{2N_c}}\ \phi_s(x); \qquad \phi_s(x) = 6x(1-x), \\ \Phi^v(x) &= \frac{f_{\chi_{c0}}}{12\sqrt{2N_c}}\ \frac{d}{dx}\phi_v(x); \quad \phi_v(x) = 6x(1-x), \\ &= \frac{f_{\chi_{c0}}}{2\sqrt{2N_c}}\ (1-2x); \quad \textit{antisym. under } x \leftrightarrow (1-x) \end{aligned}$$

- Vanishing factorized contribution:

$$< \chi_{c0}(P)|\bar{c}\gamma_\mu\ c_\beta|0 >= \frac{i}{\sqrt{2N_c}} \int_0^1 dx\ Tr[\gamma_\mu \{ \not{P}\ \Phi^v_{\chi_{c0}}(x) + m_{\chi_{c0}}\ \Phi^s_{\chi_{c0}}(x) \}] = 0$$

$$< \chi_{c0}(P)|\bar{c}\gamma_\mu\gamma_5\ c_\beta|0 >= \frac{i}{\sqrt{2N_c}} \int_0^1 dx\ Tr[\gamma_\mu\gamma_5 \{ \not{P}\ \Phi^v_{\chi_{c0}}(x) + m_{\chi_{c0}}\ \Phi^s_{\chi_{c0}}(x) \}] = 0$$

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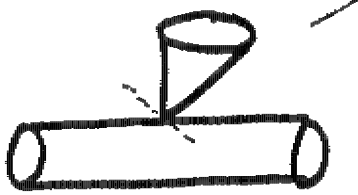
- Under the naive factorization, this process is forbidden. However when we consider radiative correction, sizable non-factorizable contribution can be appeared.

$$C_1(\mu)\, O_1(\mu) + C_2(\mu)\, O_2(\mu) = \underbrace{\left(C_2(\mu) + \frac{C_1(\mu)}{N_c}\right) O_2(\mu)}$$

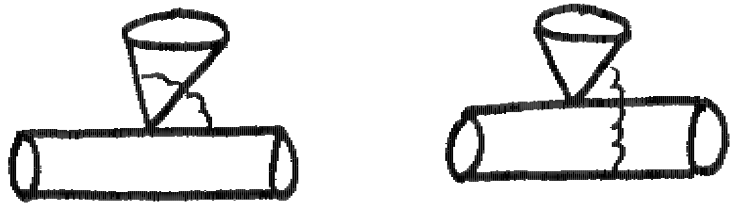
$$+\ 2C_1(\mu)[\bar{c}\Gamma_\mu T^a b][\bar{s}\Gamma^\mu T^a c]$$

$\downarrow$

$O(1)$



Factorized contribution



Non-factorized contribution

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## *Non-factorized contributions :*

Definition : $r = M_{c0}/M_B, \quad \eta = r^2/(1-r^2)$

| Hard Part $((V-A)\otimes(V-A))$ | | |
|---|---|---|
| Product of LCDAs | Left-hand side gluon | Right-hand side gluon |
| $\phi^v_{c0}\otimes\phi^A_K$ | $(1+r^2)(1-x_2)$ | $x_3+x_3(1-r^2)$ |
| $\phi^s_{c0}\otimes\phi^A_K$ | $r^2/2.$ | $r^2/2.$ |
| $\phi^v_{c0}\otimes\phi^P_K$ | $-r_K[x_3+\eta(2-x_2)]$ | $r_K[x_3+\eta x_2]$ |
| $\phi^s_{c0}\otimes\phi^P_K$ | $-2r_K\eta$ | $2r_K\eta$ |
| $\phi^v_{c0}\otimes\phi^\sigma_K$ | $r_K[x_3+\eta x_2]$ | $r_K[x_3-\eta x_2]$ |
| $\phi^s_{c0}\otimes\phi^\sigma_K$ | 0 | 0 |
| Hard Part $((S+P)\otimes(S-P))$ | | |
| Product of LCDAs | Left-hand side gluon | Right-hand side gluon |
| $\phi^v_{c0}\otimes\phi^A_K$ | $(2-x_2)-(1-x_3)(1-r^2)$ | $x_2(1+r^2)$ |
| $\phi^s_{c0}\otimes\phi^A_K$ | $-r^2/2.$ | $-r^2/2.$ |
| $\phi^v_{c0}\otimes\phi^P_K$ | $r_K[x_3+\eta(2-x_2)]$ | $-r_K[x_3+\eta x_2]$ |
| $\phi^s_{c0}\otimes\phi^P_K$ | $2r_K\eta$ | $2r_K\eta$ |
| $\phi^v_{c0}\otimes\phi^\sigma_K$ | $r_K[x_3+\eta x_2]$ | $r_K[x_3-\eta x_2]$ |
| $\phi^s_{c0}\otimes\phi^\sigma_K$ | 0 | 0 |

Table 4: Non-factorizable contribution in $B\to\chi_{c0}K$

## Experimental data :

- $B \to \chi_{c0} K$ : $< 4.8 \times 10^{-4}$ CLEO
  $(6.0 \pm 2.4) \times 10^{-4}$ Belle
  $(2.4 \pm 0.9) \times 10^{-4}$ BaBar

- $B \to \chi_{c0} \pi$ : ?

## PQCD predictions :

- $B \to \chi_{c0} K$ : $(2.1^{+1.4}_{-0.9}) \times 10^{-4} \cdot \left(\frac{f_{\chi_{c0}}}{300\,\text{MeV}}\right)^2$

- $B \to \chi_{c0} \pi$ : $(1.1^{+0.5}_{-0.3}) \times 10^{-5} \left(\frac{f_{\chi_{c0}}}{300\,\text{MeV}}\right)^2$

# Partial width CP- Asymmetry

Idea: By using the interference between NR and Resonance Amp. We can get a further constraint of the CP violating phase $\gamma$ ($\phi_3$).

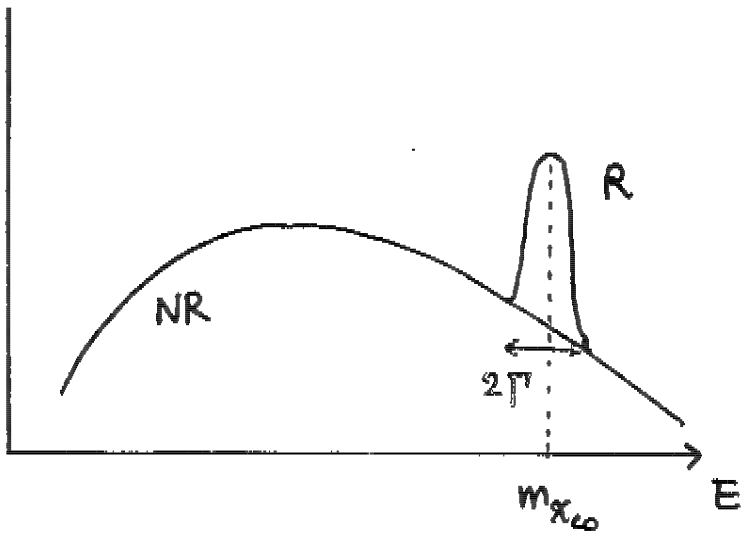
$B \to (3\pi)_{NR}$ : $\sim V_{ub}^* V_{ud}$

$B \to \chi_{c0}\pi \to (3\pi)_R$ : $\sim V_{cb}^* V_{cd}$

$$A_{tot}(B \to 3\pi) = A(3\pi)_{NR} + A(B^+ \to \chi_{c0}\pi^+ \to 3\pi)_R$$

$$= e^{i\gamma} + \text{const}$$

$$A_{cp} = \frac{\Gamma(B^+ \to 3\pi) - \Gamma(B^- \to 3\pi)}{\Gamma(B^+ \to 3\pi) + \Gamma(B^- \to 3\pi)} \simeq R \cdot \sin\gamma$$



## • Resonance Decay Amplitude :

Determined using the narrow width approximation

$$M_r(B^- \to \chi_{c0}\pi^- \to \pi^+\pi^-\pi^-) = M(B^- \to \chi_{c0}\pi^-)\,\frac{1}{s - m^2_{\chi_{c0}} + i\,\Gamma_{\chi_{c0}} m_{\chi_{c0}}}\,M(\chi_{c0} \to \pi^+\pi^-)$$

PQCD: $B_r(\chi_{c0}\pi^-) = 1.1 \times 10^{-5}$

$M(B^- \to \chi_{c0}\pi^-) = 2.34 \times 10^{-7}$ GeV

$m_{\chi_{c0}} = 3.415$ GeV

$\Gamma_{\chi_{c0}} = (0.016 \pm 0.002)$ GeV

From exp. data

$B_r(\chi_{c0} \to \pi^+\pi^-) \to M(\chi_{c0} \to \pi^+\pi^-) = 0.113$ GeV

$B_r(\chi_{c0} \to K^+K^-) \to M(\chi_{c0} \to K^+K^-) = 0.126$ GeV

$$Amp(B \to 3\pi) = M_{nr}(B \to 3\pi) + M_r(B^- \to \chi_{c0}\pi^- \to 3\pi)$$

$$M_{nr} = \left\{a_1 \langle 3\pi | O_1 | B\rangle + a_2 \langle 3\pi | O_2 | B\rangle\right\} \otimes \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}^*$$

## • Partial Decay width : $\Gamma_p(B \to 3\pi)$

$$\Gamma_p(B \to 3\pi) = \frac{1}{(2\pi)^3}\,\frac{1}{32 M_B^3}\int_{s_{min}}^{s_{max}} ds \int_{t_{min}(s)}^{t_{max}(s)} dt \;\left| M_{nr} + M_r \right|^2$$

$$s_{min} = (m_{\chi_{c0}} - 2\Gamma_{\chi_{c0}})^2$$

$$s_{max} = (m_{\chi_{c0}} + 2\Gamma_{\chi_{c0}})^2$$

## ⊙ CP non-conserving Asymmetry :

$$A_{cp}(B \to 3\pi)\Big|_{\chi_{co}} \equiv \frac{\Gamma_P - \Gamma_{\bar{P}}}{\Gamma_P + \Gamma_{\bar{P}}} \propto \sin\gamma$$

$$\Gamma_P - \Gamma_{\bar{P}} = \sin\gamma \; \frac{4\, m_{\chi_{co}} \Gamma_{\chi_{co}}}{(2\pi)^3\, 32\, m_B^3} \int_{s_{min}}^{s_{max}} ds \int_{t_{min}(s)}^{t_{max}(s)} dt$$

$$\frac{G_F}{\sqrt{2}} |V_{ub}||V_{ud}^*| \left\{ a_1 \langle 3\pi | O_1 | B \rangle_{nr} + a_2 \langle 3\pi | O_2 | B \rangle_{nr} \right\}$$

$$\otimes \; |M(B^- \to \chi_{co}\pi^-)| \; \frac{1}{(s - m_{\chi_{co}}^2)^2 + (m_{\chi_{co}}\Gamma_{\chi_{co}})^2} \; |M(\chi_{co} \to \pi^+\pi^-)|$$

$$\Gamma_P + \Gamma_{\bar{P}} = 2 \; \frac{1}{(2\pi)^3\, 32 M_B^3} \int_{s_{min}}^{s_{max}} ds \int_{t_{min}(s)}^{t_{max}(s)} dt$$

$$\otimes \left\{ |M_{nr}|^2 + |M(B^- \to \chi_{co}\pi^-) \; \frac{1}{(s - m_{\chi_{co}}^2) + i\, m_{\chi_{co}}\Gamma_{\chi_{co}}} M(\chi_{co} \to 2\pi \right.$$

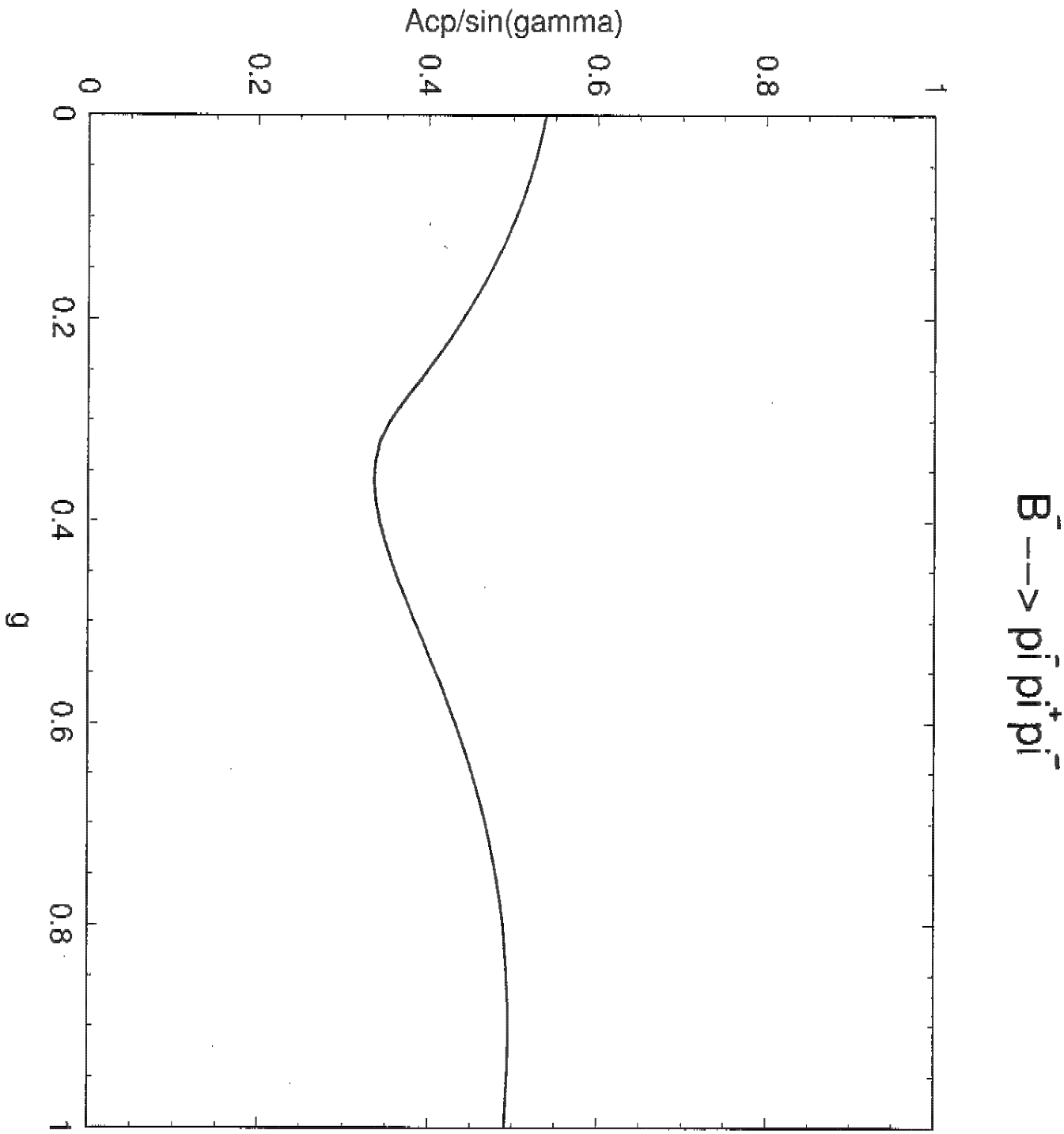
## ⊙ Results

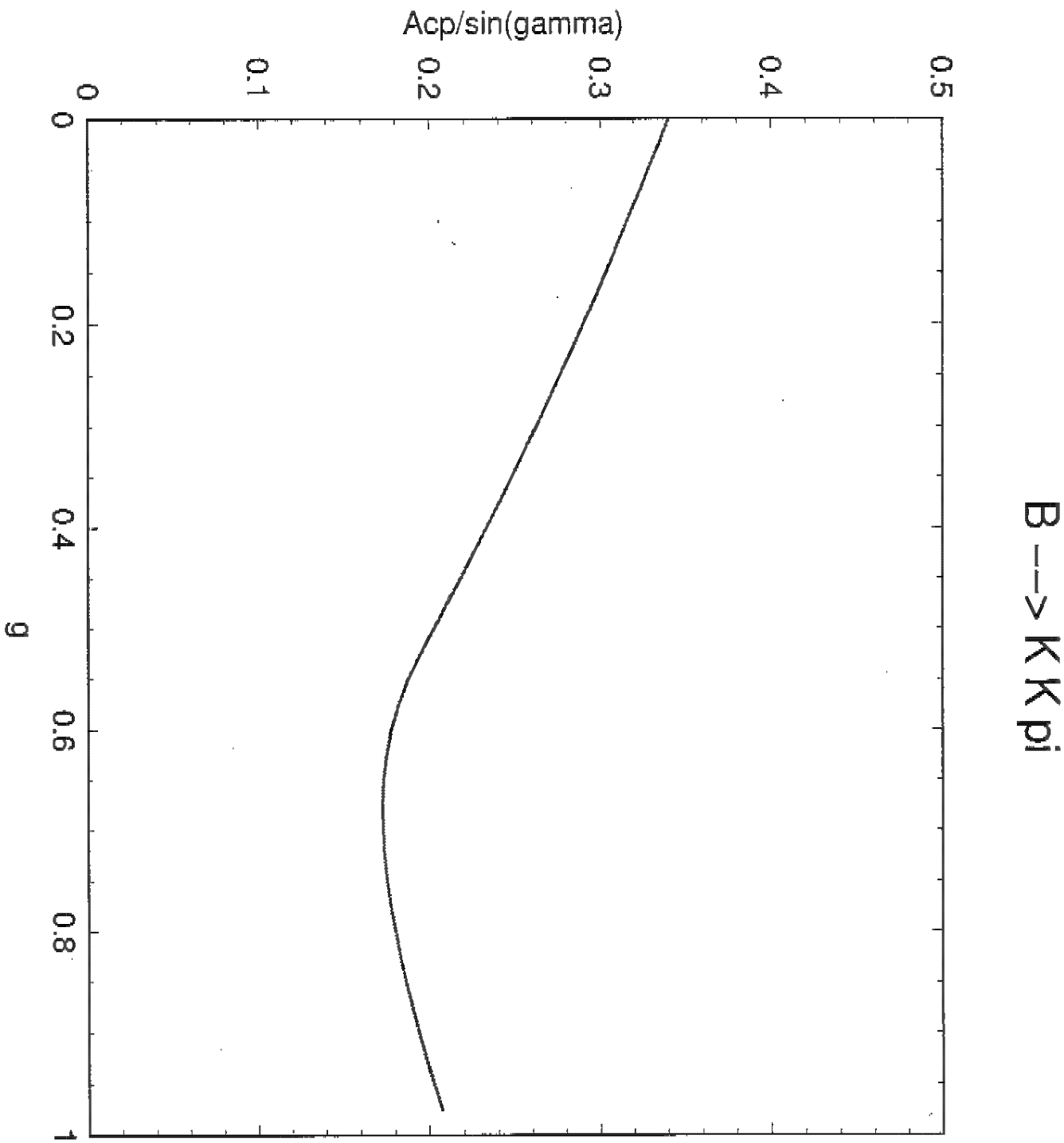
① For $B^- \to \pi^-\pi^+\pi^-$ ;

$$A_{cp} = (0.35 \sim 0.53)\sin\gamma$$

② For $B^- \to K^+K^-\pi^-$ ;

$$A_{cp} =$$


Acp/sin(gamma)
0
0.2
0.4
0.6
0.8
1
g
0
0.2
0.4
0.6
0.8
1
B- --> pi- pi0


Acp/sin(gamma)
0
0.1
0.2
0.3
0.4
0.5
0
0.2
0.4
0.6
0.8
1
g
B --> K K pi