General Introduction to Particle Accelerator Systems

2016/10/20

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What this Lecture is for

• Purpose of this particular lecture:
  – To give you a cursory overview of various accelerator systems, their subsystems and components via a short walk-through seminar in the class,
  – And a short tour in the SuperKEKB tunnel (6C → Fuji straight section)
  – So that you acquire some sense of how a real-life accelerator is like.
Tour: SuperKEKB

• We leave this room at 15:30 – Be ready
• Use decent shoes. No sandals.
• We walk over to the SuperKEKB control bldg first, and pick up hard hats and dosimeters there.
• Then we will walk over to Bldg 6C, and enter the SuperKEKB tunnel.
• We will be back by 16:30 (or 17:00).
Texbooks

• Introductory textbooks which might benefit you are as follows:


  – OHO Lecture Series (although they are mostly in Japanese).

Real Basics

- Energy / Momentum
  - Non-relativistic: \( E = \frac{1}{2}mv^2; \ p = mv \)
  - Relativistic: \( E = \sqrt{m^2c^4 + p^2c^2} = mc^2\gamma; \ p = mc\gamma\beta \)
    \[ \beta = v/c; \ \gamma = 1/\sqrt{1 - \beta^2} \]
  - Speed of light: \( c = 2.9979 \times 10^8 \text{ m/s} \)
  - Forces on charged particles: \( \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \)
    - And when the B is UP, the electrons go to the LEFT.
  - Unit of energy: eV (electron-volt)
    - keV (\( 10^3 \)), MeV (\( 10^6 \)), GeV (\( 10^9 \)), TeV (\( 10^{12} \))
  - Unit of momentum: eV/c (electron-volt/c)
### Things that Beams would Do

<table>
<thead>
<tr>
<th>Players</th>
<th>Interaction via</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material ↔ Beam</td>
<td>Q.E</td>
<td>Thermal electrons / Photo-emission / Ionization</td>
</tr>
<tr>
<td></td>
<td>EM / QED</td>
<td>Pair creation / Electro-magnetic shower</td>
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<tr>
<td></td>
<td></td>
<td>Absorption</td>
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<td></td>
<td></td>
<td>Elastic / Inelastic scattering</td>
</tr>
<tr>
<td>Environment → Beam</td>
<td>Static E-Field</td>
<td>Acceleration / Deceleration / Bending</td>
</tr>
<tr>
<td></td>
<td>Static B-Field</td>
<td>Bending / Spin rotation</td>
</tr>
<tr>
<td></td>
<td>RF Field</td>
<td>Acceleration / Deceleration / Bending</td>
</tr>
<tr>
<td>Beam → Environment</td>
<td>Transient Field</td>
<td>Fundamental mode RF / High-order mode RF</td>
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<tr>
<td></td>
<td></td>
<td>Heating</td>
</tr>
<tr>
<td>Beam ↔ Light</td>
<td></td>
<td>Synchrotron Light emission</td>
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<tr>
<td></td>
<td></td>
<td>Radiation Damping</td>
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<tr>
<td></td>
<td></td>
<td>Compton Scattering</td>
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<tr>
<td></td>
<td></td>
<td>QED Phenomena</td>
</tr>
<tr>
<td>Beam ↔ Beam</td>
<td>Space-charge forces</td>
<td>Within a single bunch</td>
</tr>
<tr>
<td></td>
<td>Beam-beam effects</td>
<td>When two bunches cross each other</td>
</tr>
<tr>
<td></td>
<td>Physics</td>
<td>Beam collision you look for</td>
</tr>
<tr>
<td>Beam → Env. → Beam</td>
<td>Longitudinal EM</td>
<td>Bunching / Debunching</td>
</tr>
<tr>
<td></td>
<td>Trasverse EM</td>
<td>Emittance growth / instabilities / beam break-up</td>
</tr>
</tbody>
</table>

**Accelerator is a device which overcomes, or takes advantages of these phenomena, to let the beam do what you like it to do.**
Accelerator System - SuperKEKB

Counter-travelling e- (8GeV) and e+ (3.5GeV) beams are made to collide repeatedly at a single collision point – Goal: Max the particle collision rate.

http://www.kek.jp/ja/activity/accl/KEKB.html
Accelerator System - KEKB
Accelerator System – Light Source Rings

http://pfwww.kek.jp/outline/pf/pf1.html

PF Ring (2.5 – 3GeV)
AR Ring (6 – 6.5 GeV)

Store only the electrons to produce SR light (X-rays) for experiments.

Goal: Max support user experiments with SR of desired characteristics.

Hard X-ray stations
Soft X-ray stations
Accelerator System – Light Source Rings
Accelerator System – J-Parc

http://www.j-parc.jp

3GeV Synchrotron
(0.4 → 3GeV; 333μA
1MW, 25Hz)

50GeV Synchrotron – 15μA

Hadron Exp

Mat / Life-science exp

Neutrino Exp Fac.

To Super-Kamiokande

High-power proton accelerator to concurrently support numerous user experiments spanning over material / life / nuclear and particle. Goal: Reliable delivery of high-current proton beams.

Accelerator-Driven Transmutation Fac.

Linac
(0.4-0.6GeV; 15μA, 500μs, 50Hz)
Accelerator System – J-Parc
Accelerate electron beam and positron beam with two linear accelerators, and have them collide head-on at the beam collision point at the center for experiments in HEP.

Goal: Max the energy reach, max the beam interaction rate.

Check the movie at http://bit.ly/dutCx0 for visual effects.
ERL is similar to light source rings in that it utilizes light emission off electron beam as it goes through undulator magnets.

Differences (and advantages), however, include:

+ one-time use of beam, i.e like linear accelerator
+ opportunities for very bright, very short light pulses
+ energy recovery

“CompactERL” prototype

E ~ 35-245 MeV; I ~ 10 mA; \( \gamma \varepsilon \sim 0.1 – 1 \text{ mm} \); \( \sigma_\tau \sim 100\text{fs rms} \)

Proposal for “Real” ERL

E ~ 5GeV; I ~ 10 - 100mA ; \( \gamma \varepsilon \sim 10 – 100 \text{ pm} \); \( \sigma_\tau < 100\text{fs rms} \)
Accelerator System – Compton-Based Light Source  

This is an exploration of another scheme for producing high-quality light beam.

The light beam here is produced via collision of high-quality electron beam, from a photo-cathode RF-gun, with laser photons, which are stored in an optical cavity.

“Quantum Beam” Project at KEK  
E ~ 25 MeV from a SC linac  
+ Laser light stored in a 4-mirror cavity  
→ X-rays at 0.2 – 70keV

http://kocbeam.kek.jp/
Accelerator System: ATF

ATF is an accelerator test facility at KEK for studies of production / control / beam focusing of ultra-low emittance beams. Applications include ILC and CLIC.
Accelerator System: STF

STF is a development area within KEK for superconducting linac technologies. Applications include future projects such as ILC and ERL.
# Accelerator Systems – Quick Survey

<table>
<thead>
<tr>
<th>Particles</th>
<th>Beam source</th>
<th>Ring</th>
<th>Bunch / energy compression</th>
<th>Linac</th>
<th>Wigglers / Undulators</th>
</tr>
</thead>
<tbody>
<tr>
<td>KEKB</td>
<td>e-, e+</td>
<td>Thermionic Gun</td>
<td>Storage rings.</td>
<td>Yes</td>
<td>Yes for controlling damping time</td>
</tr>
<tr>
<td>KEK Photon Factory Rings</td>
<td>e-</td>
<td>share with KEKB</td>
<td>Storage ring</td>
<td>share with KEKB</td>
<td>Yes for light source</td>
</tr>
<tr>
<td>J-PARC complex</td>
<td>Protons (also produce secondary particles)</td>
<td>Hydrogen ion source (H-) → proton on injection</td>
<td>Synchrotron</td>
<td>No</td>
<td>DTL + RF-Qs</td>
</tr>
<tr>
<td>Linear Collider</td>
<td>e-, e+</td>
<td>Polarized Photo-cathode Gun</td>
<td>Damping rings</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Energy Recovery Linac</td>
<td>e-</td>
<td>Hi-intensity low-emittance DC gun</td>
<td>Arc sections, but no prolonged storage</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Compton-based light source</td>
<td>e- / laser light</td>
<td>Low-emittance RF gun / laser</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>
DC Acceleration

Cockroft-Walton generator circuit.

Good for up to a few hundred KeV.

Cockroft-Walton accelerator.
KEK has one, too, for 12GeV proton synchrotron, completed in 1976.
KEK’s Cockcroft-Walton Generator
DC Acceleration

- **Van de Graaf circuit**
  - Apply electrostatic charge on a belt, at its bottom, via a comb of corona points.
  - Charge on the belt is delivered to its top, as the belt is driven on a pair of insulated pulleys.
  - Charge then, is removed from the belt at its top, via the collector comb.
  - Good up to a few MeV.

- **“Tandem” Van de Graaf**
  - Negatively charged particle (ions) is accelerated from the left to the central point.
  - If the particle charge can be flipped at the middle, by using a metallic foil etc, then the particle continues to be accelerated toward the right.
DC Acceleration

- Acceleration with a system with static electric field (from static potential - V) can be made once, but not be repeatedly made, because the integral of $E_s$ over a closed loop becomes

$$\oint_C E_s ds = \oint_C E \cdot ds = \int_S (\nabla \times E) \cdot n dxdy = \int_S (\nabla \times \nabla V) \cdot n dxdy = 0$$

With an AC field, because of the relation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

we can be freed from the problem above.

$$\oint_C E_s ds = \oint_C E \cdot ds = \int_S (\nabla \times E) \cdot n dxdy = -\frac{\partial}{\partial t} \int_S B \cdot n dxdy = -\frac{\partial}{\partial t} \Phi$$
Acceleration with AC Field -

Radio frequency alternating voltage

Hollow metal drift tubes

Figs from M.W. Poole

If non-relativistic,
\[ F = evB = \frac{mv^2}{r} \]
\[ eB = \frac{mv}{r} \]

and orbital period \( T \) stays constant
\[ T = \frac{2\pi r}{v} = \frac{2\pi m}{eB} \]

If \( T \) is set as the RF period \( \rightarrow \) Cyclotron!

Limitation of cyclotrons: (relatively) non-relativistic particles, large magnets.
Acceleration with AC Field -

Wideroe’s accelerator:

If we drive many drift tubes with a common AC source (microwave source), from the downstream part of this linac where the particles is low with $\beta = \frac{v}{c} \ll 1$, toward the upstream part where $\beta \sim 1$, the lengths of the drift tubes must be arranged to match the particles’ travel times.

Good for, for instance, protons, up to several hundred MeVs of kinetic energies.
Acceleration - Synchrotron

When acceleration is desired, sweep the field strength of magnets in sync with acceleration.

When acceleration is not needed, maintain the constant field strength

Note: $3.3356\rho \text{(GeV/c)} = B \text{(T)} \rho \text{(m)}$
# LHC Operation Log

- **Date & Time:** 06-Apr-2010 17:27:13  
  **Fill #:** 1023  
  **Energy:** 297.4 GeV  
  **I(B1):** 1.55e+08  
  **I(B2):** 7.01e+07

### Experiment Status

<table>
<thead>
<tr>
<th>Experiment</th>
<th>ATLAS</th>
<th>ALICE</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status</td>
<td>STANDBY</td>
<td>NOT READY</td>
<td>STANDBY</td>
<td>STANDBY</td>
</tr>
</tbody>
</table>

### Instantaneous Luminosity

<table>
<thead>
<tr>
<th>BRAN Count Rate</th>
<th>BKGD 1</th>
<th>BKGD 2</th>
<th>BKGD 3</th>
<th>BKGD 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.002</td>
<td>0.014</td>
<td>0.002</td>
<td>0.131</td>
</tr>
</tbody>
</table>

### Performance over the last 12 Hrs

**Graphs showing intensity and energy over time with background plots for ATLAS, ALICE, CMS, and LHCb.**
Acceleration - Synchrotron

Repetitive nature of beam passage leads to MANY interesting issues, generally called instabilities and resonances.

But before that, let us look at some individual components.

Note: $3.3356p \, (\text{GeV/c}) = B \, (\text{T}) \rho \, (\text{m})$
Example – Proton synchrotron (30GeV) at J-PARC
Acceleration with Resonant Cavities

- Single-cell “pillbox” cavity: by solving Maxwell’s equation one obtains –

\[
\begin{align*}
E_r &= 0 \\
E_\theta &= 0 \\
E_z &= E_0 J_0 (\chi_{01} r/b) \cos (\omega_{010} t) \\
H_r &= 0 \\
H_\theta &= -H_0 J_1 (\chi_{01} r/b) \sin (\omega_{010} t) \\
H_z &= 0
\end{align*}
\]

- Where,
  - \( \chi_{01} = 2.4048 \)
  - \( \omega_{010} = \chi_{01} \frac{c}{b} \)

- This is called TM01 mode (fundamental mode, TM010 to be more exact) of a pillbox cavity.
Q1: We need a single-cell cavity for $f = 714\text{MHz}$. What is the order of “b”? 

A1:

Remember: $\chi_{01} = 2.4048$; $\omega_{010} = \chi_{01} c / b$

$$b = \chi_{01} c / \omega_{010}$$
$$c = 2.9989\times10^8 \text{ m/s}$$

$$\omega_{010} = 2\pi f_{\text{RF}} = 2 \times 3.14 \times 714\times10^6 = 4.48\times10^9$$
$$b = \chi_{01} c / \omega_{010} = 2.405 \times 2.9989\times10^8 / 4.48\times10^9 = 0.16 \text{ m}$$

Check this out at the ATF damping ring.
RF Frequencies

- **L band** 1 - 2 GHz
- **S band** 2 - 4 GHz
- **C band** 4 - 8 GHz
- **X band** 8 - 12 GHz
- **K band** 12 - 40 GHz
- **Q band** 30 - 50 GHz
- **U band** 40 - 60 GHz
- **V band** 50 - 75 GHz
- **E band** 60 - 90 GHz
- **W band** 75 - 110 GHz
- **F band** 90 - 140 GHz
- **D band** 110 - 170 GHz

The naming of frequency ranges on the left came apparently from the convention which was developed in Rader-related research effort in Allied countries during WWII. Other than that, no one seems to know for sure why this pattern, except perhaps L stands for LONG and S stands for SHORT.
Resonant Cavities

- Quality factor (Q-value) of a cavity:
  - \[
  \frac{\text{stored energy in the cavity}}{\text{energy loss over 1 rad of RF oscillation}}
  \]

- In case of a TM01 mode, the stored energy in a pillbox cavity is
  \[
  U = \frac{1}{2} \varepsilon_0 \int E^2 dV = \frac{1}{2} \varepsilon_0 E_0^2 V J_1^2(\chi_{01}) \sim \frac{1}{2} \varepsilon_0 E_0^2 V \times 0.25
  \]

- Surface current in the cavity is
  \[
  J = \frac{1}{\mu_0} B_\theta = \frac{1}{\mu_0} B(r)e^{i\omega t} = \frac{1}{\mu_0} \frac{E_0}{c} J_1\left(\frac{\omega}{c} r\right)e^{i\omega t}
  \]

- Ohmic energy loss is surface integral of \(\rho_s J^2/2\) -
  \[
  \Delta P = \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c}\right)^2 \left[ 2 \cdot 2\pi \int_0^b J_1^2\left(\frac{\omega r}{c}\right) r dr + 2\pi bd J_1^2\left(\frac{\omega b}{d}\right) \right]
  \]
  \[
  = \frac{1}{2} \rho_s \frac{E_0^2}{Z_0^2} 2\pi bd \left[ 1 + \frac{b}{\chi_{01} d} \right] J_1^2(\chi_{01})
  \]
Resonant Cavities

• From the previous page, the Q value that we seek is given by

\[ Q = \frac{\omega}{\Delta P} U = \frac{\chi_{01} \mu_0 c}{2 \rho_s \left( 1 + \frac{b}{\chi_{01} d^2} \right)} \]

• In case of copper Q \sim 5000, in case of superconducting Nb Q \sim 10^{10}
Resonant Cavities

- **Shunt impedance** $R_s$ of an accelerator cavity is defined as

$$R_s = \frac{(\text{Energy gain } V_0 \text{ by a particle across the cavity})^2}{\Delta P}$$

- For a pillbox cavity the $R_s$ is given by

$$R_s = \frac{Z_0^2}{\pi \rho_s} \frac{d}{b} \frac{1}{\left(1 + \frac{b}{\lambda_{01}^2} \frac{1}{d}\right) J_1^2(\lambda_{01})} T^2$$

$$T = \frac{E_0 \int_0^{d/2} \cos \left( \frac{\omega z}{v} \right) dz}{E_0 \frac{d}{2}} = \sin \left( \frac{\omega d}{2v} \right) \left/ \left( \frac{\omega d}{2v} \right) \right.$$ 

- Where “$T$” is the transient factor.
- In case of copper: $R_s = 15$-$50\text{M}\Omega/m$ for $200\text{MHz}$, $100\text{M}\Omega/m$ for $3\text{GHz}$
Superconducting Cavity at superKEKB
Accelerator Resonant Cavity (Single-cell superconducting)

This is an example of a single-cell super-conducting cavity which was used at KEKB.
If the time that it takes the beam particle to travel across one cell coincides with one half of the RF period, the particle will be continually accelerated, in case of a so-called $\pi$-mode standing-wave structure.
Accelerator Structure
(Multi-cell Normal-conducting)

Copper-made accelerator structure.

The ones on left operate in the $2\pi/3$ travelling wave mode, meaning neighboring cells have their RF phases shifted by $2/3$ of the half period.
S-band accelerator structure for use at KEK injector linac

This is a cut-away model for display.
Accelerator Cavities / Structures

- **Q2**: If $f_{RF} = 1.3\, \text{GHz}$ for a $\pi$-mode cavity, what should be the cell length?
  
  - **A2**:  
    
    $f = 1.3\times10^9 \, \text{Hz}$; $c = 2.9979\times10^8 \, \text{m/s}$  
    
    $\Rightarrow \lambda = c/f = 0.23 \, \text{m}$;  
    
    $\pi$-mode cell length $= \lambda/2 = 0.12 \, \text{m}$

- **Q3**: If $f_{RF} = 2.856\, \text{GHz}$ for a $2\pi/3$-mode structure, what should be the cell length?
  
  - **A3**:  
    
    $f = 2.856\times10^9 \, \text{Hz}$; $c = 2.9979\times10^8 \, \text{m/s}$  
    
    $\Rightarrow \lambda = c/f = 0.105 \, \text{m}$;  
    
    $2\pi/3$-mode cell length $= (2/3)\lambda/2 = 0.035\, \text{m}$
Cavities have many resonant modes. TM01 is the lowest and is most often used for beam acceleration. But,

- Higher-order modes, such as TE11, also can be excited with an external RF source,
- Or by the beam that traverses through them off axis,
- And do some harms. This may not be big problems for many applications, but it is for some.
Accelerator Resonant Cavity (Single-cell normal conducting)

This is an example of a single-cell normal conducting cavity (which is used at superKEKB. It has “slots” for removing (damping) HOM, as excited by the beam, which are harmful for multi-bunch beam operation, and an energy storage cavity to reduce the effects of beam loading on the system.
Accelerator Cavity / Structure - Comments

Design, engineering and operation of accelerator cavities / structures is a huge area.

- Exact evaluation of resonant frequencies, shunt impedance and quality factor have to be made through numerical calculations (still, an intuitive understanding important).

- Size variation → variation of resonance frequencies → mechanical tolerance and tuning issues. Similarly, temperature stabilization issues. (Q: if ΔT = 5deg, how much energy shift for an X-band structure?)

- “Couplers” to let the external RF power into the cavities and vice versa.

- In pulsed mode operation, the accelerator cavities exhibit transient behaviors at the beginning and end of RF pulses.

- Even in continuous wave operation, the accelerator cavities see transient effects when the beams pass through them.
Microwave (RF) Source (Klystrons)

• For brevity, the solenoid beam focusing magnet is omitted from the illustration above.
• Klystron, in a way, is a reversed accelerator, which extracts RF power from the beam.
• Klystrons, depending on the type, operate in pulsed- or continual modes.
Microwave (RF) Source (Klystrons)
Microwave (RF) Source (Modulator + Klystrons)

85MW klystron modulator

Klystrons for pulsed operation

T. Okugi
A typical electron linac

DC PS  Modulator  Klystron  Control equip/

Surface level (Klystron Gallery)

Waveguides

Accelerator structures (2m x 4)

Beam focusing magnets
Microwave (RF) Source (Modulator + Klystrons)
Microwave RF Source (Klystrons) - Comments

• As said in the previous page, klystrons are like small-scale reversed accelerator, and are more difficult ones at that.
  
  – Relatively low beam energy (up to few hundred kV).
  – High current (tens of A)
  – So large beam power, and large space-charge effect.
  – Large beam size.
  – Opportunities for a good number of resonant behaviors, besides the RF power that you intend to produce.

• Design, engineering and operation of klystrons is a huge area, too, in a way similar to those for accelerator cavities / structures.
  
  – RF calculations and simulations, under prominent effect of space-charge force.
  – Tolerance and tuning issues.
  – RF windows to isolate the klystron vacuum and outside vacuum. Sometimes a cause of trouble → another large area of studies which I cannot detail.
  – Cathode lifetime. Klystrons are a consumable component. Significantly more so than accelerator cavities / structures.
Manipulation of RF Power

- RF pulse compression with SLED is often used for linac RF source.

- Power higher than what is available from a klystron, although the effective pulse length becomes shorter.

- Many NC electron linacs use SLED (but not all).

RF voltage at the hybrid output, ignoring the power coming out of SLED.

RF voltage of the power coming out of SLED.

RF voltage of the power, combined from the klystron and the one coming out of SLED.

RF voltage of the power, as it is induced within the accelerator structure.
SLED cavity at injector linac of KEK (Tsukuba)
Dipole Magnets

- Dipole magnets are the primary tools to control the orbit of beam particles.
  - Big dipole magnets to define the orbit are often called “bend magnets”.
  - Small dipole magnets for orbit correction are often called “correctors.”
- Lorentz force formula: \( F = evB \)
  - Remember: When the field is UP, an electron goes to the left, a positron (or proton) goes to the right.
- Q4: Prove \( 3.3356 \text{p [GeV]} = B \text{[T]} \rho \text{[m]} \)
Dipole Magnets

• Q5: Have a 0.3T magnet which is 5.804m long. What is the bending angle for a 8GeV beam?

• A5:

  Use $3.3356\rho = B\rho$;
  $\rho = \frac{3.3356\rho}{B}$
  Note: B in T, p in GeV/c

  Bend angle
  $\theta = \frac{L}{\rho} = BL / 3.3356p$

  Use, for instance,
  $B = 0.3T$
  $p = 8GeV/c$
  $L = 5.804m$
  $\theta = \frac{0.3 \times 5.804}{3.3356 \times 8}$
  $= 0.06251 \text{rad}$
  $\rho = 88.95m$

Q5B: How many dipoles of this kind do you need to form a complete ring?
Dipole Magnets

Q6: LHC now has beam momentum 3.5 TeV. Ring circumference is 27km. What is the required $B$, if the whole ring is completed packed by dipole magnets?

A6:

- $\rho = \frac{C}{2\pi}$
  
  $= \frac{27000}{2 \times 3.14}$
  
  $= 4297.2 \text{ m}$

- Remember, $3.3356\rho = B\rho$

- $B = \frac{3.3356\rho}{\rho}$
  
  $= \frac{3.3356 \times 3.5e3}{4297.2}$
  
  $= 2.72\text{T}$

Q7: The LHC dipoles are actually running at ~4.2T for 3.5TeV beam. What is the “dipole packing factor?”

A7:

- Remember, $3.3356\rho = B\rho$

- Bending radius in dipoles with $B = 4.2\text{T}$ would be
  
  $\rho = \frac{3.3356\rho}{B}$
  
  $= \frac{3.3356 \times 3.5e3}{4.2}$
  
  $= 2779.7 \text{ m}$

- Total length to occupy with dipoles would be
  
  $2\pi\rho = 2 \times 3.14 \times 2779.7$
  
  $= 17.465 \text{ km}$

- Packing factor
  
  $= \frac{17.5e3}{27e3} = 65\%$

(Actually LHC has 1232 units of dipoles, each 15m long)
Quadrupole Magnets

- Quadrupole (quad) magnets are the primary tools for focusing the beam, or for stabilizing the beam orbit around its center.
- \( B_y = Gx; B_x = -Gy \)
- \( G \) = field gradient [T/m]
- If quad is focusing in \( x \), it is defocusing in \( y \).
- Pole field is \( \sim 1.4 \) \( a \) G where “\( a \)” is the aperture radius of the magnet.

- Q8. Prove that a quadrupole magnet with field gradient \( G \) and length \( L \) would provide a particle of momentum \( p \) with a focal length \( F \) that is given by: \( F = 3.3356p / (GL) \)
Quadrupole Magnets

A8.

Quad magnet with
Field gradient: G
Thickness: L

Dipole field strength
at this point: \( B = Gx \)

Bend radius: \( \rho = \frac{3.3356p}{Gx} \)

Focal length: \( F = \frac{x}{\theta} = x/\frac{GL \cdot x}{3.3356p} = \frac{3.3356p}{GL} \)

Bend angle: \( \theta = \frac{L}{\rho} = \frac{GL}{3.3356p} x \)
Single-Particle Optics

- We saw that a thin quad magnet (GL) acts like a focusing / defocusing lens with 
  \[ F = \frac{3.3356p}{(GL)}. \]
  What if you had a series of such magnets spaced apart by \( L \) ?

  (NB: That \( L \) is different from that within (GL); sorry about the confusing notation.)

- Let us characterize the particle trajectory in \( x \) with a “vector” 
  \((x, x')\).
  \( x \) is the location of the particle and 
  \( x' \) is the angle of the trajectory, i.e.
  \[ x' = \frac{dx}{ds} \]
  where \( s \) denotes the coordinate along the canonical trajectory.

  In the following discussion, we ignore particle acceleration for simplicity.

- One sees that if there are no magnetic components along the beamline, after a length of \( s \), the \((x, x')\) of the particle would be like,
  \[ x(s) = x(0) + s \ x'(0) \]
  \[ x'(s) = x'(0) \]

  So in vector notation,
  \[
  \begin{pmatrix}
  x(s) \\
  x'(s)
  \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x(0) \\
  x'(0) \end{pmatrix}
  \]

- What is the effect of a thin quad (e.g. ignore the thickness) with focal length \( F \) on \((x, x')\)?
  The quad does not change \( x \), but does change \( x' \) by \( x' \rightarrow x' - \frac{x}{F} \)

  \[
  \begin{pmatrix}
  x \\
  x'
  \end{pmatrix}_{after Q} = \begin{pmatrix} 1 & 0 \\ -1/F & 1 \end{pmatrix} \begin{pmatrix} x \\
  x' \end{pmatrix}_{before Q}
  \]

  We call matrices like these “beam transfer matrices”.

Single-Particle Optics

- Consider a system that consists of a drift L (*not to mix with quad thickness!), a thin defoc quad (-F), another drift L, a foc thin quad (F). What is this system’s beam transfer matrix $M$?

$$M = \begin{pmatrix}
1 & 0 \\
-1/F & 1
\end{pmatrix}
\begin{pmatrix}
1 & L \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
1/F & 1
\end{pmatrix}
\begin{pmatrix}
1 & L \\
0 & 1
\end{pmatrix}
= \begin{pmatrix}
1 & L \\
-1/F & 1 - L/F
\end{pmatrix}
\begin{pmatrix}
1 & L \\
+1/F & 1 + L/F
\end{pmatrix}
= \begin{pmatrix}
1 + L/F & 2L + L^2/F \\
-L/F^2 & 1 - L/F - L^2/F^2
\end{pmatrix}$$

- What if we had an N series of such “cells”? → It is equivalent to considering $M^N$. This issue is considered more transparently if we ask help of eigenvalues / vectors.

Eigenvector: $\vec{V}_1, \vec{V}_2$ of matrix $M$

Eigenvalue: $\lambda_1, \lambda_2$

Arbitrary vector can be decomposed as

$$\begin{pmatrix}
x(0) \\
x'(0)
\end{pmatrix} = c_1\vec{V}_1 + c_2\vec{V}_2$$

Effect of $M^N$ on $\begin{pmatrix} x \\ x' \end{pmatrix}$ would be

$$M^N \begin{pmatrix} x \\ x' \end{pmatrix} = c_1\lambda_1^N\vec{V}_1 + c_2\lambda_2^N\vec{V}_2$$
Single-Particle Optics

- So the question reduces to evaluating the eigenvalues of $M$. Recall the basic linear algebra.

  Eigenvalues $\lambda_1, \lambda_2$ are solutions of
  
  $|M - \lambda I| = 0$
  
  Thus,
  
  $\lambda^2 - Tr M \cdot \lambda + |M| = 0$
  
  where, $|M| = 1$.

  so
  
  $\lambda_1 + \lambda_2 = Tr M$
  
  $\lambda_1 \lambda_2 = 1$

  Hence, if $\lambda$ is real, $M^N$ is divergent.

  Note: $\det M = 1$, because all the transfer matrices of the “components” which contributed to $M$ have their determinant 1.

- So, if

  $(Tr M / 2)^2 < 1$

  $-1 < 1 - L^2 / 2F^2 < 1$, i.e.

  $|L / 2F| < 1$

  Then the eigenvalues will be complex. Since $\lambda_1 \lambda_2 = 1$, they can be written as

  $\lambda_1 = e^{-i\mu}, \lambda_2 = e^{+i\mu}$

  and

  $M^N$ will not be divergent!

  $\Rightarrow$ Principle of FoDo lattice.

Q9: We have magnets with $G = 1$T/m, thickness 0.5m. Let us set the half cell length of FoDo to be 5m (full length 10m). For which momentum range can this beamline act as a non-divergent FoDo?

Q10: How would all the analysis above change, if we do NOT ignore the thickness of the quads?
FoDo Lattice Examples

Beamline of ATF2

ATF damping ring
The Beam

- How does one characterize the transverse spread of the beam, i.e. that of a group of many particles?
  - Consider an ensemble of particles with distributed \((x, p_x)\), measured from the beam centroid.
  - Take \((x, p_x)\) as the canonically conjugate variables of motion, and take it that they constitute the “Phase Space” of the group of particles.
  - If acceleration is ignored, \((x, x')\) can serve the purpose instead of \((x, p_x)\).
  - Same goes for \((y, p_y)\) and \((z, E)\).

- So, take the area occupied by particles in the \((x, x')\) plane (it is a virtual plane), and call it - “Beam Emittance in x”.
  - Dimension of transverse beam emittance is, thus, \((m)\).

- Systematic analysis of all these in practical beamlines requires much elaboration of single-particle motions, starting from the previous pages. My lecture cannot really cover this critical topic at all! Trust that Ohmi san’s lecture covers it in detail.
Synchrotron Radiation

Rate of energy loss due to SR

\[ \frac{dE}{dt} = -\frac{2}{3} \frac{r_e c}{(mc^2)^3} \frac{E^4}{\rho^2} \]

Energy loss per revolution per particle

\[ U_0 = \frac{2}{3} \frac{r_e}{(mc^2)^3} E^4 \int \frac{ds}{\rho_{\text{dipole}}^2} \]

\[ = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \frac{E^4}{\rho_{\text{dipole}}} \]

\[ \rightarrow U_0 (\text{keV}) = 88.46 \frac{E(\text{GeV})^4}{\rho_{\text{dipole}} (\text{m})} \]

Q11: A 3km ring is using dipole magnets with \( \rho = 90 \text{m} \) and is storing 3A of 8GeV electrons. What is the “average total” power to compensate the SR energy loss, per unit time?

FYI, Critical wavelength / energy

\[ \lambda_c \equiv \frac{4\pi\rho}{3\gamma^3} \]

\[ E_c = \frac{hc}{\lambda_c} \]
Superconducting Magnets

• Iron saturates at ~ 2T. This gives limitation to the field strength achievable with normal-conducting iron-dominated (NC) magnets. For instance, dipole magnets with B > 1.5 T, or quadrupole magnets with a G > 1T (a is the “aperture” of the quad) should go “superconducting” (SC).

• SC magnets, without iron, can produce higher field. But, because of the absence of the iron poles, one has to carefully arrange the electric current distribution to produce the desired field pattern. → Lecture by Tsuchiya

• SC magnets, with iron, is, again, limited in strength, similarly to NC magnets. However, they are substantially lower power consuming.
QCS for SuperKEKB
Magnets - Comments

• Generally, Maxwell’s equation allows solutions of many higher order fields besides dipole and quadrupole fields (sextupole, octapole, decapole, dodecapole etc).

• Such fields are sometimes useful for beam controls; They are also sometimes very harmful.

• It is important to control the higher pole field. This leads to tolerance issues of the magnetic poles (NC) or current distribution (SC).

• Thermal effects and mechanical aspects are also important issues for practical magnet designs and operation.

• Survey and alignment of the magnets are critical tasks to do before starting operation of a new accelerator.

• Finally, magnet power supplies (settability, stability and lifetime) are a very important area which is sometimes overlooked.
Electron Source (Example)

Thermionic Gun: Thermal electrons are extracted from a cathode which is set at a negative HV and is being heated.

Thermionic electron gun at the KEK electron/positron injector.
Electron Source (Example)

RF gun with laser-driven photo cathode

RF electron gun at the KEK electron/positron injector linac.
Electron Source – Some Comments

- Photo-cathode material: subject of studies for material science, also. Work function, lifetime, polarization, etc.

- Simulation of electron source: a difficult matter, because of the presence of space-charge forces (important for low energy).

- Recent trend: development of low emittance (i.e. high quality) electron source.
Have to produce positrons via e+e- pair creation (threshold = 2 x 511keV) in an electromagnetic shower.

Use electromagnetic shower, initiated by high-energy electrons or photons on a heavy target, e.g. Tungsten-Rhenium alloy, and collect the positively-charged positrons out of it.

Mechanical design (rotating, etc) of the target and its heat dissipation (DC and pulsed) are major issues in the e+ system, if one desires high-current e+.
Positron Source

• Flux Concentrator (FC) is a device which is used as OMD (Optical Matching Device) in a positron production system. FC can produce strongly focused \( \sim \)solenoid field (6-10T) out of eddy current that is induced by pulsed primary coil current on its outside. However, FC, generally is limited in its pulse length (< 30 \( \mu \)s).

• Other devices to use as OMD include:
  – AMD (adiabatic matching device) – a solenoid magnet with adiabatically changing field strength;
  – QWT (quarter-wavelength transformer) – a short strong solenoid followed by a weaker solenoid;
  – LL (Lithium lens).
They can replace FC for longer pulses but at lower magnetic field, or require additional R&D.
Flux concentrator in construction / testing
Positron production system at the KEK electron/positron injector linac.
Proton Source

• Out of H₂, you can either create p or H⁻, but for
  – Ease of attaining a high current, and
  – Ease of handling during injection in the booster synchrotron,

It is advantageous to use H⁻ (p with two electrons) in the upstream end of a proton accelerator system.

• J-PARC uses H⁻
  – Filament (TaB₆) discharge H⁺ source in a Plasma ch. →
  – Extraction of H⁻ via three sets of electrodes (50kV in total) → RF-Qs

• Electrons can be stripped off H⁻ easily later, when an accelerated H⁻ passes through thin foils.
Electrons can be stripped off H⁻ easily when an accelerated H⁻ passes through thin foils immediately after injection in RCS.

Radiation effect on and lifetime of the charge-conversion foils have to be dealt with, however.
Beam Monitors

- Strip-line Type Beam Position Monitors (BPM): with 4 electrodes, for use at ATF linac and extraction beamlines.

- Generally,
  \[ x = k \frac{S_L - S_R}{S_L + S_R} \]
  \[ y = k \frac{S_U - S_D}{S_U + S_D} \]

H.Hayano et al
Beam Monitors

- BPMs in rings are rotated by 45 degrees.
- Can you tell the reason?
Beam Monitors

- Cavity Type Beam Position Monitors (BPM): Rather than seeing induced charges on electrodes, measure the transverse modes (TE-modes) that are induced by the beam when it passes off center of a cavity.
Beam Monitors

- Beam Current Monitors: BPMs can also measure the beam intensity but for better precision a dedicated beam current monitor is used, as shown on the right.

- Beam Profile Monitors with phospho-screens:
  - Direct measurement of the beam, in case of linac or beamlines.
  - Measurement of synchrotron radiation light in case of rings (on the right).
Beam Monitors

- Beam Profile Measurement with interference method of SR light in rings.
  - Take advantage of the interference of SR light which comes through a slit pair.
  - Overcomes the diffraction limit in standard SR measurement.

\[ a = \text{beam size}; \gamma = \text{“visibility” (clearness)} \]
\[ \lambda = \text{wavelength}; D = \text{slit separation} \]
\[ L_0 = \text{distance from the light emission point} \]

\[
a = \frac{\lambda L_0}{\sqrt{2\pi D}} \sqrt{\ln \frac{1}{\gamma}}, \quad \gamma = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}
\]
Beam Monitors

- Beam Profile Measurement with Wire Scanners in the linac or in the beamlines.
  - Step the wire (or step the beam orbit) whose radius is assumed smaller than the beam size.
  - Look at the signal, whose strength would be proportional to the intercepted beam.
  - Profile the signal strength as function of the “stepping”.
  - Fundamentally a multi-pulse operation. Subject to limitation due to wire breakage.

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Beam Monitors

- Beam size monitor with Compton scattering off interfering pattern of laser photon population.

- Data on left (2010/5) at ATF2:
  - Laser WL: 532 nm
  - Laser crossing: 3 deg
  - Fringe pitch: 3.81 µm
  - Modulation: 0.87
  - Beam size: 310 +/- 30 nm
Controls

- Controls systems at particle accelerators must allow you to
  - Monitor
  - Analyze
  - Compute / Simulate
  - Control
  - Log
- A wide range of synchronous and asynchronous signal processing / transmission must be managed.
- Nearly a unique hotline to speak to the hardware, once the tunnel is closed for operation.
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Conventional Facilities
Conventional Facilities

- Sometimes the tunnel length becomes too big to stay inside a lab campus.
  - Going underground.
  - Going off site.
  - Safety issues.

- At any rate, you have to
  - Fit all required beamline components in wherever they have to go.
  - Connect them all either electronically, mechanically and vacuum-wise.
  - Allow enough space for installation and maintenance activities, plus, survey and alignment sighting.
  - Bring in fresh air and provide adequate cooling (taking the heat the out).
  - Ensure safety for personnel and equipment.
Conventional Facilities
## Things that Beams would Do

<table>
<thead>
<tr>
<th>Players</th>
<th>Interaction via</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material $\leftrightarrow$ Beam</td>
<td>Q.E</td>
<td>Thermal electrons / Photo-emission / Ionization</td>
</tr>
<tr>
<td></td>
<td>EM / QED</td>
<td></td>
</tr>
<tr>
<td>Environment $\rightarrow$ Beam</td>
<td>Static E-Field</td>
<td>Acceleration / Deceleration / Bending</td>
</tr>
<tr>
<td></td>
<td>Static B-Field</td>
<td>Bending / Spin rotation</td>
</tr>
<tr>
<td></td>
<td>RF Field</td>
<td>Acceleration / Deceleration / Bending</td>
</tr>
<tr>
<td>Beam $\rightarrow$ Environment</td>
<td>Transient Field</td>
<td>Fundamental mode RF / High-order mode RF Heating</td>
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<tr>
<td>Beam $\leftrightarrow$ Light</td>
<td></td>
<td>Synchrotron Light emission</td>
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<tr>
<td></td>
<td></td>
<td>Radiation Damping</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Compton Scattering</td>
</tr>
<tr>
<td></td>
<td></td>
<td>QED Phenomena</td>
</tr>
<tr>
<td>Beam $\leftrightarrow$ Beam</td>
<td>Space-charge forces</td>
<td>Within a single bunch</td>
</tr>
<tr>
<td></td>
<td>Beam-beam effects</td>
<td>When two bunches cross each other</td>
</tr>
<tr>
<td></td>
<td>Physics</td>
<td>Beam collision you look for</td>
</tr>
<tr>
<td>Beam $\rightarrow$ Env. $\rightarrow$ Beam</td>
<td>Longitudinal EM</td>
<td>Bunching / Debunching</td>
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<tr>
<td></td>
<td>Trasverse EM</td>
<td>Emittance growth / instabilities / beam break-up</td>
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</tbody>
</table>
# Accelerator Subsystems – Particle Sources

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<tr>
<th>Particles</th>
<th>Technique</th>
<th>Note</th>
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</thead>
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<td>Ions / Protons</td>
<td>Ionization / Plasma</td>
<td>Both +/- Charges</td>
</tr>
<tr>
<td>Anti-protons</td>
<td>p-pbar production</td>
<td></td>
</tr>
<tr>
<td>Electrons</td>
<td>Thermionic Photo-cathode</td>
<td>Polarized / Unpolarized Ultra-low emittance</td>
</tr>
<tr>
<td>Positrons</td>
<td>EM shower</td>
<td>e- - initiated photon-initiated</td>
</tr>
<tr>
<td>Photons</td>
<td>SR from orbit bending</td>
<td>SR rings (high inten, low-emittance)</td>
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<tr>
<td></td>
<td>Compton scattering</td>
<td>ERL (very short pulses)</td>
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<tr>
<td>Neutrons</td>
<td>Nuclear reactor</td>
<td>Slow</td>
</tr>
<tr>
<td></td>
<td>Proton on target</td>
<td>Fast</td>
</tr>
<tr>
<td>Hadrons (π, K, …)</td>
<td>Proton on target</td>
<td></td>
</tr>
<tr>
<td>Muons</td>
<td>π, K decays</td>
<td>Slow / Fast</td>
</tr>
<tr>
<td>Neutrinos</td>
<td>π, K decays</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nuclear reactors</td>
<td></td>
</tr>
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</table>
## Accelerator Subsystems – Acceleration Section

<table>
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<tr>
<th>Particles</th>
<th>Energy (Kinetic) Range</th>
<th>Technique</th>
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</thead>
<tbody>
<tr>
<td>Ions / Protons</td>
<td>0 up to 1 ~ a few MeV</td>
<td>Cockroft Ven de Graaf</td>
</tr>
<tr>
<td></td>
<td>&lt; a few 100 MeV</td>
<td>RF Qs Drift-tube linac low-β SC cavities</td>
</tr>
<tr>
<td></td>
<td>&gt; a few 100 MeV</td>
<td>linac (NC, SC) Synchrotron</td>
</tr>
<tr>
<td></td>
<td>&lt; several 10 MeV</td>
<td>Cyclotron</td>
</tr>
<tr>
<td>Electrons</td>
<td>0 up to few hundred KeV</td>
<td>Thermionic gun</td>
</tr>
<tr>
<td></td>
<td>0 up to a few MeV</td>
<td>RF gun</td>
</tr>
<tr>
<td></td>
<td>&gt; a few 100 MeV</td>
<td>linac (NC, SC) Synchrotron</td>
</tr>
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## Accelerator Subsystems – Typical Beam Manipulation

<table>
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<th>Key Elements</th>
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<td>Accelerator cavity</td>
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<tr>
<td>Deceleration</td>
<td>Accelerator cavity</td>
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<tr>
<td>Bunching</td>
<td>Accelerator cavity</td>
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<tr>
<td>Beam transport</td>
<td>FoDo lattice</td>
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<tr>
<td>Beam focussing</td>
<td>FD doublet, triplet + corrections</td>
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<tr>
<td>Emittance reduction</td>
<td>Radiation damping + Acc Cavity</td>
</tr>
<tr>
<td>Bunch compression</td>
<td>Bend + Acc cavity</td>
</tr>
<tr>
<td>Energy compression</td>
<td>Bend + Acc cavity</td>
</tr>
</tbody>
</table>
# Accelerator Subsystems – Beam Instrumentation

<table>
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<th>Quantity to Meas.</th>
<th>Beamline Hardware</th>
<th>Notes</th>
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<tr>
<td>Beam position</td>
<td>Striplines</td>
<td>Mostly typical</td>
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<td></td>
<td>Buttons</td>
<td></td>
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<tr>
<td></td>
<td>Cavities</td>
<td>New and growing</td>
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<tr>
<td>Beam intensity</td>
<td>BPMs</td>
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<td></td>
<td>Wall–current Mon</td>
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<tr>
<td></td>
<td>Current Transformer</td>
<td></td>
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<tr>
<td>Beam profile</td>
<td>Screen</td>
<td>Beam–intercepting, invasive</td>
</tr>
<tr>
<td></td>
<td>Metallic wire</td>
<td>Semi–invasive</td>
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<tr>
<td></td>
<td>Laser wire</td>
<td>Fairly non–invasive</td>
</tr>
<tr>
<td></td>
<td>SR profile</td>
<td>Diffraction limit</td>
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<tr>
<td></td>
<td>SR interference</td>
<td></td>
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<tr>
<td></td>
<td>Laser–fringe</td>
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</tr>
<tr>
<td>Beam timing</td>
<td>BPMs</td>
<td>Fast electronics</td>
</tr>
<tr>
<td></td>
<td>Prof Mons w, w/o SRs</td>
<td>Steak camera</td>
</tr>
</tbody>
</table>
Accelerators – Differences and Similarities

• You saw that accelerators vary widely in their organization, in accordance with their application, optimization, beam particles and size, but they also share similarities.

  – Differences of accelerators come from
    • differences of what you like to do with/to the beam.
  – Similarities of accelerators come from
    • common nature of particle beams that you deal with.

• Learning particle accelerators means for us to learn,

  – To identify what you like to do with the beam and
  – Find adequate technical solutions if they already exist and adopt them appropriately, or
  – Develop new technical solutions if they do not exist.
  – Now you have seen some such techniques.
Comments

• Particle accelerators
  – Consist of a number of subsystems for specific beam manipulation;
  – Use many components that do specific things with the beam.
  – Particle accelerators are a complex, almost organic system.

• Performance of particle accelerators
  – Is determined by the performance of accelerator subsystems,
  – Whose performance is determined by that of individual components,
  – And the systems design which puts them together.

• Beam
  – Is the object that you try to control and tame, but
  – The beam is also a friend who tells you what and how you are doing right or wrong.
  – Be prepared to work with the beam.
Conclusions

- We tried a rather coarse, quick walk-through of subsystems of particle accelerators.

- If you wish to become expert on anything, all the subject matters introduced here must be more systematically and seriously re-surveyed.

- You have to read some textbooks (examples mentioned earlier), come to subsequent lectures in this course, solve problems on your own and so on.

- My sincere thanks go to Drs. K.Takata, T.Okugi, E. Kako, H. Hayano, J.Urakawa, M.Poole and many others, whose work I am freely taking advantage of, to create this note.