SAD:

Some details

1. Optimization of dynamic aperture with constraints.
2. Effect of F1 on emittance calculation.
3. Some recent changes
4. Discussion

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Optimization of dynamic aperture

- Constraints on $\partial \beta^*/\partial \delta$, $\partial \alpha^*/\partial \delta$, $\partial \mu/\partial \delta$ are necessary for operation of a collider maintaining the luminosity.

- $\partial R^*/\partial \delta$ are also important in the case of SuperKEKB.

This example causes a shift of vertical waist $\Delta s(\delta) \approx 1$ mm for $\delta = \pm 0.1\%$: Reduction of luminosity may appear even at zero intensity. It may have something to do with the luminosity observed in the simulation with lattice (D. Zhou).

Optimized DA without constraints at the IP
Constraints at the IP

The momentum dependence of the Twiss parameters at the IP:

\[
\frac{1}{\beta^*} \frac{\partial \beta^*}{\partial \delta} \propto \sum_i k_{2i} \beta_i \eta_{xi} \cos(2|\psi_i - \psi^*| - \mu)
\]

\[
\frac{\partial \alpha^*}{\partial \delta} \propto \sum_i k_{2i} \beta_i \eta_{xi} \sin(2|\psi_i - \psi^*| - \mu)
\]

Thus if we keep the condition

\[
\sum_i \Delta k_{2i} \beta_i \eta_{xi} \exp(2|\psi_i - \psi^*|) = 0
\]

together with tune chromaticity condition

\[
\sum_i \Delta k_{2i} \beta_i \eta_{xi} = 0
\]

during the optimization of \(k_{2i}\), then the chromatic dependences of \(\beta^*, \alpha^*\) do not change. In other words, the sextupoles must satisfy

\[
\sum_i C_{ki} k_{2i} = v_k, (k = 1, 6) .
\] (1)
fit IP; ax 0 24; ay 0 24; bx bx0 24; by by0 24; set chromatic alpha&beta at the IP to be zero
fit nx nx0 24; ny ny0 24; set ring tune chromaticity zero.

DP=0.0001; with a small momentum width
free s*; change all sextuples
go; do matching; results are satisfactory.

Residual = 2.7186E-12  DP = 0.00010  DP0 = 0.00000  ExponentOfResidual = 2.0  OffMomentumWeight = 1.000

IP.1       AX   0      24 -1E-7 -1E-7 -1E-7 -9E-8 -8E-8 -7E-8 -5E-8 -2E-8 -1E-8 5E-14 1.E-8 2.E-8 2.E-8 3.E-8 3.E-8 3.E-8 3.E-8
IP.1       BX   .5     24  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500  .500
IP.1       AY   0      24 -2E-7 4.E-7 -8E-8 -5E-7 -4E-8 5.E-7 4.E-7 5.E-7 2.E-7 -0.2.E-7 5.E-7 -4E-8 5.E-7 -4E-8 5.E-7 -4E-8 5.E-7
IP.1       BY   .001   24  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001  .001

ol@SetupChroma["S*",{0,0}]; call a library routine (next page) to remember the result.

Please see the library file Optimize.n uploaded to INDICO
Example (cont’d)

```mathematica
Module[
{pe = Element["POSITION", elm], p = LINE["POSITION", elm],
 \[iP] = LINE["POSITION", "IP"],
 bx, by, ex, bxex, byex, sxe, sye, ep, psix, psiy, mux, muy,
 cpsix, spsix, cpsiy, spsiy, scx, ssx, scy, ssy,
 sscx, sssx, sscy, sssy},

cchoelm = elm; which element do we care (usually S*)
sexts = pe;

{bx, by, ex, psix, psiy} = Plus@@[
  Twiss["BX", "BY", "EX", "NX", "NY"], {p, p + 1}, {1}]/2;
{psixip, psiyip} = Twiss["NX", "NY", "IP"];
{mux, muy} = Twiss["NX", "NY"] /@ "$$";
psix = psixip;
psiy = psiyip;
psix = 2*(If[# < 0, # + mux, #] & /@ psix);
psiy = 2*(If[# < 0, # + muy, #] & /@ psiy);
{cpsix, spsix} = {Cos[psix], Sin[psix]};
{cpsiy, spsiy} = {Cos[psiy], Sin[psiy]};
bxex = bx * ex;
byex = by * ex;
sxe[\_] = 0;
sye[\_] = 0;
scx[\_] = 0;
ssx[\_] = 0;
scy[\_] = 0;
ssy[\_] = 0;
ep = Element["POSITION", LINE["Element", p]];

MapThread[
(sxe[#] += #2;
sye[#] += #3;
scx[#] += #4;
ssx[#] += #5;]

Chroma = Join[ch, {0, 0, 0, 0}] * 4 * Pi
+ Plus@@(Element["K2", pe] * trsbe);

sbex = sxe[#]/@pe;
sbey = sye[#]/@pe;
sscx = scx[#]/@pe;
sscy = scy[#]/@pe;
sssy = ssy[#]/@pe;

sbe = {sbex, -sbey, sccx, sssx, -sscy, -sssy};
trsbe = Transpose[sbe]; The condition matrix Cki in Eq. (1).

The goal values, \(v_k\).
```
Example (cont’d)

If[chroma==Null,
   FFS["RESET "://chroelm//";EXPAND;"];
   MapThread[SetVar,{var,v}];
   AdjustChroma[],
   ___

AdjustChroma[]:=If[adjustch,
   Module[{k2=Element["K2",sexts],ch0,dk2},
      ch0=Plus@@(k2*trsbe);
      dk2=LinearSolve[sbe,chroma-ch0];
      Element["K2",sexts]=k2+dk2];
   do this when a new vertex is tried
   calculations of lhs Eq. (1).
   calculate the correction dk2
With the constraints, the momentum dependencies are kept zero after the optimization of the DA.
Treatment of F1 in the emittance calculation

- In the calculation of BEND and QUAD, the radiation has been always calculated assuming a linear field profile at the edge:

- Then it fails in the calculation of radiation when a BEND is sliced into thin pieces (A. Morita):

\[
\int B^2 ds / L = \frac{2}{3} B_0^2 \\
\int B^3 ds / L = \frac{1}{2} B_0^3
\]
A short fix:

- F1, FB1, FB2 are ignored in the emittance calculation, if the end of a BEND or QUAD touches to another BEND or QUAD, with zero-length separation.
- Focusing effect of F1 is calculated as before.
- MULT has been always ignoring F1, etc., in the emittance calculation.

- You can discuss about more consistency between elements...
Some recent changes

* X-y coupling parameters when $|R|$ is large.

The transformation matrix from the physical coordinate $(x, p_x, y, p_y)$ to the $x$-$y$ decoupled coordinate $(X, P_x, Y, P_y)$ is written as

$$R = \begin{pmatrix} \mu I & J^t r J \\ r & \mu I \end{pmatrix},$$

(1)

with the submatrix

$$r \equiv \begin{pmatrix} R_1 & R_2 \\ R_3 & R_4 \end{pmatrix},$$

(2)

where $\mu^2 + |r| = 1$ and

$$I \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad J \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$ 

(3)

The value of function DETR is equal to $|r|$ in this case.

Let $T$ stand for the physical transfer matrix from location 1 to location 2, then the transformation in the decoupled coordinate is diagonalized as

$$R_2 T R_1^{-1} = \begin{pmatrix} T_X & 0 \\ 0 & T_Y \end{pmatrix}.$$ 

(4)

The Twiss parameters are defined for the matrices $T_X$ and $T_Y$.

If $|r| \geq 1$, the above condition for $\mu$ is violated. In such a case, an alternative form of $R$ is used:

$$R = \begin{pmatrix} J^t r J & \mu I \\ \mu I & r \end{pmatrix},$$

(5)

where $\mu^2 + |r| = 1$. The function DETR shows a number $a - |r|$, where $a = 1.375$. Thus the alternative form is used when $|r| \geq 0.625$. 
Some recent changes (2)

- Transformation of a combined function dipole was changed:
  - It used to be a transformation alternating flat dipole and transverse kick, including the linear (quadrupole) part.
  - Now it has been changed to alternating linear combined function dipole and nonlinear correction. The nonlinear correction includes the rest of sort term and nonlinear horizontal kicks to satisfy the Maxwell equation in curved coordinate.
  - The new one is something similar to the transformation for quadrupoles.
  - The betatron tunes become exact with the new transformation for the design orbit.
Some recent changes (3)

- The calculation of microwave instability for a pure-resistive wake was recently re-confirmed by K. Bane (SLAC).
- Since the original code in 1995 does not run today, it was re-coded in SAD in around 2009.
- The SAD environment is easier for scripting, graphics, file I/O, etc. (at least for myself).
- The calculated growth rate reproduces the published results very well.
- Also agrees with K. Bane’s recent code, written independently.
- Some internal scalings are changed this time.

Figure 2: Growth rates of unstable modes with the pure-resistive wake $W(q) = R\delta(q)$ obtained from the matrix in Eq. 22. The parameter $m$ specifies the nearest integer of the frequency of each mode. It is seen that the growth rate is roughly proportional to $k^2$.

Discussions:
An example script has been shown for optimization of DA with some chromatic conditions at the IP.

It may solve the drop of luminosity simulated with the SuperKEKB lattice.

A short fix on F1 will be provided for the emittance calculation in sliced BENDs.

Changes are made for the representation of x-y coupling and transformation of a combined dipole.