# AdS/CFT：応用の10年

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# はじめに

AdS/CFT (holography) : 強結合の場の理論（ゲージ理論）を解く手法（詳細は後で）

2005年ごろから「現実世界」に盛んに応用されてきた.

本講演の目的：

- AdS/CFTの初歩
- 主な応用

# 1つのきっかけ

American Physical Society annual meeting 2005

RHIC 実験のプレス・リリース (April 18, 2005)

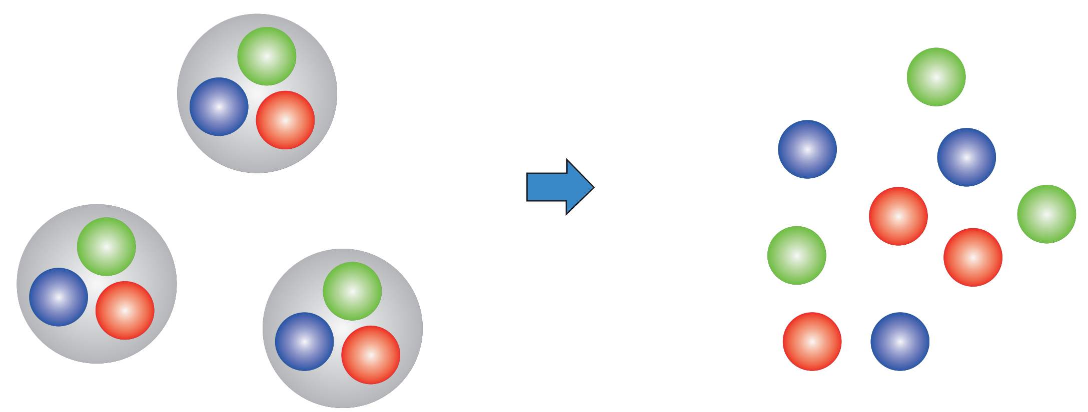
"The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating,"
(Director of the DOE Office of Science)

- 大型実験の報告で超弦理論が言及されたのは初めて
- AdS/CFTの結果が実験とよく一致

# クォーク・グルーオン・プラズマ

陽子・中性子等（ハドロン）は基本的ではなく，クォークやグルーオンからなる

通常の状況下では強い力によって「閉じ込め」られている



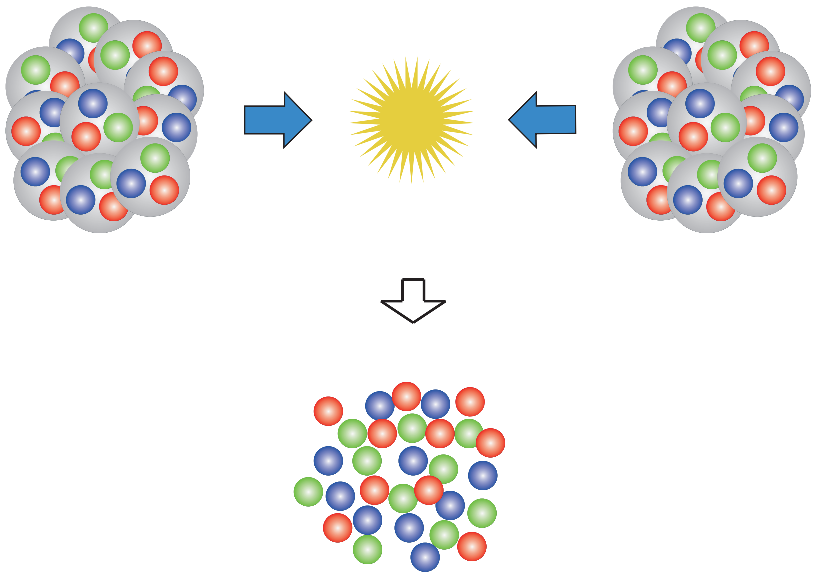
しかし，十分高温（約2兆度）では閉じ込めから解放され，クォークやグルーオンの自由度が顕在化

⇨ クォーク・グルーオン・プラズマ (QGP)

# 重イオン実験

重イオン実験では (e.g. RHIC & LHC), 重イオン同士 (e.g. $^{197}$Au) を衝突させる.

十分高温であれば, ごく短時間 (<10fm/c) QGPが生成される



# RHICの発見

重イオン実験によると

> QGP は流体として振るまう．それも
> 粘性ゼロの完全流体に近い．

注：より正確には，小さいのは“η/s”

η: ずり粘性 (shear viscosity)

s: エントロピー密度

$l_{mfp}$: 平均自由行程

一般に　　　$\eta \propto l_{mfp}$

↓強結合で短くなる

→ 完全流体は強結合極限．摂動論？

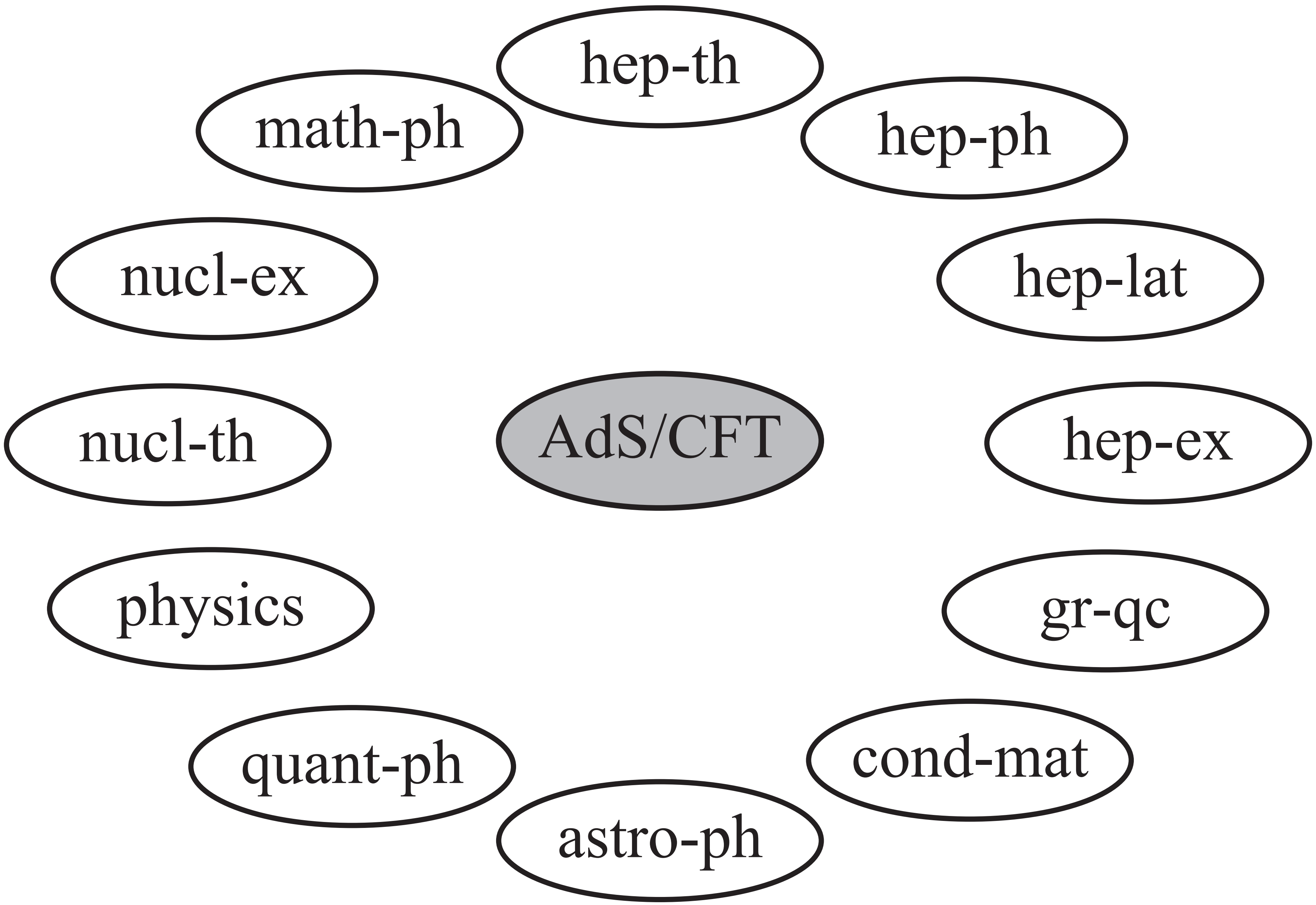
→ AdS/CFT?

# 主な応用分野

RHICが1つの契機となり，AdS/CFTは「現実世界」に盛んに応用されてきた．例えば

- クォーク・グルーオン・プラズマ (QGP)/QCD
- 非平衡物理（流体力学）
- 非線形物理（乱流，カオス）
- 物性物理
- 量子情報

# Discussed in ALL physics arXivs!



# AdS/CFT

| | | |
|---|---|---|
| 有限温度 SU(Nc) ゲージ理論の強結合極限 | ＝ | AdSブラックホール時空での重力理論 |

↓ 有限温度系

↓ ホーキング放射により有限温度系

4次元時空 “Boundary”

5次元時空 “Bulk”

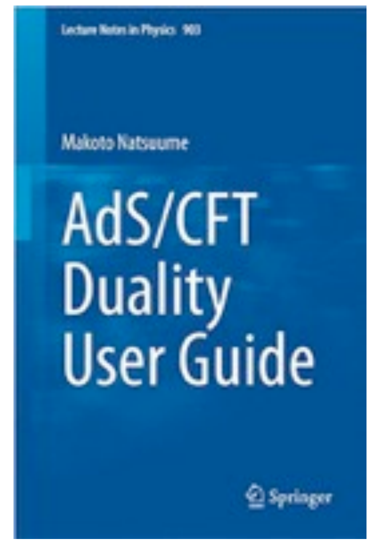
“holography”

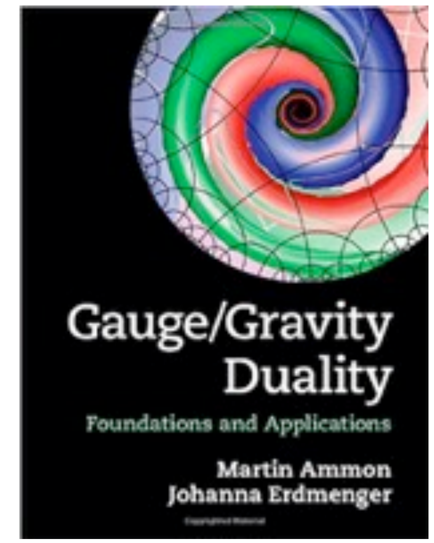
# コメント

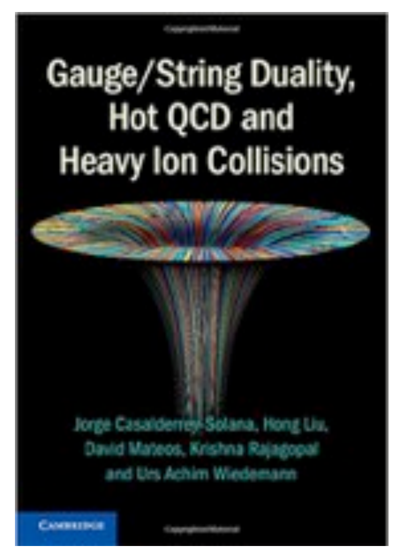
- 初心者には馴染みのない専門用語や概念が多数登場しますが，現時点では気にしないで下さい．徐々に説明します．
- とりあえず頭の片隅に留めておいてもらいたいことは
  - AdS/CFT はゲージ理論の強結合極限を計算（弱結合では重力理論には見えないから）
  - ゲージ理論と重力理論は違う時空次元に住んでいる
  - 有限温度だからブラックホール

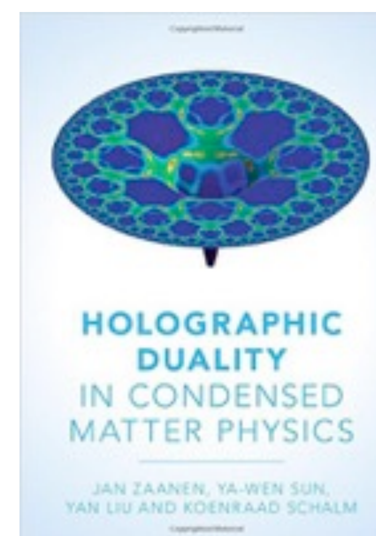
# Many textbooks available by now

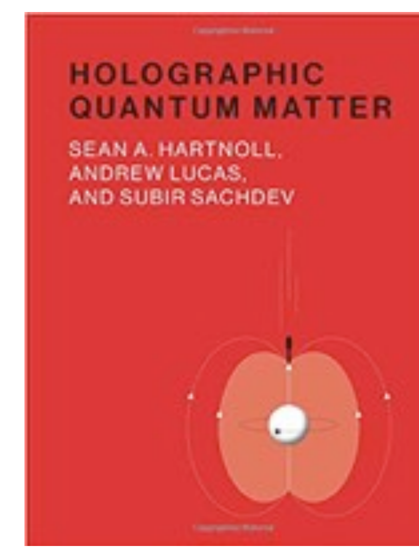
- *"AdS/CFT duality user guide"*
- *"Gauge/Gravity duality: foundations and applications"*
- *"Gauge/String duality, hot QCD and heavy ion collisions"*
- *"Holographic duality in condensed matter physics"*
- *"Holographic quantum matter"*



# Are they same?

Sometimes people use different names for AdS/CFT:

holography
gauge/gravity duality
gauge/string
bulk/boundary
Maldacena's conjecture
...

They basically mean the same thing.
I use AdS/CFT & holography interchangeably.

Note: Even if we call "AdS/CFT", it may include non-AdS/non-CFT

# アウトライン

- イントロ：QGP & AdS/CFT　} 15 min
- AdS/CFTの初歩
  ブラックホールの物理を中心として　} 20 min
- 応用例
  - QGP
  - 物性系　} $15\,\mathrm{min} \times 2$

「輸送」と「相図」の話とも言える

# アウトライン

- カバーしないこと
  - AdS/CFTの詳細な証拠
  - 具体的な計算手法
  - 超弦理論との関係
  - 包括的なレビュー：関連論文はおそらく数千

# “Intuitive holography”

# Motivating holography

There are many reasons to trust AdS/CFT.

- Black hole thermodynamics
- Large-Nc gauge theories
  perturbative expansion = genus expansion ~ string theory expansion
- D-branes
  D-branes reduce to YM and BHs in appropriate limits

more convincing,
but more technical

In addition, many circumstantial evidences
comparing various quantities both from gravity and from gauge theory pt of view

Yet no complete proof

I'll cover only BH thermodynamics in this talk.

Main focus is the following question:

**Can BHs describe standard statistical systems?**

# BH thermodynamics

If BH can ever describe statistical systems, BH must be a thermodynamic system above all things.

| Statistical system | ⇔ | Black Hole |
|---|---|---|
| Thermodynamics | | "BH thermodynamics" |
| Temp | | Hawking temp |
| Energy | | BH mass |
| Entropy | | BH entropy |

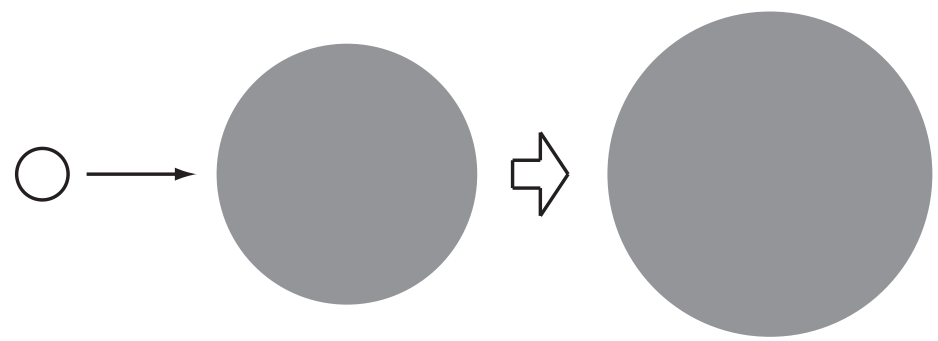
- BH has the notion of temperature due to Hawking radiation.
- BH satisfies thermodynamic-like laws (0th-3rd)

# 2nd law

BH horizon (Schwarzschild): located at

$$R = \frac{2GM}{c^2}$$

If matter falls in, BH horizon area $A = 4\pi R^2$ increases



Nothing comes out from the horizon (classically), $A$ is a nondecreasing quantity ↔ entropy?

$$S_{BH} \propto A\,?$$

BH is a thermodynamics object?

# Dimensional analysis

In fact, Hawking ('74) shows that BH radiates (due to matter quantum effect) and has temperature.

For Schwarzschild BH, thermodynamic quantities can be derived from dimensional analysis.
Below $\hbar = c = k_B = 1 \Rightarrow \mathrm{L} = \mathrm{s} = \mathrm{M}^{-1}$

- *T*: only length scale *R*

$$T \propto 1/R$$

- *E*: *M*
- *S*: 1st law

$$dE = TdS$$

$$\to \frac{1}{G} dR \sim \frac{1}{R} dS \qquad R = 2GM$$

$$\to \quad S \sim \frac{R^2}{G} \sim \frac{A}{G}$$

w/ numerical factors and restoring units,

$$T = \frac{\hbar c}{4\pi R k_B}$$

$$S = \frac{A}{4G\hbar} k_B c^3$$

“area law”

We use Schwarzschild as an example,
but BH entropy is always given by $S_{BH} = A/(4G)$ if the gravitational action is written by Einstein-Hilbert action i.e.

$$\frac{1}{16\pi G_d} \int d^d x \sqrt{-g} R$$

e.g. Wald textbook (1994)

Area law is universal

cf. The above form of $T$ is not universal

# Back to the question

"Can BH describe standard statistical system?"

- BH entropy in 4d

$$S_{BH} \propto A = V_2$$

- Statistical systems

$$S \propto V_3$$

4d BHs cannot describe 4d statistical systems.

# 5d BHs

But notice

| | BH | | statistical |
|---|---|---|---|
| 4d spacetime | $S_{BH} \propto A = V_2$ | $\overset{\times}{\longleftrightarrow}$ | $S \propto V_3$ |
| 5d | $\propto A = V_3$ | $\swarrow\!\!\!\nearrow$ | |

If a BH can ever describe a usual statistical system, the BH must live in five-dimensional spacetime.

⇨ holography

# Prob of specific heat

However, "usual" BH (Schwarzschild) has a negative specific heat

$$T \propto 1/R \propto 1/M$$

$$C = \frac{\partial M}{\partial T} < 0$$

⇨ no stable equilibrium

This behavior differs from standard statistical systems
e.g. Stefan-Boltzmann

$$\epsilon \propto T^4 \Rightarrow c = \frac{\partial \epsilon}{\partial T} > 0$$

# AdS BH

Schwarzschild BHs live in flat spacetime.

But BHs do exist even in curved spacetime.
AdS BHs (BHs in anti-deSitter spacetime) are special and do have positive specific heat.

**Usual BHs cannot describe statistical systems, but AdS BHs can**

# Scale-inv geometry

Curved spacetime is fine to consider if it has a 4-dim flat "interpretation"
→ 4-dim Poincare inv.

AdS spacetime is such a geometry w/ a special property

$$ds^2 = r^2 \underbrace{(-dt^2 + dx^2 + dy^2 + dz^2)}_{\text{4d Minkowski}} + \frac{dr^2}{r^2}$$

- Symmetry
  - Lorentz SO(1,3) & translations
  - Scale inv. $t \to at, \vec{x} \to a\vec{x}, r \to r / a$
  - Actually, they combine into a larger sym, SO(2,4)

# AdS BH

$$ds^2 = r^2(-f dt^2 + \underbrace{dx^2 + dy^2 + dz^2}_{\text{“planar” horizon}}) + \frac{dr^2}{r^2 f}$$

$$f = 1 - \left(\frac{r_0}{r}\right)^4$$

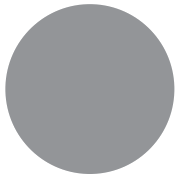
$r = r_0$ : horizon

cf. 4d Schwarzschild:

$$ds^2 = \cdots + r^2(\underbrace{d\theta^2 + \sin^2\theta d\varphi^2}_{\text{spherical horizon}})$$

- planar horizon ↔ gauge theory on $R^3$

- 4-dim flat BH: horizon topology must be $S^2$
  → another reason why curved spacetime



- It obeys Stefan-Boltzmann: $\epsilon \propto T^4$

# Summary so far

Can a BH describe a usual statistical system?

- "Area law" → necessity of higher dim.
- BH in flat spacetime: not suitable due to negative specific heat → curved spacetime
- BH in curved spacetime must have 4d flat spacetime interpretation
- 4d Poincare & scale inv. → AdS5
- AdS5 actually has a larger sym. SO(2,4)
  AdS BH obeys Stefan-Boltzmann

# But what kind of statistical systems?

Let's accept the argument so far.
What kind of statistical systems AdS BHs describe?

Holography claims AdS BHs describe
large-Nc gauge theories

# Large-Nc gauge theories

U(1): QED
SU(3): QCD
⋮
SU(Nc)

2 “parameters”:

- coupling constant $g_{YM}$ → instead use ‘t Hooft coupling $\lambda := g_{YM}^2 N_c$
- # of “colors” Nc

$N_c \gg 1 \to 1/N_c$ expansion

$\lambda \ll 1$ → weak coupling: perturbative expansion in λ

$\lambda \gg 1$ → strong coupling

→ AdS BHs describe this limit

# Scale-inv gauge theory

Since AdS is scale inv, our gauge theory should be so as well.

Pure gauge theory in 4d (e.g. Maxwell)

→ Classical: scale inv
Quantum: not inv ($\beta \neq 0$)

Special class of gauge theories w/ $\beta=0$ exist.

simplest: $\mathcal{N}$=4 super-Yang-Mills (SYM)
↑: # of supersym.

gauge field + 4 fermions + 6 scalars
(all in adjoint reps)

→ scale & Poincare inv combine into a larger sym SO(2,4),
4d conformal inv. conformal field theory (CFT)

$\mathcal{N}$=4 SYM has the same sym as AdS5

# QGP application
# (Transports)

# Transports

Next simplest situation:
add perturbs. & see how they decay (relaxation)

→ Nonequilibrium statistical mechanics or hydrodynamics

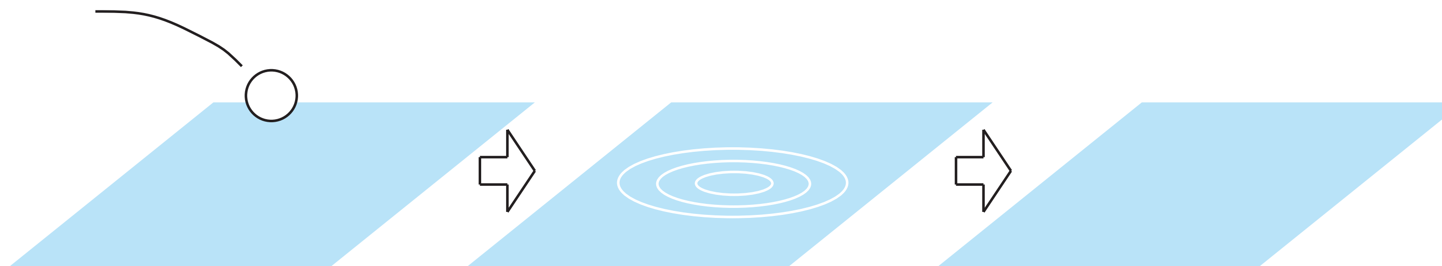
→ important quantities: transport coefficients

e.g. (bulk & shear) viscosity
speed of sound
conductivity
. . .

If AdS/CFT is correct, BHs and hydrodynamic systems should behave similarly.

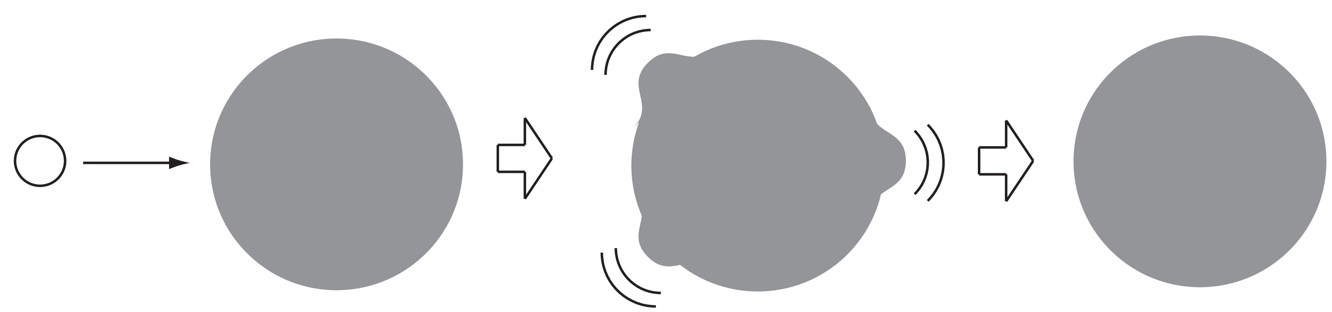
# BH and hydrodynamics

Water pond:



The dissipation: consequence of viscosity

BH:



The dissipation: consequence of BH absorption
→ horizon: origin of dissipation

One can compute transports thru such a process.

# One lesson

Universal relations can exist for transports in strong coupling limit (of large-Nc theories).

From BH pt of view, this comes from universal nature of horizon.

e.g. $\eta / s$, chaos bound...

η: shear viscosity
s: entropy density

# Universality of η/s

According to AdS/CFT

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

in large-Nc limit

Only violations:
**Natsuume - Ohta, 1008.4142**
Erdmenger et al., 1011.5912

- ☑ conformal plasma ($\mathcal{N}$=4 SYM) — Policastro - Son - Starinets, 0104066
- ☑ nonconformal plasmas — Kovtun - Son - Starinets, 0309213; Buchel - J.Liu, 0311175
- ☑ Plasmas in different dimensions — Herzog, 0210126; Kovtun - Son - Starinets, 0309213
- ☑ Plasmas at finite chemical potential — Mas, 0601144; Son - Starinets, 0601157; Saremi, 0601159
- ☑ Plasmas w/ fund. reps. — **Maeda - Natsuume - Okamura, 0602010**; Mateos - Myers - Thomson, 0610184
- ☑ Time-dependent plasma — Janik, 0610144
- ☑ "General proof" — Kovtun - Son - Starinets, 0405231; Benincasa - Buchel - Naryshkin, 0610145

# QGP viscosity

- The result is universal
- If large-Nc limit is a good approx to QCD, it may hold even to QGP

So, let's compare.

- AdS/CFT: $$\frac{\eta}{s} = \frac{1}{4\pi}$$
- RHIC: $$1 < 4\pi\frac{\eta}{s} < 2.5$$ for Tc < T < 2Tc

  Song - Bass - Heinz - Hirano -Shen, 1011.2783 [nucl-th]
- Lattice (pure SU(3) YM): $$1 < 4\pi\frac{\eta}{s} < 2$$ for 1.2 Tc < T < 1.7 Tc

  Meyer, 0704.1801 [hep-lat]

# Why universal?

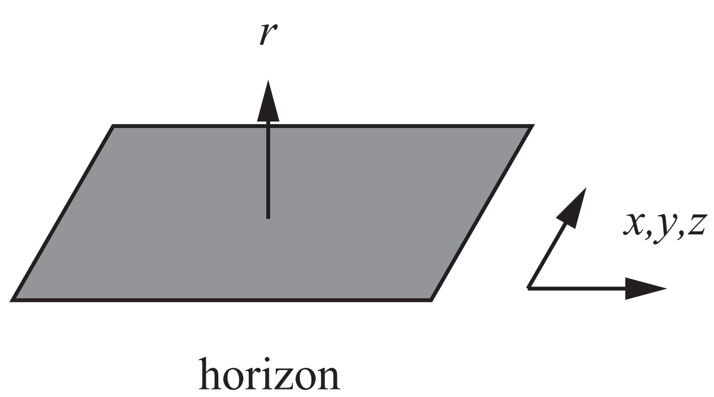
Related to the "universal" nature of BH

e.g. Black hole entropy:

$$S_{BH} = \frac{A}{4G_5}$$

"Planar" horizon → entropy density

$$s_{BH} = \frac{a}{4G_5}$$




$a$: "horizon proper area density"
$A_3 = a V_3$, $V_3$: bdy volume

One can show $\eta \propto a$, so $\eta / s = \text{const}$

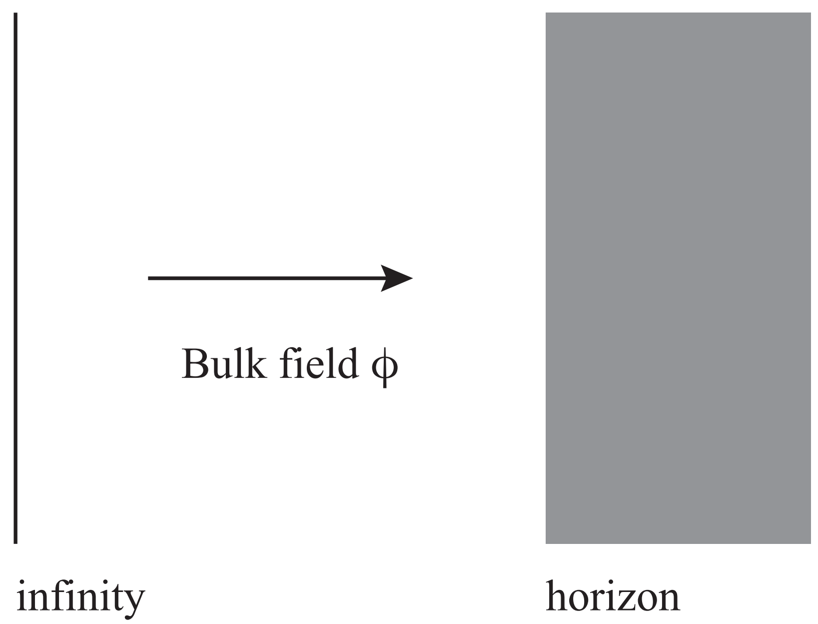
**Similarly, many universal results are related to physics @ horizon**

# Why universal?

I haven't explain how to compute transports, but

- Choose appropriate bulk pureturbs.
- Add them at infinity
- Solve bulk EOM (under appropriate BCs)

In principle, whole AdS spacetime would matter to computations. But in some cases, only near-horizon region matters. When this happens, one has universal results.



# Viscosity bound

In strong coupling limit,

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

At finite coupling, one expects a larger η/s $\eta \propto l_{mfp}$

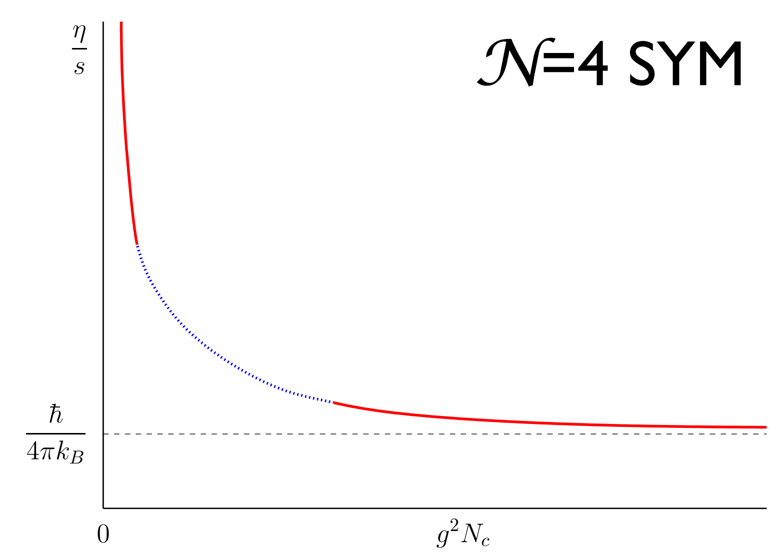
(finite 't Hooft coupling λ)

Holography tells perfect fluid cannot exist.

Conjecture: any fluid satisfies

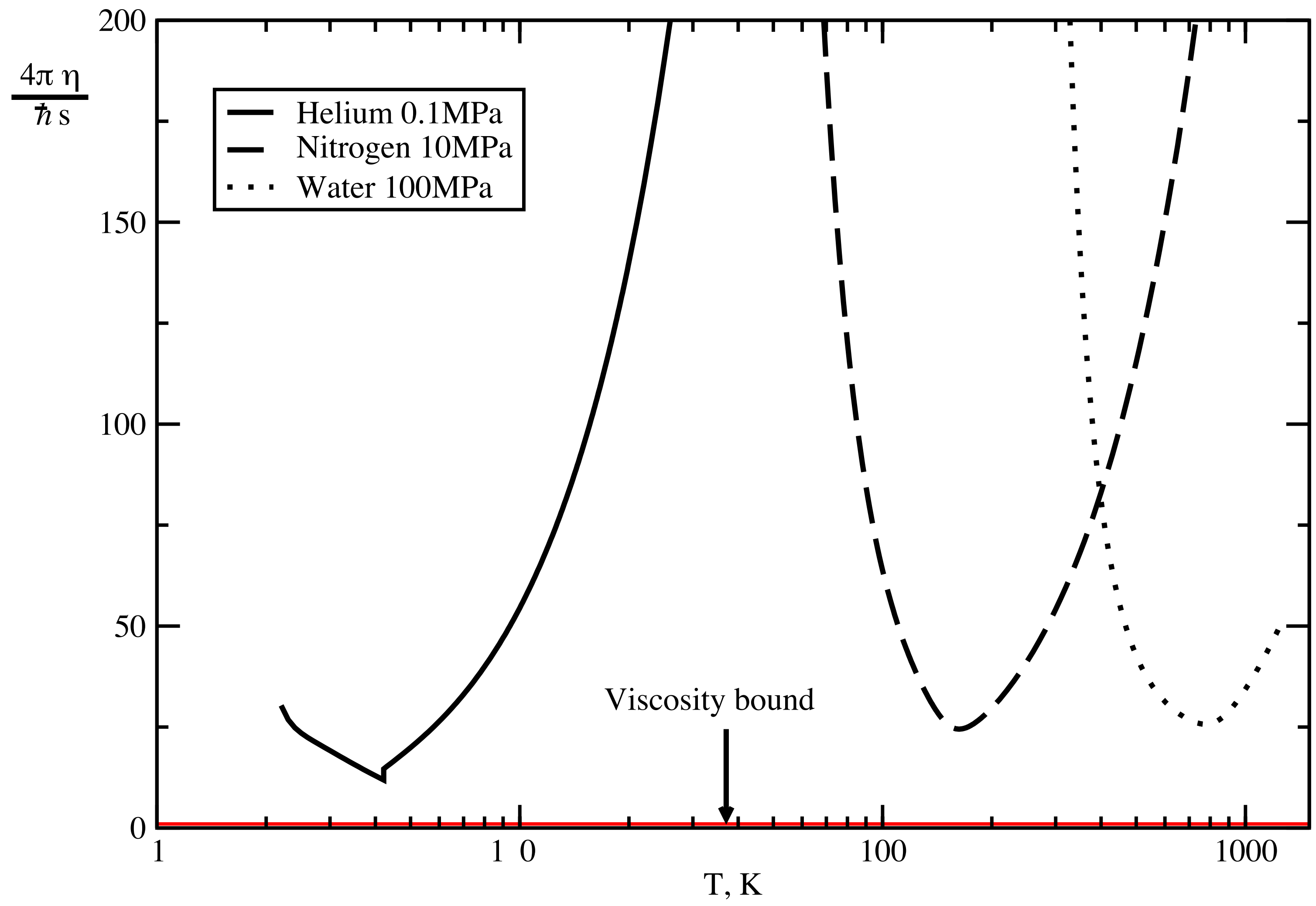
Kovtun - Son - Starinets, 0405231

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$



# Viscosity of real materials

Kovtun - Son - Starinets, 0405231




Note: Helium has nonzero viscosity (due to superfluid+normal components)

# Rough argument

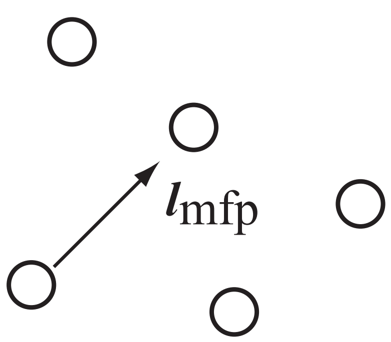
$$\begin{cases} \eta \sim \rho v l_{mfp} \\ s \sim \dfrac{\rho}{m} \end{cases}$$

ρ: mass density

Then, the bound should hold if quasiparticle picture is valid:

$$\frac{\eta}{s} \sim m v l_{mfp} > \hbar$$

$$\Rightarrow l_{mfp} > \lambda_{deBroglie}$$




Note: Counterexamples are known, so the bound is not strictly true, but the existence of a bound is physically natural.

e.g. Cremonini, 1108.0677

# CM applications (phase diagram)

# Condensed-matter in holography?

- Often strongly-correlated systems in condensed-matter physics (e.g. High-Tc), nice if one could have dual gravity descriptions
- Unclear if any large-Nc theory is hidden beyond those condensed-matter systems
- Our approach:
  - Put aside the correspondence w/ real systems for the time being.
  - Simply realize interesting CM behaviors in large-Nc theories.

# One lesson

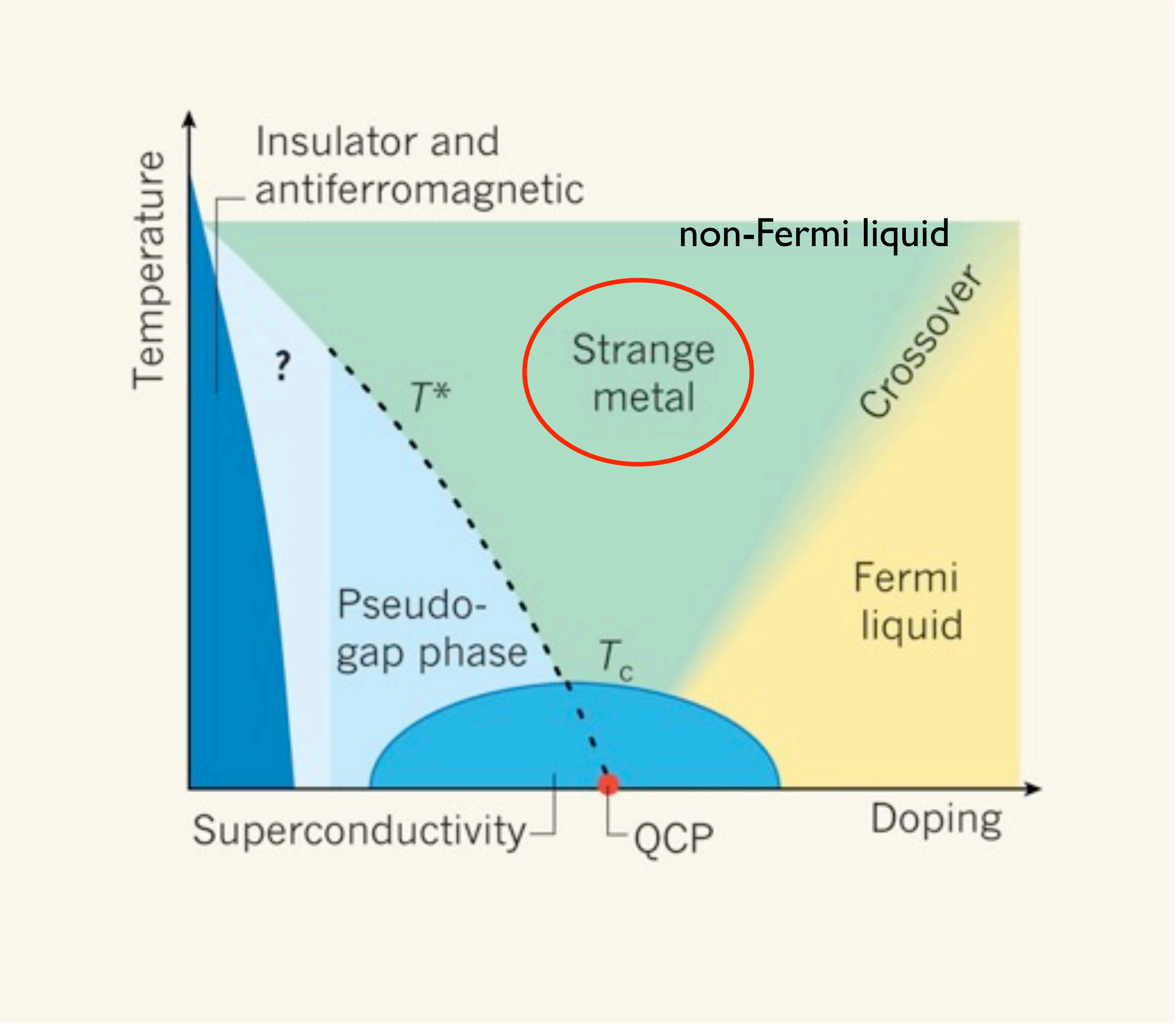
The phase diagram of (strongly-coupled) gauge theories are as rich as condensed-matter systems

Not surprising, people have known this thru QCD studies.
(cd. Itakura-san's talk?)

Conversely, AdS BH phase diagrams: richer than usual BHs

# Typical high-Tc phase diagram

One goal is to understand high-Tc




Varma, Nature 468, 184 (2010)

- We do not understand many regions of phase diagram:
  - non-Fermi liquid (strange metal)
  - quantum critical pt
  - pseudo-gap region
- Holographic description for (some of) each phases separately:
  - superconductor
  - Fermi liquids
  - non-Fermi liquids
- But we are still far

# Adding matters

$\mathcal{N}$=4 SYM is describe by pure gravity:

$$\mathcal{L} = \sqrt{-g}(R - 2\Lambda)$$

Λ: cosmological const.

It has only plasma phase (due to scale inv), phase diagram: not very interesting

Adding matter fields: more interesting phase diagram

↔adding relevant/marginal perturbs.

In condensed-matter, one typically considers charged matter (e.g. $e^-$), so consider Einstein-Maxwell

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - F_{MN}^2\right]$$

BH solution: charged BH

Reissner-Nordstrom AdS BH

M,N...: bulk indices

μ,ν...: bdy indices

# Adding matters

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - F_{MN}^2\right]$$

Further add matter (often as perturbs.)

- Complex scalar → holographic superconductor/superfluid

Hartnoll - Herzog - Horowitz, 0803.3295; 0810.1563
Gubser, 0801.2977

$$\mathcal{L}_M = -\sqrt{-g}\left[|D_M\psi|^2 + m^2|\psi|^2\right] \qquad D_M = \partial_M - ieA_M$$

- Fermion → holographic Fermi/non-Fermi liquids

Lee, 0809.3402
Liu - McGreevy - Vegh, 0903.2477
Cubrovic - Zaanen - Schalm, 0904.1993

$$\mathcal{L}_M = -\sqrt{-g}i\left[\bar{\psi}\Gamma^M D_M\psi - m\bar{\psi}\psi\right]$$

$$D_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - ieA_M$$

# Holographic superconductors

Typically, Einstein-Maxwell-complex scalar system:

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - \frac{1}{q^2}\left(F_{MN}^2 + |\nabla_M \psi - iA_M \psi|^2 + m^2 |\psi|^2\right)\right]$$

Hartnoll - Herzog - Horowitz, 0803.3295; 0810.1563
Gubser, 0801.2977

- Phase structure
  - T>Tc: ψ = 0
  - T<Tc: ψ ≠ 0 → ψ: order parameter
    ~ dual to "macroscopic wave fn"
- Dual to superconductor/superfluids
  - electromagnetic responses: diverging DC conductivity, London eq.

# Interesting "theoretical lab"

- critical phenomena
  phase transition: 2nd order
  → critical exponents: $\Phi^4$ mean-field theory
  **Maeda - Natsuume - Okamura, 0904.1914**

  dual Ginzburg-Landau theory can be derived including numerical coeffs.
  → "first-principle" derivation of GL theory
  **Natsuume - Okamura, 1801.03154**

- defect formation
  Cool critical system
  → defect formation (vortices)
  → Kibble-Zurek scaling
  Sonner - del Campo - Zurek, 1406.2329
  Chesler- Garcia-Garcia - H. Liu, 1407.1862
  **Natsuume - Okamura, 1703.00933**

- Superfluid turbulence
  → Kolmogorov scaling
  Adams - Chesler - H. Liu, 1212.0281

# Other applications

- **2nd-order hydrodynamics**

  initiated by
  Baier - Romatschke - Son - Starinets - Stephanov, 0712.2451
  Bhattacharyya - Hubeny - Minwalla - Rangamani, 0712.2456
  **Natsuume - Okamura, 0712.2916**

  hydrodynamics: effective theory or derivative expansion
  standard hydro: 0th or 1st order in expansion
  Not just a minor improvement, mandatory for many reasons

- **Turbulences (classical & quantum)**

  Kolmogorov scaling observed

  $$E(k) \propto k^{-5/3}$$

  Adams - Chesler - H. Liu, 1307.7267
  1212.0281

- **Many-body quantum chaos**

  quantum counterpart of Lyapunov exponent → bound

  Maldacena - Shenker - Stanford, 1503.01409

# Chaos

**Classical:**

Sensitivity on initial conditions "butterfly effect"

$$\delta x(t) \sim \delta x(0) e^{\lambda t}$$

λ: Lyapunov exponent

**Quantum:**

Certain correlators ("out-of-time ordered correlators") behave as

$$C(t) \sim e^{2\lambda t}$$

in large-Nc thermal systems, and one would define λ.

$$C(t) = -\left\langle [W(t), V(0)]^2 \right\rangle$$

$\langle \ \rangle$ : thermal expectation value

e.g. $W(t)=x(t)$, $V=p(0)$, semiclassically

$$[x(t), p(0)] \to i\hbar \{x(t), p(0)\} = i\hbar \frac{\partial x(t)}{\partial x(0)} \sim i\hbar e^{\lambda t}$$

# Chaos bound

Maldacena - Shenker - Stanford, 1503.01409

$$\lambda \leq \frac{2\pi T}{\hbar}$$

Conjecture: Systems w/ BH duals saturate the bound.
"BH is maximally chaotic"

e.g. SYK model & its variations?

# Summary

- Some “killer applications”
  - η/s
  - non-Fermi liquids
- Some lessons
  - Universal relations can exist for transports in strong coupling limit. Universal results are usually related to horizon properties.
  - The phase diagram of (strongly-coupled) gauge theories are as rich as condensed-matter systems.
- Hopefully, AdS/CFT will be a standard tool for theoretical physics.