Coherent Beam-beam instability in collision with a large crossing angle

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Accelerator Physics seminar

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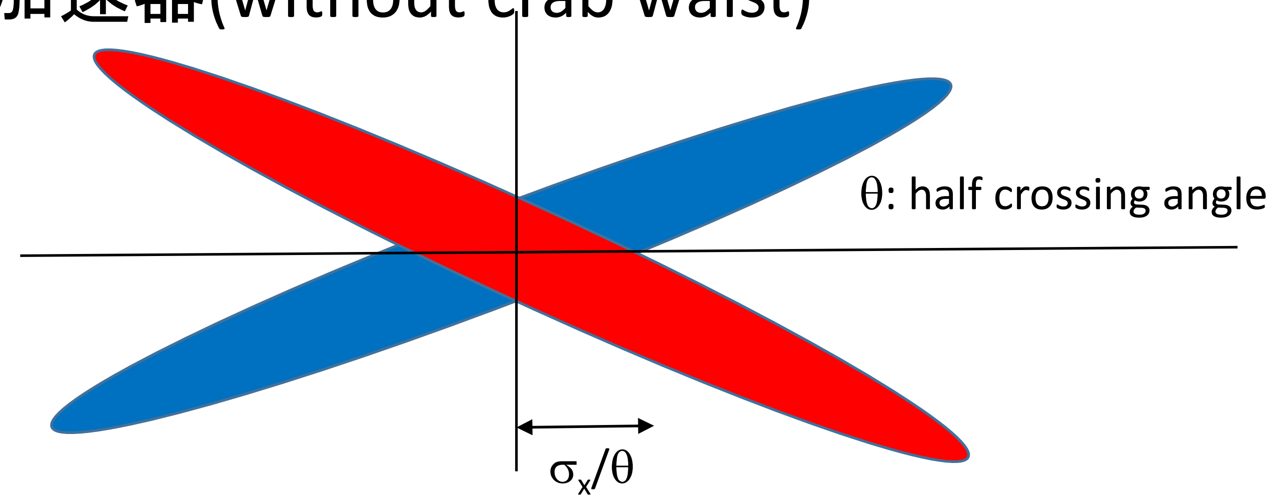
Thanks to D. Shatilov, K. Oide , N. Kuroo, D. Zhou,
F. Zimmermann

Collision with a large crossing angle

- クラブウェストと組み合わせて、最近の円形電子陽電子衝突加速器の設計に広く使われるようになった。(P. Raimondi)
- 特徴づける量、Piwinski角$\sigma_z\theta/\sigma_x$. 衝突領域に対するバンチ長
- 実証実験DAFNA $\sigma_z\theta/\sigma_x$=2, with crab waist
- SuperKEKB $\sigma_z\theta/\sigma_x$=20、この方式での初めての本格的な加速器(without crab waist)



この衝突方式に死角はないか？

- DAFNEの実験はPiwinski角が2と小さい。KEKBは1
- Beam-beam simulationによる検討がされてきたが、ほとんどはweak-strong simulationだった。
- その理由は後述するが、バンチを進行方向にスライスするがその数が大きくなる。$N_{sl}=10\sigma_z\theta/\sigma_x$
- SuperKEKBのstrong-strong simulationは衝突当たりN_{sl}^2=200x200=40,000回のポテンシャル計算。
- Weak-strongではN_{sl}回、複素エラー関数からガウス分布によるビームビーム力を計算。数分でルミノシティ計算ができる。
- クラブウェストと組み合わせると、weak-strongではビームビームパラメータ、ξ=0.1は簡単に越えられる。
- これは本当か

Beam-beam limit

- Luminosity

$$L = \frac{N^2 f_{rep}}{4\pi\sigma_x\sigma_y} R\left(\frac{\sigma_z}{\beta_y} \text{or } \frac{\sigma_x}{\theta_c\beta_y}, \frac{\theta_c\sigma_z}{\sigma_x}\right)$$

N=N_+=N_-: bunch population
f_{rep}: collision freq.
θ_c: half crossing angle

- $\frac{\sigma_z}{\beta_y}$ or $\frac{\sigma_x}{\theta_c\beta_y}$: hourglass (衝突領域とβ_yの比),
  $\frac{\theta_c\sigma_z}{\sigma_x}$: normalized crossing angle (Piwinski angle)
- Tune shift $\xi_y = \Delta\nu_y = \frac{Nr_e}{2\pi\gamma}\frac{\beta_y}{\sigma_y(\sigma_x+\sigma_y)} R\left(\frac{\sigma_z}{\beta_y} \text{or } \frac{\sigma_x}{\theta_c\beta_y}, \frac{\theta_c\sigma_z}{\sigma_x}\right)$
- Nを増やすとビームサイズ特にyが大きくなりtune shiftは飽和し、ルミノシティはN^2で増えなくなる。この状態をBeam-beam limit.

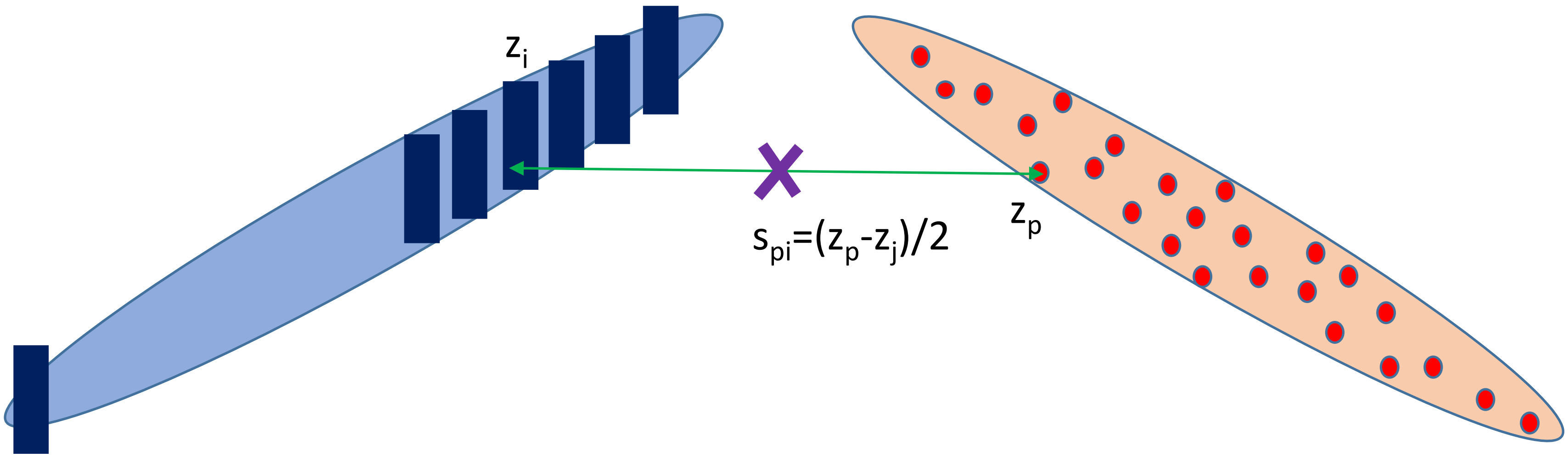
$$L = \frac{N\gamma f_{rep}}{2r_e\beta_y}\xi_y \qquad \sigma_x \gg \sigma_y$$

- この式はまたアワーグラスが効かなければ、β_yが小さいほどルミノシティが大きくできることを示す。大衝突角
- SuperKEKBはcrab waistを使わない。IR非線形が強すぎて、crab waist sextupoleの非線形がIRでキャンセルできず、DAが小さくなってしまう。

Weak-strong and strong-strong simulation

- Weak-strong simulation
  - One (strong) beam is assumed to be fixed charge distribution, and the other (weak) beam is represented by macro-particles.
  - Beam-beam interaction is evaluated by tracking the macro-particles in the electro-magnetic field induced by the fixed charge distribution.
  - The strong beam is assumed to be Gaussian distribution in most cases.
- Strong-strong simulation - Both beams are represented by macro-particles.
  - Beam distribution is represented on meshed space using Particle In Cell method. Arbitrary and self-consistent distribution of two beams are treated.
  - Statistical noise of macro-particles induces an fluctuation in potential calculated by PIC. The unphysical emittance growth by the noise is cared in the strong-strong simulation.
    - As an approximation, two beams are represented by Gaussian whose sizes are determined turn-by-turn. It is called Soft Gaussian approximation.
    - Strong-strong simulation based on PIC is more popular than the soft Gaussian approximation.
- Quasi-strong-strong simulation
  - Repeat weak-strong simulation with keeping self-consistency.

Weak-strong simulation for Large crossing angle

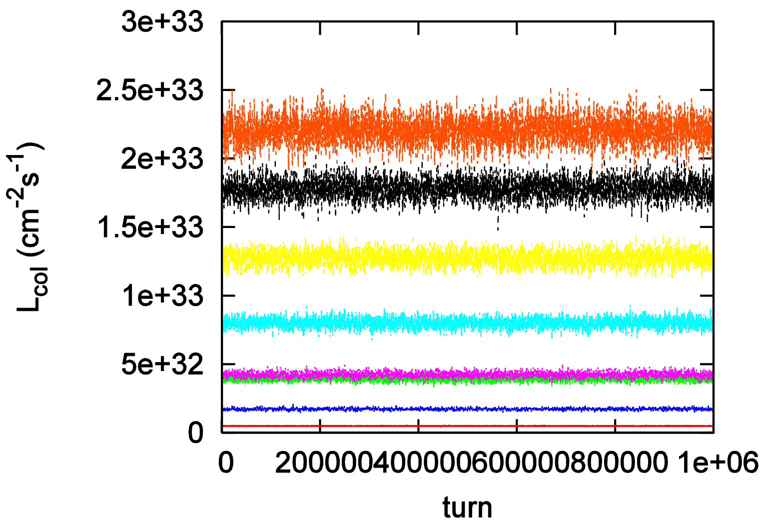


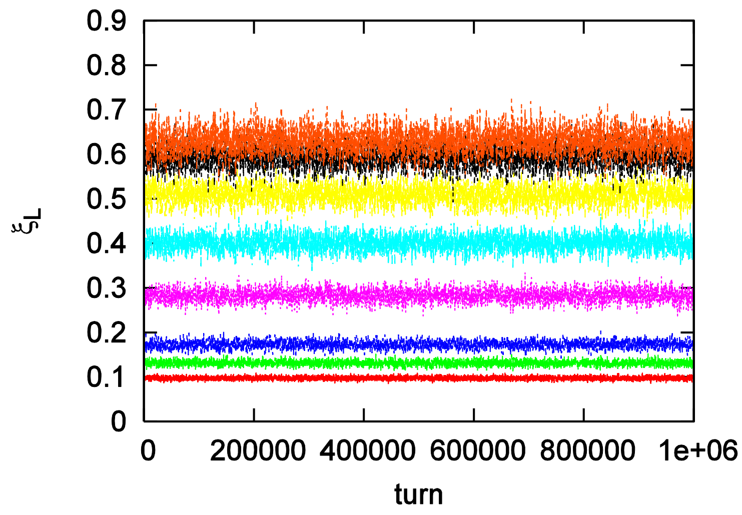
- Two colliding bunches are divided into many slices, N_{sl}~$10x\sigma_z\theta/\sigma_x$.
- Calculate slice-particle collision at $s_{pi}=(z_p-z_j)/2$.
- Crab waist transformation at IP.

Large crossing angle and crab waist weak-strong simulation

- Beam-beam parameter ξ_L=0.6 is achieved for collision with crab waist in weak-strong simulation, (ν_x, ν_y=0.51,0.55).
- Beam-beam parameter is saturated at ξ_L=0.1 without crab waist.

Luminosity evolution for scanning bunch population

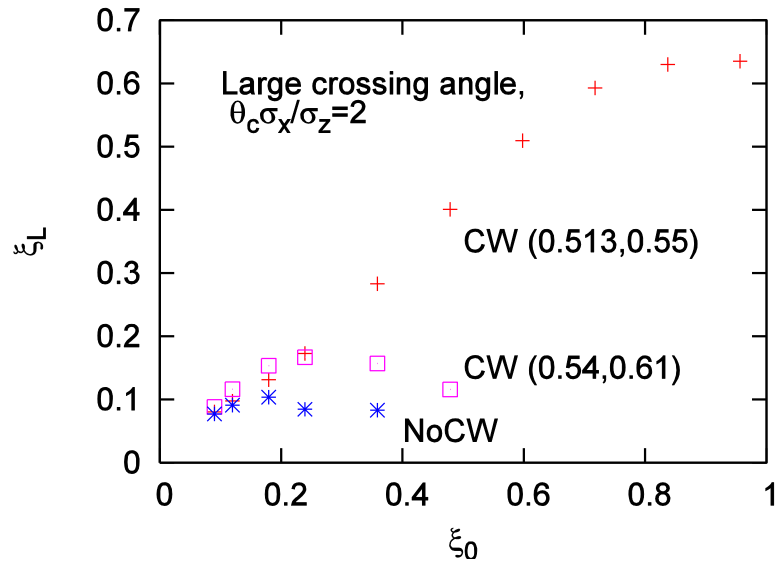


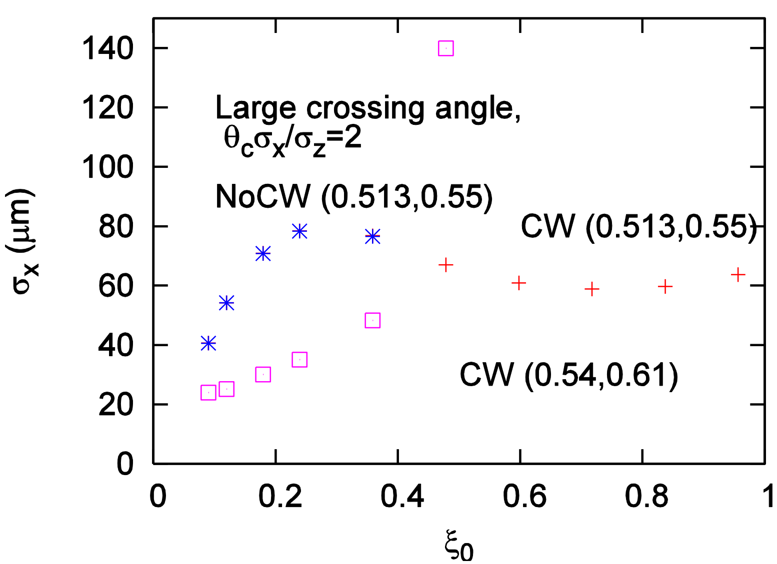


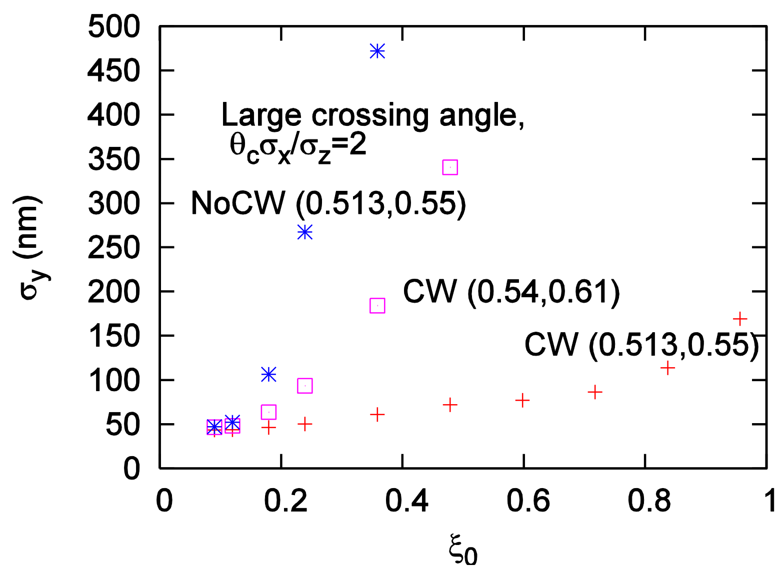
Equilibrium beam-beam parameter and beam size in weak-strong simulation

ξ_{max}~0.6 for (ν_x, ν_y)=(0.51,0.55)
ξ_{max}~0.2 for (ν_x, ν_y)=(0.54,0.61)
σ_y behavior correlates to Luminosity.

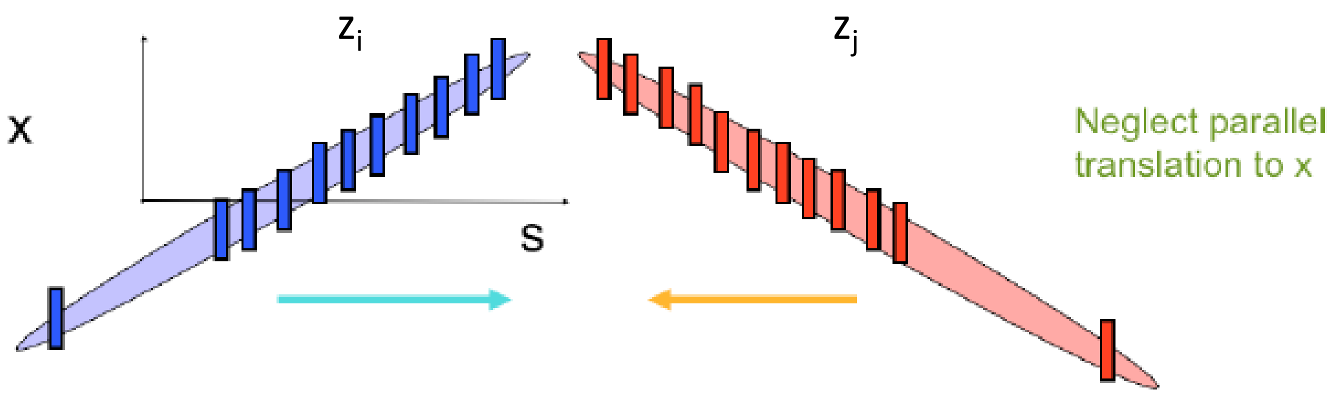
ξ_{max}チューンによって、ほとんど天井知らず







Strong-strong simulation for Large crossing angle



- Two colliding bunches are divided into many slices, N_{sl}~$10x\sigma_z\theta/\sigma_x$.
- Sort slices with their positons z_i+z_j, collision order.
- Each slice contains >10,000 macro-particles
- Solve potential slice-by-slice collision, or Gaussian approx.

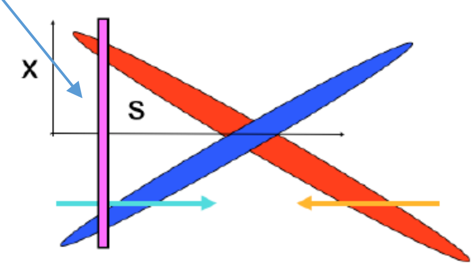
Several option of Strong-strong simulation

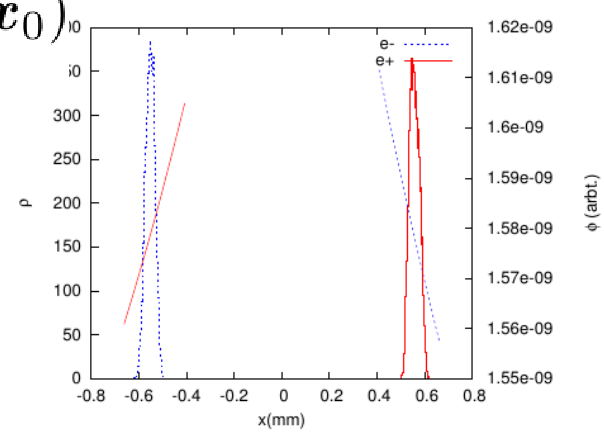
- Gaussian approximation using turn-by-turn RMS values.
- Gaussian approximation using turn-by-turn Gaussian fitting.
- PIC for core part and Gaussian approximation for slice collision with large offset.
- Complete PIC using shifted Green function

Example of shifted potential for collision with large offset.

Shifted Green function (J. Qiang)

$$\phi(\boldsymbol{x}) = -\frac{2Nr_e}{\gamma}\int d\boldsymbol{x}'G(\boldsymbol{x}-\boldsymbol{x}'-\boldsymbol{x}_0)$$





Coherent beam-beam instability

- A coherent beam-beam instability in head-tail mode was found to start beam-beam studies using strong-strong simulation.
- In Strong-strong simulation, both beams which are represented by macro-particles, interacts with each other in their classical EM field.
- The instability is cross-checked by D. Shatilov using quasi-strong-strong simulation.

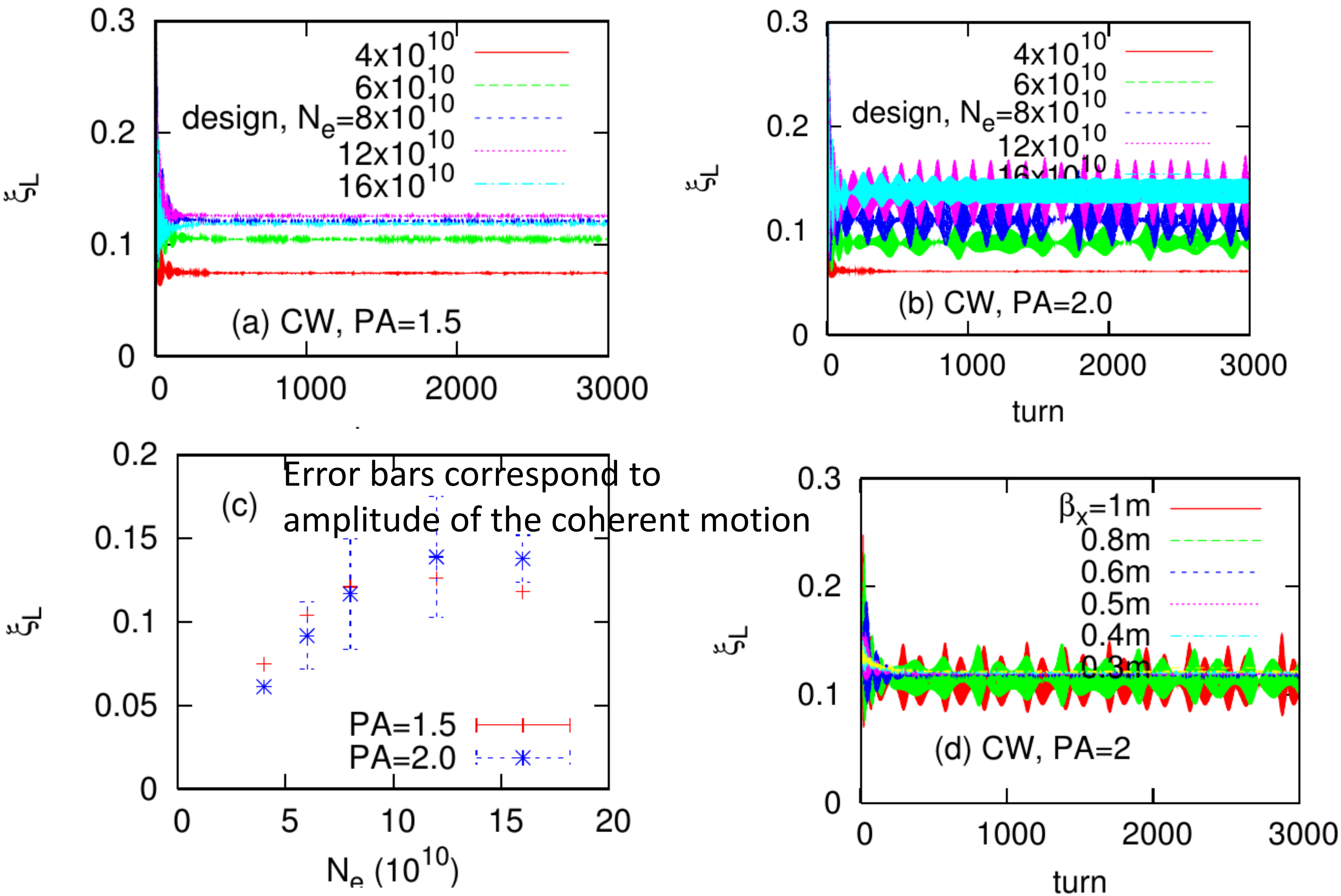
Parameters studied by early 2017

Parameter		SuperKEKB		FCC-ee-Z		H
		design	commissioning	HiLum	base	
Energy	$E_{+/-}$ (GeV)	4/7		45.5	45.5	120
Bunch population	$N_{+/-}(10^{10})$	9/6.5	6.3/5	10	3.3	8
Emittance	$\varepsilon_{x/y}$ (nm/pm)	3.2/8.64	3.2/44	0.2/1	0.09/1	0.61/1.2
Beta at IP	$\beta^*_{x/y}$ (m/mm)	0.032/0.27	0.25/2.2	0.5/1	1/2	1/2
Bunch length	σ_z (mm)	6		6.7	3.8	2.4
Energy spread	σ_δ (%)	0.08		0.22	0.09	0.12
Damping time	τ_x/T_0	4000		3000		150
Synchrotron tune	ν_z	0.025		0.036	0.025	0.056
Luminosity per IP	L (10^{34} cm^{-2}s^{-1})	80	-	207	90	5.1
Beam-beam	$\xi_{x/y}$	0.0028/0.088	-	0.025/0.16	0.05/0.13	0.08/0.14
Piwinski angle	$\sigma_z\theta_c/\sigma_x$	20	8.7	10	6	1.5

Simulation for H

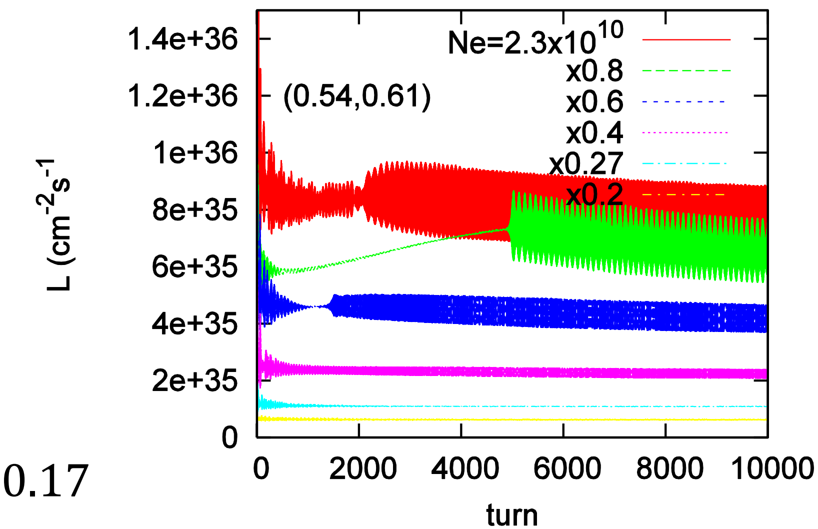
$$\xi_L = \frac{2 r_e \beta_y}{N \gamma f_{rep}} L$$

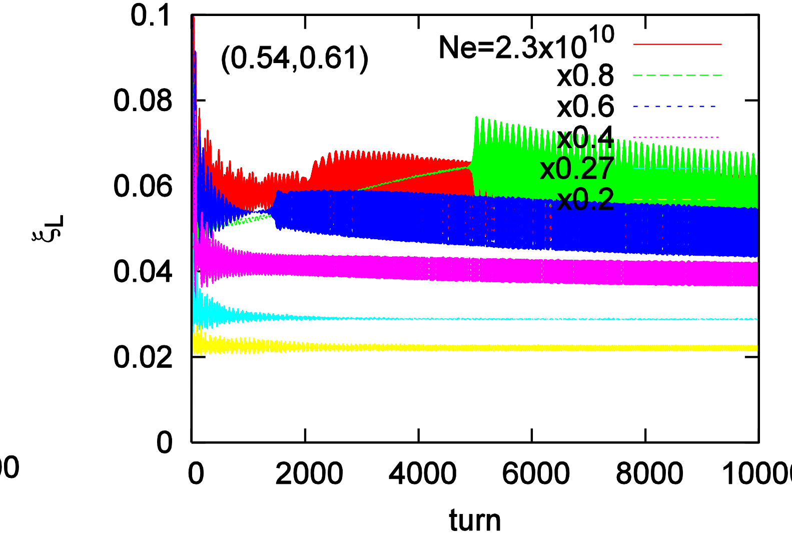
- PA=1.5 in the design. Safe for the instability



Strong-strong simulation for Z factory

$L_{\text{target}} = 2.2 \times 10^{36}\ \text{cm}^{-2}\text{s}^{-1}$

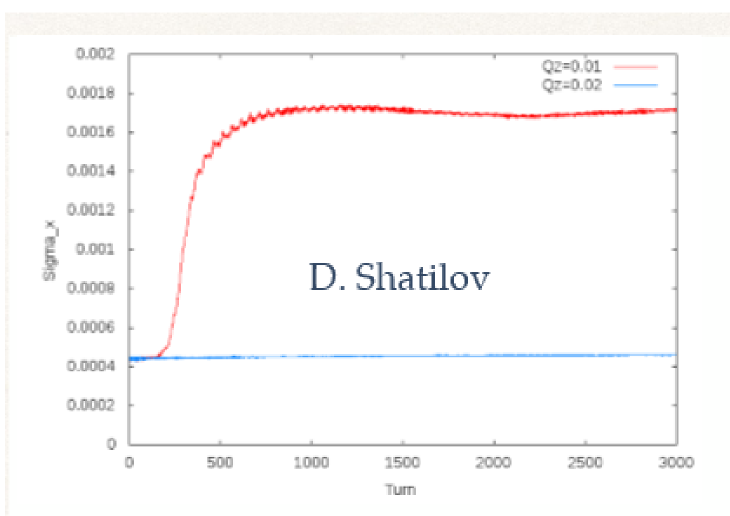


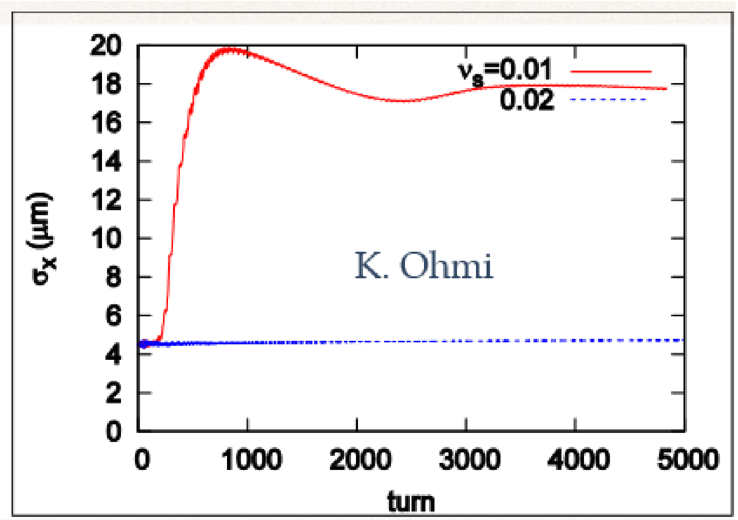


$\xi_{\text{target}} = 0.17$

ξ limit is around 0.06-0.07.

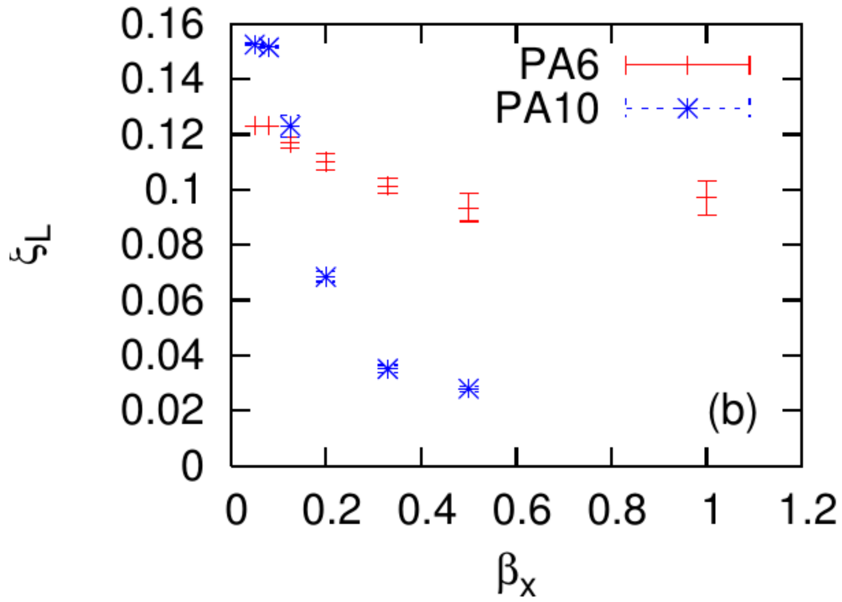
Coherent instability is strong.

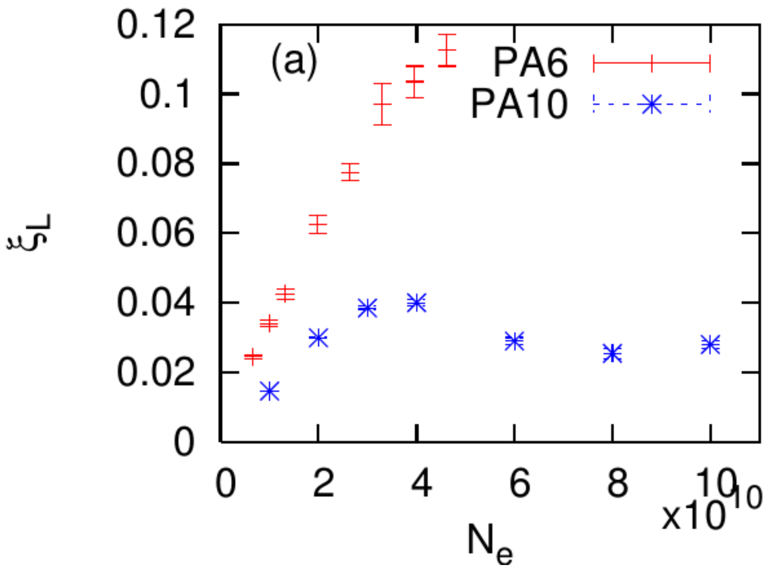


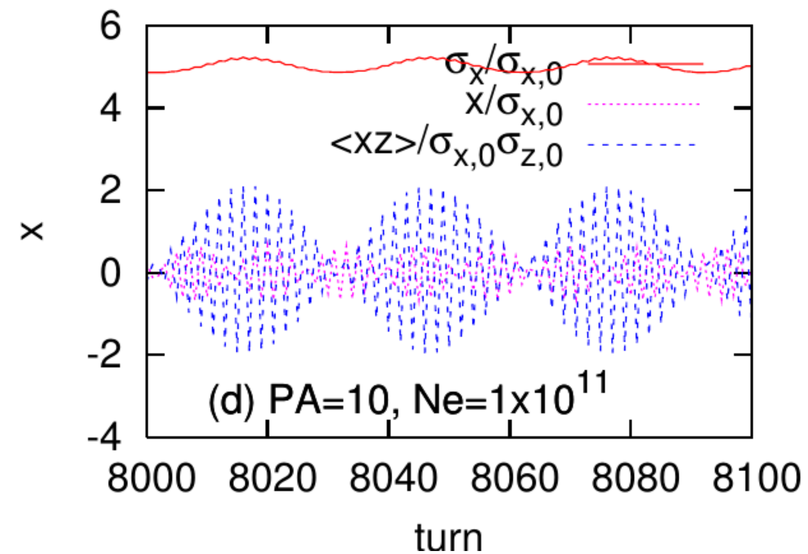


Simulation for Z

- Larger PA is more serious
- σ mode of head-tail motion, in which head-tail phases of two beams are in phase, is seen.

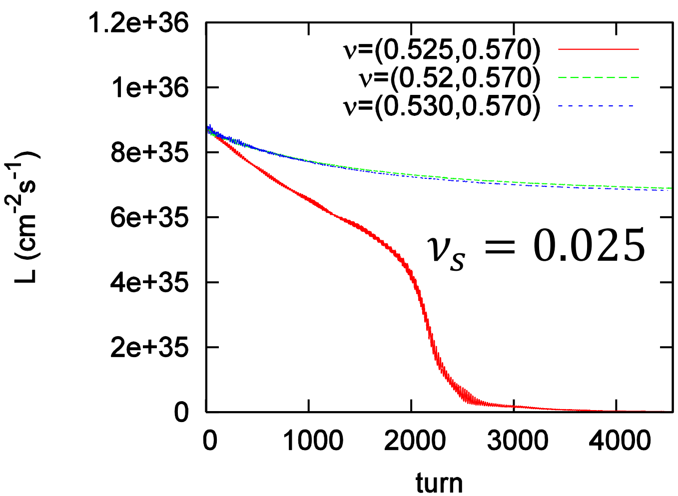


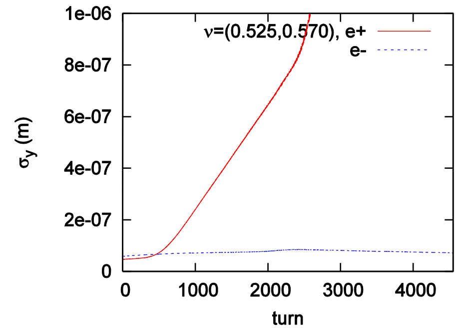


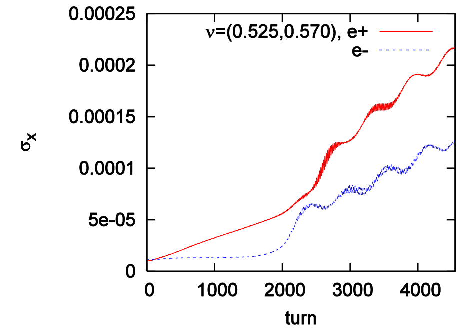


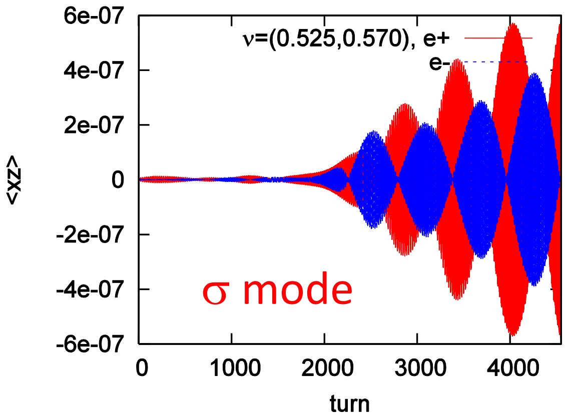
Strong-strong simulation in SuperKEKB

$$\frac{\theta_c \sigma_z}{\sigma_x} = 20 \qquad \xi_{x/y} = 0.0028/0.088$$

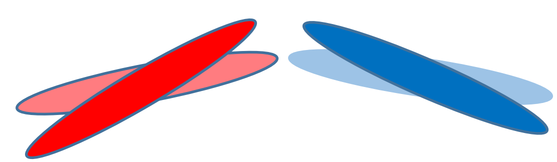








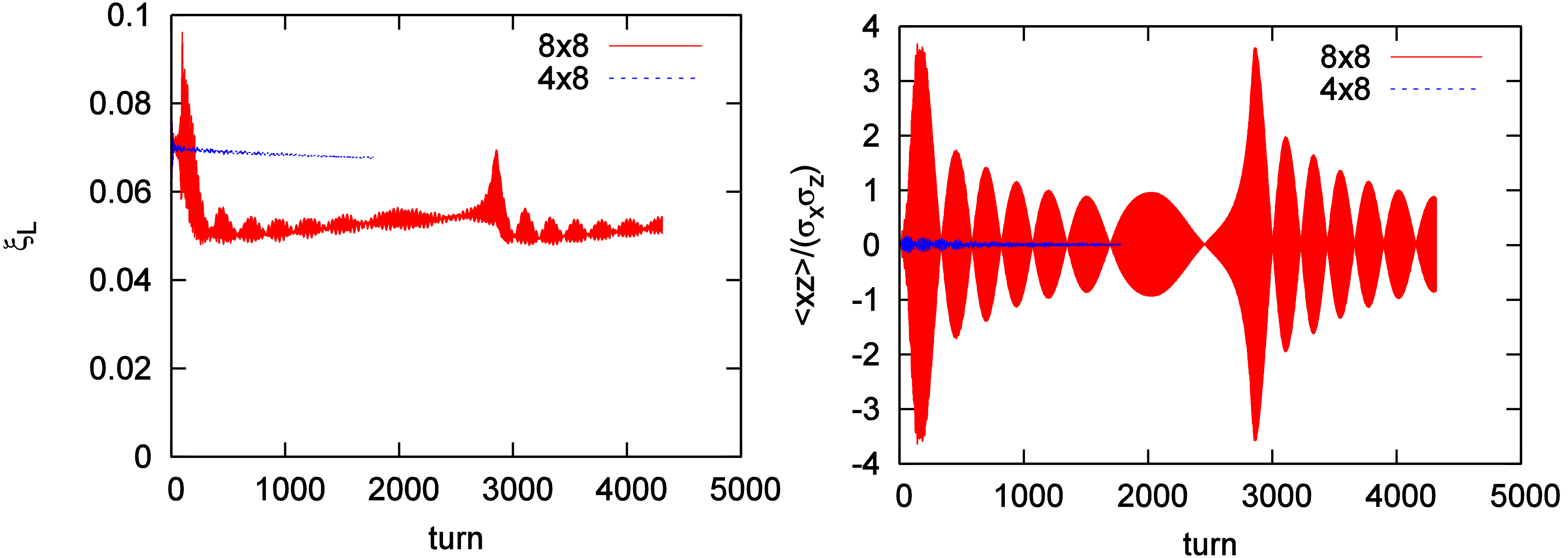
Strong-head-tail instability is seen only in limited tune. The stopband seems narrow.



SuperKEKB Phase 2

β_x=8xβ_{x0}, β_y=8xβ_{y0} and β_x=4xβ_{x0}, β_y=8xβ_{y0}

I_+=1mA, I_-=0.8mA, Crab waist

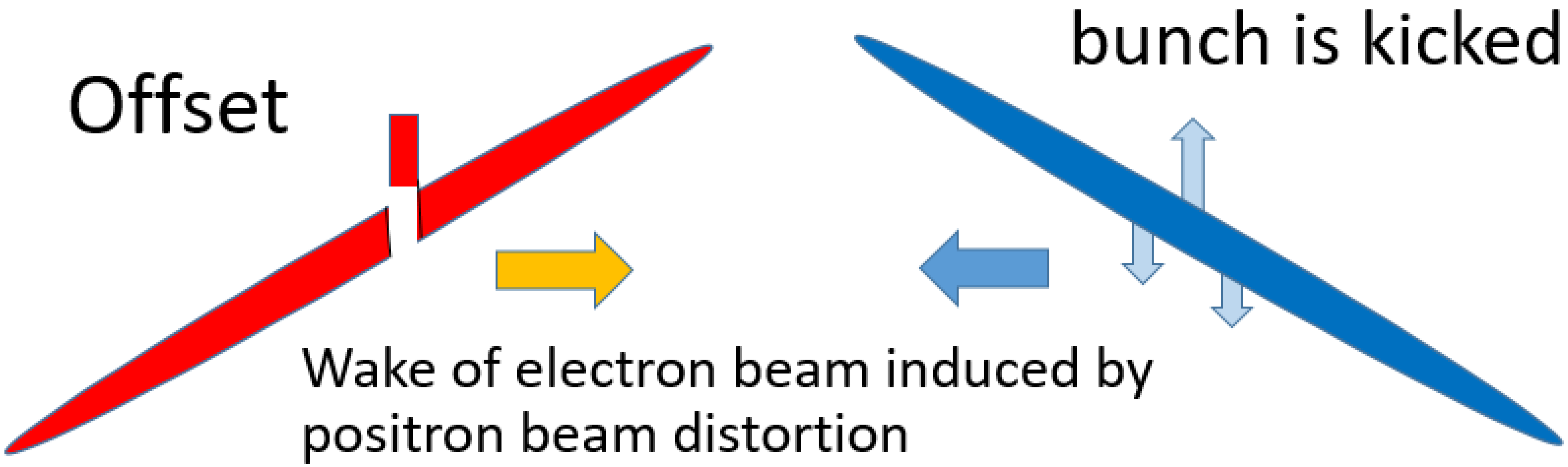


β_{x0}=0.03m, β_{y0}=0.3mm,

This instability can be observed in SuperKEKB Phase II commissioning. Phase II starts from 2018.

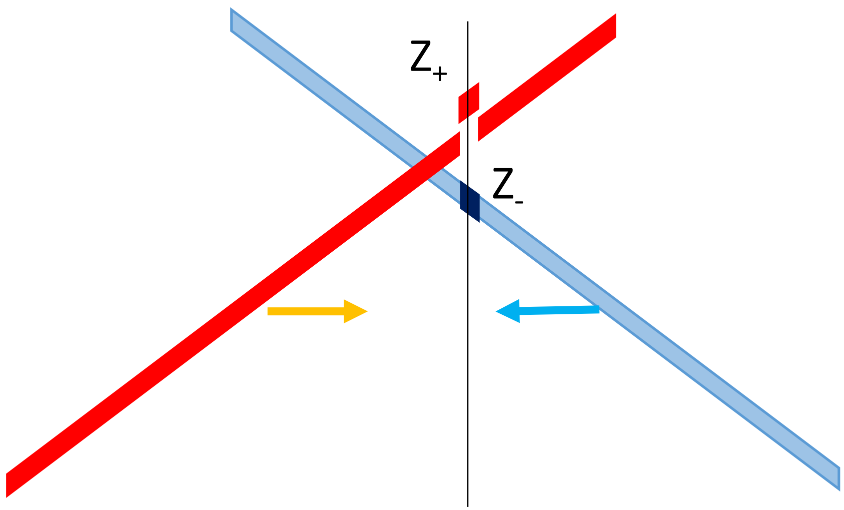
Study of the mechanism of the instability

- Wake force during collision



Analytic expression of the wake force

- Slice-slice force $$\Delta p_x^{(-)} = \frac{N_+ \rho_0(z_+) r_e}{\gamma} (F(x_- - x_+ - \Delta x) - F_x(x_- - x_+))$$



$$F(x,y) = F_y + iF_x = \frac{2\sqrt{\pi}}{\Sigma}\left[w\left(\frac{x+iy}{\Sigma}\right)\right.$$

$$- \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) w\left(\frac{\sigma_y x/\sigma_x + i\sigma_x}{\Sigma}\right.$$

$$\Sigma = \sqrt{2(\sigma_x^2 - \sigma_y^2)} \qquad \sigma_{x(y)} = \sqrt{\sigma^2_{x(y),-} + \sigma^2_{x(y),+}}$$

$$F_x((z_- - z_+)\theta_c - \Delta x, 0) - F_x((z_- - z_+)\theta_c, 0)$$

$$= - \left.\frac{\partial F_x(x,0)}{\partial x}\right|_{x=(z_- - z_+)\theta_c} \Delta x$$

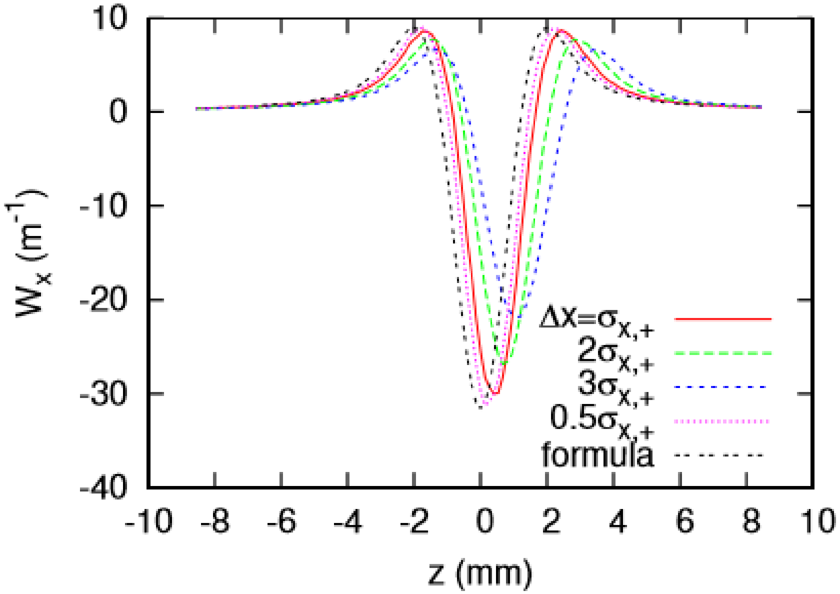
$$x_\pm \approx z_\pm \theta_c$$

θc: half crossing angle

Wake force due to beam-beam collision

$$\Delta p_{x,\pm}(z_\pm) = -\int_{-l}^{l} W_x(z_\pm - z'_\mp)\rho_x(z'_\mp)dz'_\mp \qquad l \sim 3\sigma_z$$

$$\rho_x(z_+) = \rho_0(z_+)\delta(z'_+ - z_+)\Delta x$$



$$\Delta p_x^{(-)} = -W_x(z_- - z_+)\rho_0(z_+)\Delta x$$

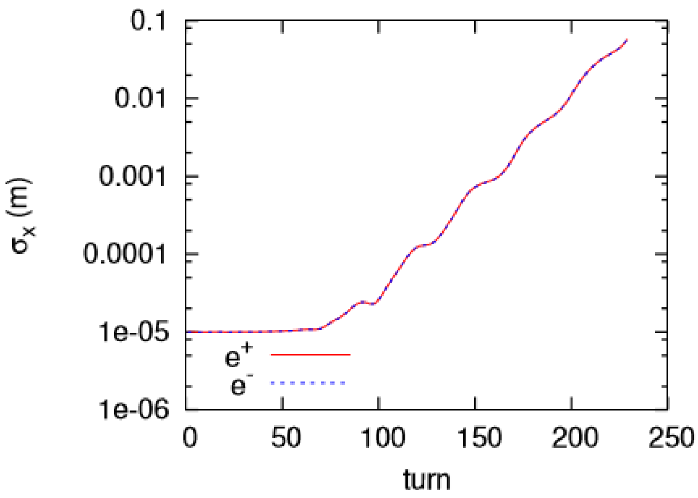
$$W_x(z_- - z_+) = \frac{N_+ r_e}{\gamma} \left.\frac{\partial F_x(x,0)}{\partial x}\right|_{x=(z_- - z_+)\theta_c}$$

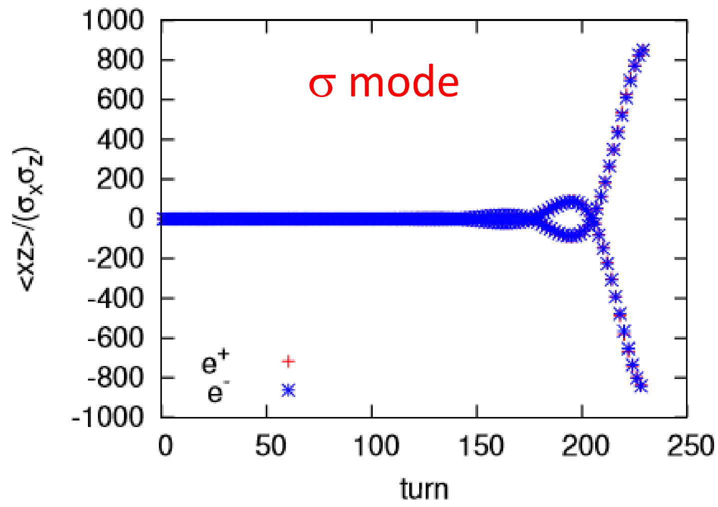
Minimum $W_x(0) = \frac{N_+ r_e}{\gamma}\frac{2}{\sigma_x(\sigma_x+\sigma_y)}$ $\qquad \sigma_{x(y)} = \sqrt{\sigma^2_{x(y),-} + \sigma^2_{x(y),+}}$

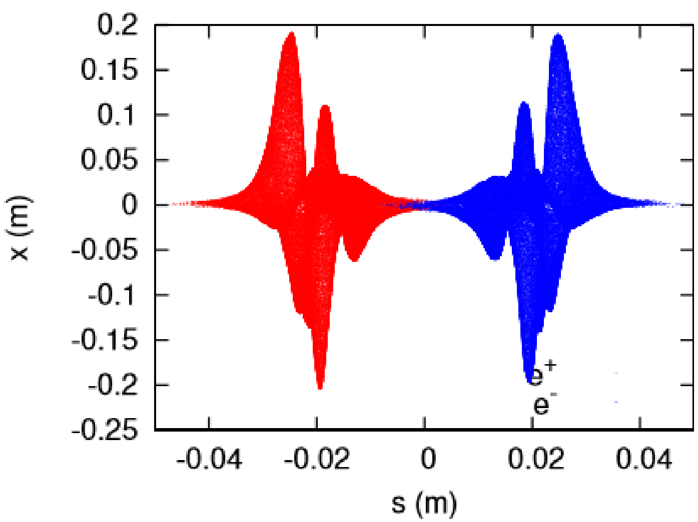
$W(z) = 0$ at $z \approx \pm 1.3\sigma_x/\theta_c$

Maximum W $\approx 0.28|W_x(0)|$ at $z \approx \pm 2.2\sigma_x/\theta_c$

Simulation result using the wake







Correlated wake simulation, not beam-beam simulation.

Both beams have the same distribution. σ mode oscillation.

Instability theory

- Two beams had the same (identical) distribution in the simulation, σ mode head-tail.
- The two beam wake force is treated as a single beam wake force for σ mode.

$l \sim 3\sigma_z$

$$\Delta p_{x,\pm}(z_\pm) = -\int_{-l}^{l} W_x(z_\pm - z'_\pm)\rho_x(z'_\pm)dz'_\pm$$

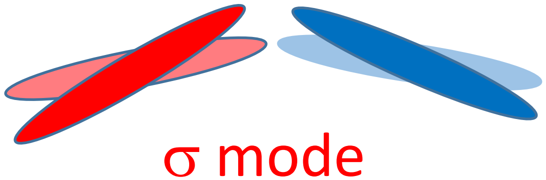
σ mode

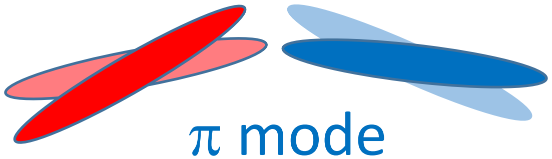
- For π mode, the sign of wake is inversed.

$$\Delta p_{x,\pm}(z_\pm) = +\int_{-l}^{l} W_x(z_\pm - z'_\pm)\rho_x(z'_\pm)dz'_\pm$$

π mode

- Conventional instability theory can be applicable.

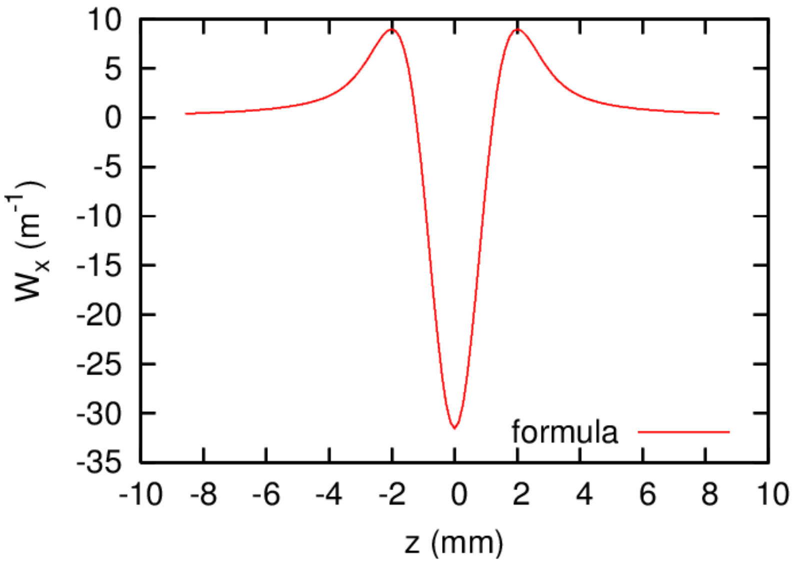


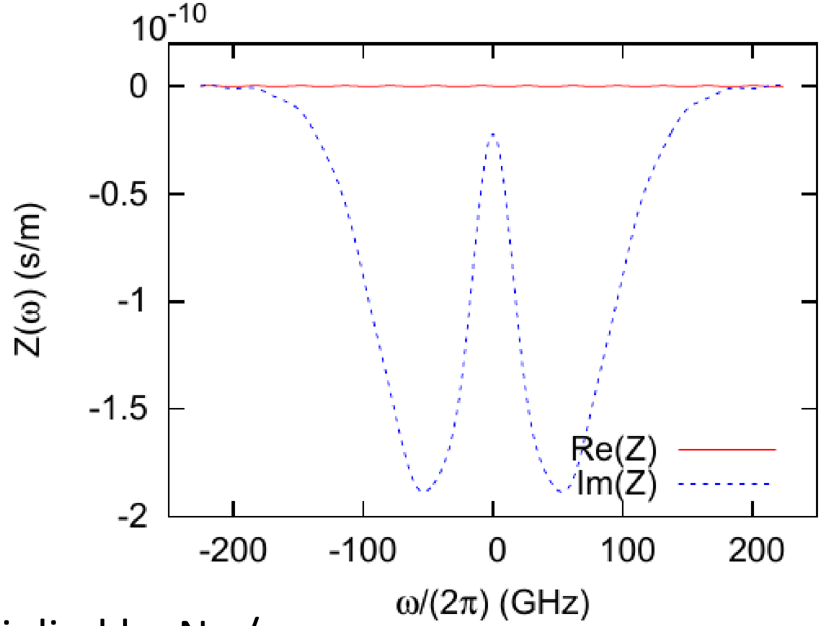


Impedance

$$Z_x(\omega) = i \int_{-\infty}^{\infty} W_x(z) e^{-i\omega z/c} \frac{dz}{c}$$

- The wake is symmetric for z.
- The impedance is pure imaginary and symmetric for ω.





W and Z are multiplied by Nr_e/γ.

Mode coupling theory

$$(\mu - \mu_x - l\mu_z)a_{k\ell} = \sum_{k'l'} M_{kl,k'l'}a_{k'l'}$$

$$M_{k\ell,k'\ell'} = \frac{\beta_x}{2} i^{l-l'-1}\omega_0 \sum_{p=-\infty}^{\infty} Z(\omega')g_{kl}(\omega')g_{k'l'}(\omega')$$

- Neglect off-diagonal component, the effective impedance

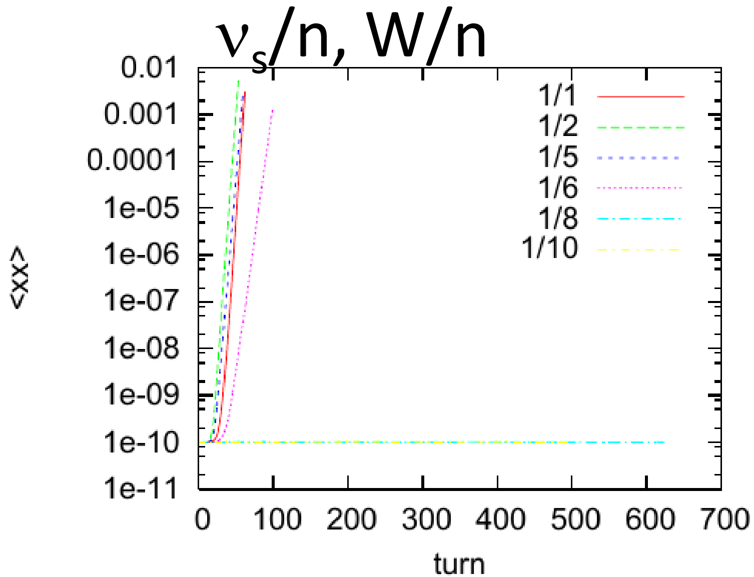
$$M_{k\ell,k\ell} = -i\frac{\beta_x}{2}\omega_0 \sum_{p=-\infty}^{\infty} Z(\omega')g_{kl}(\omega')^2 \approx -i\frac{\beta_x}{2}\int_{-\infty}^{\infty} d\omega' Z(\omega')g_{kl}(\omega')^2$$

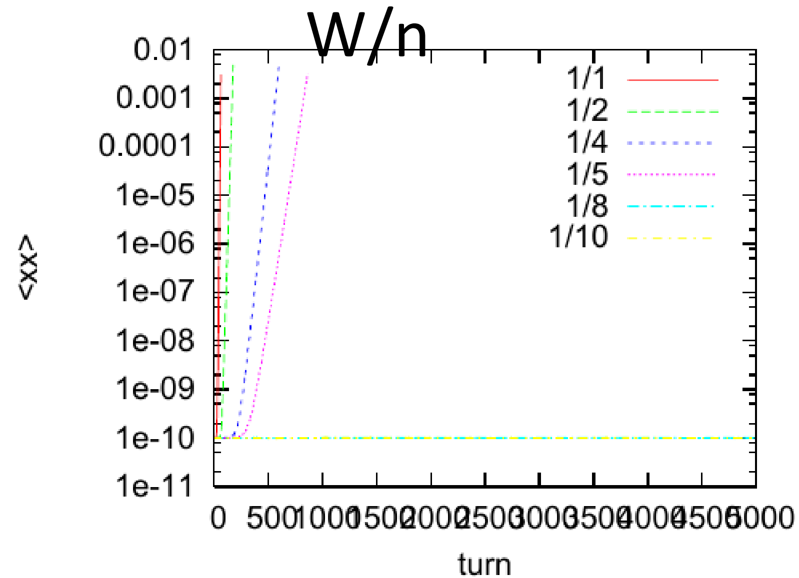
$$g_{kl}(\omega) = \frac{1}{\sqrt{2\pi k!(|l|+k)!}}\left(\frac{\omega\sigma}{\sqrt{2}c}\right)^{|l|+2k} e^{-\omega^2\sigma^2/2c^2}$$

- Diagonal M, which has only real part, induces tune shifts for *l*-th modes.
- The impedance is symmetric for ω. Terms with l+l'=odd is zero. No coupling between 0-1, 2-3… modes.
- Ordinary theory based on a distributed wake shows weak instability for this type of wake/impedance.

Simulation of single beam instability using the wake force

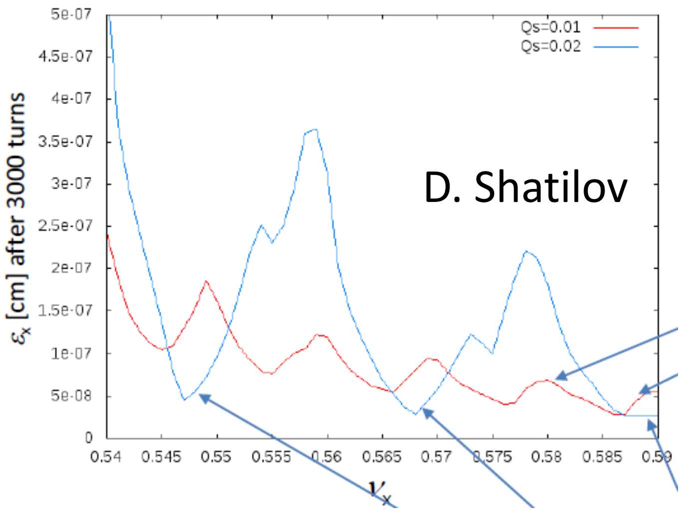
The growth disappears ν_s/n, W/n, n->∞



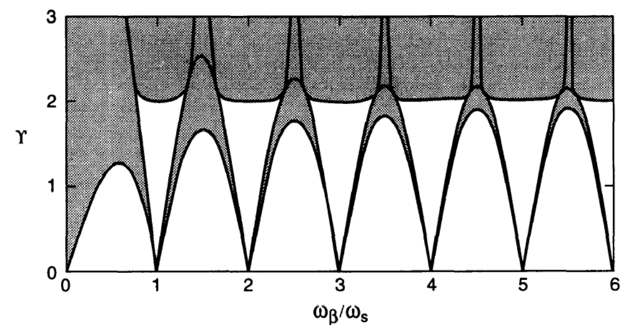


- W/8 is stable independent of ν_s. Strength of the localized W is essential.
- The wake with opposite sign is stable. π mode head-tail is stable.
- Growth is not sensitive for Z_{peak} at z=0 or not.

Localized wake force due to beam-beam interaction

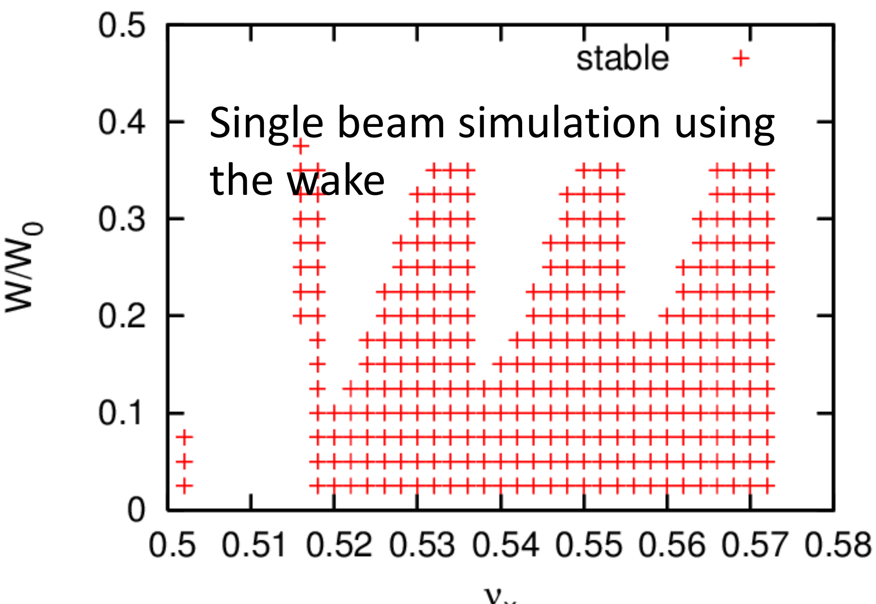


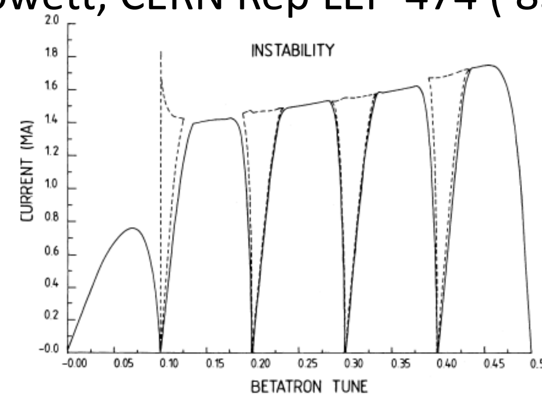
- Synchro-beta structure should be seen.



A. Chao, Phys. Collective Instability …

J. Jowett, CERN Rep LEP-474 ('83)





F. Ruggiero, PA20, 45 (1986)

Theory for instability due to a localized wake force

based on the bunch lengthening theory by K. Oide, Part.Accel. 51, 43 (1995)

- Dipole moments on the synchrotron phase space, J,ϕ.

$$x_{ij} = x(J_i, \phi_j) \qquad p_{ij} = p(J_i, \phi_j) \qquad \psi_i = \psi(J_i)$$

$$J_i = i\Delta J \qquad \phi_j = 2\pi\nu_s j \qquad z_{ij} = \sqrt{2\beta_z J_i}\cos\phi_j$$

Synchrotron motion

$j \rightarrow j+1$。 $1/\nu_s = n_s$

- Revolution of the dipole moments

$$\begin{pmatrix} x_{ij} \\ p_{ij} \end{pmatrix} = \sum_{j'=1}^{n_s} M_{ij,ij'} \begin{pmatrix} x_{ij'} \\ p_{ij'} \end{pmatrix} = \sum_{j'=1}^{n_s} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix} \delta_{j-1,j'} \begin{pmatrix} x_{ij'} \\ p_{ij'} \end{pmatrix}$$

- Wake force

$$\begin{pmatrix} x_{ij} \\ p_{ij} \end{pmatrix} = \sum_{i'j'} W_{ij,ij'} \begin{pmatrix} x_{i'j'} \\ p_{i'j'} \end{pmatrix} = \sum_{i'j'=1} \begin{pmatrix} 1 & 0 \\ -W(z_{ij} - z_{i'j'})\psi_{i'} & 1 \end{pmatrix} \begin{pmatrix} x_{i'j'} \\ p_{i'j'} \end{pmatrix}$$

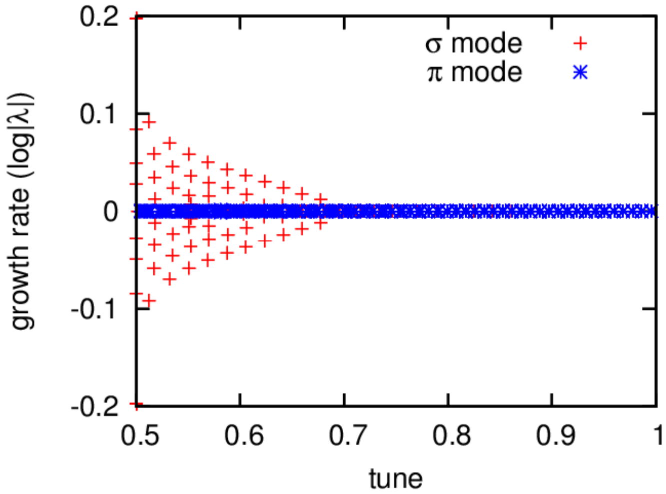
- Solve eigenvalue problem

$$M_W = \begin{pmatrix} \delta_{i,i'}\delta_{j,j'+1} & 0 \\ -\beta_x W(z_{i,j} - z_{i',j'+1})\psi_{i'}\Delta J\Delta\phi & \delta_{i.i'}\delta_{j,j'+1} \end{pmatrix} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix}$$

- Real matrix, $2 \mathrm{x} n_J \mathrm{x} n_s$

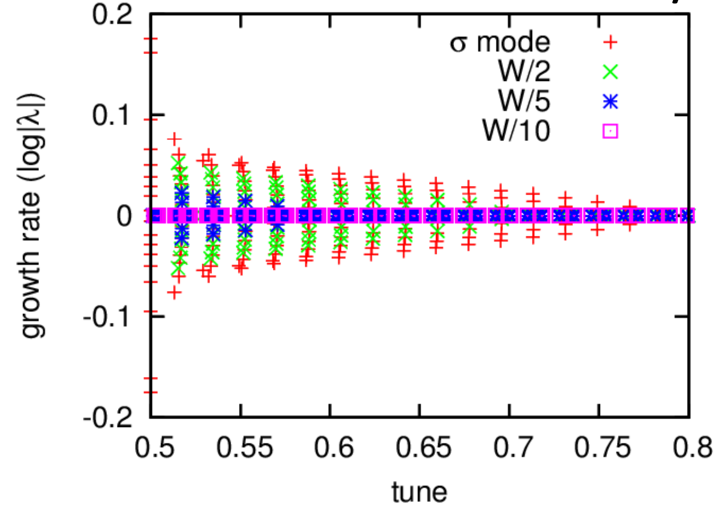
Eigenvalues and eigenvectors

σ/π modes, all π modes are stable



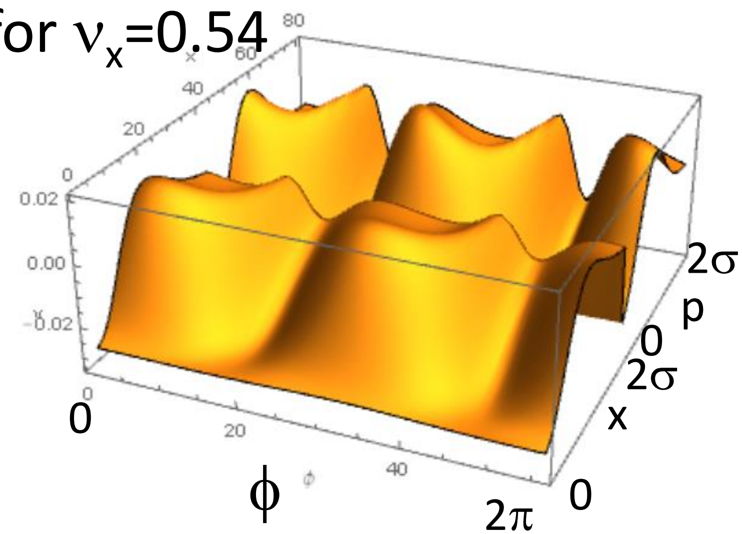
Wake strength scan,

all modes are stable at W/10



Eigenvector with largest growth

for ν_x=0.54



- σ modes are unstable at ν=0.5+ν_s.
- All π modes are stable.
- Threshold exists for strength of the wake.
- Everything is consistent with the single beam simulation
- π modes are unstable in pp collision.

Beam-beam simulations using the latest parameters

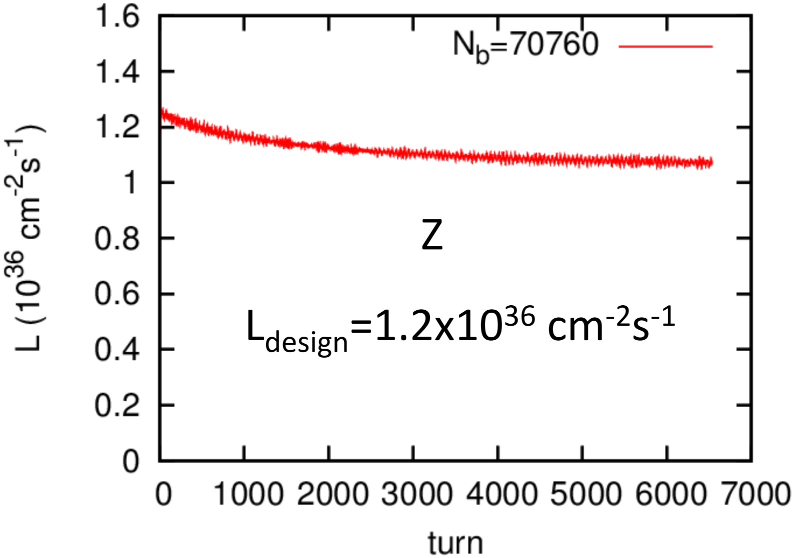
Table 1: FCC-ee baseline parameters.

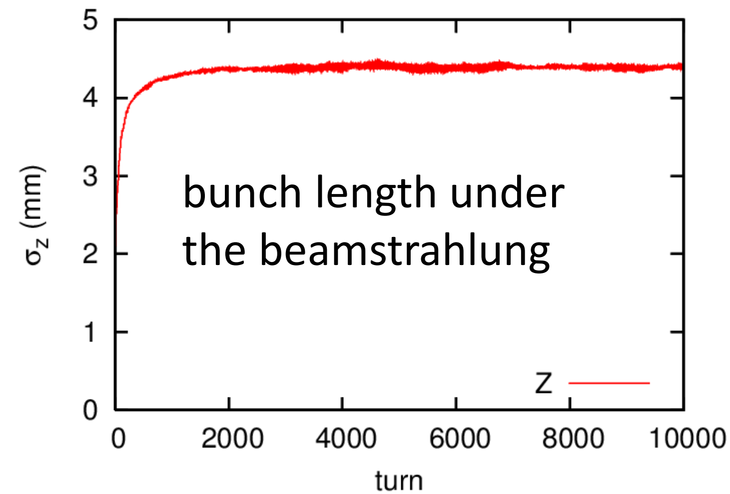
	Z	W	H	tt
Circumference [km]	97.750			
Bending radius [km]	10.747			
Beam energy [GeV]	45.6	80	120	175
Beam current [mA]	1399	147	29	6.4
Bunches / beam	71200	7500	740	62
Bunch spacing [ns]	2.5 and 5.0	40	400	5000
Bunch population [10^{11}]	0.4	0.4	0.8	2.11
Horizontal emittance ε [nm] Vertical emittance ε [pm]	0.267 1.0	0.26 1.0	0.61 1.2	1.33 2.66
Momentum comp. [10^{-6}]	14.79	7.31	7.31	7.31
Arc sextupole families	208	292	292	292
Betatron function at IP - Horizontal β* [m] - Vertical β* [mm]	 0.15 1	 1 2	 1 2	 1 2
Energy spread [%] - Synchrotron radiation - Total (including BS)	 0.038 0.064	 0.066 0.074	 0.10 0.11	 0.145 0.169
Bunch length [mm] - Synchrotron radiation - Total	 2.1 3.6	 2.0 2.3	 2.0 2.3	 2.38 2.77
Energy loss / turn [GeV]	0.0356	0.34	1.71	7.72
Luminosity/IP for 2IPs [10^{34} $cm^{-2}s^{-1}$]	158	16.4	5.0	1.46
Beam-beam parameter - Horizontal - Vertical	 0.010 0.118	 0.08 0.13	 0.09 0.14	 0.09 0.14

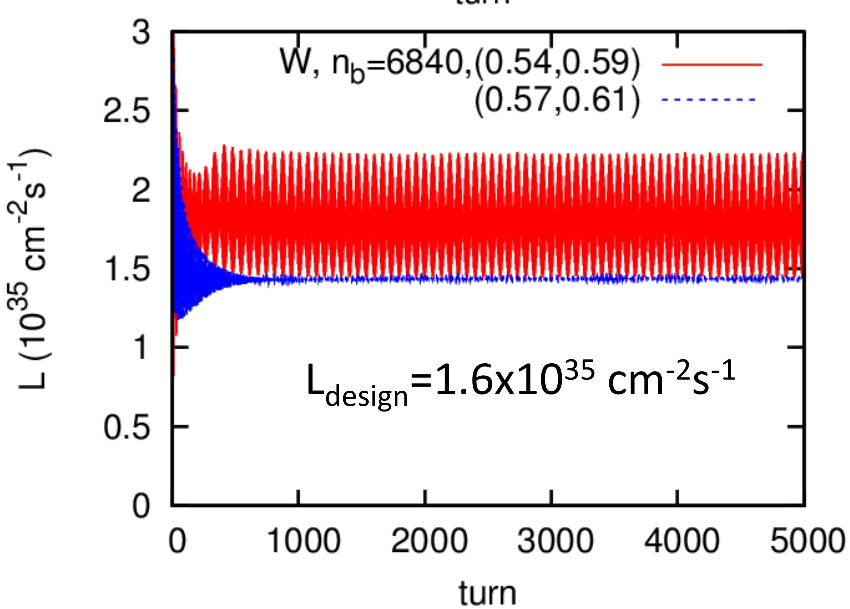
K Oide, May 24

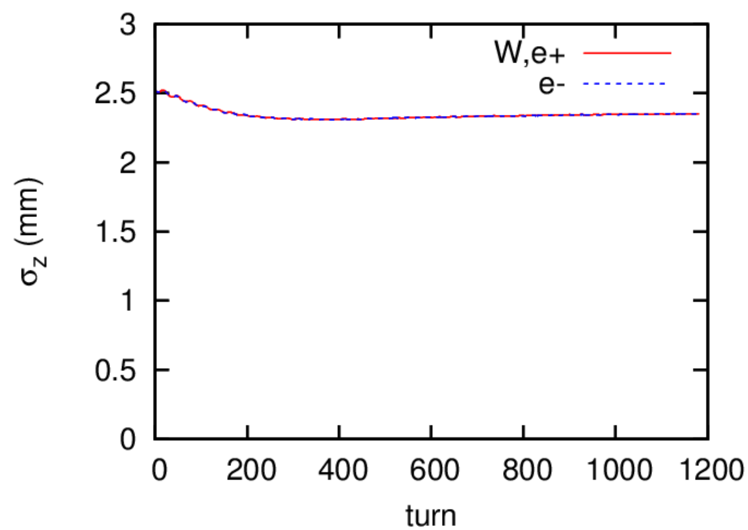
Design		2017			
Circumference	[km]	97.750			
Arc quadrupole scheme		**twin aperture**			
Bend. radius of arc dipole	[km]	10.747			
Number of IPs / ring		2			
Crossing angle at IP	[mrad]	30			
Solenoid field at IP	[T]	±2			
ℓ^*	[m]	2.2			
Local chrom. correction		y-plane with crab-sext. effect			
RF frequency	[MHz]	400			
Total SR power	[MW]	100			
Beam energy	[GeV]	45.6	80	120	175
SR energy loss/turn	[GeV]	0.036	0.34	1.72	7.80
Long. damping time	[ms]	414	76.8	22.9	7.49
Current/beam	[mA]	1390	147	29.0	6.4
Bunches/ring		70760	6840 (3860)	750 (560)	62
Particles/bunch	[10^{10}]	4.0	4.4 (7.8)	7.9 (10.5)	21.1
Arc cell		60°/60°	90°/90°		
Mom. compaction α_p	[10^{-6}]	**14.79**	7.31		
Horizontal tune ν_x		269.14	389.08		
Vertical tune ν_y		267.22	389.18		
Arc sext. families		208	292		
Horizontal emittance ε_x	[nm]	**0.267**	0.28	0.63	1.34
$\varepsilon_y/\varepsilon_x$ at collision	[%]	0.38	0.36	0.2	0.2
β_x^*	[m]	**0.15**	1 (0.5)		1
β_y^*	[mm]	1	2 (1)		2
Energy spread by SR	[%]	0.038	0.066	0.099	0.147
Energy spread SR+BS	[%]	0.083	0.078 (0.109)	0.114 (0.140)	0.193
RF Voltage	[MV]	**255**	696	2620	9500
Bunch length by SR	[mm]	2.1	2.1	2.0	2.4
Bunch length SR+BS	[mm]	4.6	2.5 (3.5)	2.3 (2.8)	3.2
Synchrotron tune ν_z		−0.0413	−0.0340	−0.0499	−0.0684
RF bucket height	[%]	3.8	3.7	2.2	10.3
Luminosity/IP	[$10^{34}/\text{cm}^2\text{s}$]	**121**	16.4 (30.0)	4.4 (7.9)	1.32

Strong-strong simulation for FCCee-Z & W



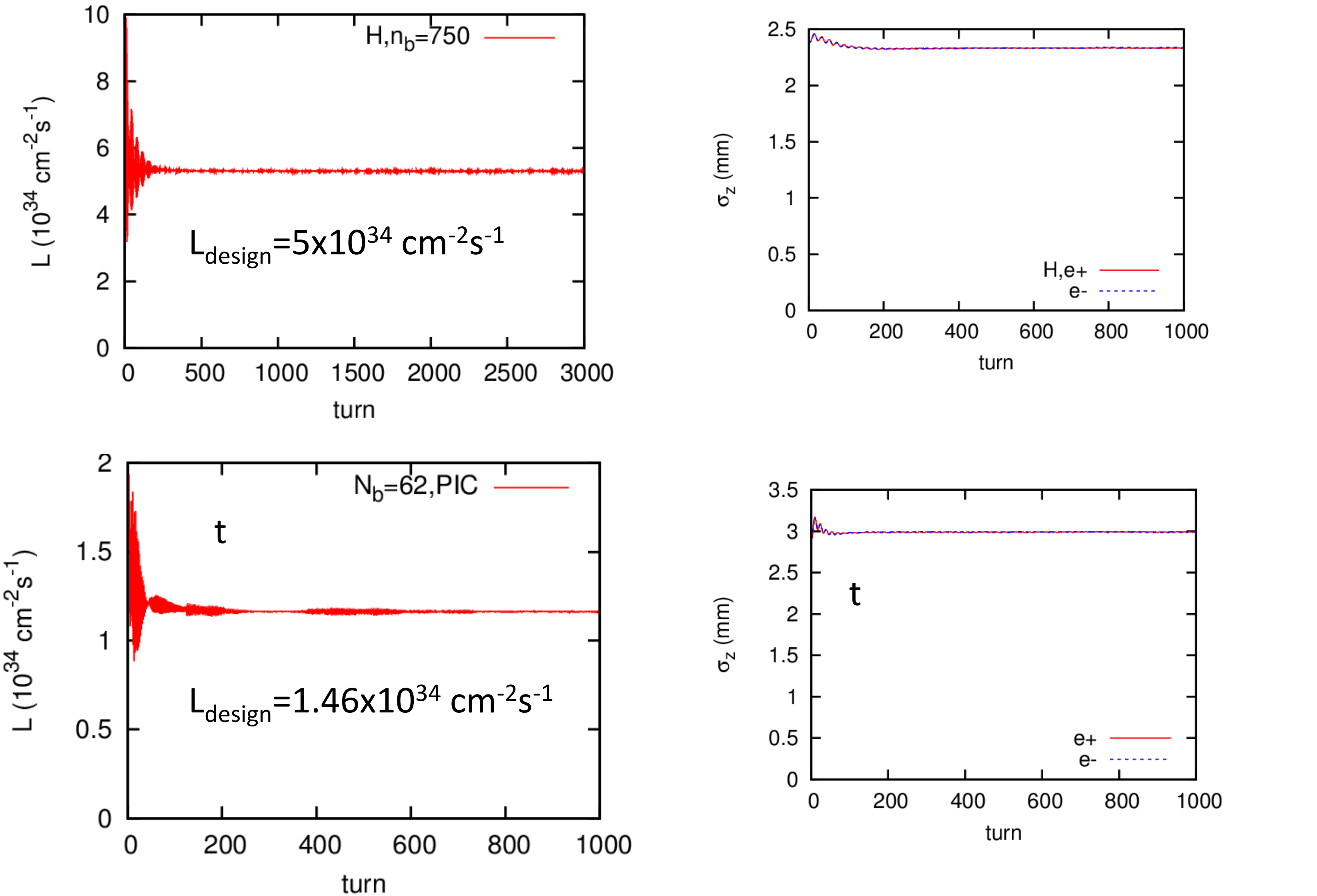






- ½ model, "turn" is ½ of the actual number

Strong-strong simulation for FCCee-H & t

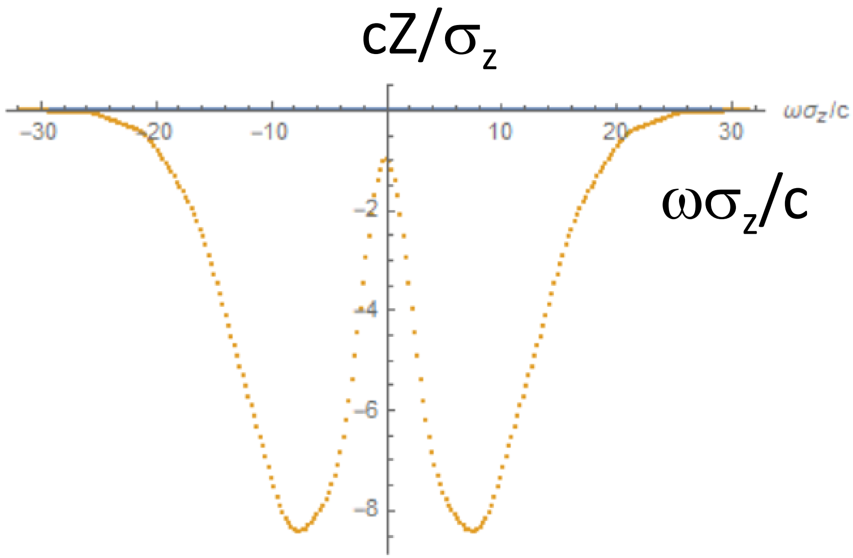


Summary

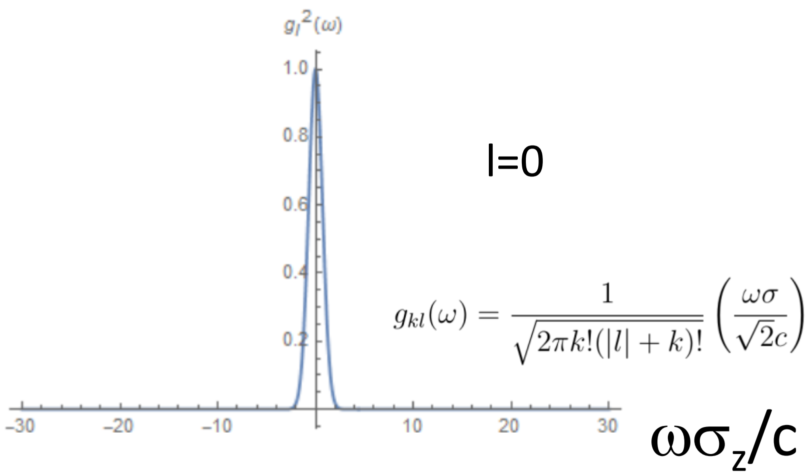
- Strong-strong beam-beam simulations showed a coherent beam-beam instability in head-tail mode.
- The instability was serious in collision with a large crossing (Piwinski) angle.
- FCC parameters were revised to suppress the instability. Now the parameters for Z-t work well.
- The instability is explained by a wake force for correlation between two beams.
- It is important that the wake is localized.
- Theory with mode analysis was completed to explain this instability .

Thank you for your attention

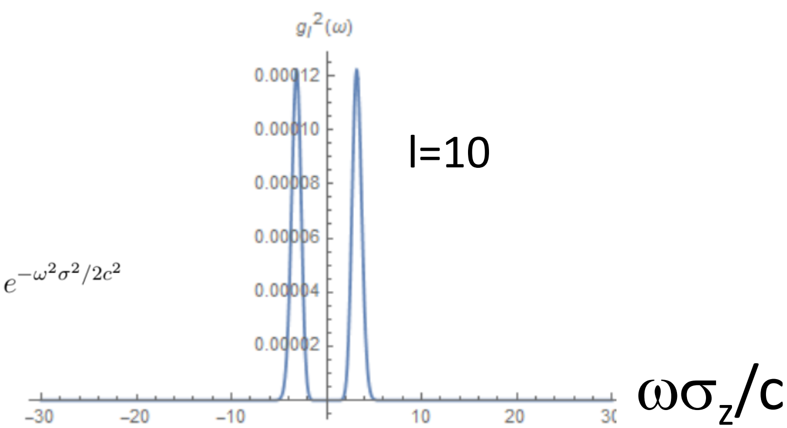
Tune shift



l	$\Delta\nu/\nu_s$
0	-0.84
1	-0.31
2	-0.15
3	-0.077
4	-0.039
5	-0.020
6	-0.0098
7	-0.0049
8	-0.0024



$$g_{kl}(\omega) = \frac{1}{\sqrt{2\pi k!(|l|+k)!}} \left(\frac{\omega\sigma}{\sqrt{2}c}\right)^{|l|+2k} e^{-\omega^2\sigma^2/2c^2}$$



Fourier expansion of the dipole moments

$$x(J,\phi) = \sum_{l=-\infty}^{\infty} x_l(J) e^{il\phi} \qquad p(J,\phi) = \sum_{l=-\infty}^{\infty} p_l(J) e^{il\phi}$$

- Revolution

$$\begin{pmatrix} x_l \\ p_l \end{pmatrix} = e^{2\pi i l \nu_s} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix} \begin{pmatrix} x_l \\ p_l \end{pmatrix}$$

- Wake force

$$\Delta p_l(J) = -\frac{1}{2\pi} \sum_{l'} \int dJ' W_{ll'}(J,J') x_{l'}(J') \psi(J')$$

$$\begin{aligned} W_{ll'}(J,J') &= \int\int d\phi d\phi' e^{-il\phi + il'\phi'} W(z-z') \\ &= 2\pi i^{l'-l-1} \omega_0 \sum_{p=-\infty}^{\infty} Z(\omega) J_l\left(\frac{\omega' r}{c}\right) J_{i'}\left(\frac{\omega' r'}{c}\right) \end{aligned}$$

- Eigenvalue problem

$$M_W = e^{2\pi i l \nu_s} \begin{pmatrix} 1 & 0 \\ -\beta_x W_{ll'}(J_i, J_{i'}) \psi_{i'} \Delta J_{i'}/2\pi & 1 \end{pmatrix} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix}$$

Laguerre expansion for the radial modes

$$x_l(J) = \sum_k x_{kl} \sqrt{\frac{k!}{(|l|+k)!}} \hat{J}^{|l|/2} L_k^{(|l|)}(\hat{J})$$

- Wake force

$$\Delta p_{kl} \;=\; -\sum_{k'l'} x_{k'l'} i^{l-l'-1} \omega_0 \sum_{p=-\infty}^{\infty} Z_1(\omega) g_{kl}(\omega') g_{k'l'}(\omega')$$

$$g_{kl}(\omega') = \sqrt{\frac{1}{2\pi k!(|l|+k)!}} \left(\frac{\omega'\sigma}{\sqrt{2}c}\right)^{2k+|\ell|} \exp\left(-\frac{\omega'^2\sigma^2}{2c^2}\right)$$

- Eigen value problem

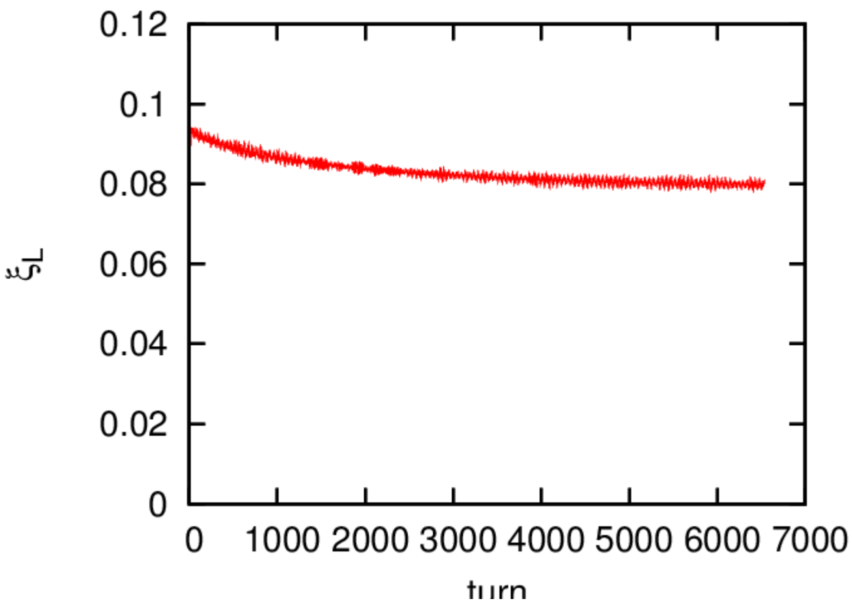
$$M_W = e^{2\pi i l \nu_s} \begin{pmatrix} 1 & 0 \\ -2M_{klk'l'} & 1 \end{pmatrix} \begin{pmatrix} \cos\mu_x & \sin\mu_x \\ -\sin\mu_x & \cos\mu_x \end{pmatrix}$$

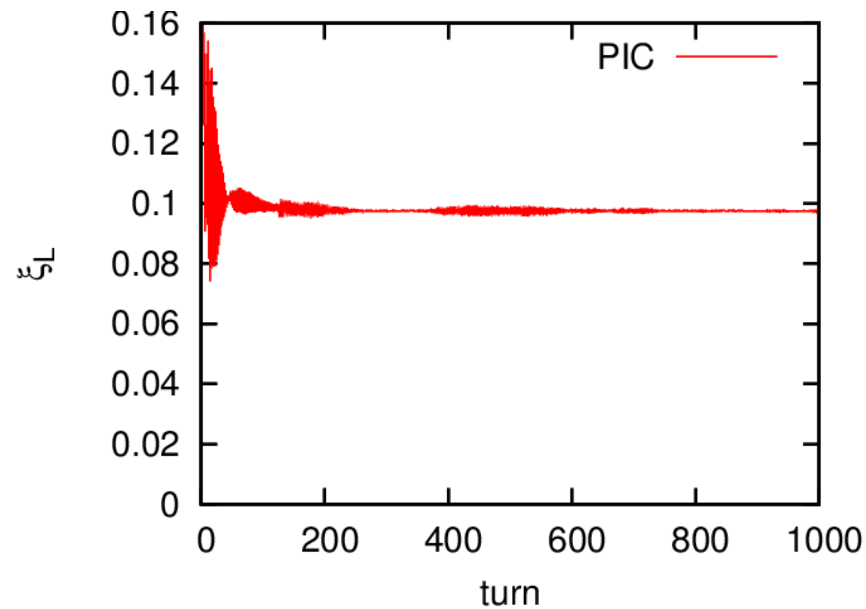
$$M_{k\ell,k'\ell'} = \frac{1}{2}\beta_x i^{l-l'-1} \omega_0 \sum_{p=-\infty}^{\infty} Z(\omega') g_{kl}(\omega') g_{k'l'}(\omega')$$

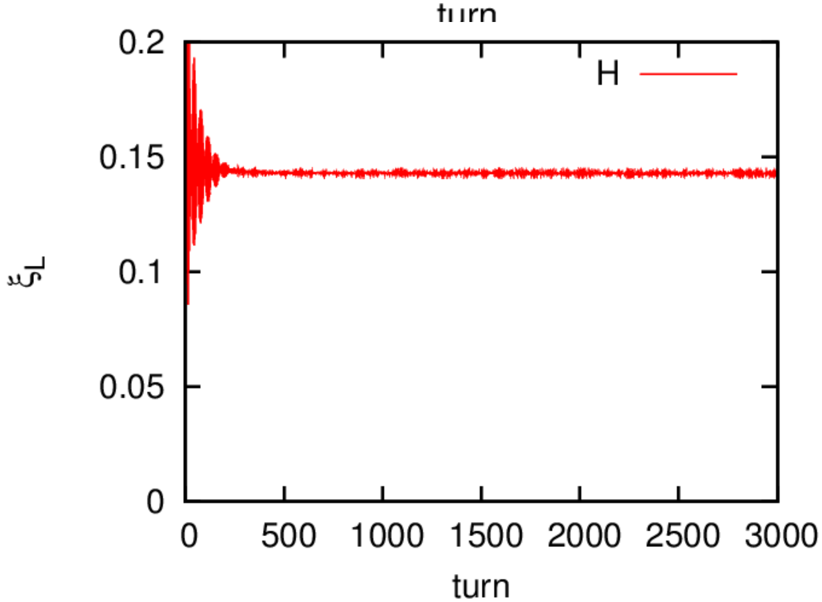
- $\Delta\nu = K/4\pi$ gives usual dispersion relation, $K=M_{21}$.
- Luguerre expansion is not goof for high frequency wake/impedance, $\omega\sigma_z/c \gg 1$

- Strong-strong beam-beam simulation
- Single beam simulation using multi-turn wake
- Two beam simulation using two beam wake
- Single beam simulation using two beam wake, σ or π modes.
- They gave similar results.

σ_z for beamstrahlung







Luminosity for 60 degree lattice of FCC-ee-Z

K. Ohmi, May. 25, 2017

Parameters given by K. Oide (Feb. 17)

Design momentum P0 = 45.600000 GeV Revolution freq. f0 = 6133.6491 Hz
Energy loss per turn U0 = 17.203330 MV Effective voltage Vc = 44.392690 MV
Equilibrium position dz = -.0014254 mm Momentum compact. alpha = 1.4654E-5
Bucket height dV/P0 = .0159296
Emittance X = 2.5520E-10 m Emittance Y = .00000000 m
Emittance Z = 1.32712E-6 m Energy spread = 3.68724E-4
Bunch Length = 3.59923315 mm Beam tilt = .00000000 rad
Beam size xi = .22479962 mm Beam size eta = .00000000 mm
Real tune:-0.4250112 -0.3900373 -0.0116833
betax* = 15 cm, betay* = 1 mm.

Ne=4e10, Nbunch=91500