FCC-ee Machine Layout and Beam Optics + Matching with synchrotron motion

International FCC collaboration (CERN as host lab) to study:

- **$pp$-collider ($FCC-hh$)**
  - main emphasis, defining infrastructure requirements
  - $\sim 16 \, T \Rightarrow 100 \, \text{TeV} \, pp$ in 100 km

- **80-100 km tunnel infrastructure in Geneva area**

- **$e^+e^-$ collider ($FCC-ee$)** as potential first step

- **$p$-$e$ ($FCC-he$) option**

- **HE-LHC** with $FCC-hh$ technology
Future Circular Collider Study
Michael Benedikt
2nd FCC Week, Rome, April 2016

CERN Circular Colliders & FCC


Constr.  Physics  LEP
Design  Proto  Construction  Physics  LHC
Design  Construction  Physics  HL-LHC

20 years
Now is the time to plan for the period 2035 – 2040
<table>
<thead>
<tr>
<th>Year</th>
<th>Quarter</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
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<td>2018</td>
<td>Q4</td>
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</table>

**Study plan, scope definition**
- **Explore options**
- **FCC Week 2015:** work towards baseline

**Conceptual study of baseline develop baseline <|> detailed studies**
- **FCC Week 2016**
  - Progress review

**FCC Week 17 & Review**
- Cost model, LHC results
- study re-scoping?

**Elaboration, consolidation**
- **FCC Week 2018**
  - contents of CDR

**Report**
- **CDR ready**
Future Circular Collider Study
Michael Benedikt
2nd FCC Week, Rome, April 2016

CDR Study Time Line

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<tr>
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<tr>
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# lepton collider parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>FCC-ee (400 MHz)</th>
<th>CEPC</th>
<th>LEP2</th>
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<tbody>
<tr>
<td>Physics working point</td>
<td>Z</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>energy/beam [GeV]</td>
<td>45.6</td>
<td>120</td>
<td>105</td>
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<tr>
<td>bunches/beam</td>
<td>30180</td>
<td>50</td>
<td>4</td>
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<tr>
<td>bunch spacing [ns]</td>
<td>7.5</td>
<td>3600</td>
<td>22000</td>
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<tr>
<td>bunch population [10^{11}]</td>
<td>1.0</td>
<td>3.8</td>
<td>4.2</td>
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<tr>
<td>beam current [mA]</td>
<td>1450</td>
<td>16.6</td>
<td>3</td>
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<tr>
<td>luminosity/IP x 10^{34}cm^{-2}s^{-1}</td>
<td>210</td>
<td>2.0</td>
<td>0.0012</td>
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<tr>
<td>energy loss/turn [GeV]</td>
<td>0.03</td>
<td>3.1</td>
<td>3.34</td>
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<tr>
<td>synchrotron power [MW]</td>
<td>100</td>
<td>103</td>
<td>22</td>
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<tr>
<td>RF voltage [GV]</td>
<td>0.4</td>
<td>6.9</td>
<td>3.5</td>
</tr>
</tbody>
</table>

identical FCC-ee baseline optics for all energies

FCC-ee: 2 separate rings  CEPC, LEP: single beam pipe
Future increase with squeeze to
\( \beta_y^* = 1 \text{ mm}, \beta_x^* = 0.5 \text{ m} \)

new baseline
crab waist with 2 IPs
\( \beta_y^* = 2 \text{ mm}, \beta_x^* = 1 \text{ m} \)
Design constraints & assumptions

- $C = 100$ km, fits to the FCC-hh tunnel as much as possible.
- 2 IPs / ring.
- 30 mrad crossing angle at the IP with crab waist.
- Common lattice for all energies.
- $\varepsilon_x \leq 1.3$ nm @ 175 GeV.
- $\pm 2\%$ momentum acceptance at 175 GeV.
- Vertical emittance less than 1 pm at 175 GeV.
- $\beta_{x,y}^* = (1$ m, 2 mm) at 175 GeV, (0.5 m, 1 mm) at 45.6 GeV.
- Suppress the synchrotron radiation to the IP below 100 keV, up to 500 m upstream (as suggested by H. Burkhardt).
- “tapering” to cure the sawtooth at high energy.
## Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Circumference [km]</td>
<td>99983.76</td>
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<tr>
<td>Number of IPs / ring</td>
<td>2</td>
</tr>
<tr>
<td>Crossing angle at IP [mrad]</td>
<td>30</td>
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<tr>
<td>Solenoid with compensation at IP</td>
<td>$\pm 2 , \text{T} \times 1 , \text{m}$</td>
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<tr>
<td>$l^*$ [m] (asymmetric version)</td>
<td>2.2 / 2.9</td>
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<tr>
<td>Critical energy of photons to IP</td>
<td>$&lt; 100 , \text{keV} @ 175 , \text{GeV}$, up to 510 m upstream</td>
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<tr>
<td>IR Optics</td>
<td>asymmetric</td>
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<tr>
<td>Local chromaticity correction</td>
<td>Y</td>
</tr>
<tr>
<td>Crab sexts</td>
<td>integrated with LCCS</td>
</tr>
<tr>
<td>Arc cell</td>
<td>FODO, $90^\circ/90^\circ$</td>
</tr>
<tr>
<td>Arc sextuple families</td>
<td>292 (paired)</td>
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<tr>
<td>mom. comp. $[10^{-5}]$</td>
<td>0.70</td>
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<tr>
<td>Tunes $(x/y)$</td>
<td>387.08 / 387.14</td>
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<table>
<thead>
<tr>
<th>Ebeam [GeV]</th>
<th></th>
<th>175</th>
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<tbody>
<tr>
<td>SR energy loss per turn [GeV]</td>
<td>0.0346</td>
<td>7.47</td>
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<td>Current / beam [mA]</td>
<td>1450</td>
<td>6.6</td>
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<tr>
<td>$P_{SR,\text{tot}}$ [MW]</td>
<td>100.3</td>
<td>98.6</td>
</tr>
<tr>
<td>$\varepsilon_x$ [nm]</td>
<td>0.86</td>
<td>1.26</td>
</tr>
<tr>
<td>$\beta^*_x$ [m]</td>
<td>0.5 (1)</td>
<td>1 (0.5)</td>
</tr>
<tr>
<td>$\beta^*_y$ [mm]</td>
<td>1 (2)</td>
<td>2 (1)</td>
</tr>
<tr>
<td>RF frequency [MHz]</td>
<td></td>
<td>400</td>
</tr>
<tr>
<td>$\sigma_{6,SR}$ [%]</td>
<td>0.038</td>
<td>0.141</td>
</tr>
<tr>
<td>$\sigma_{Z,SR}$ [mm]</td>
<td>$2.8 @ V_c = 78 , \text{MV}$</td>
<td>$2.4 @ V_c = 9.04 , \text{GV}$</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>$-0.0158 @ V_c = 78 , \text{MV}$</td>
<td>$-0.0657 @ V_c = 9.04 , \text{GV}$</td>
</tr>
</tbody>
</table>
The separation of 3(4) rings is about 12 m: wide tunnel and two tunnels are necessary around the IR, for ±1.2 km.

A more compact layout/optics around the IP is also possible (A. Bogomyagkov).

Beams must cross over through the common RF (@ tt) to enter the IP from inside. Only a half of each ring is filled with bunches.
Ring Optics

- Above are the optics for tt, $\beta^{*}_{x/y} = 1$ m / 2 mm.
- 2 IP/ring.
- The optics for straight sections except for the IR are tentative, customizable for infection/extraction/collimation, etc.
• The optics in the interaction region are asymmetric.
• The synchrotron radiation from the upstream dipoles are suppressed below 100 keV up to 450 m from the IP.
• The crab sextuples are integrated in the local chromaticity correction in the vertical plane.
Synchrotron radiation toward the IP @ 175 GeV

$\nu_c$ (keV) $\begin{bmatrix} 1062 & 930 \\ 16.6 & 15.3 \end{bmatrix}$

$P_{SR}$ (kW) $\begin{bmatrix} 204 & 449 & 292 & 9.1 \\ 1.7 & 5.4 & 2.3 & 0.003 \end{bmatrix}$

$\nu_c < 100$ keV up to 510 m from the IP.
A more compact layout / optics (AB Lattice) has been developed by A, Bogomyagkov.

The deviation from FCC-hh is reduced to 5 m (9.5 m), the maximum excursion 7.8 m (11.9 m), the wide tunnel region ±730 m (1,200 m).

Local chromaticity correction for both X and Y can be installed.

A stronger dipoles are necessary for upstream of the IP (100 keV up to ~200 m, 200 keV up to ~300 m).
Solenoid compensation / shielding at the IR

- Favoured design at the moment. (it is not clear that a luminometer can fit inside the compensating solenoid)

- The effect of the solenoids are locally compensated within ±2 m around the IP.
- The final quads are shielded.

M. Koratzinos
The effect of the solenoids are locally compensated within ±2 m around the IP.
- The final quads are shielded.
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M. Koratzinos
Main contributors are Ivan Okunev and Pavel Vobly

Two versions of the FF twin-aperture iron yoke quad prototype with 2 cm aperture and 100 T/m gradient are in production.

Saddle-shaped coils, complicated in production, the first coil failed. New winding device is in development.

Straight coil, successfully wound and tested (650 A instead of the nominal 400 A)
Main contributors are Ivan Okunev and Pavel Vobly

Two versions of the FF twin-aperture iron yoke quad prototype with 2 cm aperture and 100 T/m gradient are in production.

Saddle-shaped coils, complicated in production, the first coil failed. New winding device is in development.

Also prototyping of CCT quadrupoles has started at CERN (M. Koratzinos, G. Kirby).

Straight coil, successfully wound and tested (650 A instead of the nominal 400 A).
Another QD0 prototype

A new version of QD0 was developed at BINP recently and a single-aperture prototype was manufactured.

Main parameters:
Max. gradient 100 T/m
Max. current 1100 A
Length 40 cm
Aperture 2 cm
NbTi 1.8 x 1.4 mm²
Saddle-type coils

During the first cryo-test (01.02.16) the current of 1060 A was achieved after 3 quenches.

A. Bogomyagkov, E. Levechev
HOM trapping by the cavity structure at IP

- HOM is trapped in the IP beam pipe, if all beam pipes are narrower than the IP, which needs to be larger than 40 mm (M. Sullivan).
- Heating, esp. at Z.
- Leak of HOM to the detector, through the thin Be beam pipe at the IP.
Asymmetric $L^*$: larger outgoing beam pipe & thinner final quads

- The HOM can escape to the outside through the outgoing beam pipe, which has a diameter not smaller than IP.
- The outgoing final quad becomes thinner and stronger (E. Levichev, S. Sinyatkin).

![Diagram with dimensions]

- $L_{\text{out}}^* = 2.9 \text{ m}$
- $L_{\text{in}}^* = 2.2 \text{ m}$
- $177.2 \text{ T/m}$
- $98.2 \text{ T/m}$
- $40 \text{ mm}$
- $26 \text{ mm}$
- $1.6 \text{ m}$
- $3.2 \text{ m}$

[Diagram showing dimensions and magnitudes]
Even with the asymmetric L*, the optics, so as the chromaticity, look similar.

The solenoid compensation is unchanged: locally compensated up to 2.2 m from the IP.

Longer L* downstream may give a space for a luminometer.
Basically a 90/90 degree FODO cell.

The quadrupoles QF/QD are 3.5 m/1.8 m long, respectively, to reduce the synchrotron radiation. They also depend on the design of quads and the beam pipe (A. Milanese, F. Zimmermann).

All sextupoles are paired with \(-I\) transformation.

292 sextupole pairs per half ring.
The RF section (175 GeV)

- The usage of the straights on the both sides of the RF is to be determined.
- If the nominal strengths of quads are symmetrical in the common section, it matches to the optics of both beam.
- This section is compatible with the RF staging scenario. For lower energy, the common RF and cross over will not be necessary.

An electrostatic separator, combined with a dipole magnet

Beams cross over through the RF section.
The change of the orbit due to energy loss along the arc causes serious deformation on the optics, causing the loss of the dynamic aperture.

Everything can be cured almost completely by “tapering”, i.e. scaling the strengths of all magnets along the local energy of the beam: this is one of the best merits of a double-ring collider (F. Zimmermann).
Dynamic Aperture satisfies the requirements.

175 GeV, $\beta^*_{x,y} = (1 \text{ m}, 2 \text{ mm})$

45.6 GeV, $\beta^*_{x,y} = (0.5 \text{ m}, 1 \text{ mm})$

Requirements assuming the same horizontal emittance as the collider and 1% coupling from the booster:

$\Delta p/p > \pm2\%$, $\Delta x > 15\sigma_x$, $\Delta y > 15\sigma_y$ @ 175 GeV,
$\Delta p/p > \pm2\%$, $\Delta x > 15\sigma_x$, $\Delta y > 18\sigma_y$ @ 45.6 GeV (See M. Aiba’s talk).
## Effects included in the dynamic aperture survey

<table>
<thead>
<tr>
<th>Effects</th>
<th>included?</th>
<th>significance for DA in FCC-ee @ 175 GeV</th>
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<tbody>
<tr>
<td>synchrotron motion</td>
<td>yes</td>
<td>essential</td>
</tr>
<tr>
<td>radiation damping (turn by turn)</td>
<td>yes</td>
<td>essential</td>
</tr>
<tr>
<td>radiation damping (each element, esp. quads)</td>
<td>yes (no fluctuation yet)</td>
<td>essential (aperture↑)</td>
</tr>
<tr>
<td>“tapering”</td>
<td>yes</td>
<td>essential</td>
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<tr>
<td>crab waist</td>
<td>yes</td>
<td>yes, aperture↓</td>
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<tr>
<td>solenoids</td>
<td>yes</td>
<td>minimal, if locally compensated</td>
</tr>
<tr>
<td>Maxwellian fringe field</td>
<td>yes</td>
<td>small</td>
</tr>
<tr>
<td>kinematical terms</td>
<td>yes</td>
<td>small</td>
</tr>
<tr>
<td>beam-beam</td>
<td>yes (weak-strong)</td>
<td>yes, esp. on lifetime (D. Zhou)</td>
</tr>
<tr>
<td>errors/misalignments</td>
<td>not yet</td>
<td>essential, correction schemes must be developed</td>
</tr>
</tbody>
</table>
The dynamic aperture for the AB lattice is under optimization, and looks promising so far.

P. Piminov, A. Bogomyagkov
A negative field gradient in the main dipole of the unit cell provides:

- longer cell length for a given emittance / better packing factor
- larger momentum compaction (longer bunch length for a same RF voltage)
- larger energy spread
- larger dispersion
- weaker sextupoles

Suggested by E. Levechev
An example of combined function: $J_z = 0.6 \ @ 175 \ GeV$

<table>
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<th>$J_z$</th>
<th>0.6</th>
<th>2</th>
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<tbody>
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<td># of FODO cells</td>
<td>1062</td>
<td>1442</td>
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<tr>
<td>Length of dipole (m)</td>
<td>33.9</td>
<td>23.1</td>
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<tr>
<td>H dispersion at SF (cm)</td>
<td>29.6</td>
<td>16.3</td>
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<tr>
<td>1 turn energy loss (GV)</td>
<td>7.09</td>
<td>7.74</td>
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<tr>
<td>momentum spread (%)</td>
<td>0.24</td>
<td>0.14</td>
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<tr>
<td>momentum compaction ($10^{-6}$)</td>
<td>12.8</td>
<td>7.2</td>
</tr>
<tr>
<td>bunch length (mm)</td>
<td>5.0</td>
<td>2.4</td>
</tr>
<tr>
<td>RF voltage (GV)</td>
<td>9.6</td>
<td>9.4</td>
</tr>
<tr>
<td>synchrotron tune</td>
<td>-0.10</td>
<td>-0.068</td>
</tr>
</tbody>
</table>
Dynamic aperture of combined function lattice.

175 GeV, $\beta^{*}_{x,y} = (0.5 \, \text{m}, 1 \, \text{mm})$

- The dynamic aperture is comparable to the flat-dipole lattice.
- Looking for beam-beam simulation and hardware solution of the dipole.
Several effects on the dynamic aperture

- **No RF, No radiation**: ±2%
- **RF, No radiation**: ±2%
- **RF, radiation damping each turn**:
- **RF, radiation in each element**:
Several effects on the dynamic aperture (2)

$\beta^*_x, y = (0.5 \, \text{m}, \, 1 \, \text{mm})$

$\beta^*_x, y = (1 \, \text{m}, \, 2 \, \text{mm})$

The reduction of the vertical aperture for $\beta^*_y = 1 \, \text{mm}$ is due to the synchrotron radiation in the final quads.

K. Oide
Less chromaticity $\neq$ better dynamic aperture

$\beta^*_{x,y} = (0.5 \text{ m}, 1 \text{ mm})$, no radiation damping
Consider a transverse transfer matrix along a synchrotron motion:

\[ M(\Delta_s) = M(\delta_n) \cdot M(\delta_{n-1}) \cdots M(\delta_0) , \]  

(1)

where \( \delta_k \) is the momentum deviation at the \( k \)-th turn of the synchrotron motion with an amplitude \( \Delta_s \):

\[ \delta_k = \Delta_s \sin(k \mu_s) , \]  

(2)

assuming a simple sinusoidal synchrotron motion. If we can approximate the synchrotron tune \( \nu_s = \mu_s/2\pi \) by a rational number \( m/n \), the transfer matrix \( M(\delta_n) \) becomes periodic over \( m \) times the synchrotron period, then we can calculate Twiss parameters in a similar way to a usual periodic optic. Such an approximation can be done with a continued fraction of the synchrotron tune:

\[ \nu_s = n_0 + \frac{1}{n_1 + \frac{1}{n_2 + \cdots}} . \]  

(3)

In these example optics we use,

\[ \nu_s = 0.0328756 \approx \frac{1}{30 + \frac{1}{\frac{1}{2} + \frac{1}{\frac{1}{2}}} } = \frac{5}{152} = 0.0328947 . \]  

(4)
The synchrotron optics becomes unstable at $\Delta p/p = 1.7 - 1.8\%$ in this chromaticity-optimized solution.
Optimized synchrotron-optics

\[ \beta^*_{x,y} = (0.5 \, \text{m}, 1 \, \text{mm}) \], no radiation damping

- A synchrotron-optimized solution can be obtained like above.
- Strong 3rd+ orders appear in chromaticity.
- The rf frequency is reduced to 100 MHz (actual: 400 MHz) for a more constant synch. freq.
DA-optimized synchrotron-optics

\[ \beta_{x,y}^* = (0.5 \text{ m}, 1 \text{ mm}), \text{no radiation damping} \]

The stability of synchrotron-optics becomes worse for DA-optimized optics.
Any correlation?

<table>
<thead>
<tr>
<th>Optimization</th>
<th>&gt;3rd order chromaticity</th>
<th>Synchrotron optics</th>
<th>Dynamic aperture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromaticity</td>
<td>small</td>
<td>unstable @ 1.7-1.8%</td>
<td>thin</td>
</tr>
<tr>
<td>Synchrotron optics</td>
<td>large</td>
<td>stable up to 1.9%</td>
<td>thin+</td>
</tr>
<tr>
<td>Dynamic aperture</td>
<td>large</td>
<td>largely unstable</td>
<td>thick</td>
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</tbody>
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Summary

- **Optics for FCC-ee are presented, considering:**
  - 2 IPs/ring, with 30 mrad crossing angle.
  - Local chromaticity correction with crab waist.
  - Suppression of synchrotron radiation in the IR below 100 keV up to 510 m from the IP.
  - Solenoid at IP & its compensation.
  - Possible asymmetric L* for wider outgoing beam pipe at the IP.
  - Element-by-element synchrotron radiation.
  - Tapering of all magnets according to the local beam energy to suppress sawtooth.
  - Common RF sections with cross-over of two beams (at least at tt).
  - Optimization of dynamic aperture with hundreds of sextuple families.
  - Geometrical fitting to the FCC-hh tunnel.
  - Combined function dipole in the arc will bring a number of merits, if realized.

- **Resulting dynamic aperture almost satisfies the requirements.**

- **Things need further investigation:**
  - Field quality, more realistic profile of magnetic field.
  - Tolerances / tuning scheme for machine errors, misalignments.
  - 4 IPs.
  - and more…
Optics calculation over synchrotron periods is tried

- A better chromaticity does not provide a better dynamic aperture.
- DA-optimized solutions show large 3rd+ orders chromaticity.
- Synchrotron-optimized solutions indeed increase such 3rd+ orders.
- The correlation between the synchrotron optics and dynamic aperture has not been clear.
Backups
Crab waist sextuple within CCS

The crab waist scheme shifts the vertical waist of a beam by

$$\Delta s = -\frac{x^*}{2\theta_x}.$$  \hspace{1cm} (1)

Thus the associated transformation is

$$y^* \rightarrow y^* - p_y^* \Delta s = y^* + \frac{p_y^* x^*}{2\theta_x},$$ \hspace{1cm} (2)

which is performed by a Hamiltonian at the IP:

$$H^* = \frac{x^* p_y^{*2}}{4\theta_x}.$$ \hspace{1cm} (3)

If there are the phase relations between the IP and the sextupoles:

$$\Delta \psi_x = 2\pi \quad \text{and} \quad \Delta \psi_y = 2.5\pi,$$ \hspace{1cm} (4)

then the variables at the IP $$(x^*, p_y^*)$$ are expressed in those at the sext $$(x, y)$$:

$$x^* = \sqrt{\frac{\beta_x^*}{\beta_x}} x, \quad p_y^* = \frac{y}{\sqrt{\beta_y^* \beta_y}}.$$ \hspace{1cm} (5)

Thus the Hamiltonian at the IP is equivalent to a Hamiltonian at the sext:

$$H = \frac{xy^2}{4\theta_x \beta_x^* \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}},$$ \hspace{1cm} (6)

which can be approximated by a Hamiltonian of a sextupole:

$$H_s = \frac{k_2}{6} (x^3 - 3xy^2),$$ \hspace{1cm} (7)

with

$$k_2 = -\frac{1}{2\theta_x \beta_y^* \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}}.$$ \hspace{1cm} (8)
Chromogeometric aberration of CCS

Consider the transfer matrix between sexts of YCCS with momentum dependence:

\[ m_y = \begin{pmatrix} -1 + 8\delta^2 & -2\delta\ell(-4 - 4\sqrt{2} + (6 + 5\sqrt{2})\delta) \\ \delta/\ell(-4 - 4\sqrt{2} + (14 + 9\sqrt{2})\delta) & -1 + 8\delta^2 \end{pmatrix} + O(\delta^3), \]

(1)

and similar for \( m_x \). Then the emittance increment due to the sextupole kick is calculated as:

\[ R\delta^2 \equiv \frac{\Delta\varepsilon_y}{\varepsilon_y} = \frac{256\delta^2}{\beta_x\beta_y^2\eta_x^2\ell^2} (2\sqrt{2}\beta_y + \ell\xi_y)^2 \]

\[ \times \left( (6 - 4\sqrt{2})\beta_y^2\varepsilon_x\ell^2 + ((9 - 6\sqrt{2})\beta_y^2 - 2\ell^2)\beta_x\beta_y \varepsilon_y \right) \]

\[ + ((2\sqrt{2} - 1)\beta_y^2 + (14 + 8\sqrt{2})\ell^2)\beta_x^2 \varepsilon_x \],

(2)

(3)

(4)

where \( \beta_x, \beta_y \), and \( \eta_x \) are the values at the sextupole, \( \ell \) the separation of quads, and we have assumed

\[ -k_2\beta_y\eta_x = \xi_y + 2\sqrt{2}\beta_y/\ell. \]

(5)

If we plug in the numbers:

\[ \beta_x = 15 \text{ m}, \varepsilon_x = 1.2 \text{ nm}, \varepsilon_y = 2.4 \text{ pm}, \eta_x = 0.16 \text{ m}, \ell = 58 \text{ m}, \xi_y = 3,500 \text{ }, \]

(6)

we get the following graph.

Although the optimum is at around \( \beta_y = 600 \text{ m} \), the increment is small up to \( \beta_y \leq 8,000 \text{ m} \).

- Above are just tentative optics.
- Usage of these sections is to be determined.
A rough estimation of radiation by arc quads

- The radiation power:
  \[ P \propto \gamma^2 B^2 \ell \]

- Ratio of powers by dipoles and quadrupoles per unit cell:
  - dipole: \[ P_d \propto \gamma^2 \left( \frac{B \ell_{\text{cell}}}{B \rho} \right)^2 \left( \frac{B \rho}{\ell_{\text{cell}}} \right)^2 \ell_{\text{cell}} \propto \gamma^4 \frac{\theta^2}{\ell_{\text{cell}}} \]
  - quadrupole: \[ P_q \propto \frac{\gamma^2}{2} \left( \frac{B' \Delta x \ell_q}{B \rho} \right)^2 \left( \frac{B \rho}{\ell_q} \right)^2 \ell_q \propto \frac{\gamma^4 k_1^2 \Delta x^2}{2 \ell_q} \]
  - ratio: \[ \frac{P_q}{P_d} = \frac{(k_1 \ell_{\text{cell}})^2 \beta_{xq} n^2 \varepsilon_x}{\ell_{\text{cell}} \theta^2 \ell_q}, \quad \Delta x^2 = n^2 \beta_{xq} \varepsilon_x \]

- In the case of a 90° cell, \( k_1 \ell_{\text{cell}} = 2\sqrt{2}, \beta_{xq}/\ell_{\text{cell}} = 1 + \frac{1}{\sqrt{2}}, \) then:
  \[ \frac{P_q}{P_d} = (4 + 2\sqrt{2}) \frac{n^2 \varepsilon_x}{\theta^2 \ell_q} \]

- or a particle with an amplitude of \( n \sigma_x \) will receive an energy loss per every turn:
  \[ \frac{\Delta p_1}{p_0} = \frac{P_q}{P_d} \times \frac{U_0}{E} = (4 + 2\sqrt{2}) \frac{n^2 \varepsilon_x}{\theta^2 \ell_q} \alpha_\varepsilon \quad (\alpha_\varepsilon: \text{long. damping per turn}) \]

- which causes a synchrotron motion with a momentum amplitude \( \pm \Delta p/p_0 \):
  \[ \frac{\Delta p}{p_0} = \frac{1}{2\pi \nu_s \sigma_x} \frac{\Delta p_1}{p_0} = \left( 2 + \sqrt{2} \right) \frac{n^2 \varepsilon_x}{\pi \theta^2 \ell_q} \frac{\alpha_\varepsilon}{\nu_s} \]
A rough estimation of radiation by arc quads (cont’d)

\[ \varepsilon_x = 2 \text{ nm}, \theta = \frac{2\pi}{1240}, \frac{\alpha_{\varepsilon}}{\nu_s} = 0.41 \text{ gives} \]

\[ \frac{\Delta p}{p_0} = 0.58\% \left( \frac{n}{10} \right)^2 \left( \frac{0.6 \text{ m}}{\ell_q} \right) \]

* If we plug-in the number for FCC-ee-tt:

\[
\begin{align*}
\Delta p_x/\sigma_{p_x} & \sim 8\sigma_\varepsilon \\
\Delta p_y/\sigma_{p_y} & = 1.1\% \\
\delta/\sigma_\varepsilon & = 1.1\% \\
\Delta x/\sigma_x & \sim 8\sigma_\varepsilon \\
\Delta y/\sigma_y & = 1.1\% \\
\Delta z/\sigma_z & \sim 8\sigma_\varepsilon
\end{align*}
\]

* only damping, no fluctuation, is taken into account in simulations in these slides.

The effect on the dynamic aperture

\[ \Delta x / \sigma_x \]

\[ \ell_q = 0.6 \text{ m} \]

\[ \ell_q = 3 \text{ m} \]