KEK連携コロキウム・研究会エディション
「量子多体系の素核・物性クロスオーバー」2019.1.14-16
高エネ研つくばキャンパス・4号館セミナーホール

高精度量子多体数値計算と機械学習が描き出すギャップ/質量形成、高温超伝導と暗黒フェルミ粒子

January 15, 2019, KEK, Tsukuba

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東京大学 THE UNIVERSITY OF TOKYO

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科研費 KAKENHI

Outline

1. Introduction to a grand challenge in condensed matter physics; high T_c
2. Progress in numerical methods
3. Rejuvenated understanding of cuprate superconductors
4. How does a gap/mass emerge in physics?
   Cases of condensed matter and particle physics
   insulating gap, pseudogap and superconducting gap
5. How does strong-coupling superconductivity emerge?
   Dark fermion theory and machine learning
6. Summary and outlook

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Introduction:
single-particle Green's function and self-energy

Nambu representation for superconductors

$$G = (\omega - H)^{-1} = \begin{pmatrix} \omega - \varepsilon_{\mathbf{k}} - \Sigma^{\mathrm{nor}} & -\Sigma^{\mathrm{ano}} \\ -\Sigma^{\mathrm{ano}} & \omega + \varepsilon_{\mathbf{k}} + \Sigma^{\mathrm{nor}} \end{pmatrix}^{-1} = \begin{pmatrix} G_{11}{}^{\mathrm{nor}} & F \\ F^{*} & G_{22}{}^{\mathrm{nor}} \end{pmatrix}$$

$$G_{11}{}^{\mathrm{nor}}(\mathbf{k},\omega) = \left[\omega + \mu - \varepsilon_{\mathbf{k}} - \left(\Sigma^{\mathrm{nor}}(\mathbf{k},\omega) + W(\mathbf{k},\omega)\right)\right]^{-1}$$

$\langle cc \rangle = \mathrm{Im}\, F$ 秩序パラメタ

ギャップ関数

$\Delta(k,\omega) = z(k,\omega)\Sigma^{\mathrm{ano}}(k,\omega)$

$$W(\mathbf{k},\omega) = \frac{\Sigma^{\mathrm{ano}}(\mathbf{k},\omega)^2}{\omega - \mu + \varepsilon_{\mathbf{k}} + \Sigma^{\mathrm{nor}}(\mathbf{k},-\omega)^{*}}$$

$A(k,\omega) = \mathrm{Im}\, G_{11}(k,\omega)$ 光電子分光

Re Σ is given from Im Σ through K.K. transformation

$$A(k,\omega) \Leftrightarrow \mathcal{A}\left(\left\{\mathrm{Im}\,\Sigma^{\mathrm{nor}}(k,\omega), \mathrm{Im}\,\Sigma^{\mathrm{ano}}(k,\omega)\right\}\right)$$

$A(k,\omega)$ is a functional of Im $\Sigma^{\mathrm{nor}}(k,\omega)$ and Im $\Sigma^{\mathrm{ano}}(k,\omega)$

Experimentally, only $A(k,\omega)$ is known

How to estimate (infer)

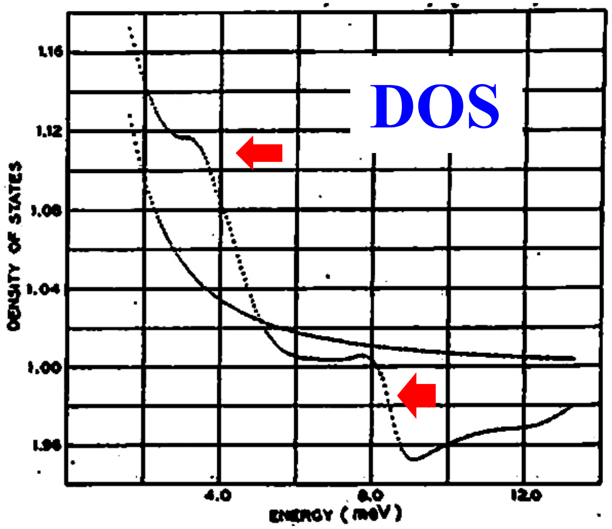
Im $\Sigma^{\mathrm{nor}}(k,\omega)$ and Im $\Sigma^{\mathrm{ano}}(k,\omega)$ separately?

inverse problem

Why is Σ^{ano} peak important?: How did the BCS el-ph mechanism become convincing?

strong-coupling superconductivity

Pb

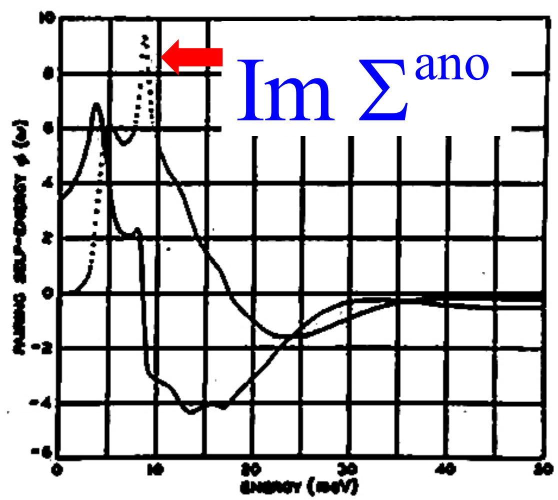


tunneling

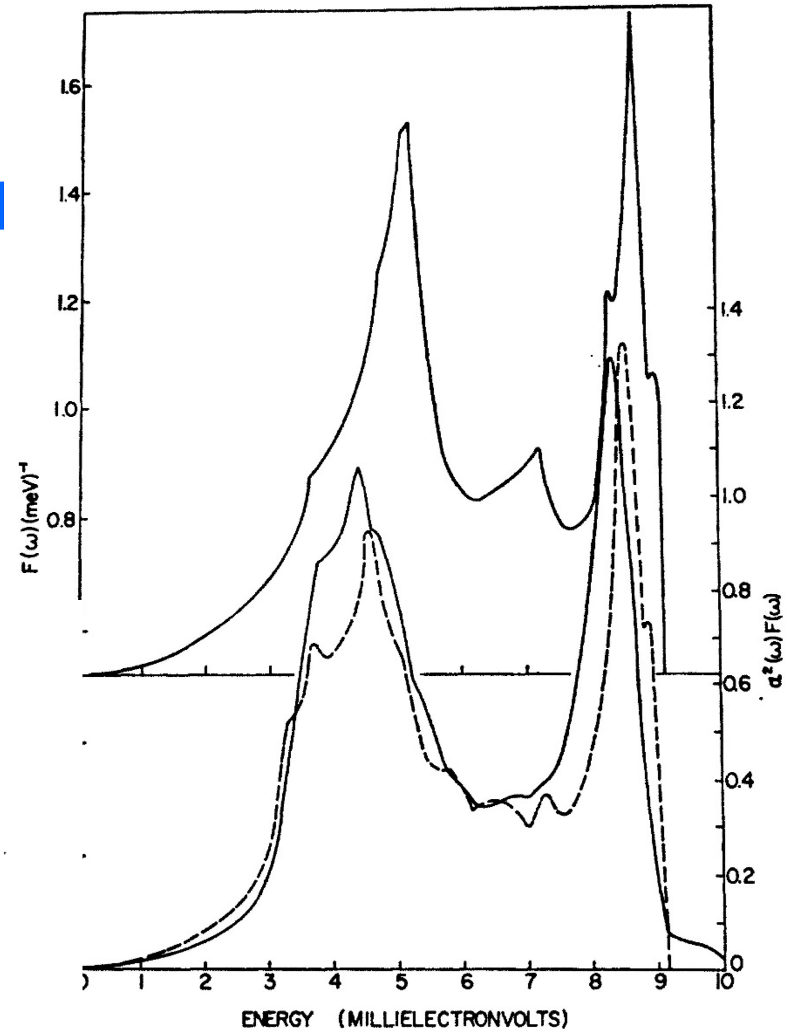
Eliashberg eq.

anomalous self-energy

$$\Delta(E) = z\Sigma^{ano}$$



Real (——) and imaginary (···) parts of the computed pairing self-energy $\phi(\omega)$ for Pb vs. $\omega - \Delta_0$.



Bosonic (phonon) glue makes peaks

McMillan-Rowell 1965, 1968

Scalapino, Schrieffer Wilkins 1968

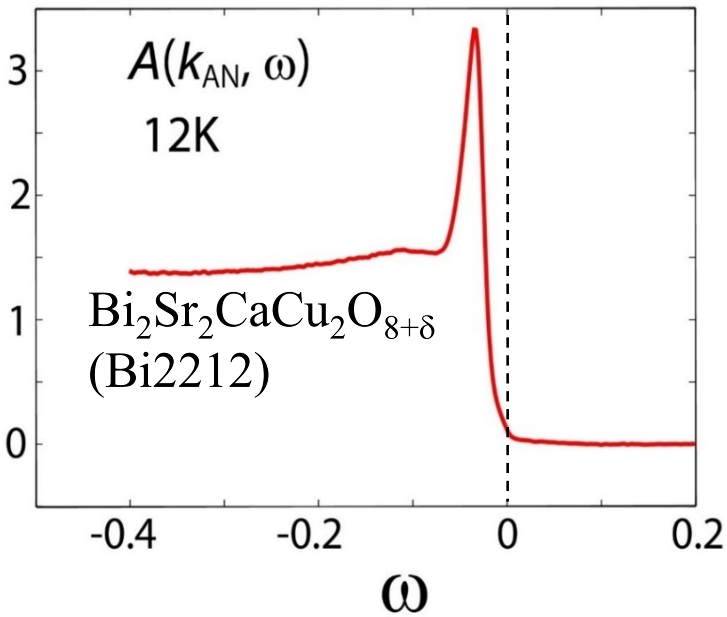
How about high T_c cuprate superconductors?

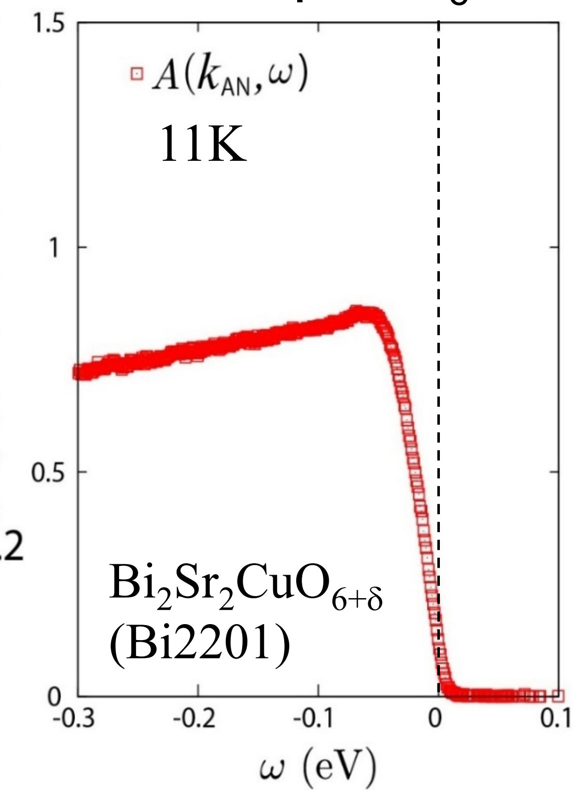
ARPES data **角度分解＝運動量分解**

spectral func. $A(k,\omega)=\mathrm{Im}\ G(k,\omega)$

Kondo *et al.*
Nature 457, 296 (2009)
Nat. Phys. 5, 21 (2010)

optimal doping T_c~90K underdoped T_c~29K





antinodal point
mild peak-dip-hump

Can we infer SC mechanism from *A*(*k*,ω)?
Thirty-years-long puzzle & challenge

One more puzzle; pseudogap

spin excitation NMR T_1 Yasuoka *et al.* (1989)

Knight shift Takigawa *et al.* (1991)

susceptibility Johnston, Nakano *et al.*

neutron Rossat-Mignod *et al.*

optical conductivity

specific heat Loram *et al.*

μSR

resistivity, Hall coefficient

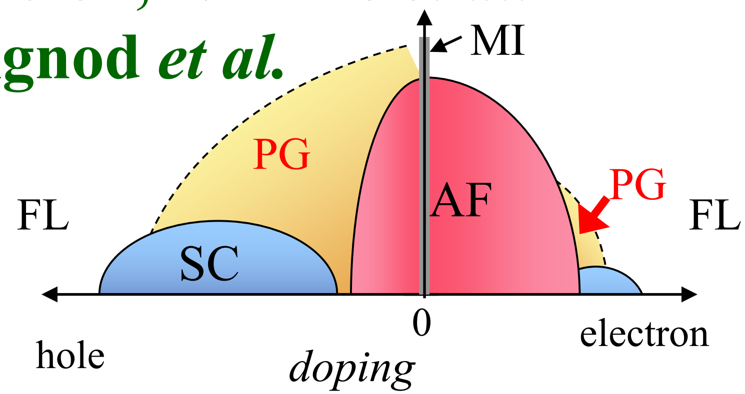
Raman

photoemission, ARPES Shen *et al.*

STM

tunnel conductance

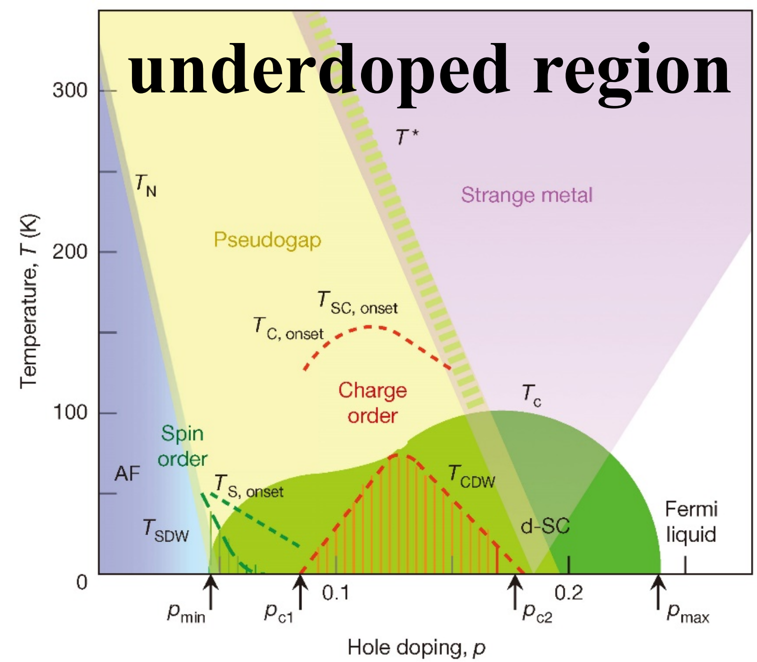
....



Mott gap beyond AF gap

strange metal $\rho \sim T$

Mott insulator



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Variational Monte Carlo

Tahara, MI
JPSJ 77 (2008), 114701

$$|\psi\rangle = \mathcal{L}_L \mathcal{P}_{\mathrm{J}} \mathcal{P}_{\text{d-h}}^{\text{ex.}} \mathcal{P}_{\mathrm{G}} \mathcal{L}^{S=0} |\phi_{\mathrm{pair}}\rangle$$

$$\left|\Phi_{\mathrm{pair}}\right\rangle = \left[\sum_{ij\sigma\tau} f_{ij\sigma\tau} c_{i\sigma}^{\dagger} c_{j\tau}^{\dagger}\right]^{N/2} |0\rangle$$

Pfaffian

f_{ij} : pair-dependent variational parameter

optimization of 1,000-100,000 variables to overcome bias

Represent strong entanglement in the real space representation

M. IMADA

FTTN combined with VMC

Zhao *et al.* PRB 96, 085103 (2017)

$$\left|\Psi\right\rangle = \mathcal{P}(\mathcal{L}^{K=0}\mathcal{L}^{C_4}\mathcal{M})\mathcal{L}^{K=0}\mathcal{L}^{S=0}\left|\phi_{\text{pair}}\right\rangle$$

$$\Rightarrow \mathcal{P}(\mathcal{L}^{K=0}\mathcal{L}^{C_4}\mathcal{M}) \sum_{\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N} \left|\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N\right\rangle\left\langle\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N\right|\mathcal{L}^{K=0}\mathcal{L}^{S=0}\left|\phi_{\text{pair}}\right\rangle$$

1. exact contraction, variational principle

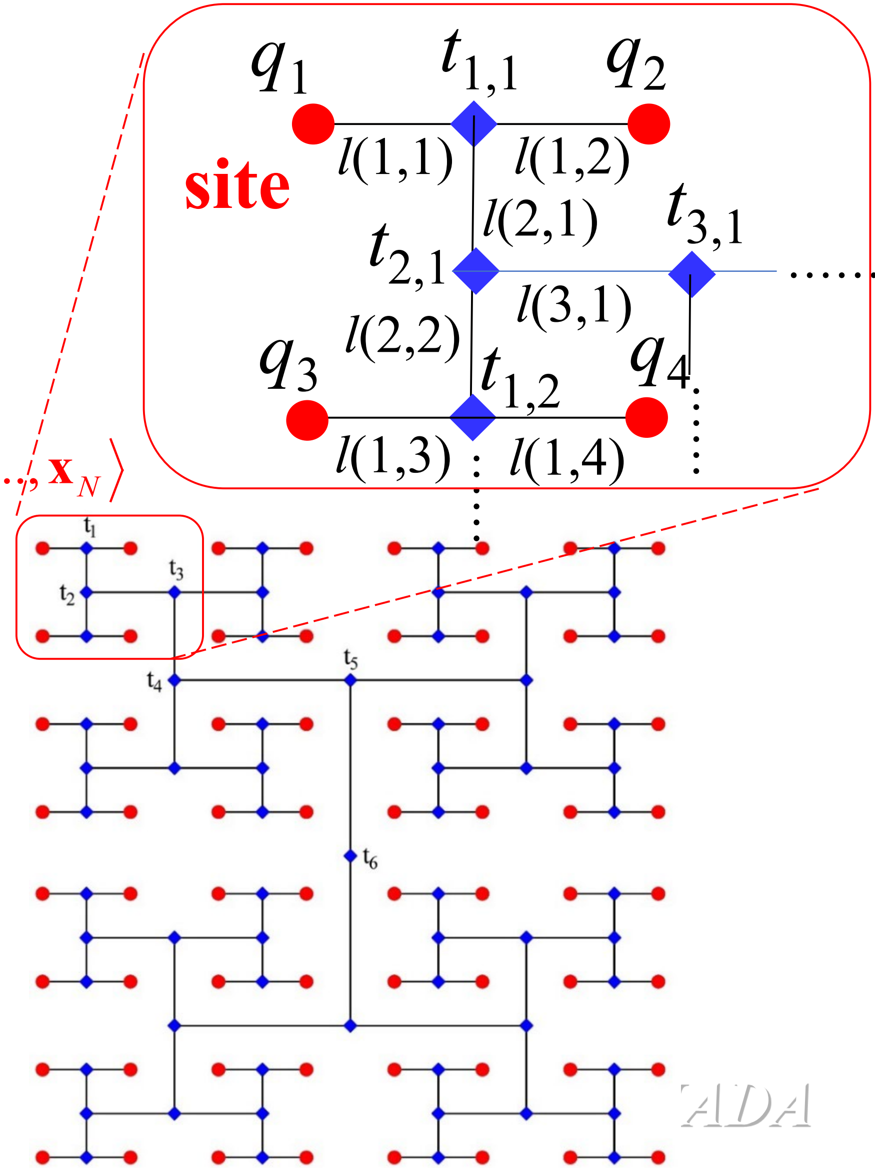
2. lattice symmetry preserving

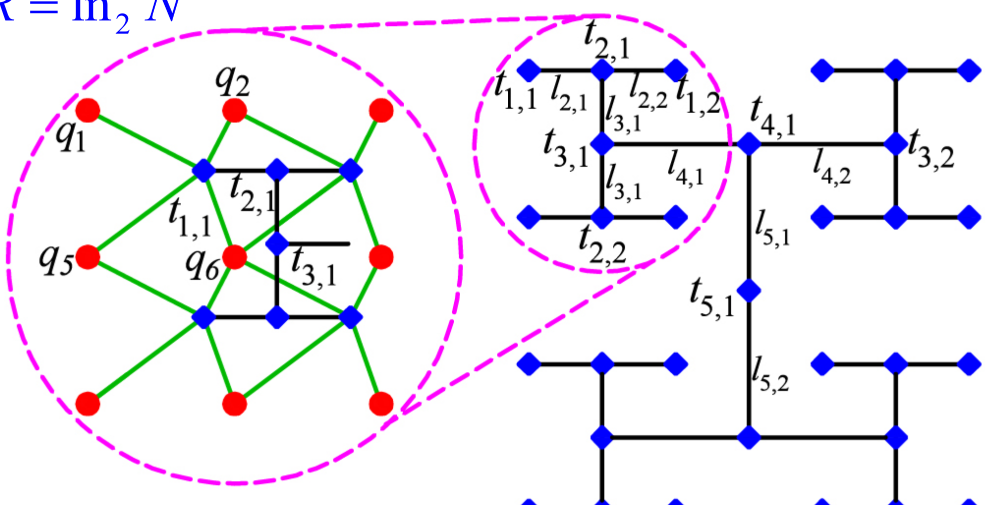
3. guaranteed convergence for $D\rightarrow\infty$

$$\mathcal{M}\left|q_1,q_2,\ldots,q_N\right\rangle = \sum_{\{l(j,i)\}=1}^{D}\prod_{i=1}^{N/2}\left[t_{1,i}(q_{2i-1},q_{2i},l_{2,i})\right] \times\prod_{j=2}^{R-1}\prod_{i=1}^{N/2^j}\left[t_{j,i}(l_{j,2i-1},l_{j,2i},l_{j+1,i})\right]t_{R,1}(l_{R,1},l_{R,2})\left|\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_N\right\rangle$$

$R = \ln_2 N$

TTN

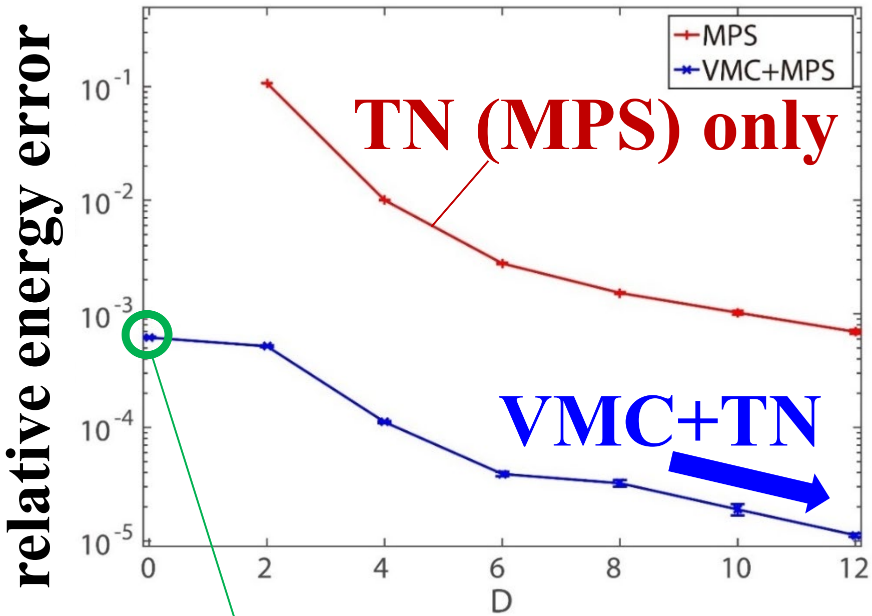




Fat tree tensor network (FTTN)

Efficiency and accuracy of TN+VMC

1D Hubbard,
U=10, L=16, N_e=10

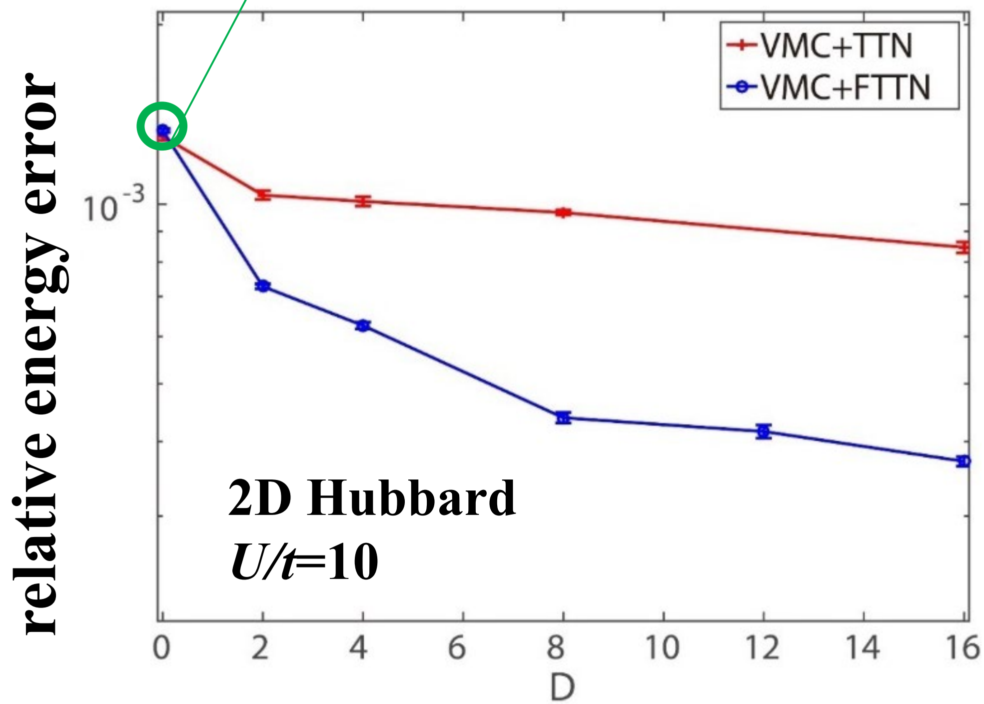


VMC only

tensor dimension

2D Hubbard, 4x4
U=10, N_e=10

VMC only



tensor dimension

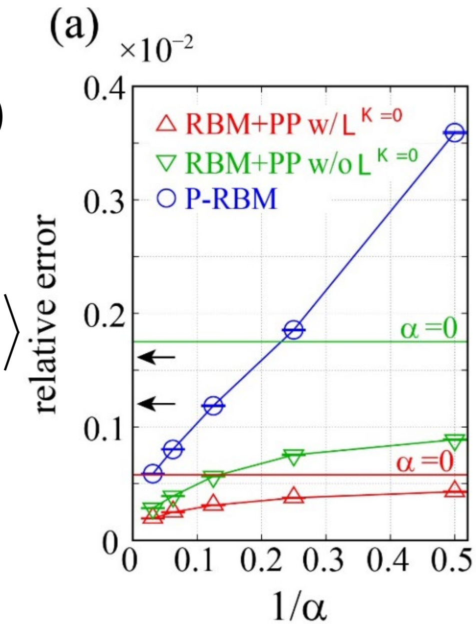
Zhao *et al.*
PRB 96, 085103 (2017)

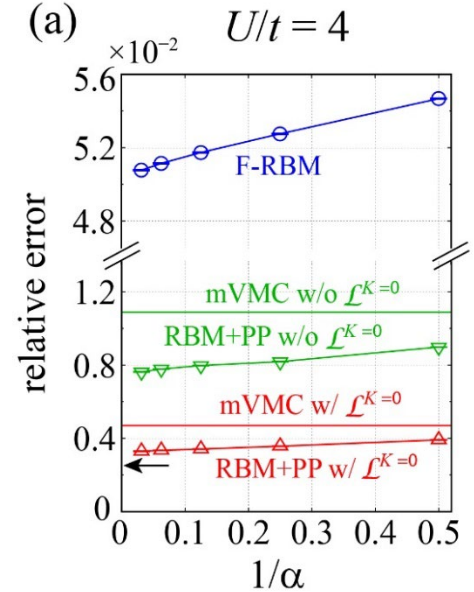
accuracy beyond each single method

cf. Chou *et al.*(2012)
Sikora *et al.* (2015)

Combined VMC and Neural Network

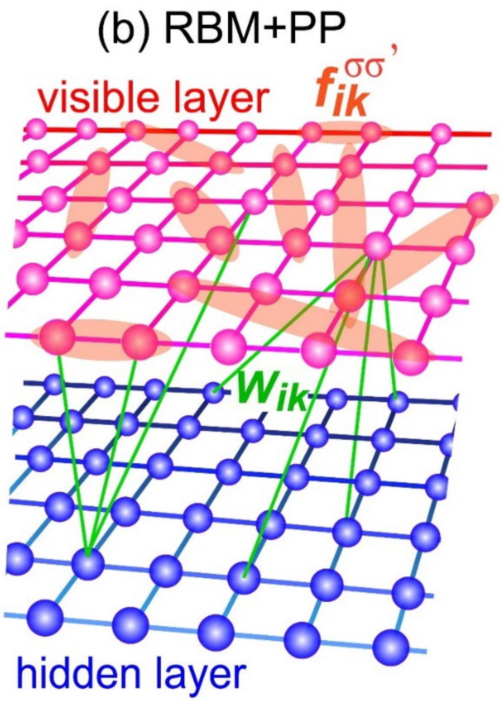
Nomura, Darmawan, Yamaji, Imada
PRB 96, 205152 (2017)





variational Monte Carlo +
neural network (machine learning)
restricted Boltzmann machine

$$\left|\Psi\right\rangle = \mathcal{P}(\mathcal{L}^{K=0}\mathcal{L}^{C_4}\mathcal{N})\sum_x |x\rangle\langle x|\mathcal{L}^{K=0}\mathcal{L}^{S=0}\left|\phi_{\text{pair-product}}\right\rangle$$

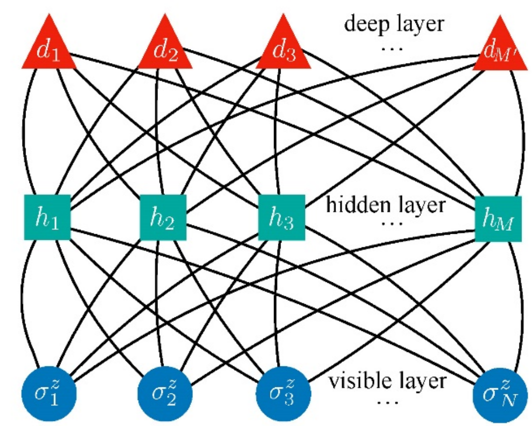


$$\mathcal{N} = \prod_{k,\sigma} 2\cosh(b_k + \sum_i W_{i\sigma k}(2n_{i\sigma} - 1))$$

$$\mathcal{N} = \prod_{k} 2\cosh(b_k + \sum_i W_{ik}\sigma_i^z)$$

$$\left|\phi_{\text{pair-product}}\right\rangle = \left[\sum_{ij} f_{ij} c^{\dagger}_{i\sigma} c_{j\sigma'}\right]^{N/2} \left|0\right\rangle$$

deep BM



***b*, *W*; variational parameter**
***σ*; physical variable**

Carleo, Nomura, Imada
Nat. Commun. (2018)

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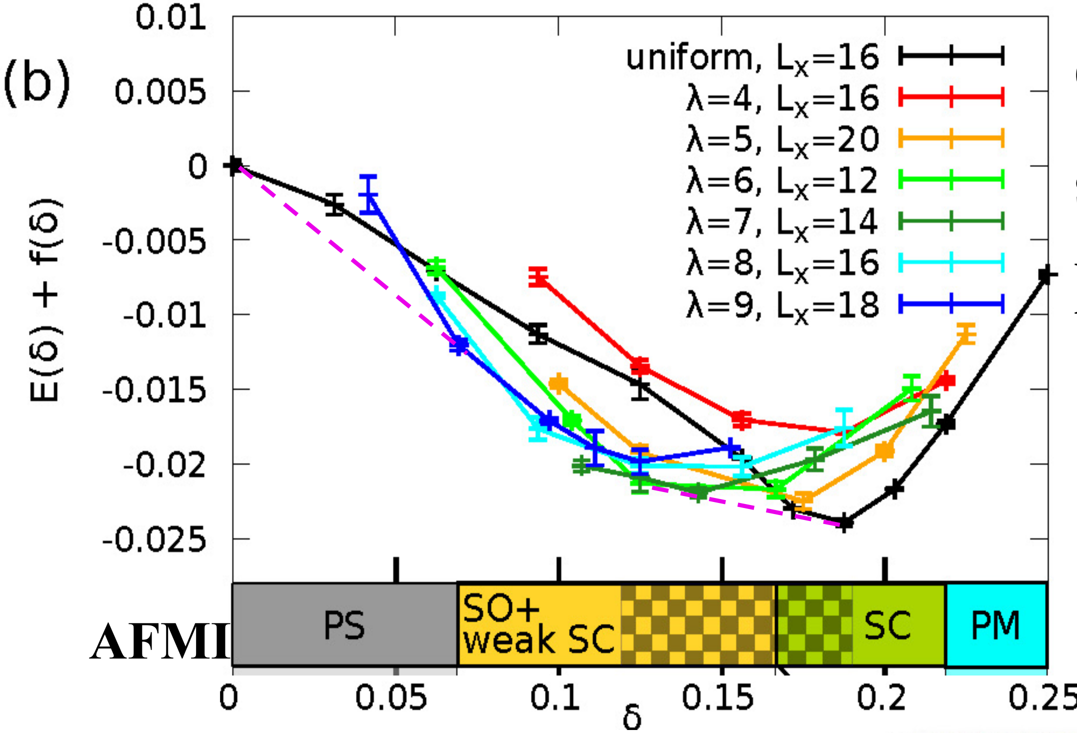
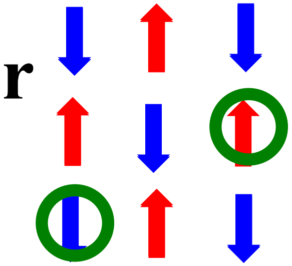
VMC+tensor network+Lanczos: Phase diagram of Hubbard model

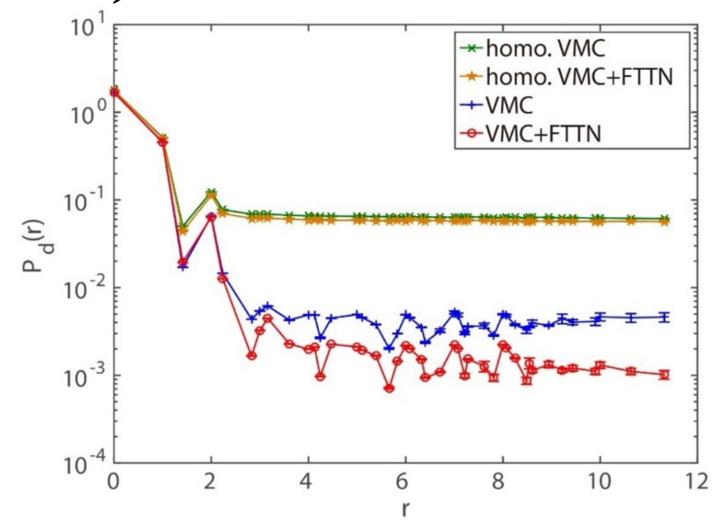
$$H = -\sum_i t_{ij}(c_{i\sigma}{}^{\dagger} c_{j\sigma} + \mathrm{h.c.}) + U \sum n_{i\uparrow} n_{i\downarrow}$$



doped Mott insulator

stripes with various periods (4~9)

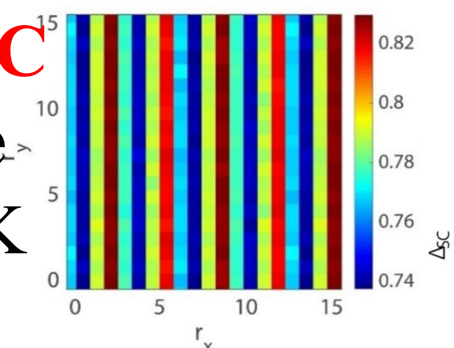




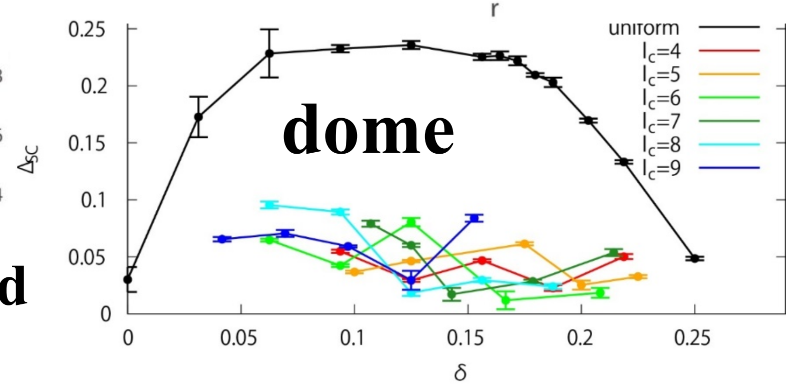
severe competitions with SC

with tiny energy difference

$\Delta E \leq 0.005t \sim 20\mathrm{K}$



Josephson coupled superconductor



SC G.S. confined in the overdoped region >0.2

Discrepancy from experiments

Darmawan, Yamaji, Nomura, Imada PRB2018

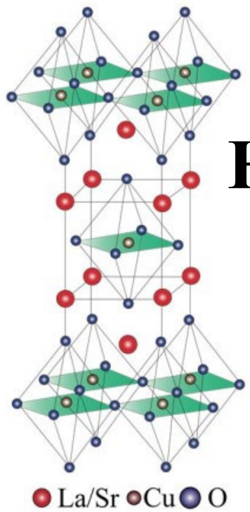
1st-principles Hamiltonian for curates

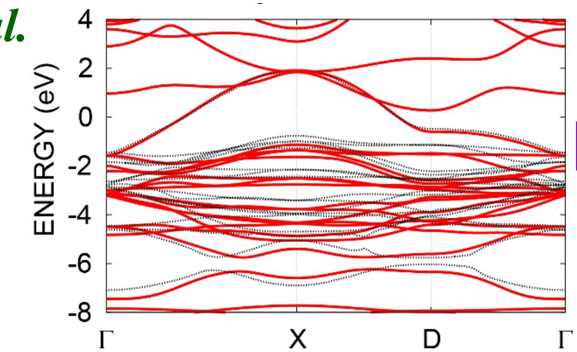
Hirayama *et al.* PRB(2018)
arXiv:1901.00763

MACE scheme for *ab initio* Hamiltonian

Aryasetiawan *et al.* PRB 2004
Hirayama *et al.* PRB 2013, 2017

$La_{2-x}Sr_xCuO_4$





RG-like partial trace: MACE

$HgBa_2CuO_4$
La_2CuO_4
1band (x^2-y^2)
2band (x^2-y^2, $3z^2$-r^2)
3band (x^2-y^2, $2p_\sigma$)
Hamiltonians

beyond cRPA

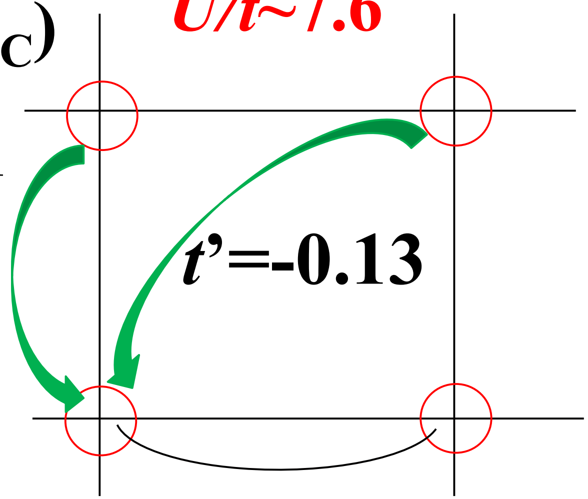
improved cGW-SIC method ($2p\sigma$ level feedback to satisfy $n_{GW} = n_{VMC}$)

$HgBa_2CuO_4$ 1-band Hamiltonian eV

One-body	t_1	t_2	t_3	t_4	t_5
parameters (eV)	0.509	-0.127	0.077	-0.018	-0.004
Two-body	U	V_1	V_2	V_3	V_4
parameters (eV)	3.846	0.834	0.460	0.318	0.271

U/t~7.6

t=0.51



U=3.85 *V*=0.83 eV

Hirayama, Misawa, Ohgoe, Yamaji, Imada

VMC+FTTN+LCZS for *ab initio* Hamiltonian of $HgBa_2CuO_4$

Mott gap ~2eV$_{\text{La based}}$
AF moment ~$0.6\mu_B$

Ohgoe, Hirayama,
Ido, Misawa, Yamaji,
Imada,

Uniform SC state is stabilized for *ab intio* case

**further neighbor
transfer and
interaction make
stripe energy higher**

experiments: $0.09<\delta<0.12$, q~0.25
W. Tabis, Y. Li, M. L. Tacon,
et al., Nat. Commun. 5, 5875 (2014).
G. Campi, A. Bianconi, et al.
S. M. Kazakov, et al., Nature 525,
359 (2015).

**V_3 partially
cancels V_1**

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Origin of gap (mass) generation

Spontaneous-symmetry-breaking gap

$$c_{k,\sigma}{}^{\dagger} c_{k+q,\sigma} d_{p,\sigma'}{}^{\dagger} d_{p-q,\sigma'} \Rightarrow \left\langle d_{p\sigma'}{}^{\dagger} c_{p\sigma} \right\rangle c_{k\sigma}{}^{\dagger} d_{k\sigma'}$$

emergent hybridization

$$H = \sum_{k\sigma\sigma'} [\varepsilon_c(k) c_{k\sigma}{}^{\dagger} c_{k\sigma} + \varepsilon_d(k) d_{k\sigma'}{}^{\dagger} d_{k\sigma'} - \Lambda(k)(c_{k\sigma}{}^{\dagger} d_{k\sigma'} + h.c)]$$

BCS **el-ph. int.**

$$c^{\dagger}{}_{k,\sigma} c_{k+q,\sigma} \left(b_q + b^{\dagger}{}_{-q} \right)$$

$$\Rightarrow c^{\dagger}{}_{k,\sigma} c_{k+q,\sigma} c^{\dagger}{}_{p,-\sigma} c_{p-q,-\sigma}$$

AF, CO: $d_{k,\sigma} = c_{k+Q,\sigma}$, SC: $d_{k,\sigma} = c_{-k,-\sigma}{}^{\dagger}$

➧ **hybridization gap of two-component fermions**

$$G(\omega) = (\omega - H)^{-1} = \begin{pmatrix} \omega - \varepsilon_c & -\Lambda \\ -\Lambda & \omega - \varepsilon_d \end{pmatrix}^{-1} = \frac{1}{(\omega - \varepsilon_c)(\omega - \varepsilon_d) - \Lambda^2} \begin{pmatrix} \omega - \varepsilon_d & \Lambda \\ \Lambda & \omega - \varepsilon_c \end{pmatrix}$$

$$G_c = \frac{\omega - \varepsilon_d}{(\omega - \varepsilon_c)(\omega - \varepsilon_d) - \Lambda^2} = \frac{1}{(\omega - \varepsilon_c) - \dfrac{\Lambda^2}{\omega - \varepsilon_d}} \Longleftrightarrow G_c^{\text{bare}} = \frac{1}{\omega - \varepsilon_c}$$

$$\Sigma_c^{\text{nor}}(k, \omega) = \frac{\Lambda_k{}^2}{\omega - \varepsilon_d}$$

pole of G_d^{bare} ⇔ pole of Σ_c^{nor} ⇔ zero of G_c

if $\varepsilon_d = -\varepsilon_c \Rightarrow \omega = \sqrt{\varepsilon_c(k)^2 + \Lambda_k{}^2}$ **mass generation**

Origin of gap (mass) generation

Nambu-Jona Lasinio mechanism PR 122, 345 (1961)

Case of QCD **strong interaction**

$$L = \bar{q}(x)(i\gamma^{\mu} D_{\mu} - M)q(x) - \frac{1}{2}\mathrm{Tr}\left[G_{\mu\nu}(x)G^{\mu\nu}(x)\right]$$

$$q(x) = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad M = \begin{pmatrix} m_u, 0, & 0 \\ 0, & m_d, 0 \\ 0, & 0, & m_s \end{pmatrix}, \; D_{\mu} = \partial_{\mu} - ig_s G_{\mu} T,$$

quark interaction through gluon *G*

$$c^{\dagger} d \left(b + b^{\dagger}\right) \Rightarrow c^{\dagger} d c^{\dagger} d$$

$T; SU(3)$ generator

quark mass $\Rightarrow$ **chiral symmetry breaking**

$$\left\langle \bar{q}_L(x) q_R(x) \right\rangle \neq 0$$

gluon condensation;

nonperturbative effect

quark-antiquark condensate

$$c^{\dagger} d \left(b + b^{\dagger}\right) \Rightarrow c^{\dagger} d \left(\langle b \rangle + \langle b^{\dagger} \rangle\right)$$

vacuum condensation

similar for weak interaction

W-, Z-boson condensation

Higgs mechanism

All require SSB.

How about the pseudogap and Mott gap if SSB is absent?

How about Mott gap?

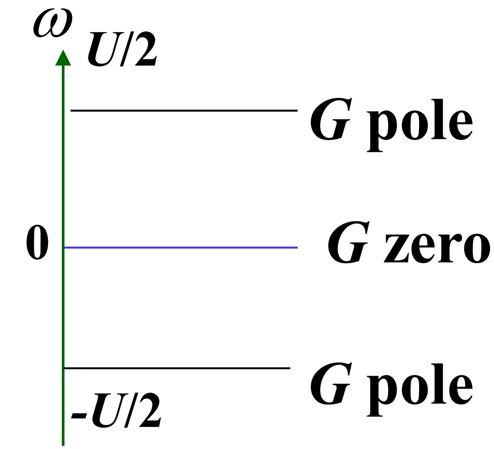
Zhu, Zhu PRB 87, 085120 (2013)
Sakai *et al.* PRB 94, 115130 (2016)

$$H = U(c_\sigma{}^\dagger c_\sigma - \frac{1}{2})(c_{-\sigma}{}^\dagger c_{-\sigma} - \frac{1}{2}) = -\frac{U}{2}\sum_\sigma c_\sigma{}^\dagger c_\sigma + U n_\sigma n_{-\sigma}$$

***t*→0 atomic limit; exact**

$$G = \frac{1}{2}\frac{1}{\omega + \frac{U}{2}} + \frac{1}{2}\frac{1}{\omega - \frac{U}{2}} \Leftrightarrow G = \frac{1}{\omega - \Sigma}, \quad \Sigma = \frac{\frac{1}{4}U^2}{\omega}$$

lower Hubbard **upper Hubbard**



$$f_\sigma \equiv c_\sigma (1 - 2n_{-\sigma})$$

$$-c_\sigma{}^\dagger f_\sigma = -c_\sigma{}^\dagger c_\sigma (1 - 2n_{-\sigma})$$
$$= -n_\sigma + 2n_\sigma n_{-\sigma}$$

$$H = \sum_\sigma [\varepsilon_c c_\sigma{}^\dagger c_\sigma + \varepsilon_f f_\sigma{}^\dagger f_\sigma + \Lambda(c_\sigma{}^\dagger f_\sigma + h.c)]$$

$$\varepsilon_c = \varepsilon_f = \Lambda = -\frac{U}{8}$$

***Λ*, a constant -*U*/8, has *nothing to do* with the symmetry breaking**

contrast to AF;
Slater mechanism

$$f_{k,\sigma} \equiv c_{k+Q,\sigma}, \quad \Lambda = U\left\langle c_{k,\sigma}{}^\dagger c_{k+Q,\sigma}\right\rangle$$

Composite fermion for Mott gap

c, f : fermion operator,

orthogonal as an average at half filling $\langle n_\sigma \rangle = 1$:

exact fermion anticommutation of c and f

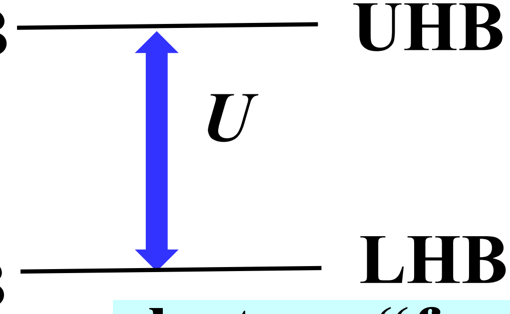
$$\left[c_1, f^\dagger\right]_+ = 0,\ \left[f, f^\dagger\right]_+ = 1,\ .....$$

diagonalization of $H = \sum_\sigma [\varepsilon_c c_\sigma{}^\dagger c_\sigma + \varepsilon_f f_\sigma{}^\dagger f_\sigma + \Lambda(c_\sigma{}^\dagger f_\sigma + h.c)], \varepsilon_c = \varepsilon_f = \Lambda = -\frac{U}{8}$

$\Rightarrow$

$$\frac{1}{2}(c_\sigma{}^\dagger - f_\sigma{}^\dagger) = c_\sigma{}^\dagger n_{-\sigma}$$ **UHB**

$$\frac{1}{2}(c_\sigma{}^\dagger + f_\sigma{}^\dagger) = c_\sigma{}^\dagger (1 - n_{-\sigma})$$ **LHB**



antibonding of *c* and *f*

bonding

electron "fractionalization"

$$c_\sigma = c_\sigma(n_{-\sigma} + (1 - n_{-\sigma}))$$

UHB ($n_{-\sigma}$) **LHB** ($1 - n_{-\sigma}$)

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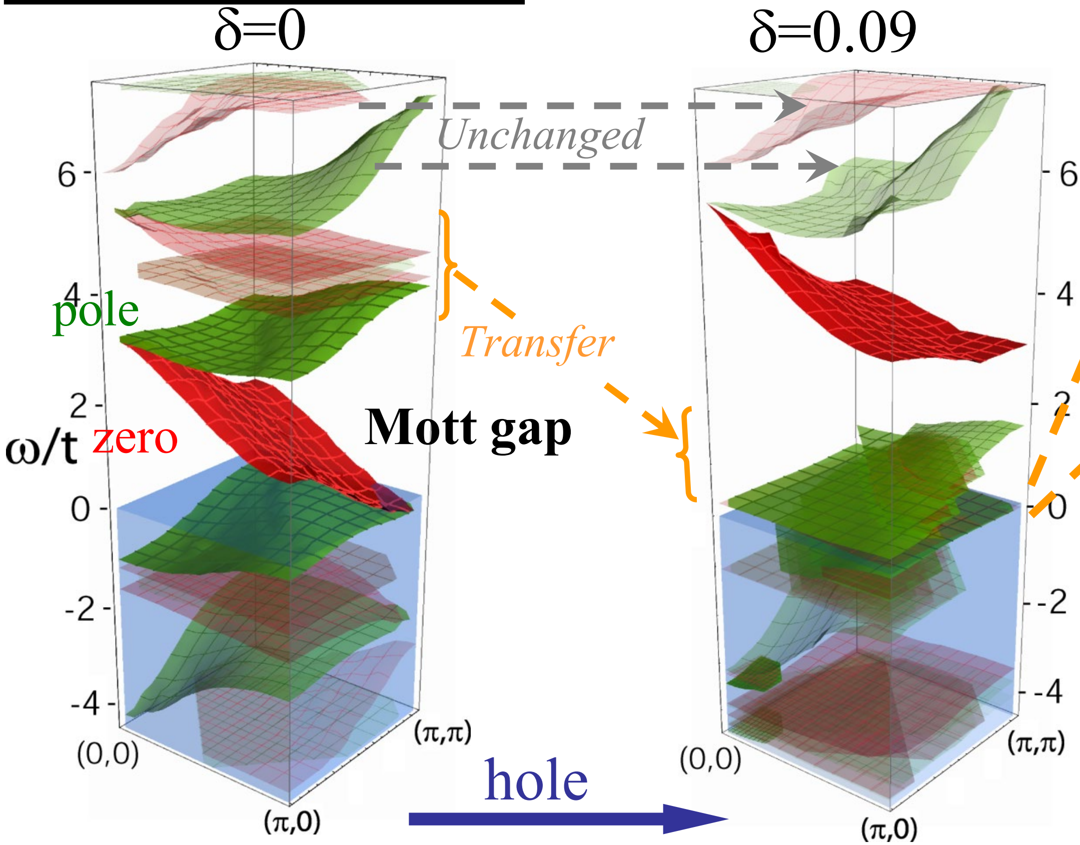
M. IMADA

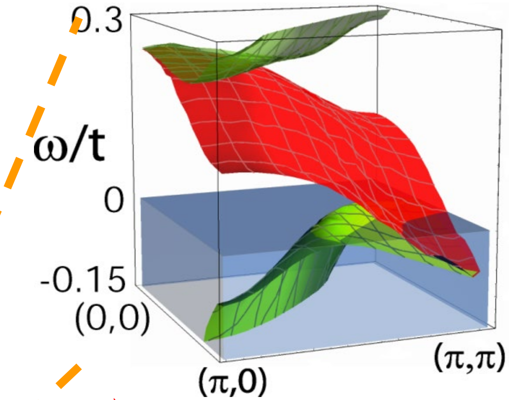
Mott gap and pseudogap, itinerancy and carrier doping

- **CDMFT+ED**
- ***U*=8*t*, *t'*=0, *T*=0**

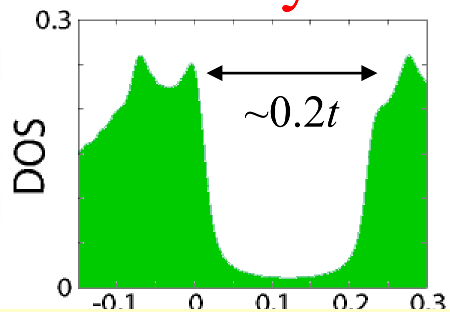
keep paramagnetic



A small-energy zero surface



Pseudogap
w/o symmetry breaking



Sakai, Motome, Imada
PRL 102 (2009) 056404

suggest emergence of hybridizing d ; two-component fermion

$$H = \sum_k [\varepsilon_c(k) c_{k\sigma}^{\dagger} c_{k\sigma} + \Lambda_k (c_{k\sigma}^{\dagger} d_{k\sigma} + \mathrm{H.c.}) + \varepsilon_d(k) d_{k\sigma}^{\dagger} d_{k\sigma}]$$

Central question: What is *d*?

Smoking gun for the hidden fermion

Two-component fermion model at $T<T_c$

$$H = \sum_k [\varepsilon_c(k) c_{k\sigma}^\dagger c_{k\sigma} + \Lambda_k (c_{k\sigma}^\dagger d_{k\sigma} + \text{H.c.}) + \varepsilon_d(k) d_{k\sigma}^\dagger d_{k\sigma}$$

$$+(\Delta_c c_{k\sigma}^\dagger c_{k\sigma}^\dagger + \Delta_d d_{k\sigma}^\dagger d_{k\sigma}^\dagger + \text{H.c.})]$$

Nambu representation
4x4 matrix

$$\Sigma_c^{\text{nor}}(k,\omega) = G_d \Lambda_k^2 = \frac{\Lambda_k^2}{(\omega - \varepsilon_d) - \dfrac{\Delta_d^2}{\omega + \varepsilon_d}}$$

$$\Sigma_c^{\text{ano}}(k,\omega) = \Delta_c + G_d \Lambda_k^2 = \Delta_c + \frac{\Lambda_k^2 \Delta_d (\omega + \varepsilon_d)}{(\omega - \varepsilon_d) - \dfrac{\Delta_d^2}{(\omega + \varepsilon_d)}}$$

$$G(\mathbf{k},\omega) = \left[\omega + \mu - \varepsilon_\mathbf{k} - \Sigma_c^{\text{nor}}(\mathbf{k},\omega) - W(\mathbf{k},\omega)\right]^{-1}$$

The poles cancel in $\Sigma^{\text{nor}}+W$

$$W(\mathbf{k},\omega) = \frac{\Sigma_c^{\text{ano}}(\mathbf{k},\omega)^2}{\omega - \mu + \varepsilon_\mathbf{k} + \Sigma_c^{\text{nor}}(\mathbf{k},-\omega)^*}$$

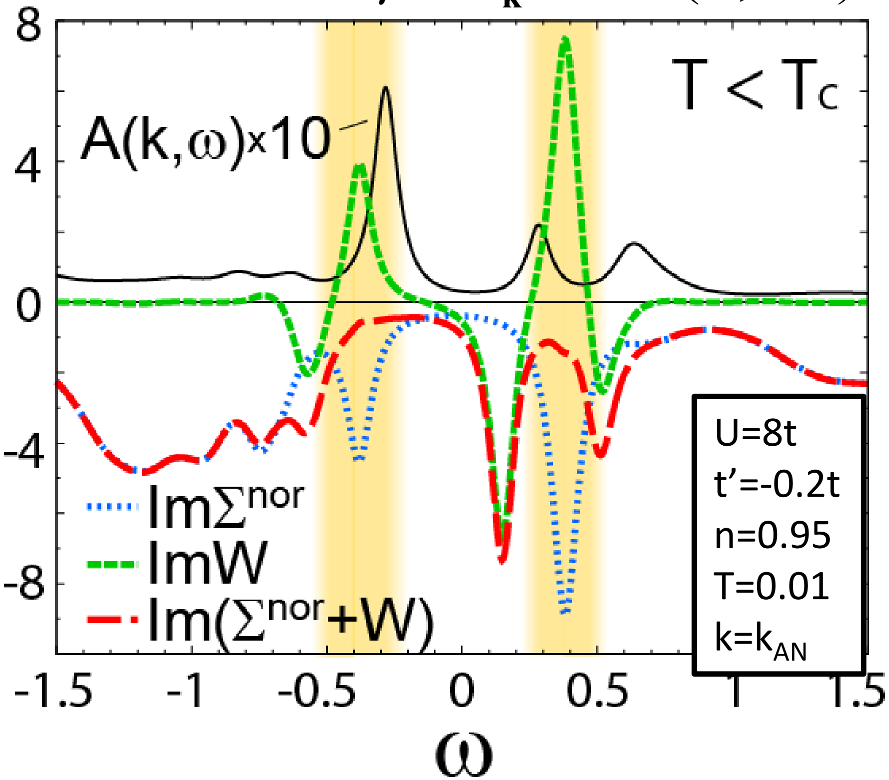
smoking gun

Pole cancellation between Σ^{nor} and Σ^{ano}

$$G(\mathbf{k},\omega)=\left[\omega+\mu-\varepsilon_{\mathbf{k}}-\Sigma^{nor}(\mathbf{k},\omega)-W(\mathbf{k},\omega)\right]^{-1}$$

Sakai *et al.*
Phys. Rev. Lett.
116 (2016) 057003

$$W(\mathbf{k},\omega)=\frac{\Sigma^{ano}(\mathbf{k},\omega)^2}{\omega-\mu+\varepsilon_{\mathbf{k}}+\Sigma^{nor}(\mathbf{k},-\omega)^*}$$



$$A(\mathbf{k},\omega)=-\frac{1}{\pi}\mathrm{Im}\,G(\mathbf{k},\omega)$$

Σ^{nor} cancels with W !

Perfect agreement with TCFM

Poles of $\Sigma^{nor/ano}$ are invisible in $A(\mathbf{k},\omega)$. = The reason why overlooked in experiments

If the peak directly comes from bosonic excitations such as spin fluctuations, the cancellation does not happen.

Evidence for hidden fermion

Why is the peak important?

Im Δ peak generates high T_c

cluster DMFT
2x2 cluster

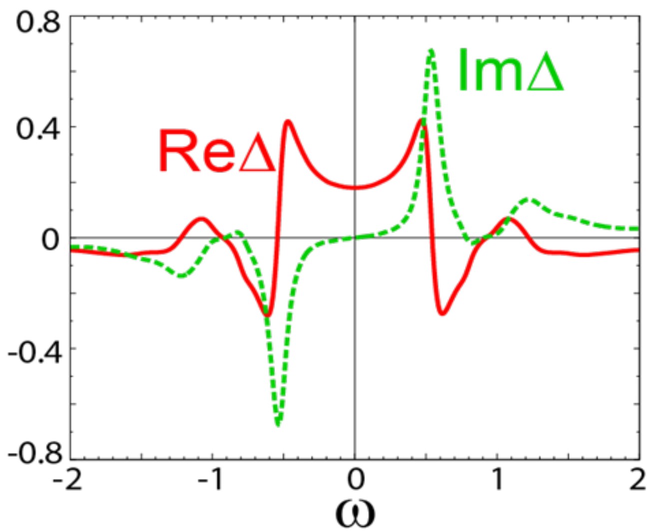
Sakai, Civelli, Imada
PRL116 (2016) 057003

Gap function

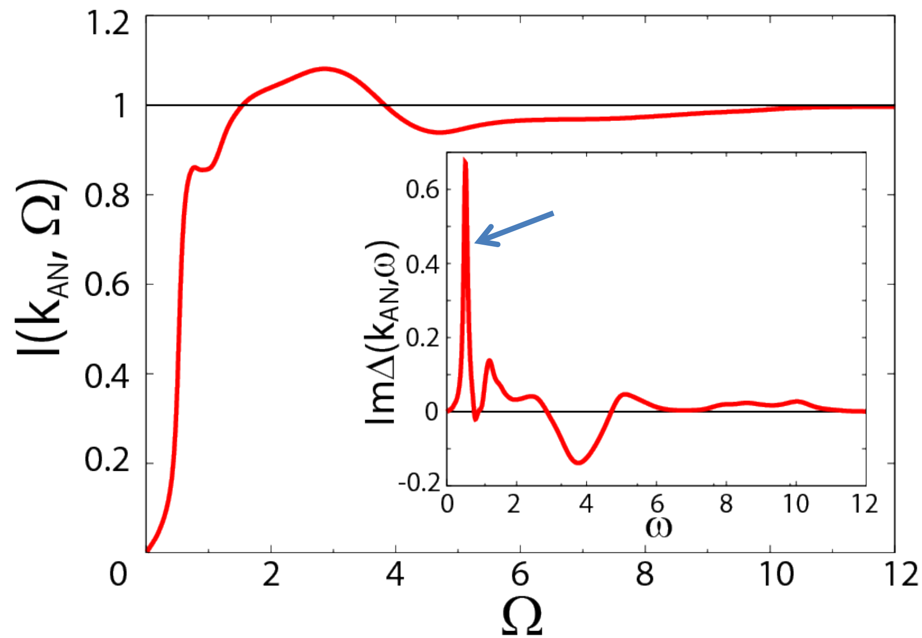
$\Delta(\omega) = z\,\Sigma^{ano}$

$$\mathrm{Re}\,\Delta(\mathbf{k},\omega=0) = \frac{2}{\pi}\int_0^{\infty} \frac{\mathrm{Im}\,\Delta(\mathbf{k},\omega')}{\omega'} d\omega'$$ **Kramers Kronig**

$$I(\mathbf{k},\Omega) = \frac{2}{\pi\,\mathrm{Re}\,\Delta(\mathbf{k},\omega=0)}\int_0^{\Omega} \frac{\mathrm{Im}\,\Delta(\mathbf{k},\omega')}{\omega'} d\omega'$$



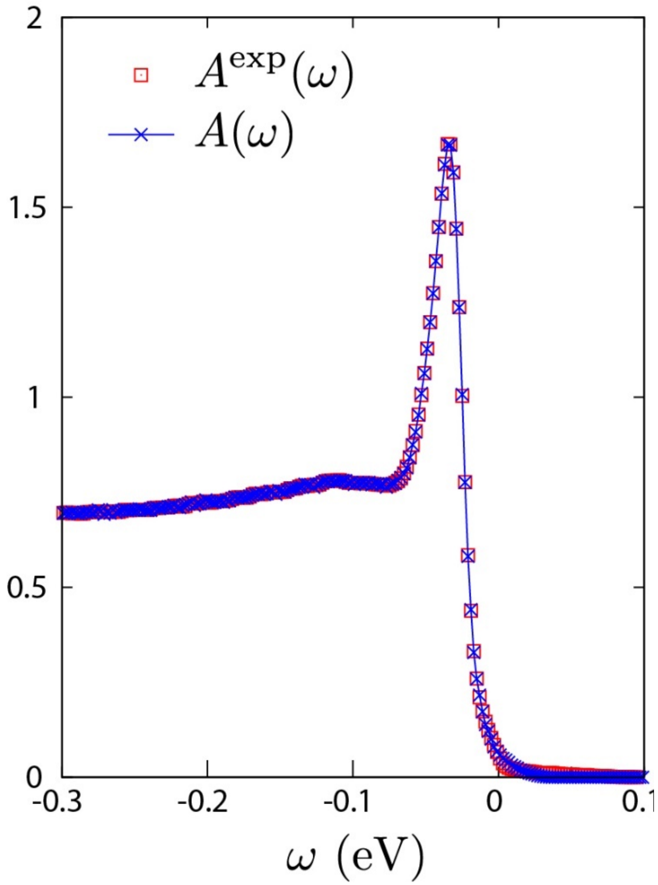
cf. Maier, Poiblanc, Scalapino, PRL'08

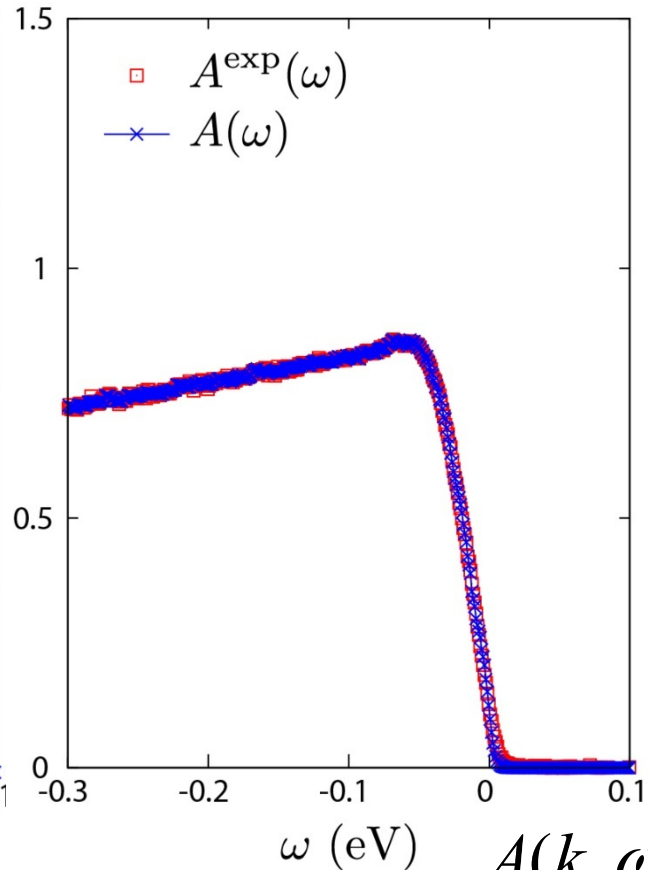


80% of the gap is attributed to the peak!

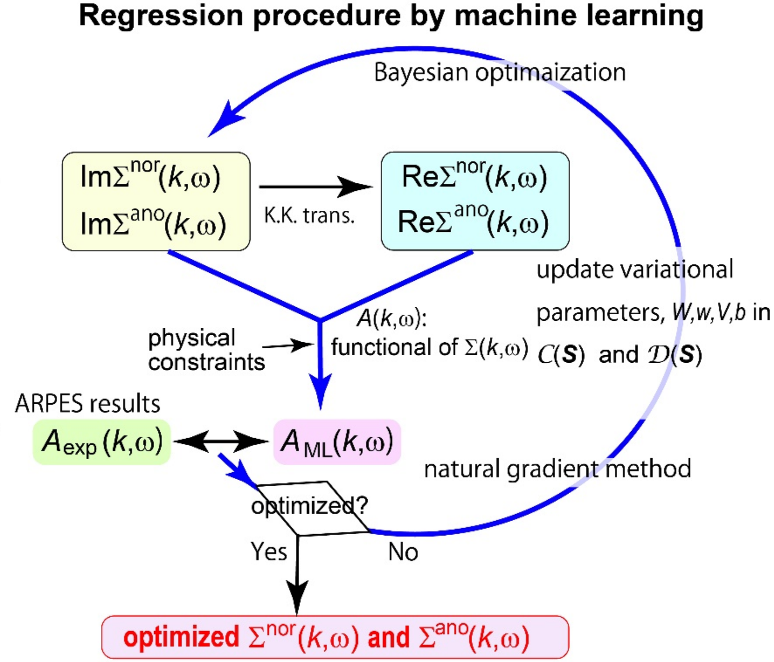
What makes this peak? = What is the hidden fermion?

$A(k,\omega)$; comparison between ARPES data and machine learning





antinodal point



$$A(k,\omega) \Leftrightarrow \mathcal{A}\left(\left\{\mathrm{Im}\,\Sigma^{\mathrm{nor}}(k,\omega), \mathrm{Im}\,\Sigma^{\mathrm{ano}}(k,\omega)\right\}\right)$$

optimal Bi2212

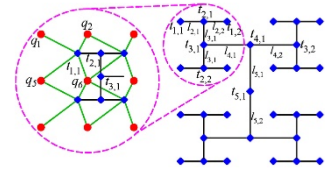
underdoped Bi2201

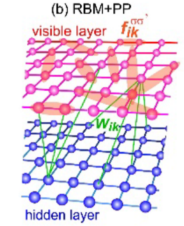
solve inverse problem by machine learning (BM)

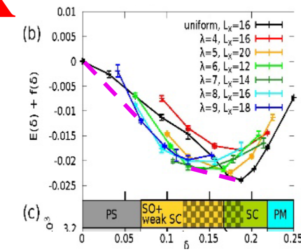
perfect agreement

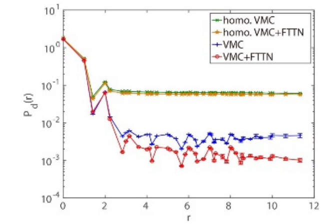
Summary

1. VMC, tensor network, neural network
   as quantum many-body solvers
   without sign problem

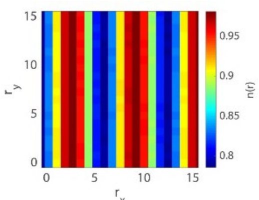


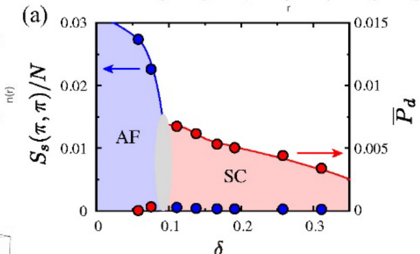




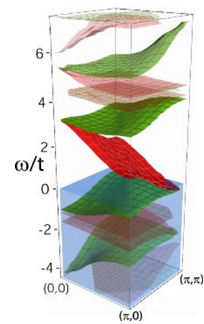


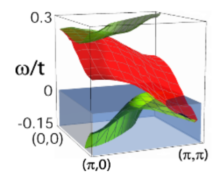
2. Severe competition between SC and charge inhomogeneity (PS and stripes)

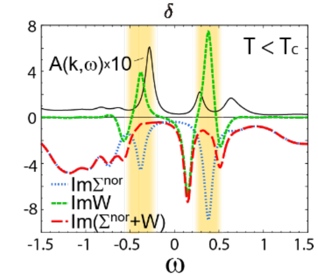




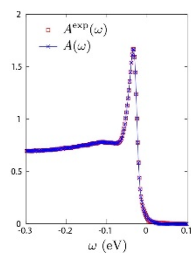
3. *Ab initio* Hamiltonian reproduces
   the phase diagram of cuprates







4. Universal gap (mass) generation by
   hybridization gap without SSB
   (Mott gap, pseudogap)

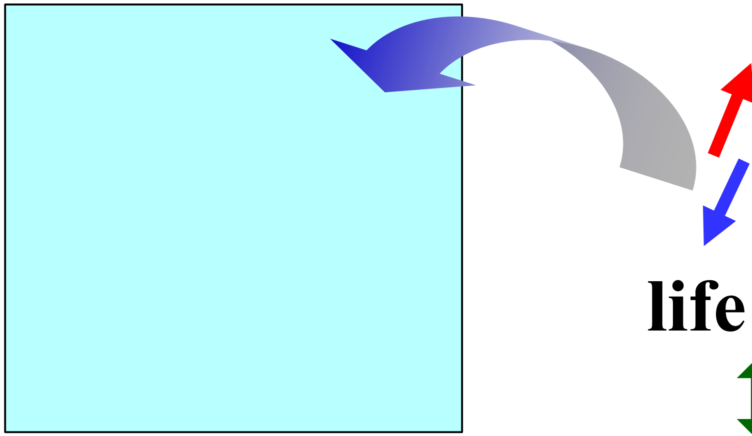


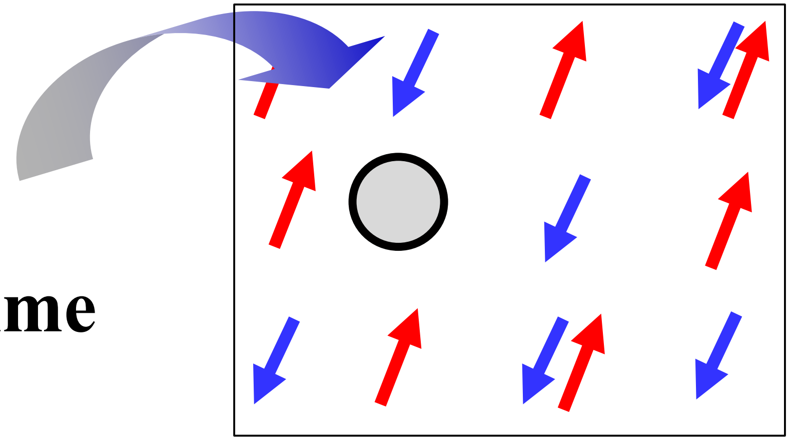
5. Prominent self-energy peak generating SC
   with cancellation ⇒ dark fermion theory

6. Machine learning of ARPES supports
   dark fermion theory and Planckian dissipation

Candidate of hidden fermion: Quasiparticle vs. Composite fermion

Yamaji & Imada
PRL101 (2011) 016404
PRB 83 (2011) 214522





life time

⬍

hybridization

quasiparticle:

kinetic energy gain

composite fermion:

interaction energy gain

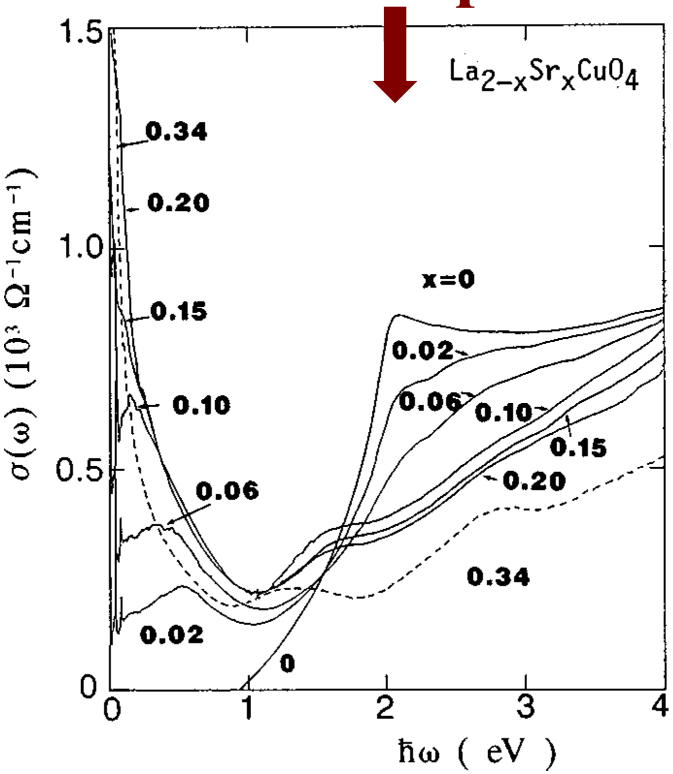
natural extension of exciton

incoherent part of fermion

Excitons ⇔ Mott gap

Uchida *et al.* 1989

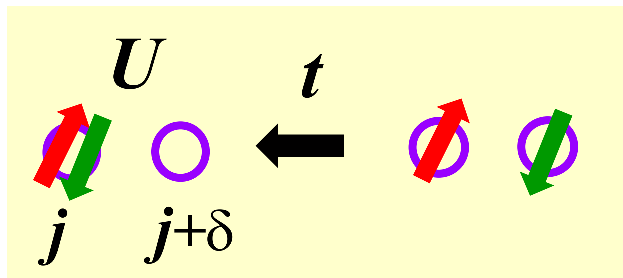
exciton peak



exciton creation

doublon-holon pair

$$b^{\dagger}_{j,\delta,\sigma} \equiv c^{\dagger}_{j\sigma} c_{j+\delta,\sigma} n_{j,-\sigma} (1 - n_{j+\delta,-\sigma})$$



$$\varsigma \sum_{j,\delta} (b_{j,\delta}^{\dagger} + b_{j,\delta})$$

$$\varsigma \sim t$$

upper Hubbard band

$$\langle b_{j,\delta}^{\dagger} \rangle = \langle b_{j,\delta} \rangle^{*} \neq 0$$

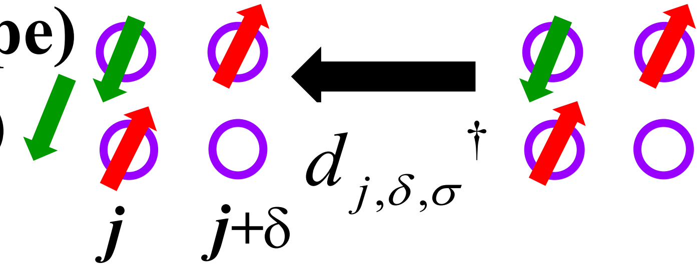
Bose condensation, without spontaneous symmetry breaking

contrast with el.-photon and el.-phonon coupled system, W, Z bosons and gluons

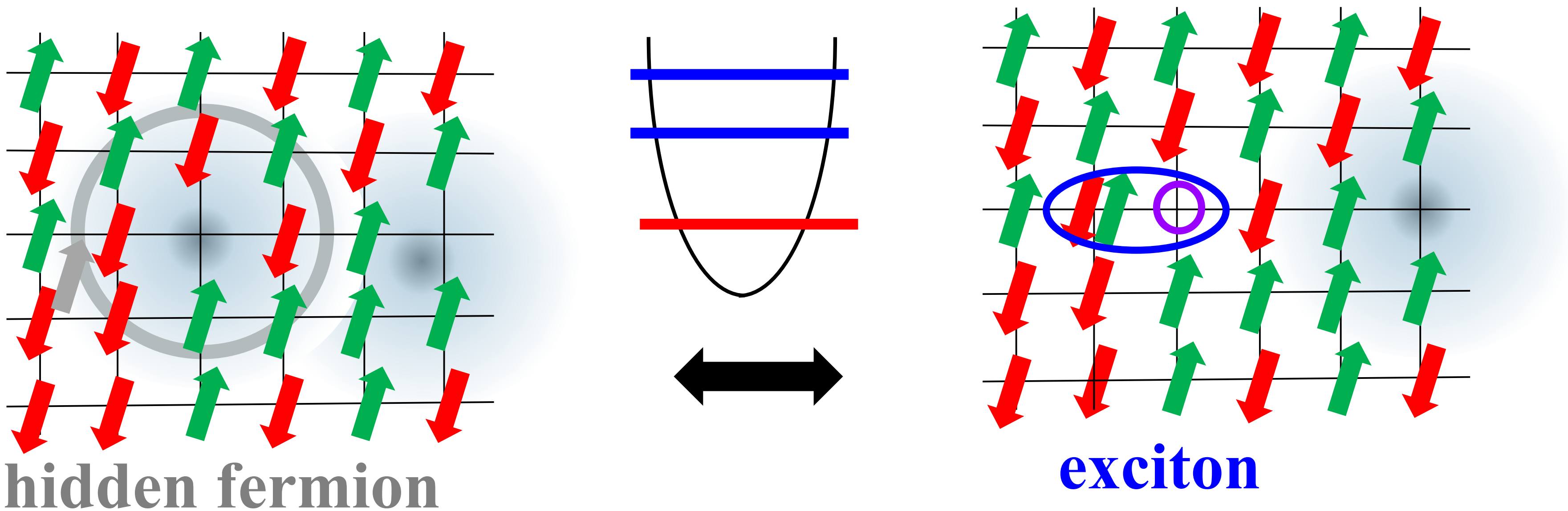
Exciton and hidden fermion

electron weakly bound to a hole (Wannier type)

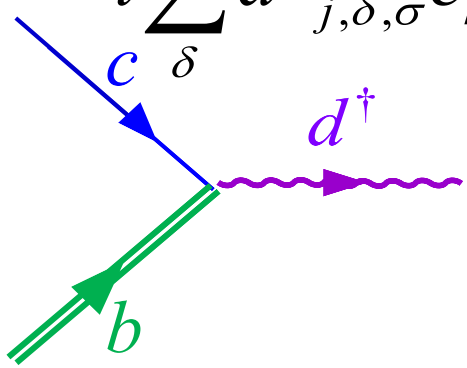
$$d_{j,\delta,\sigma}^{\dagger} = g_{d\delta} c_{j\sigma}^{\dagger}(1-\alpha_\delta n_{j+\delta,-\sigma})(1-\beta_\delta n_{j+\delta,\sigma})$$



hidden fermion creation $\alpha_\delta \sim 1, \beta_\delta \sim 1$



$$t\sum_{\delta} d^{\dagger}{}_{j,\delta,\sigma} c_{k\sigma} b_{j,\delta} + \text{H.c.} \Longrightarrow t\sum_{\delta} d^{\dagger}{}_{j,\delta,\sigma} c_{k\sigma} \langle b_{j,\delta} \rangle + \text{H.c.}$$



hybridization gap $\sim t^2/U$

Imada, Suzuki, JPSJ (2019)