Coherent Beam-beam instability in collision with a large crossing angle

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Accelerator Physics seminar
June 29, 2017, KEK

Thanks to D. Shatilov, K. Oide, N. Kuroo, D. Zhou, F. Zimmermann
Collision with a large crossing angle

-クラブウェストと組み合わせて、最近の円形電子陽電子衝突加速器の設計に広く使われるようになった。(P. Raimondi)

-特徴づける量、Piwinski角$\sigma_z \theta/\sigma_x$。衝突領域に対するバンチ長

-実証実験DAFNA $\sigma_z \theta/\sigma_x = 2$, with crab waist

-SuperKEKB $\sigma_z \theta/\sigma_x = 20$、この方式での初めての本格的な加速器(without crab waist)

\[ \theta: \text{half crossing angle} \]

\[ \sigma_x/\theta \]
この衝突方式に死角はないか？
• DAFNEの実験はPiwinski角が2と小さい。KEKBは1
• Beam-beam simulationによる検討がされてきたが、ほとんどはweak-strong simulationだった。
• その理由は後述するが、バンチを進行方向にスライスするがその数が大きくなる。\( N_{sl} = 10\sigma_z\theta/\sigma_x \)
• SuperKEKBのstrong-strong simulationは衝突当たり\( N_{sl}^2 = 200 \times 200 = 40,000 \)回のポテンシャル計算。
• Weak-strongでは\( N_{sl} \)回、複素エラー関数からガウス分布によるビームビーム力を計算。数分でルミノシティ計算ができる。
• クラブウェストと組み合わせると、weak-strongではビームビームパラメータ、\( \xi = 0.1 \)は簡単に越えられる。
• これは本当か
Beam-beam limit

• Luminosity

\[ L = \frac{N^2 f_{\text{rep}}}{4\pi \sigma_x \sigma_y} R \left( \frac{\sigma_z}{\beta_y} \text{ or } \frac{\sigma_x}{\theta_c \beta_y}, \frac{\theta_c \sigma_z}{\sigma_x} \right) \]

\( N = N_+ = N_- : \) bunch population
\( f_{\text{rep}} : \) collision freq.
\( \theta_c : \) half crossing angle

• \( \frac{\sigma_z}{\beta_y} \text{ or } \frac{\sigma_x}{\theta_c \beta_y} : \) hourglass (衝突領域と\( \beta_y \)の比),

• \( \frac{\theta_c \sigma_z}{\sigma_x} : \) normalized crossing angle (Piwinski angle)

• Tune shift

\[ \xi_y = \Delta \nu_y = \frac{N r_e \beta_y}{2\pi \gamma \sigma_y (\sigma_x + \sigma_y)} R \left( \frac{\sigma_z}{\beta_y} \text{ or } \frac{\sigma_x}{\theta_c \beta_y}, \frac{\theta_c \sigma_z}{\sigma_x} \right) \]

• \( N \)を増やすとビームサイズ特に\( y \)が大きくなりtune shiftは飽和し、ルミノシティは\( N^2 \)で増えなくなる。この状態をBeam-beam limit.

\[ L = \frac{N \gamma f_{\text{rep}}}{2r_e \beta_y} \xi_y \quad \sigma_x \gg \sigma_y \]

• この式はまたアワーグラスが効かなければ、\( \beta \)が小さいほどルミノシティが大きくできることを示す。大衝突角

• SuperKEKBはcrab waistを使わない。IR非線形が強くすぎて、crab waist sextupoleの非線形がIRでキャンセルできず、DAが小さくなってしまう。
Weak-strong and strong-strong simulation

• **Weak-strong simulation**
  • One *(strong)* beam is assumed to be *fixed charge distribution*, and the other *(weak)* beam is represented by macro-particles.
  • Beam-beam interaction is evaluated by tracking the macro-particles in the electro-magnetic field induced by the fixed charge distribution.
  • The *strong beam* is assumed to be *Gaussian distribution in most cases*.

• **Strong-strong simulation** - *Both beams are represented by macro-particles*.
  • Beam distribution is represented on meshed space using *Particle In Cell* method. Arbitrary and self-consistent distribution of two beams are treated.
  • Statistical noise of macro-particles induces an fluctuation in potential calculated by PIC. The unphysical emittance growth by the noise is cared in the strong-strong simulation.
    • As an approximation, two beams are represented by Gaussian whose sizes are determined turn-by-turn. It is called *Soft Gaussian approximation*.
    • Strong-strong simulation based on PIC is more popular than the soft Gaussian approximation.

• **Quasi-strong-strong simulation**
  • Repeat weak-strong simulation with keeping self-consistency.
Weak-strong simulation for Large crossing angle

• Two colliding bunches are divided into many slices, $N_{sl} \sim 10\times \sigma_z \theta / \sigma_x$.
• Calculate slice-particle collision at $s_{pi} = (z_p - z_j)/2$.
• Crab waist transformation at IP.
Large crossing angle and crab waist weak-strong simulation

• Beam-beam parameter $\xi_L = 0.6$ is achieved for collision with crab waist in weak-strong simulation, $(\nu_x, \nu_y = 0.51, 0.55)$.

• Beam-beam parameter is saturated at $\xi_L = 0.1$ without crab waist.

Luminosity evolution for scanning bunch population
Equilibrium beam-beam parameter and beam size in weak-strong simulation

\[ \xi_{\text{max}} \sim 0.6 \text{ for } (v_x, v_y) = (0.51, 0.55) \]
\[ \xi_{\text{max}} \sim 0.2 \text{ for } (v_x, v_y) = (0.54, 0.61) \]

\( \sigma_y \) behavior correlates to Luminosity.

\[ \chi_{\text{max}} \]チューンによって、ほとんど天井知らず
Strong-strong simulation for Large crossing angle

- Two colliding bunches are divided into many slices, $N_{sl} \sim 10x\sigma_z\theta/\sigma_x$.
- Sort slices with their positions $z_i + z_j$, collision order.
- Each slice contains $>10,000$ macro-particles
- Solve potential slice-by-slice collision, or Gaussian approx.
Several option of Strong-strong simulation

- Gaussian approximation using turn-by-turn RMS values.
- Gaussian approximation using turn-by-turn Gaussian fitting.
- PIC for core part and Gaussian approximation for slice collision with large offset.
- Complete PIC using shifted Green function

Example of shifted potential for collision with large offset.

\[ \phi(x) = -\frac{2N\epsilon}{\gamma} \int dx' G(x, x' - x_0) \]

Shifted Green function (J. Qiang)
Coherent beam-beam instability

• A coherent beam-beam instability in head-tail mode was found to start beam-beam studies using strong-strong simulation.

• In Strong-strong simulation, both beams which are represented by macro-particles, interacts with each other in their classical EM field.

• The instability is cross-checked by D. Shatilov using quasi-strong-strong simulation.
## Parameters studied by early 2017

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SuperKEKB</th>
<th>FCC-ee-Z</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>design</td>
<td>commissioning</td>
<td>HiLum base</td>
</tr>
<tr>
<td>Energy</td>
<td>$E_{+/−}$ (GeV)</td>
<td>4/7</td>
<td>45.5</td>
</tr>
<tr>
<td>Bunch population</td>
<td>$N_{+/−}(10^{10})$</td>
<td>9/6.5 / 6.3/5</td>
<td>10</td>
</tr>
<tr>
<td>Emittance</td>
<td>$\varepsilon_{x/y}$ (nm/pm)</td>
<td>3.2/8.64 / 3.2/44</td>
<td>0.2/1</td>
</tr>
<tr>
<td>Beta at IP</td>
<td>$\beta^*_{x/y}$ (m/mm)</td>
<td>0.032/0.27 / 0.25/2.2</td>
<td>0.5/1</td>
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<tr>
<td>Bunch length</td>
<td>$\sigma_z$ (mm)</td>
<td>6</td>
<td>6.7</td>
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<tr>
<td>Energy spread</td>
<td>$\sigma_\delta$ (%)</td>
<td>0.08</td>
<td>0.22</td>
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<tr>
<td>Damping time</td>
<td>$\tau_x/T_0$</td>
<td>4000</td>
<td>3000</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>$\nu_z$</td>
<td>0.025</td>
<td>0.036</td>
</tr>
<tr>
<td>Luminosity per IP</td>
<td>$L ,(10^{34} \text{ cm}^{-2}\text{s}^{-1})$</td>
<td>80 -</td>
<td>207</td>
</tr>
<tr>
<td>Beam-beam</td>
<td>$\xi_{x/y}$</td>
<td>0.0028/0.088</td>
<td>-</td>
</tr>
<tr>
<td>Piwinski angle</td>
<td>$\sigma_z\theta_c/\sigma_x$</td>
<td>20 / 8.7</td>
<td>10</td>
</tr>
</tbody>
</table>
Simulation for $H$

$\xi_L = \frac{2r_e \beta_y}{N \gamma f_{rep}} L$

- PA=1.5 in the design. Safe for the instability.

Error bars correspond to amplitude of the coherent motion.
Strong-strong simulation for Z factory

$L_{\text{target}} = 2.2 \times 10^{36} \text{ cm}^{-2} \text{s}^{-1}$

$\xi_{\text{target}} = 0.17$

$\xi$ limit is around 0.06-0.07.

Coherent instability is strong.
Simulation for $Z$

- Larger PA is more serious
- $\sigma$ mode of head-tail motion, in which head-tail phases of two beams are in phase, is seen.
Strong-strong simulation in SuperKEKB

\[ \frac{\theta_c \sigma_z}{\sigma_x} = 20 \]

\[ \xi_{x/y} = 0.0028/0.088 \]

\[ \nu_S = 0.025 \]

Strong-head-tail instability is seen only in limited tune. The stopband seems narrow.

\( \sigma \) mode
SuperKEKB Phase 2

\[ \beta_x = 8x\beta_{x0}, \beta_y = 8x\beta_{y0} \text{ and } \beta_x = 4x\beta_{x0}, \beta_y = 8x\beta_{y0} \]

\( I_+ = 1\text{mA}, I_- = 0.8\text{mA}, \text{Crab waist} \)

\[ \beta_{x0} = 0.03\text{m}, \beta_{y0} = 0.3\text{mm}, \]

This instability can be observed in SuperKEKB Phase II commissioning. Phase II starts from 2018.
Study of the mechanism of the instability

- Wake force during collision

Offset

Wake of electron beam induced by positron beam distortion

bunch is kicked
Analytic expression of the wake force

• Slice-slice force

\[ \Delta p_{x}^{(-)} = \frac{N_{+}\rho_{0}(z_{+})r_{e}}{\gamma} (F(x_{-}-x_{+}-\Delta x)-F_{x}(x_{-}-x_{+})) \]

\[ F(x, y) = F_{y} + iF_{x} = \frac{2\sqrt{\pi}}{\Sigma} w \left( \frac{x + iy}{\Sigma} \right) \]
\[ -\exp \left( -\frac{x^{2}}{2\sigma_{x}^{2}} - \frac{y^{2}}{2\sigma_{y}^{2}} \right) w \left( \frac{\sigma_{y}x/\sigma_{x} + i\sigma_{y}}{\Sigma} \right) \]
\[ \Sigma = \sqrt{2(\sigma_{x}^{2} - \sigma_{y}^{2})} \]
\[ \sigma_{x(y)} = \sqrt{\sigma_{x(y),-}^{2} + \sigma_{x(y),+}^{2}} \]

\[ F_{x}((z_{-}-z_{+})\theta_{c} - \Delta x, 0) - F_{x}((z_{-}-z_{+})\theta_{c}, 0) \]
\[ = - \frac{\partial F_{x}(x, 0)}{\partial x} \bigg|_{x=(z_{-}-z_{+})\theta_{c}} \Delta x \]

\[ x_{\pm} \approx z_{\pm} \theta_{c} \]

\( \theta_{c} \): half crossing angle
Wake force due to beam-beam collision

\[
\Delta p_{x,\pm}(z_{\pm}) = -\int_{-l}^{l} W_x(z_{\pm} - z'_{\mp}) \rho_x(z'_{\mp}) dz'_{\mp} \quad l \sim 3\sigma_z
\]

\[
\rho_x(z_+) = \rho_0(z_+) \delta(z'_+ - z_+) \Delta x
\]

\[
\Delta p_x^{(-)} = -W_x(z_- - z_+) \rho_0(z_+) \Delta x.
\]

\[
W_x(z_- - z_+) = \frac{N_w r_e}{\gamma} \frac{2}{\sigma_x (\sigma_x + \sigma_y)} \frac{\partial F_x(x, 0)}{\partial x} \bigg|_{x=(z_--z_+)\theta_c}
\]

Minimum \( W_x(0) = \frac{N_w r_e}{\gamma} \frac{2}{\sigma_x (\sigma_x + \sigma_y)} \)

\[ W(z) = 0 \text{ at } z \approx \pm 1.3\sigma_x / \theta_c. \]

Maximum \( W \approx 0.28|W_x(0)| \text{ at } z \approx \pm 2.2\sigma_x / \theta_c \)
Simulation result using the wake

Correlated wake simulation, not beam-beam simulation.

Both beams have the same distribution. $\sigma$ mode oscillation.
Instability theory

• Two beams had the same (identical) distribution in the simulation, \( \sigma \) mode head-tail.

• The two beam wake force is treated as a single beam wake force for \( \sigma \) mode.

\[
\Delta p_{x,\pm}(z_{\pm}) = - \int_{-l}^{l} W_x(z_{\pm} - z'_{\pm}) \rho_x(z'_{\pm}) dz'_{\pm}
\]

\( l \sim 3\sigma_z \)  \( \sigma \) mode

• For \( \pi \) mode, the sign of wake is inversed.

\[
\Delta p_{x,\pm}(z_{\pm}) = \int_{-l}^{l} W_x(z_{\pm} - z'_{\pm}) \rho_x(z'_{\pm}) dz'_{\pm}.
\]

\( \pi \) mode

• Conventional instability theory can be applicable.
Impedance

- The wake is symmetric for $z$.
- The impedance is pure imaginary and symmetric for $\omega$.

\[ Z_x(\omega) = i \int_{-\infty}^{\infty} W_x(z) e^{-i\omega z/c} \frac{dz}{c} \]

$W$ and $Z$ are multiplied by $N r_e / \gamma$. 
Mode coupling theory

\[(\mu - \mu_x - l\mu_z) a_{kl} = \sum_{k'l'} M_{kl,k'l'} a_{k'l'}\]

\[M_{kl,k'l'} = \frac{\rho_x}{2} (l-l')^{-1} \omega_0 \sum_{p=-\infty}^{\infty} Z(\omega') g_{kl}(\omega') g_{k'l'}(\omega')\]

- Neglect off-diagonal component, the effective impedance

\[M_{kl,kl} = -i \frac{\beta_x}{2} \omega_0 \sum_{p=-\infty}^{\infty} Z(\omega') g_{kl}(\omega')^2 \approx -i \frac{\beta_x}{2} \int_{-\infty}^{\infty} d\omega' Z(\omega') g_{kl}(\omega')^2\]

\[g_{kl}(\omega) = \frac{1}{\sqrt{2\pi k!(|l|+k)!}} \left( \frac{\omega\sigma}{\sqrt{2c}} \right)^{|l|+2k} e^{\omega^2\sigma^2/2c^2}\]

- Diagonal M, which has only real part, induces tune shifts for \(l\)-th modes.

- The impedance is symmetric for \(\omega\). Terms with \(l+l'=\text{odd}\) is zero. No coupling between 0-1, 2-3... modes.

- Ordinary theory based on a distributed wake shows weak instability for this type of wake/impedance.
Simulation of single beam instability using the wake force

The growth disappears \( \nu_s/n, \ W/n, \ n->\infty \)

- \( \nu_s/n, \ W/n \)

- **W/8** is stable independent of \( \nu_s \). Strength of the localized \( W \) is essential.

- The wake with opposite sign is stable. \( \pi \) mode head-tail is stable.

- Growth is not sensitive for \( Z_{\text{peak}} \) at \( z=0 \) or not.
Localized wake force due to beam-beam interaction

- Synchro-beta structure should be seen.

D. Shatilov

A. Chao, Phys. Collective Instability ... J. Jowett, CERN Rep LEP-474 (‘83)

F. Ruggiero, PA20, 45 (1986)
Theory for instability due to a localized wake force

- Dipole moments on the synchrotron phase space, $J, \phi$.

$$
\begin{align*}
  x_{ij} &= x(J_i, \phi_j) \\
  p_{ij} &= p(J_i, \phi_j) \\
  \psi_i &= \psi(J_i)
\end{align*}
$$

$$J_i = i \Delta J \\
\phi_j = 2 \pi \nu_s j \\
z_{ij} = \sqrt{2\beta_z J_i \cos \phi_j}$$

- Revolution of the dipole moments

$$
\begin{pmatrix}
  x_{ij} \\
  p_{ij}
\end{pmatrix} = \sum_{j'=1}^{n_s} M_{ij,ij'} \begin{pmatrix}
  x_{ij'} \\
  p_{ij'}
\end{pmatrix} = \sum_{j'=1}^{n_s} \begin{pmatrix}
  \cos \mu_x & \sin \mu_x \\
  -\sin \mu_x & \cos \mu_x
\end{pmatrix} \delta_{j-1,j'} \begin{pmatrix}
  x_{ij'} \\
  p_{ij'}
\end{pmatrix}
$$

- Wake force

$$
\begin{pmatrix}
  x_{ij} \\
  p_{ij}
\end{pmatrix} = \sum_{i'j'} W_{ij,i'j'} \begin{pmatrix}
  x_{i'j'} \\
  p_{i'j'}
\end{pmatrix} = \sum_{i'j'=1}^{n_s} \begin{pmatrix}
  1 & 0 \\
  -W(z_{ij} - z_{i'j'}) \psi_{i'} & 1
\end{pmatrix} \begin{pmatrix}
  x_{i'j'} \\
  p_{i'j'}
\end{pmatrix}
$$

- Solve eigenvalue problem

$$M_W = \begin{pmatrix}
\delta_{i,i'} \delta_{j,j'+1} & 0 \\
-\beta_z W(z_{i,j} - z_{i',j'+1}) \psi_{i'} \Delta J \Delta \phi & \delta_{i,i'} \delta_{j,j'+1}
\end{pmatrix} \begin{pmatrix}
\cos \mu_x & \sin \mu_x \\
-\sin \mu_x & \cos \mu_x
\end{pmatrix}$$

- Real matrix, $2 \times n_s \times n_s$
Eigenvalues and eigenvectors

\( \sigma/\pi \) modes, all \( \pi \) modes are stable

- All \( p \) modes are stable.
- Threshold exists for strength of the wake.
- Everything is consistent with the single beam simulation.
- \( p \) modes are unstable in pp collision.

Wake strength scan, all modes are stable at \( W/10 \)

- \( \sigma \) modes are unstable at \( \nu = 0.5 + \nu_s \).
- All \( \pi \) modes are stable.
- Threshold exists for strength of the wake.
- Everything is consistent with the single beam simulation.
- \( \pi \) modes are unstable in pp collision.
Beam-beam simulations using the latest parameters

<table>
<thead>
<tr>
<th></th>
<th>Z</th>
<th>W</th>
<th>H</th>
<th>tt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference [km]</td>
<td>97.750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending radius [km]</td>
<td>10.747</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam energy [GeV]</td>
<td>45.6</td>
<td>80.0</td>
<td>120.0</td>
<td>175.0</td>
</tr>
<tr>
<td>Beam current [mA]</td>
<td>1399.0</td>
<td>147.0</td>
<td>29.0</td>
<td>6.4.0</td>
</tr>
<tr>
<td>Bunches / beam</td>
<td>71200.0</td>
<td>7500.0</td>
<td>740.0</td>
<td>62.0</td>
</tr>
<tr>
<td>Bunch spacing [ns]</td>
<td>2.5 and 5.0</td>
<td>40.0</td>
<td>400.0</td>
<td>5000.0</td>
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<tr>
<td>Bunch population [10^{11}]</td>
<td>0.4</td>
<td>0.4</td>
<td>0.8</td>
<td>2.11</td>
</tr>
<tr>
<td>Horizontal emittance (\varepsilon) [nm]</td>
<td>0.267</td>
<td>0.26</td>
<td>0.61</td>
<td>1.33</td>
</tr>
<tr>
<td>Vertical emittance (\varepsilon) [pm]</td>
<td>1.0</td>
<td>1.0</td>
<td>1.2</td>
<td>2.66</td>
</tr>
<tr>
<td>Momentum comp. [10^{-6}]</td>
<td>14.79</td>
<td>7.31</td>
<td>7.31</td>
<td>7.31</td>
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<tr>
<td>Arc sextupole families</td>
<td>208</td>
<td>292</td>
<td>292</td>
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<tr>
<td>Betatron function at IP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Horizontal (\beta^*) [m]</td>
<td>0.15</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>- Vertical (\beta^*) [mm]</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Energy spread [%]</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>- Synchrotron radiation</td>
<td>0.038</td>
<td>0.066</td>
<td>0.10</td>
<td>0.145</td>
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<tr>
<td>- Total (including BS)</td>
<td>0.064</td>
<td>0.074</td>
<td>0.11</td>
<td>0.169</td>
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<tr>
<td>Bunch length [mm]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Synchrotron radiation</td>
<td>2.1</td>
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<tr>
<td>- Total</td>
<td>3.6</td>
<td>2.3</td>
<td>2.3</td>
<td>2.77</td>
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<tr>
<td>Energy loss / turn [GeV]</td>
<td>0.0356</td>
<td>0.34</td>
<td>1.71</td>
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<td>Luminosity/IP for 2IPs [10^{34} cm^{-2}s^{-1}]</td>
<td>158</td>
<td>16.4</td>
<td>5.0</td>
<td>1.46</td>
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<tr>
<td>Beam-beam parameter</td>
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<tr>
<td>- Horizontal</td>
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<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
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<td>- Vertical</td>
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<td>0.13</td>
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<td>Design</td>
<td>2017</td>
<td></td>
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</tr>
<tr>
<td>--------------------------------------------</td>
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<tr>
<td>Circumference</td>
<td>97.750</td>
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<td></td>
<td></td>
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<tr>
<td>Arc quadrupole scheme</td>
<td>twin aperture</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bend. radius of arc dipole [km]</td>
<td>10.747</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of IPs / ring [mrad]</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crossing angle at IP</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solenoid field at IP [T]</td>
<td>±2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f^*$ [m]</td>
<td>2.2</td>
<td></td>
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<td></td>
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<tr>
<td>Local chrom. correction y-plane</td>
<td>with crab-sext. effect</td>
<td></td>
<td></td>
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<tr>
<td>RF frequency [MHz]</td>
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<tr>
<td>Total SR. power [MW]</td>
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<td>Beam energy [GeV]</td>
<td>45.6</td>
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<tr>
<td>SR energy loss/turn [GeV]</td>
<td>0.036</td>
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<td>Long. damping time [ms]</td>
<td>414</td>
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<td>Current/beam [mA]</td>
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<td>Bunches/ring</td>
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<tr>
<td>Particles/bunch [10^10]</td>
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<td>Arc cell 60°/60°</td>
<td>90°/90°</td>
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<tr>
<td>Moment. compaction $\alpha_p$ [10^{-6}]</td>
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<tr>
<td>Horizontal tune $\nu_x$</td>
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<tr>
<td>Vertical tune $\nu_y$</td>
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<td>Arc sext. families</td>
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<tr>
<td>Horizontal emittance $\varepsilon_x$ [nm]</td>
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<tr>
<td>$\varepsilon_y/\varepsilon_x$ at collision [%]</td>
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<tr>
<td>$\beta_x^*$ [m]</td>
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<td>$\beta_y^*$ [mm]</td>
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<tr>
<td>Energy spread by SR [%]</td>
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<tr>
<td>Energy spread SR+BS [%]</td>
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<td>RF Voltage [MV]</td>
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<td>Bunch length by SR [mm]</td>
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<tr>
<td>Bunch length SR+BS [mm]</td>
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<td>Synchrotron tune $\nu_z$</td>
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<td>RF bucket height [%]</td>
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<tr>
<td>Luminosity/IP $[10^{34}/cm^2s]$</td>
<td>121</td>
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</table>
Strong-strong simulation for FCCee-Z & W

\[ \text{\( L_{\text{design}} = 1.2 \times 10^{36} \text{ cm}^{-2}\text{s}^{-1} \)} \]

\[ \text{\( L_{\text{design}} = 1.6 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1} \)} \]

- ½ model, “turn” is ½ of the actual number
Strong-strong simulation for FCCee-H & t

$L_{\text{design}} = 5 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$

$L_{\text{design}} = 1.46 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
Summary

- Strong-strong beam-beam simulations showed a coherent beam-beam instability in head-tail mode.
- The instability was serious in collision with a large crossing (Piwinski) angle.
- FCC parameters were revised to suppress the instability. Now the parameters for Z-t work well.
- The instability is explained by a wake force for correlation between two beams.
- It is important that the wake is localized.
- Theory with mode analysis was completed to explain this instability.
Thank you for your attention
Tune shift

cZ/σz

ωσz/c

\[
g_{kl}(\omega) = \frac{1}{\sqrt{2\pi k!(|l| + k)!}} \left( \frac{\omega \sigma}{\sqrt{2c}} \right)^{|l|+2k} e^{-\omega^2 \sigma^2 / 2c^2}
\]

<table>
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<tr>
<th>l</th>
<th>Δν/νs</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>-0.15</td>
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<tr>
<td>3</td>
<td>-0.077</td>
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<tr>
<td>4</td>
<td>-0.039</td>
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<tr>
<td>5</td>
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<td>6</td>
<td>-0.0098</td>
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<tr>
<td>7</td>
<td>-0.0049</td>
</tr>
<tr>
<td>8</td>
<td>-0.0024</td>
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</table>

l=0

ωσz/c

l=10

ωσz/c
Fourier expansion of the dipole moments

\[ x(J, \phi) = \sum_{l=-\infty}^{\infty} x_l(J)e^{il\phi} \quad p(J, \phi) = \sum_{l=-\infty}^{\infty} p_l(J)e^{il\phi} \]

- **Revolution**
  \[ \begin{pmatrix} x_l \\ p_l \end{pmatrix} = e^{2\pi il\nu_s} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x_l \\ p_l \end{pmatrix} \]

- **Wake force**
  \[ \Delta p_l(J) = -\frac{1}{2\pi} \sum_\nu \int dJ' W_{lw}(J, J') x_\nu(J') \psi(J') \]
  \[ W_{lw}(J, J') = \int \int d\phi d\phi' e^{-il\phi + il'\phi'} W(z - z') \]
  \[ = 2\pi i^{l'} - l - 1 \omega_0 \sum_{p=-\infty}^{\infty} Z(\omega) J_{l} \left( \frac{\omega' r}{c} \right) J_{l'} \left( \frac{\omega' r'}{c} \right) \]

- **Eigenvalue problem**
  \[ M_W = e^{2\pi il\nu_s} \begin{pmatrix} 1 & 0 \\ -\beta_x W_{lw}(J_i, J_{i'}) \psi_{i'} \Delta J_{i'}/2\pi & 1 \end{pmatrix} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \]
Laguerre expansion for the radial modes

\[ x_l(J) = \sum_k x_{kl} \sqrt{\frac{k!}{(|l| + k)!}} \hat{J}^{|l|/2} L_k^{(|l|)}(\hat{J}) \]

- **Wake force**

\[ \Delta p_{kl} = - \sum_{k' l'} x_{k' l'} j^{l-l'-1} \omega_0 \sum_{\nu=-\infty}^{\infty} Z_1(\omega') g_{kl}(\omega') g_{k' l'}(\omega') \]

\[ g_{kl}(\omega') = \sqrt{\frac{1}{2\pi k!(|l| + k)!}} \left( \frac{\omega' \sigma}{\sqrt{2}c} \right)^{2k+|l|} \exp \left( -\frac{\omega'^2 \sigma^2}{2c^2} \right) \]

- **Eigen value problem**

\[ M_W = e^{2\pi iv_\nu s} \begin{pmatrix} 1 & 0 \\ -2M_{klk'l'} & 1 \end{pmatrix} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \]

\[ M_{kl,k'l'} = \frac{1}{2} \beta_x j^{l-l'-1} \omega_0 \sum_{\nu=-\infty}^{\infty} Z(\omega') g_{kl}(\omega') g_{k' l'}(\omega') \]

- \( \Delta v = K/4\pi \) gives usual dispersion relation, \( K = M_{21} \).
- Laguerre expansion is not goof for high frequency wake/impedance, \( \omega \sigma_z/c >> 1 \).
• Strong-strong beam-beam simulation
• Single beam simulation using multi-turn wake
• Two beam simulation using two beam wake
• Single beam simulation using two beam wake, \( \sigma \) or \( \pi \) modes.
• They gave similar results.
\( \sigma_z \) for beamstrahlung
Luminosity for 60 degree lattice of FCC-ee-Z

K. Ohmi, May. 25, 2017

Parameters given by K. Oide (Feb. 17)

Design momentum $P_0 = 45.600000$ GeV
Revolution freq. $f_0 = 6133.6491$ Hz
Energy loss per turn $U_0 = 17.203330$ MV
Effective voltage $V_c = 44.392690$ MV
Equilibrium position $d_z = -0.0014254$ mm
Momentum compact. alpha = 1.4654E-5
Bucket height $dV/P_0 = 0.0159296$

Emittance X = 2.5520E-10 m
Emittance Y = 0.00000000 m
Emittance Z = 1.32712E-6 m
Energy spread = 3.68724E-4
Bunch Length = 3.59923315 mm
Beam tilt = 0.00000000 rad
Beam size xi = 0.22479962 mm
Beam size eta = 0.00000000 mm

Real tune: -0.4250112 -0.3900373 -0.0116833
betax* = 15 cm, betay* = 1 mm.

Ne=4e10, Nbunch=91500