Fundamental Concepts of Particle Accelerators III: High-Energy Beam Dynamics (2)

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Contents

The Idea of the Collider

- High-energy physics experiment had been done by colliding high energy beams against a target fixed on the accelerator, i.e. fixed target experiment.
- The particle reaction, however, depends not on the laboratory energy of the projectile particle, but on the center of mass energy of it and a target particle.
- History of colliding-beam machines (collider)
  - The idea was first conceived by Rolf Wideröe in 1943.
  - A brief history is given by C. Bernardini. *
- Construction of the first full-fledged collider AdA by B. Touschek *et al*, Frascati Lab., Italy (1960).

$$200\,\mathrm{MeV}\ e^- \Rightarrow\Leftarrow 200\,\mathrm{MeV}\ e^+$$

- Hence-forward, the collider scheme has become the paradigm for high energy accelerators today.

* "AdA: The first Electron-Positron Collider", Phys. perspect. 6 (2004) 156 - 183, Birkhäuser Verlag, Basel.

- Consider the collision of particles of the same rest mass m.
- In the rest frame:
  - particle energy in the beam is γmc^2, while that in the target is mc^2.
    - total energy of the two particles: $E_T = (1+\gamma)mc^2$.
    - total momentum: $p_T = \beta\gamma mc = \sqrt{\gamma^2 - 1}\, mc$.
- Since $E^2 - c^2 p_T^2$ is a Lorentz invariant, the total energy

  in the center of mass frame is

$$E_{CM}(\text{fixed target}) = \sqrt{2\gamma + 2}\, mc^2 \approx \sqrt{2\gamma}\, mc^2$$

- But if we use a collider, it becomes

$$E_{CM}(\text{collider}) = 2\gamma mc^2 \gg E_{CM}(\text{fixed target}).$$

- In colliders, very thin beams collide each other.
  - If we put the beam cross section as S and the reaction cross section as σ, the probability of reaction at a single collision is given by

$$\sigma/S$$

  - When each beams comprise N_+ and N_- particles, respectively, and collide at a rate of f times per second, the number of reaction per second is

$$f \times \frac{N_+ \times N_-}{S}\,\sigma$$

- Coefficient of σ is called the luminosity $\mathcal{L}$

$$\mathcal{L} = f \times \frac{N_+ \times N_-}{S}$$

- In every collider, a great effort is continually paid to maximize the luminosity both at the design stage and at the beam commissioning stage.

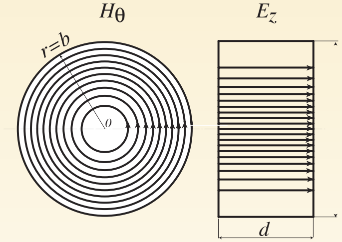
- The first RF accelerator tried by Rolf Wideröe used electric fields in the gap between drift tubes which are in open space.
- But this simple scheme has a problem of the radiation loss of the RF power.
- The gap may be considered as a kind of oscillating electric dipole having a moment $p = qd\, e^{j\omega t}$, where q is the charge induced on the drift tube surface and d the gap length.
- And the radiation power may be estimated like

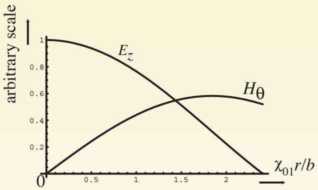
$$\frac{\omega^4}{12\pi}\mu_0\sqrt{\varepsilon_0\mu_0}|p|^2.$$

- It is therefore necessary to enclose the acceleration gap with a metal wall, when both RF field and frequency become ever higher.

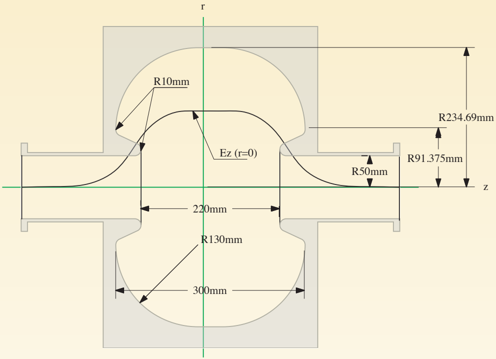
$\rightarrow$ Acceleration by Resonant Cavities

- There are a great variety of RF cavities for the beam acceleration. But each one is a variation of the cylindrical cavity resonating with the lowest resonant mode TM_{010}.
  - The cavity of this type is often called "pillbox cavity."
  - $E_z(r) \propto J_0(\xi_{01} r/b)$ and $H_\theta(r) \propto J_1(\xi_{01} r/b)$, where b is the cavity radius, J_n the Bessel function of n th order, and $\xi_{01} = 2.40483$ the lowest zero of J_0.
  - the resonant frequency is $\omega_{01} = \xi_{01} b/c$.

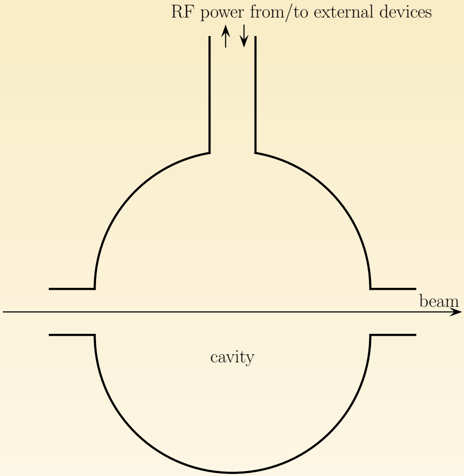




- An example: single cell cavity at KEK Photon Factory $2.5\,\mathrm{GeV}$ Storage Ring
    - modified pill-box cavity
    - specification: $f_{RF} = 500\,\mathrm{MHz}$, $V_{peak} = 0.7\,\mathrm{MV}$



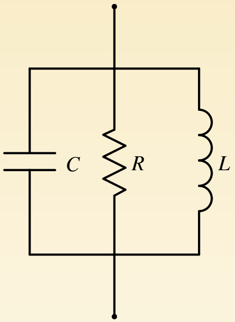
- Schematic diagram of accelerating cavity
  with beam and coupling hole to external RF circuits.



- Global behaviors of a cavity is represented by an equivalent circuit with three parameters L, C and R.
- Two equations are straightforward to obtain by using the observed/simulated resonant frequency and Q value:

  $\omega_0 = 1/\sqrt{LC}$ and $Q = \omega_0 RC$.

- One more equation is wanted to determine the 3 parameters.
- Therefore we must define the RF acceleration voltage.

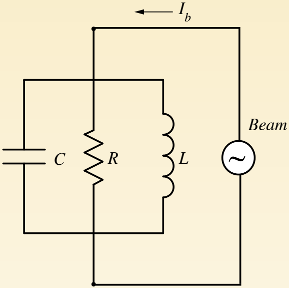


RF Cavity and Beam (6)

- In order to define the voltage for the equivalent circuit, we add the beam $I_b = |I_b|\, e^{j\omega t}$ as an external current.
- In the cavity at a stationary state, the following energy conservation law

$$\iiint_V \mathbf{J} \cdot \mathbf{E}\, dV + \iint_S (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n}\, dS = 0$$

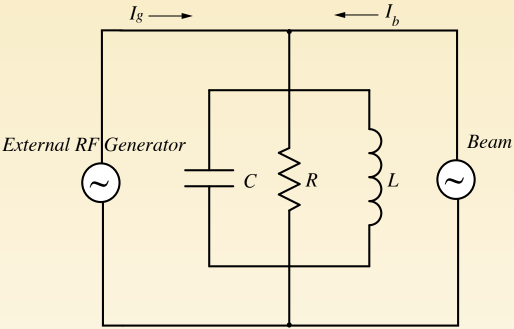
  should be satisfied, where $\mathbf{J}$ is the current density of the beam, and $\mathbf{E}$ and $\mathbf{H}$ are EM fields excited by the beam.
- This equation means that the energy lost by the beam on the orbit should be equal to the power loss on the cavity wall S.



RF Cavity and Beam (7)

- The first term of the equation in the previous page can be interpreted as the effective deceleration voltage $-V_b$ times the beam current I_b.
- Then the energy generated by the beam is represented as $I_b V_b$.
- This should be equal to the ohmic loss of the circuit: $R I_b^2$.
- Thus we can define circuit resistance by the relation $R = V_b / I_b$ with the aid of the voltage defined above and eventually can uniquely define L and C.
- The R thus defined is called shunt impedance of the accelerating cavity.

- The cavity system is excited by an external RF generator too as shown below.



- The cavity voltage is a phasor sum of voltages generated by I_b and I_g.

- Synchrotron radiation (SR) is an electric-dipole radiation due to the point charge's transverse acceleration by bending magnetic fields.
- Radiation power by a charge that is instantaneously at rest is given by Larmor's formula:

$$P = \frac{q^2}{6\pi\varepsilon_0 mc^3}\left(\frac{d\mathbf{v}}{dt}\right)^2.$$

- For an electron, in particular, it is written as

$$P = \frac{2r_e m_e}{3c}\left(\frac{d\mathbf{v}}{dt}\right)^2,$$

where r_e is the classical electron radius:

$$r_e \equiv \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.82\times 10^{-15}\mathrm{m}.$$

- Radiation pattern in the rest frame of an electric dipole oscillating in the z direction at a frequency ω:
  - the pattern is rotationally symmetric with the z axis
  - $\lambda = 2\pi c/\omega$

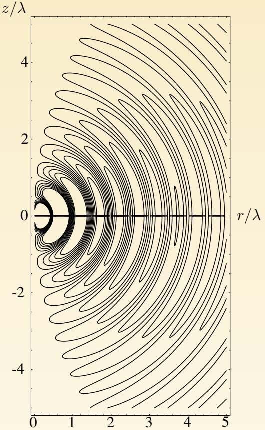


Fig: Contours of electric field lines

- Let us transform the Larmor's formula

$$P = \frac{2r_e m_e}{3c}\left(\frac{d\mathbf{v}}{dt}\right)^2 = \frac{2r_e}{3m_e c}\left(\frac{d\mathbf{p}}{dt}\right)^2.$$

  to that for the laboratory frame.
- P is the radiated energy ΔE during a time span Δt, but energy and time transform in the same manner under the Lorentz transformation.
- Thus P should have a Lorentz invariant form.
- For this end, we must find an invariant form for the right-hand side of the above equation.
- This is achieved through replacing $(d\mathbf{p}/dt)^2$ by

$$(d\mathbf{p}/d\tau)^2 - (dE/d\tau)^2/c^2,$$

  where $d\tau$ is the differential of proper time

$$d\tau = \sqrt{dt^2 - (dx^2 + dy^2 + dz^2)/c^2} = dt/\gamma.$$

- Formula in the laboratory frame is then given as follows:
  - Instantaneous radiation power of a single electron:

$$P = \frac{2r_e m_e}{3c}\gamma^2 \left\{ \left[\frac{d\,(\gamma \mathbf{v})}{dt}\right]^2 - \left[\frac{d\,(\gamma c)}{dt}\right]^2 \right\}.$$

  - Radiated energy ΔE per turn of a circular orbit with the radius ρ:

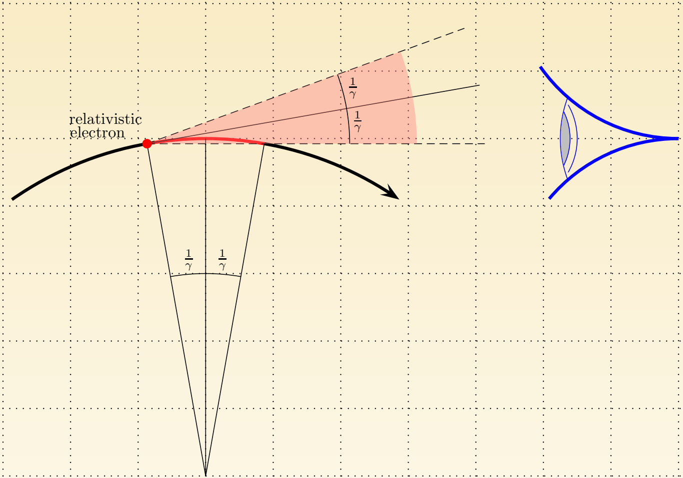
$$\frac{\Delta E}{m_e c^2} = \frac{4\pi}{3}\,\frac{r_e}{\rho}\,\beta^3\gamma^4.$$

  - Expression in practical units for an electron energy of $E_e(\mathrm{GeV})$:

$$\Delta E(\mathrm{keV}) \approx 88.5\,[E_e(\mathrm{GeV})]^4\,/\rho(m).$$

Electric-dipole Radiation (4)

- Radiation from a relativistic electron on a circular orbit



Electric-dipole Radiation (5)

- Radiation profile in the laboratory frame
  - Radiation profile in the electron's rest frame (x', y', z', ct')
    $$dP/d\Omega \propto \sin^2\theta,$$
    with Ω being the solid angle and θ the angle from the z' axis.
  - Coordinates in laboratory frame are:
    $$x' = x, \quad y' = y, \quad z' = \gamma\,(z - vt)\,, \quad ct' = \gamma\,(ct - vz/c)\,.$$
  - Projection angle of the x' axis and y' axis to the z axis:
    $$\sim 1/\gamma.$$
  - Power radiated in the forward direction is concentrated in the cone with a full angle of $\sim 2/\gamma$.
  - An observer can see the light emission only during the period when the electron is running on the arc whose length is
    $$\sim 2\rho/\gamma.$$
  - Wavelengths for the observer are shortened due to the doppler effect by a factor of $(1 - v/c) \sim \dfrac{1}{2\gamma^2}$.

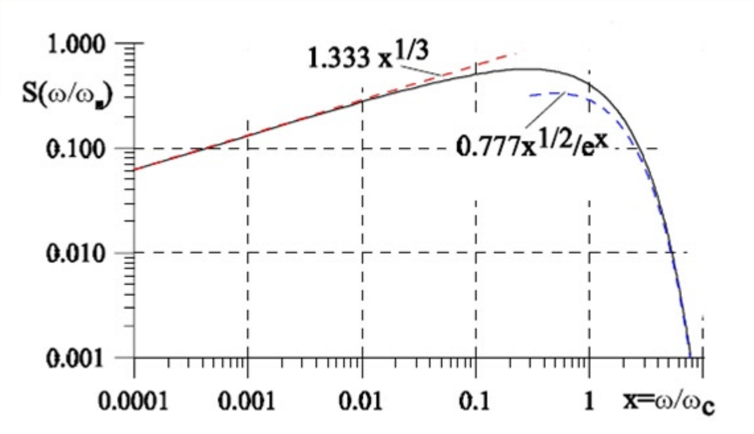
Electric-dipole Radiation (6)

- Critical Frequency ω_c and Wavelength $\lambda_c = 2\pi c/\omega_c$:
  - Power spectrum increases as ω up to around ω_c, wherefrom it sharply drops to zero.
  - The critical frequency is given by

$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$

    after Schwinger-Jackson's definition.
  - The radiation is wave-like up to ω_c, wherefrom it becomes photon-like (quantum regime).

Electric-dipole Radiation (6)



(from wikipedia: synchrotron radiation)