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From Kitaev Model to String Gas via Tensor Networks

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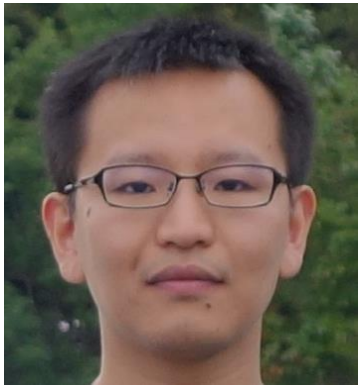

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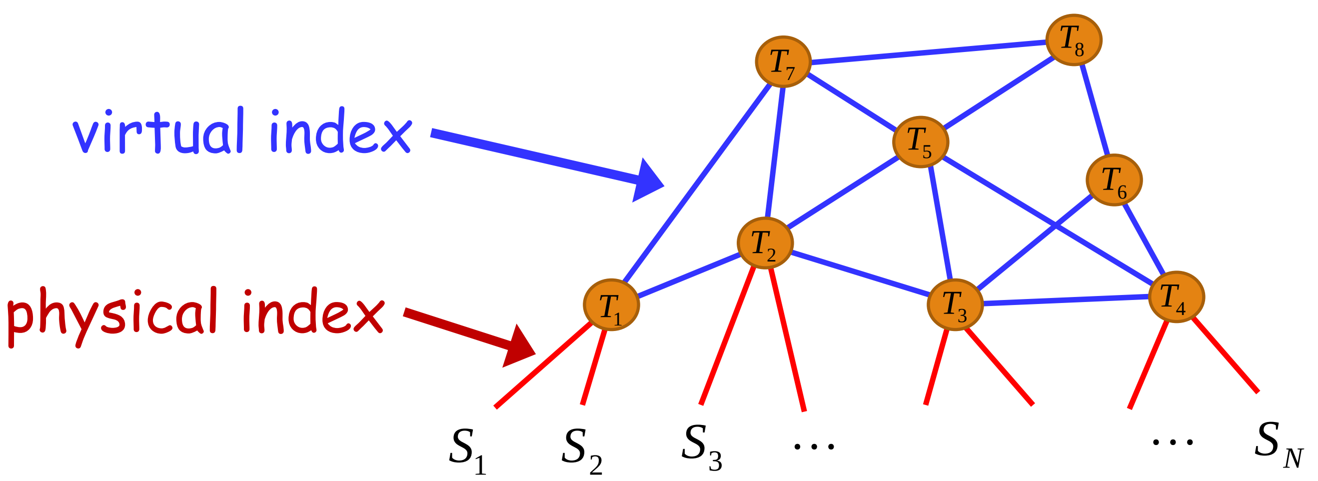
Tensor Network (TN)

$$\left|\psi(\{T_\alpha\})\right\rangle = \sum_{S_1=\pm1}\sum_{S_2=\pm1}\cdots\sum_{S_N=\pm1} \mathrm{Cont}\left(\bigotimes_\alpha T_\alpha\right)_{S_1,S_2,\cdots,S_N} \left|S_1,S_2,\cdots,S_N\right\rangle$$

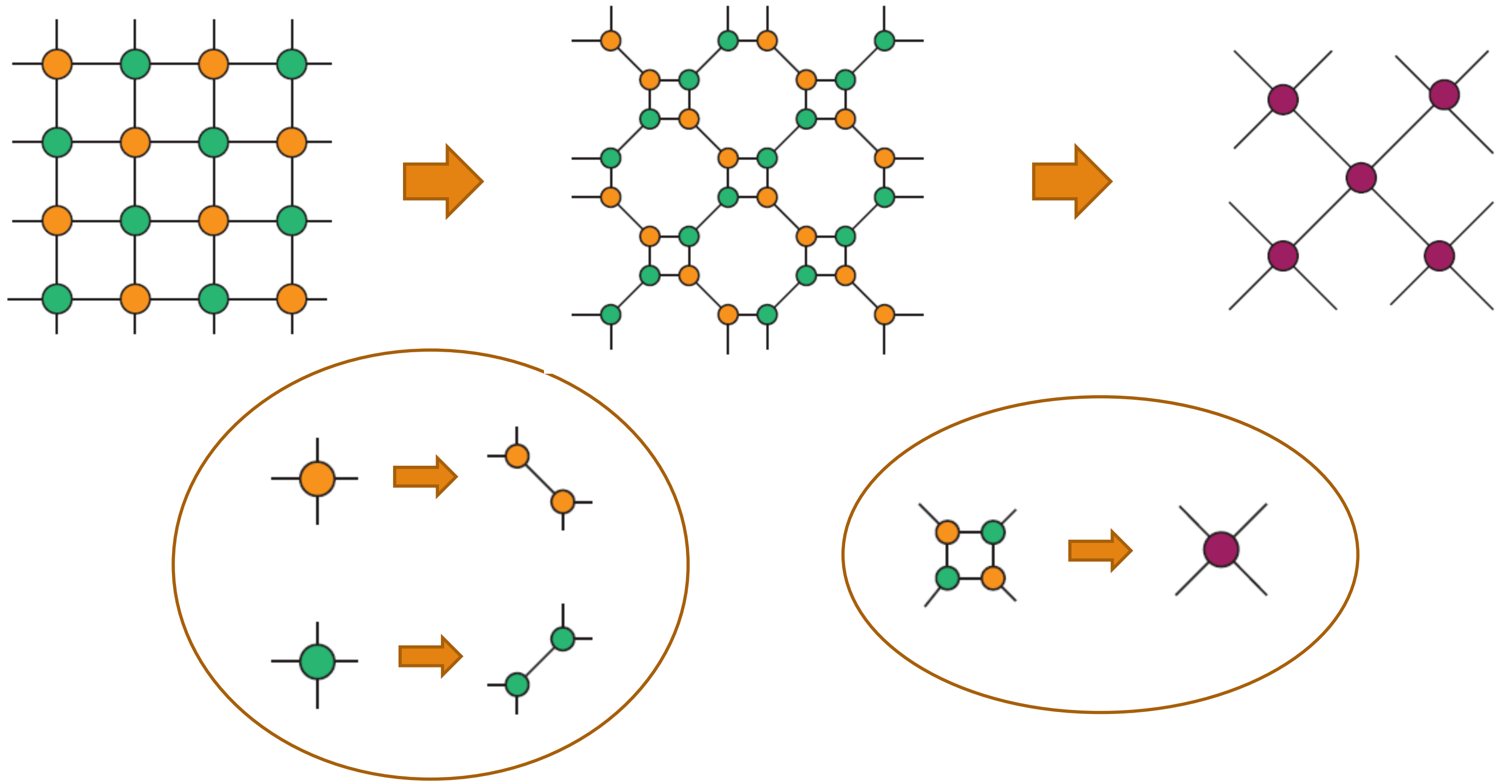


Parametrized by only O(N) tensors.

Traditional model O(1)	<<	TN model O(N)	<<	Exact model $O(e^N)$

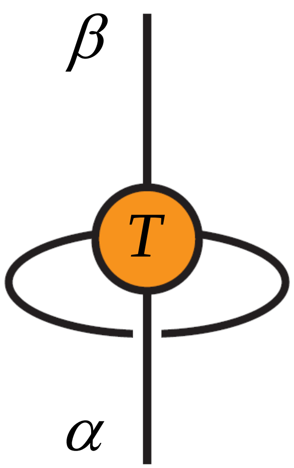
Real Space RG with TN

Gu, Levin, Wen: PRB 78 (2008); Schuch, et al: PRL 98 (2007)



CFT with TN

Gu-Wen: PRB80, 155131 (2009)



$$\lambda_\mu = e^{-2\pi\left(\Delta_\mu - \frac{c}{12}\right)}$$

eigenvalues of the partially contracted scale invariant tensor

$$\zeta_s \equiv e^{-f_s} = \sum_\mu e^{-2\pi\left(\Delta_\mu - \frac{c}{12}\right)\mathrm{Im}\,\tau + i\sigma_\mu \mathrm{Re}\,\tau}$$

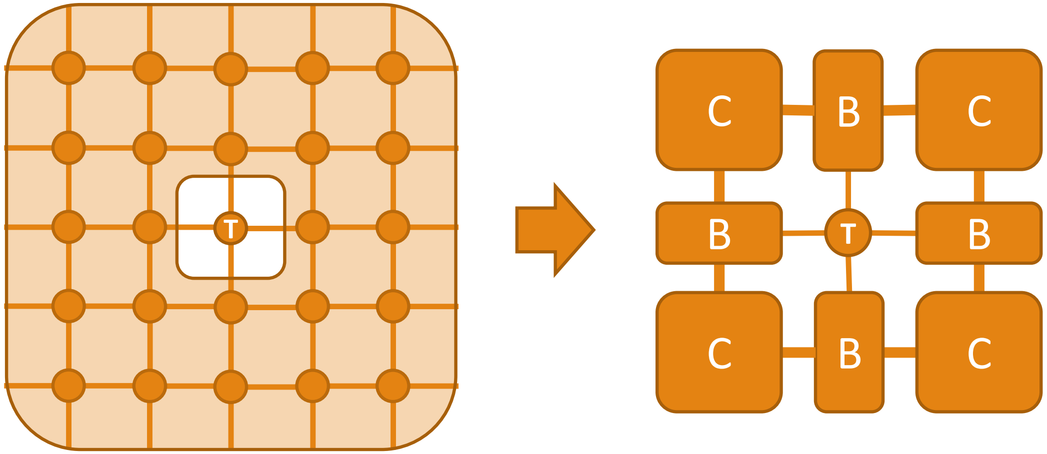
$$\Delta_\mu \equiv h_\mu^R + h_\mu^L \ , \quad \sigma_\mu \equiv h_\mu^R - h_\mu^L$$

$$\tau \equiv (\text{complex aspect ratio parameter})$$

Cardy: Nucl. Phys. B 270 (1986)

Contraction by CTM

T. Nishino and K. Okunishi, JPSJ **65**, 891 (1996)
R. Orus *et al*, Phys. Rev. B **80**, 094403 (2009)



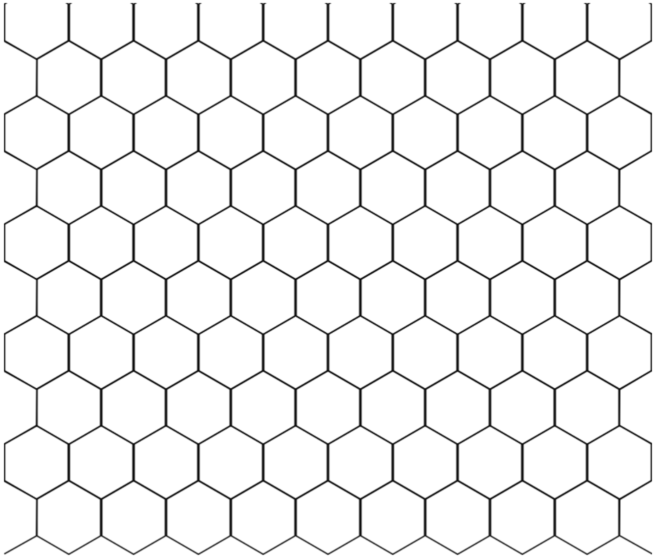
Effect of the infinite environment is approximated by B and C, which are obtained by iteration/self-consistency.

Kitaev Model

Kitaev, Ann. Phys. 321 (2006) 2

$$H = \sum_{(ij)} \sum_{\mu=x,y,z} J_{ij}^{\mu} \sigma_i^{\mu} \sigma_j^{\mu}$$

$$J_{ij}^{\mu} = \begin{cases} J & ((ij) \parallel \mu\text{-axis}) \\ 0 & (\text{otherwise}) \end{cases}$$



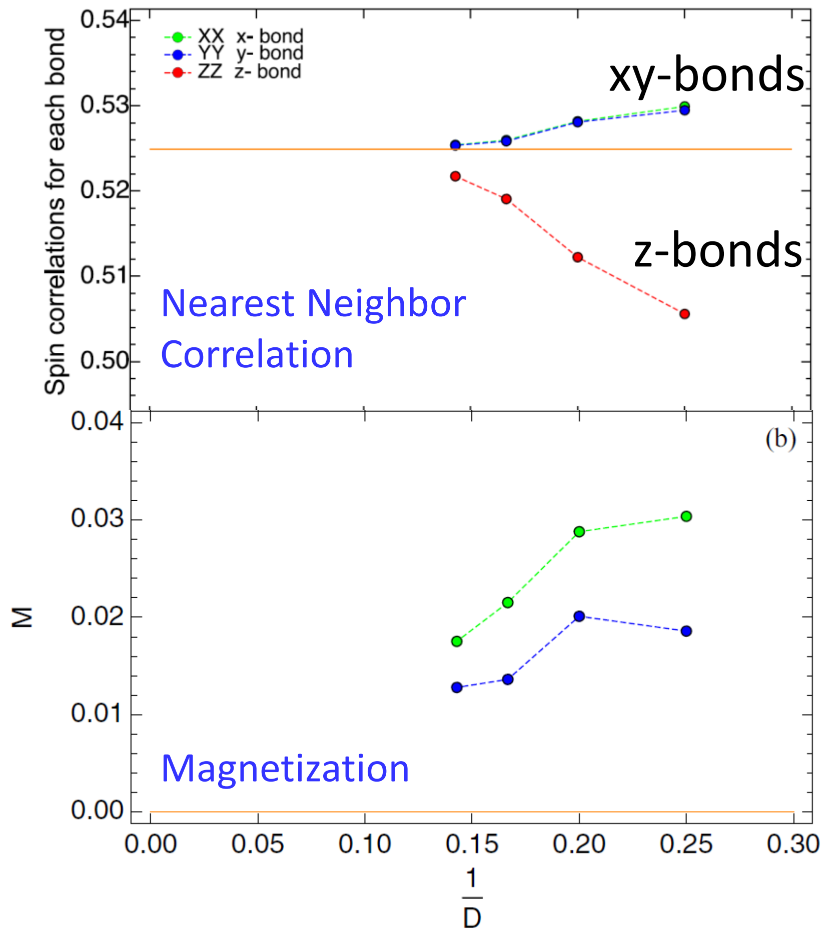
(1) Symmetries (Spin, Rotation, Traslation)
(2) Flux-free
(3) Gapless (2D Ising Universality Class)

TN Calculation is Biased

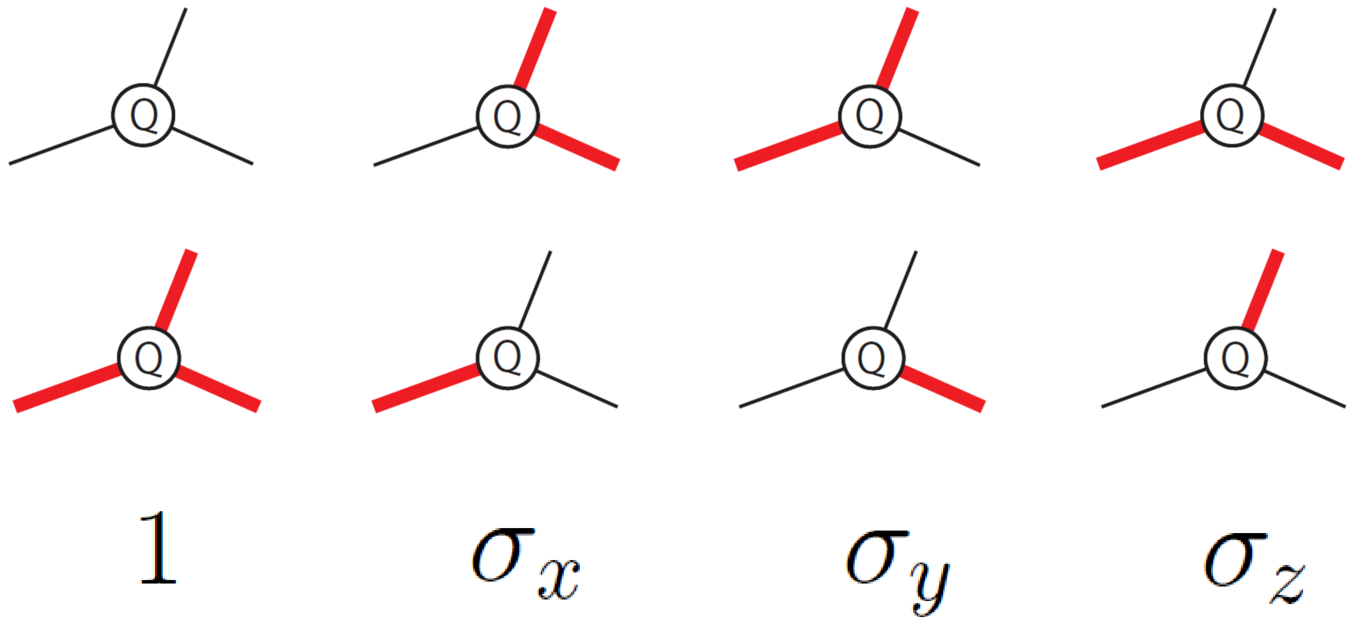
Symmetric Kitaev Model

J.O.Iregui et al., PRB.90.195102(2014)

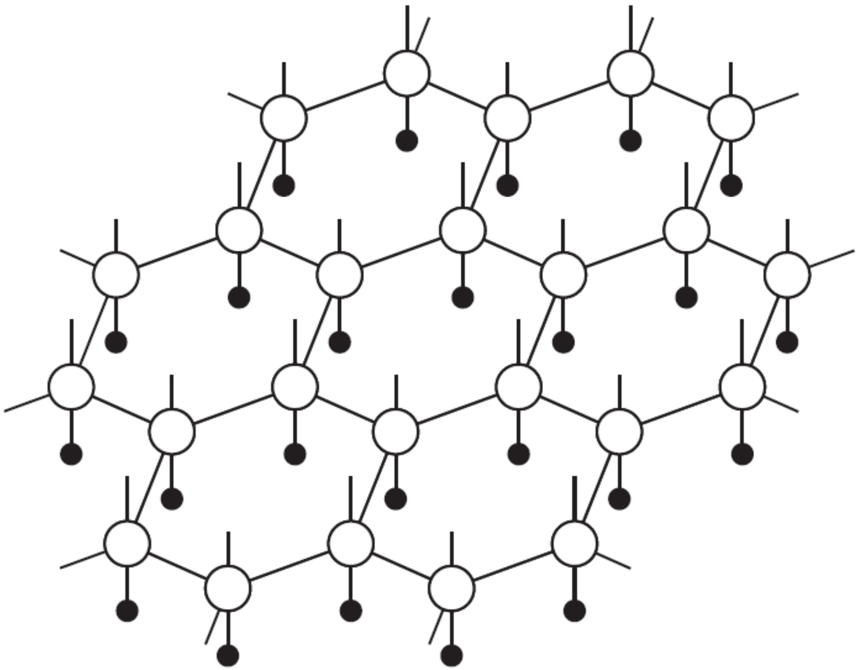
The full-update calculation is trapped by a magnetic state that also breaks Z3 spatial rotation symmetry.



Generalization of element op.



Loop Gas State (LGS)



$$|\psi_0\rangle \equiv |\mathrm{LGS}\rangle \equiv Q_{\mathrm{LG}}|(111)\rangle$$

$$|(111)\rangle \equiv \bigotimes_i |(111)\rangle_i$$

 $= |(111)\rangle_i$

fully-polarized state along (111) axis

$$\langle(111)|\sigma_i^\alpha|(111)\rangle = \frac{1}{\sqrt{3}}$$

$$(\alpha = x, y, z)$$

Z3 rotation in spin-space is guaranteed

Mapping to Classical Loop Gas

$$|\mathrm{LGS}\rangle = \sum_{G:\,\text{loop config.}} Q(G)|(111)\rangle$$

$$\begin{aligned}
\langle \mathrm{LGS}|\mathrm{LGS}\rangle &= \sum_{G',G:\,\text{loop config.}} \langle (111)|Q(G')Q(G)|(111)\rangle \\
&= \sum_{G',G:\,\text{loop config.}} \langle (111)|Q(G'\oplus G)|(111)\rangle \\
&= N_G \sum_{G:\,\text{loop config.}} \langle (111)|Q(G)|(111)\rangle \\
&= N_G \sum_{G:\,\text{loop config.}} \left(\frac{1}{\sqrt{3}}\right)^{|G|}
\end{aligned}$$

classical loop gas model
at fugacity 1/√3

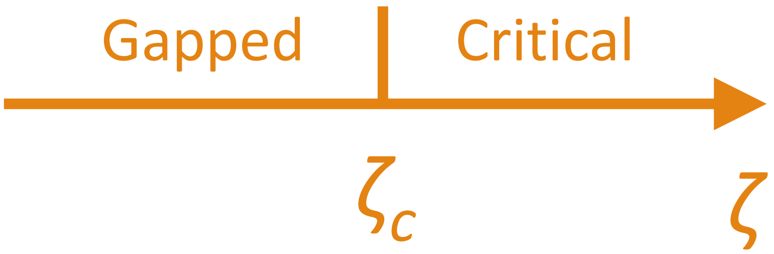
Classical Loop Gas

B. Nienhuis, Physical Review Letters 49, 1062 (1982).

$$Z_{\mathrm{LG}}(n, \zeta) \equiv \sum_{G:\,\text{loop config.}} n^{N_{\mathrm{loop}}(G)} \zeta^{|G|}$$

$$\zeta_c(n) = \frac{1}{\sqrt{2 + \sqrt{2 - n}}}$$

$$\zeta_c(1) = \frac{1}{\sqrt{3}}$$



LGS is gapless and belongs to the 2D Ising universality class (the same as the KHM ground st.)

LGO is projector to flux-free space

$$Q_{\mathrm{LG}}^2 = Q_{\mathrm{LG}}$$

$$Q_{\mathrm{LG}} W_p = W_p Q_{\mathrm{LG}} = Q_{\mathrm{LG}}$$

$$|\psi_0\rangle \equiv |\mathrm{LGS}\rangle \equiv Q_{\mathrm{LG}} |(111)\rangle$$

LGS is flux-free and non-magnetic

LGS as an approximation

$$\begin{aligned} E_0 &= N^{-1}\langle \mathrm{LGS}|H_{\mathrm{KHM}}|\mathrm{LGS}\rangle \\ &= -0.16349J \end{aligned}$$

CF: $E_{\mathrm{KHM}} = -0.19682\cdots J$

Not so good.

... not surprising since LGS is designed only to share the same symmetries as the ground state of KHM and has no variational parameter.

A way to KHM ground state

$$|\psi(\lambda)\rangle = Q_{\mathrm{LG}} X(\lambda)|(111)\rangle$$

$X(\lambda) =$ any operator preserving the symmetries (translation, rotation, σ-rotation)

Series of Ansatzes

$$|\psi_1(c_1)\rangle \equiv R_{\mathrm{DG}}(c_1)|\mathrm{LGS}\rangle$$

$$|\psi_2(c_1, c_2)\rangle \equiv R_{\mathrm{DG}}(c_2)R_{\mathrm{DG}}(c_1)|\mathrm{LGS}\rangle$$

$$\vdots$$

$$|\psi_k\rangle \equiv |\psi_k(c_1, c_2, \cdots, c_k)\rangle$$

$$\text{with } \{c_1, c_2, \cdots\} \text{ that minimizes } \langle H_{\mathrm{KHM}}\rangle$$

Summary

New series of ansatzes for Kitaev spin liquid that

(1) have the same symmetry as the KHM ground state

(2) are critical and belong to 2D Ising univ. class

(3) are flux-free

(4) connect classical loop gas to the KHM ground state

	ψ0=LGS	ψ1	ψ2	KHM gr. st.
# of DOF	0	1	2	
E/J	-0.16349	-0.19643	-0.19681	-0.19682
ΔE/E	0.17	0.02	0.00007	-

END