Quantum dynamics of continuously monitored many-body systems

Masahito Ueda

University of Tokyo, APSA
RIKEN Center for Emergent Matter Science

Outline

1. Continuously monitored many-body dynamics
   - 1-1. quantum statistics
   - 1-2. quantum criticality
   - 1-3. full-counting dynamics
   - 1-4. thermalization

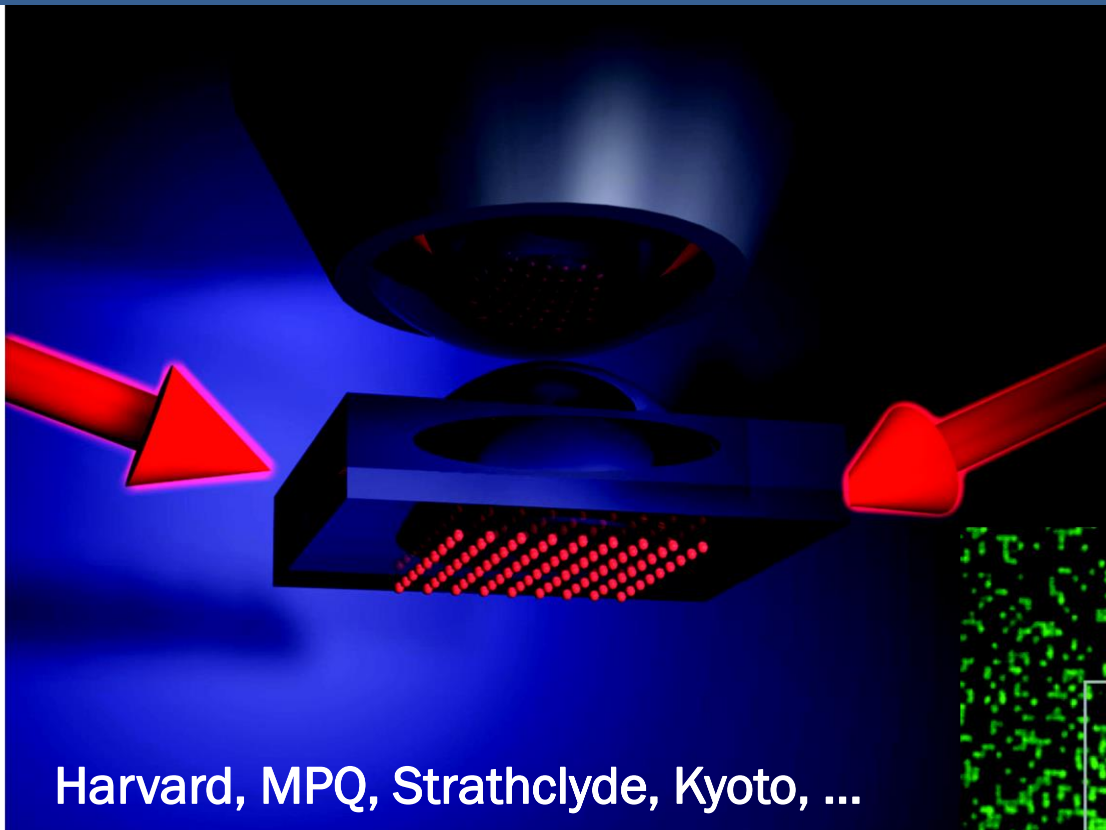
2. Quantum Zeno dynamics
   - 2-1. Zeno Hall effect
   - 2-2. retroreflection

Continuously monitored many-body dynamics

Yuto Ashida

Shunsuke Furukawa

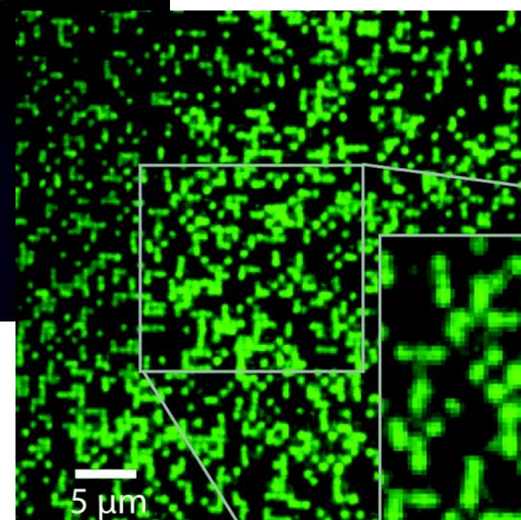
Motivation –Quantum Gas Microscopy –

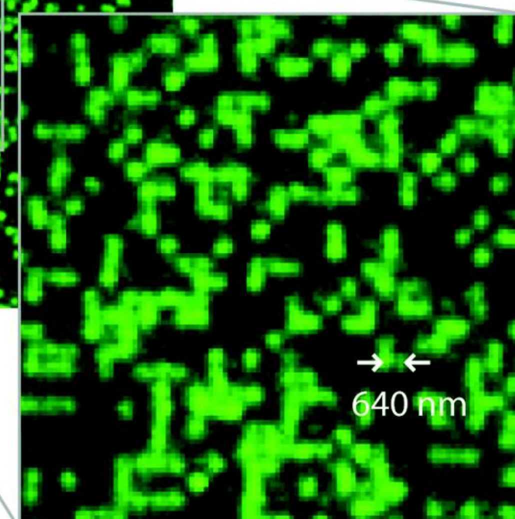


Enables observation of the many-body wave function at the single-site resolution.

↓ **Heisenberg's uncertainty relation**

The higher the resolution, the greater the backaction will be.





single-shot image

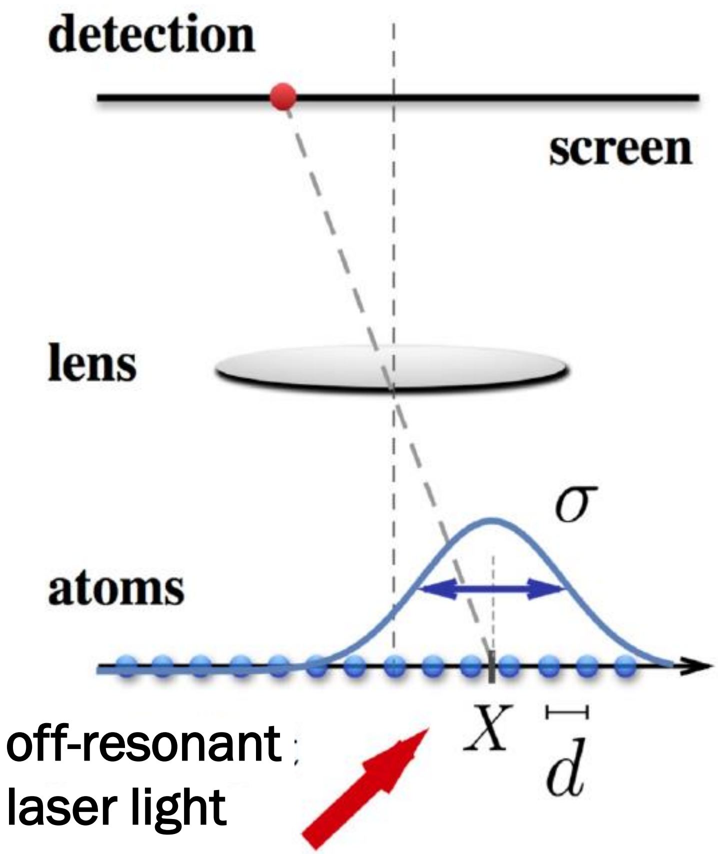
How are the many-body dynamics and quantum phase transitions altered by the measurement backaction?

Frontier of QGM requires generalization of quantum-statistical mechanics that incorporates effects of measurement backaction.

Multi-Particle Quantum Dynamics under Continuous Observation

Model

Our model is to continuously monitor atoms in an optical lattice lattice by spatially resolved measurement.



We shine off-resonant laser light on atoms in an optical lattice and measure scattered photons on the screen.

Measurement operator

strength of measurement

$$\hat{M}(X) = \sqrt{\gamma}\sum_m f\left(X - md\right)\hat{n}_m$$

point spread function

atom-number operator at lattice site *m*

Dynamics

The dynamics of atoms under continuous monitoring is governed by three terms:

$$d|\psi\rangle = -\frac{i}{\hbar}\hat{H}|\psi\rangle dt$$

unitary evolution for Bose-Hubbard Hamiltonian

$$-\frac{1}{2}\int dX\left(\hat{M}^\dagger(X)\hat{M}(X) - \langle\hat{M}^\dagger(X)\hat{M}(X)\rangle\right)|\psi\rangle dt$$

non-unitary evolution without photodetection

$$+\int dX\left(\frac{\hat{M}(X)|\psi\rangle}{\sqrt{\langle\hat{M}^\dagger(X)\hat{M}(X)\rangle}} - |\psi\rangle\right)dN(X;t)$$

jump process upon photodetection

marked point process

Y. Ashida and MU, Phys. Rev. A**95**, 022124 (2017)

Continuous-Measurement Limit

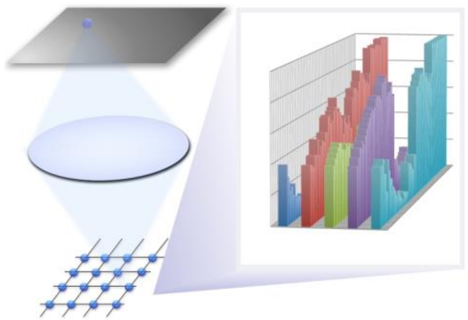
1. frequent $\gamma \gg J$

   measurement rate hopping rate of atoms

2. weak $\sigma \gg d$

   spatial resolution lattice constant

while keeping the ratio γ/σ^2 constant.

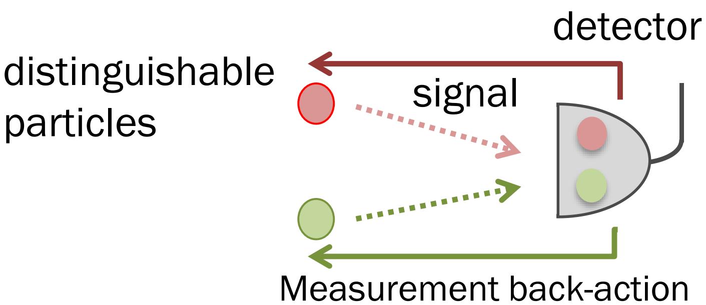


Y. Ashida and MU, PRL **115**,095301 (2015)

- The weak measurement ($\sigma >> d$) keeps the measurement disturbance small.
- With such low resolution, one can still achieve single-site resolution by means of Bayesian inference (Y. Ashida and MU, PRL **115**, 095301 (2015)).
- From the central limit theorem, the marked point process *dN*(*X*; *t*) is replaced by the Wiener process *dW(X).*

$$dN(X) = \langle \hat{M}^\dagger(X)\hat{M}(X)\rangle dt + \sqrt{\langle \hat{M}^\dagger(X)\hat{M}(X)\rangle} dW(X)$$

Distinguishable Particles



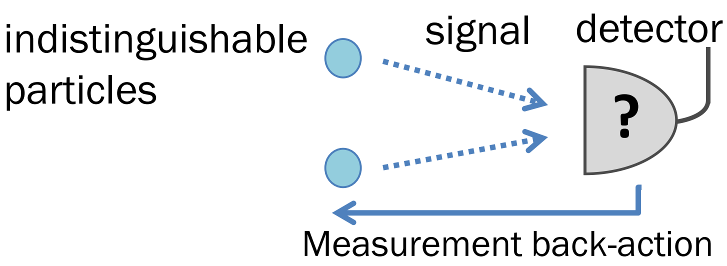
Continuous measurement of distinguishable particles can be modeled by a generalization of the single-particle model due to S. Diosi [Phys. Rev. A **40**, 1165 (1989)].

$$d\hat{\rho} = \underbrace{-\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]dt}_{\text{unitary}} \underbrace{- \frac{\gamma_{\text{tot}} d^2}{4\sigma^2}\left[\hat{X}_{CM},\left[\hat{X}_{CM},\hat{\rho}\right]\right]dt + \sqrt{\frac{\gamma_{\text{tot}} d^2}{2\sigma^2}}\left\{\hat{X}_{CM} - \langle\hat{X}_{CM}\rangle,\hat{\rho}\right\}dW}_{\text{center-of-mass decoherence}}$$

$$\underbrace{-\frac{d^2}{4\sigma^2}\sum_{i=1}^{N}\gamma_i\left[\hat{r}_i,\left[\hat{r}_i,\hat{\rho}\right]\right]dt + \sum_{i=1}^{N}\sqrt{\frac{\gamma_i d^2}{2\sigma^2}}\left\{\hat{r}_i - \langle\hat{r}_i\rangle,\hat{\rho}\right\}dW_i}_{\text{relative positional decoherence}}$$

The measurement backaction leads to decoherence of both the COM coordinate and the relative positional coordinates. ← All DOF can be distinguished and hence susceptible to measurement backaction.

Indistinguishable Particles



For indistinguishable particles, the "relative positions" cannot be distinguished.

In fact, the relative-coordinate part is canceled out due to two-particle interference!

$$d\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}, \hat{\rho}\right] dt \quad \underbrace{- \frac{\gamma_{\rm tot} d^2}{4\sigma^2}\left[\hat{X}_{CM}, \left[\hat{X}_{CM}, \hat{\rho}\right]\right] dt + \sqrt{\frac{\gamma_{\rm tot} d^2}{2\sigma^2}}\left\{\hat{X}_{CM} - \langle \hat{X}_{CM} \rangle, \hat{\rho}\right\} dW}_{\text{center-of-mass decoherence}}$$

$$\underbrace{-\frac{d^2}{4\sigma^2}\sum_{i=1}^{N}\gamma_i\left[\hat{r}_i, \left[\hat{r}_i, \hat{\rho}\right]\right] dt + \sum_{i=1}^{N}\sqrt{\frac{\gamma_i d^2}{2\sigma^2}}\left\{\hat{r}_i - \langle \hat{r}_i \rangle, \hat{\rho}\right\} dW_i}_{\text{relative positional decoherence}}$$

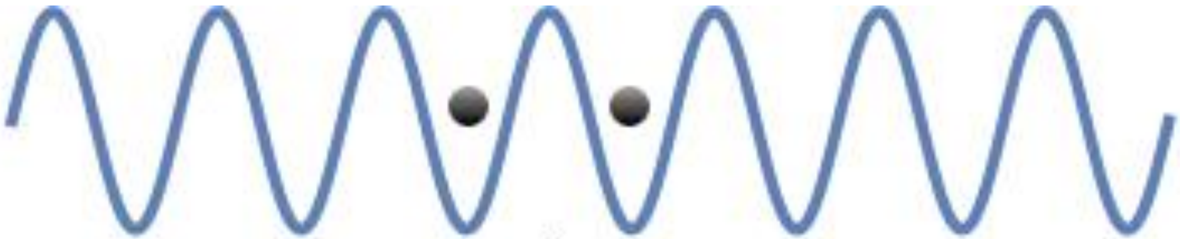
Multi-particle quantum interference suppresses the relative positional decoherence, and only the center-of-mass position decoheres.

Y. Ashida and MU, Phys. Rev. A**95**, 022124 (2017)

Numerical Simulations: Quantum Walks of Two Particles

Initial condition: two particles at adjacent sites

t=0

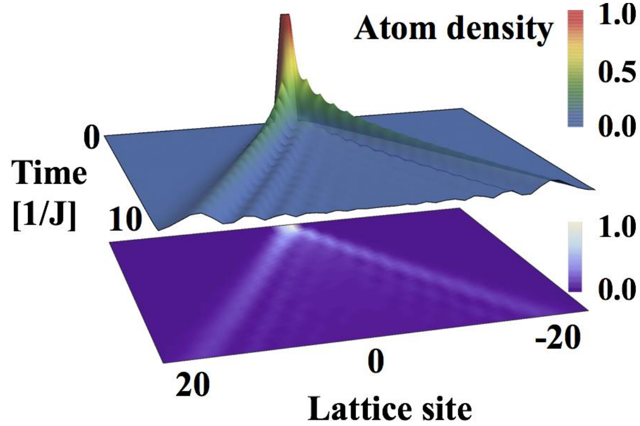


Let the system evolve freely
subject to continuous monitoring.

We compare quantum transport for
- distinguishable particles
- fermions
- bosons

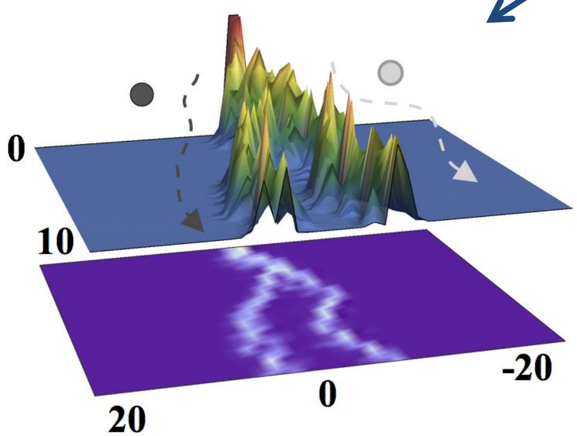
Distinguishable vs. Indistinguishable Quantum Transport

<u>Without measurement:</u>
<u>unitary evolution</u>

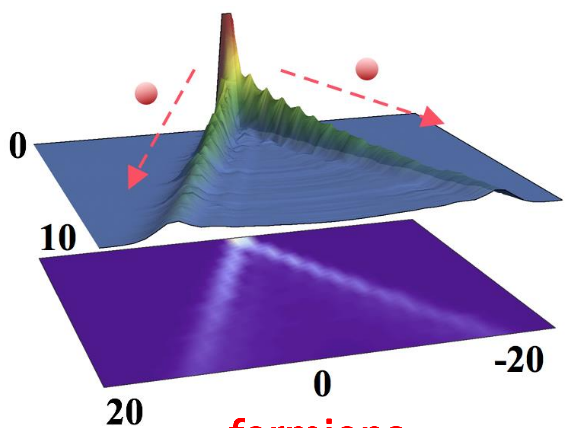


Two particles move away in opposite directions, and the density profile is almost independent of quantum statistics.

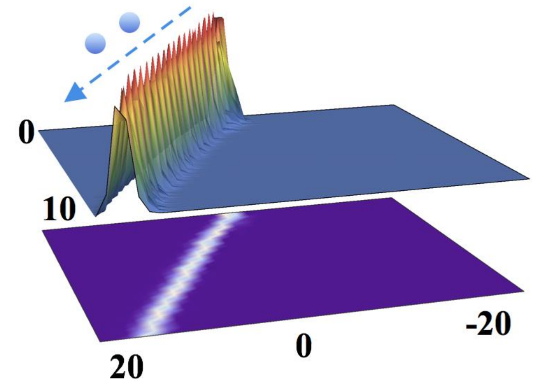
<u>Under continuous observation</u>



distinguishable
particles



fermions



bosons

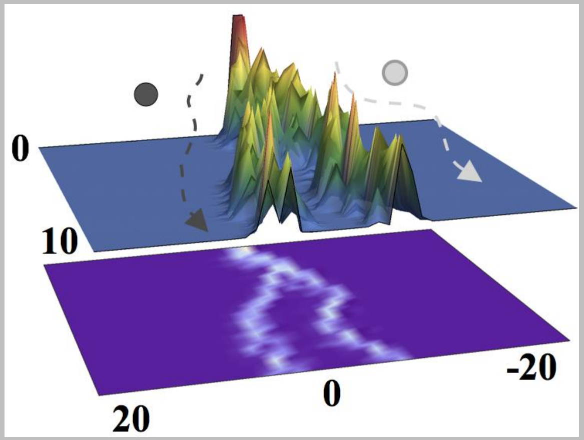
Y. Ashida and MU, Phys. Rev. A**95**, 022124 (2017)

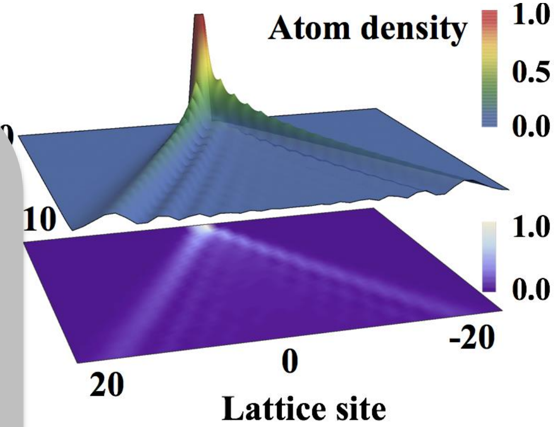
Distinguishable Particles

Distinguishable

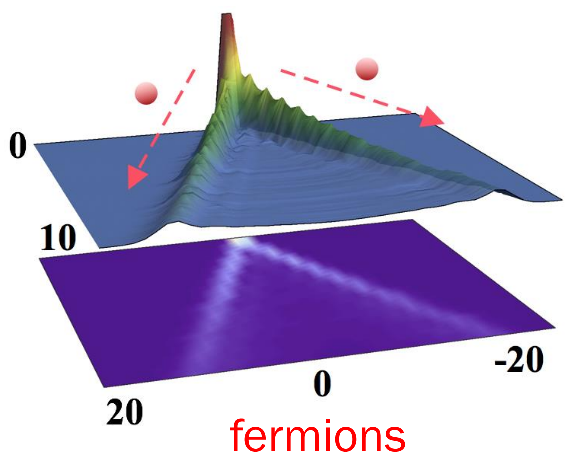
Measurement backaction
→ COM+relative positional decoherence

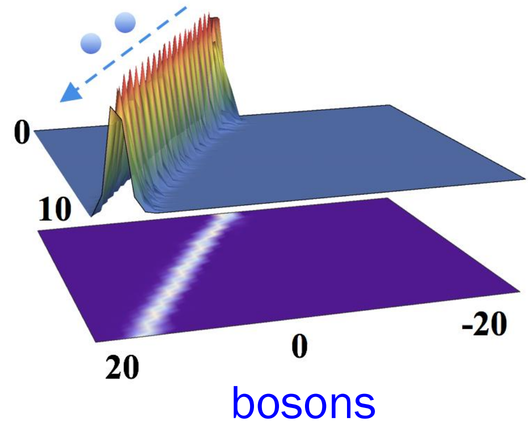
- uncorrelated
- diffusive random walk





Two particles move away in opposite directions, and the density profile is almost independent of quantum statistics.



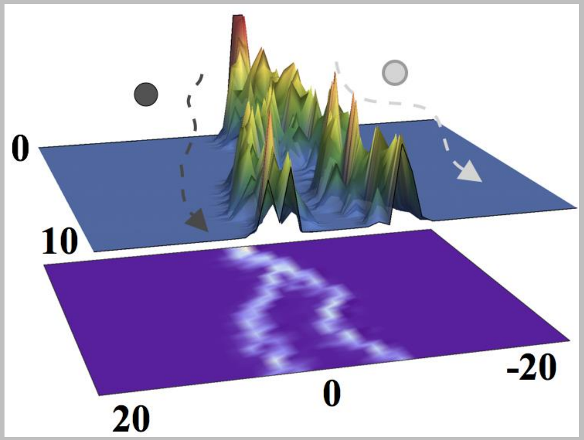


Indistinguishable Particles

Distinguishable

Measurement backaction
→ COM+relative positional decoherence

- uncorrelated
- diffusive random walk

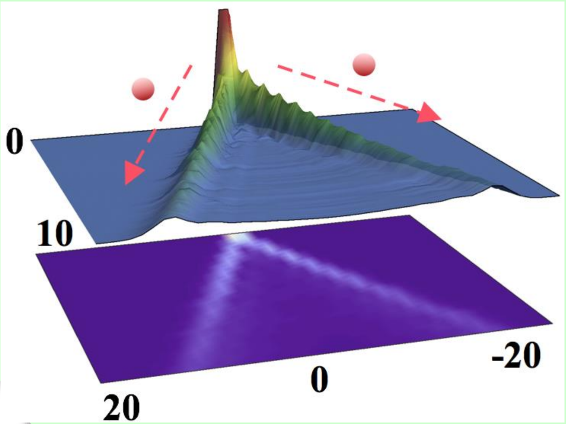


Indistinguishable

COM+relative position decoherence
(no relative positional decoherence)

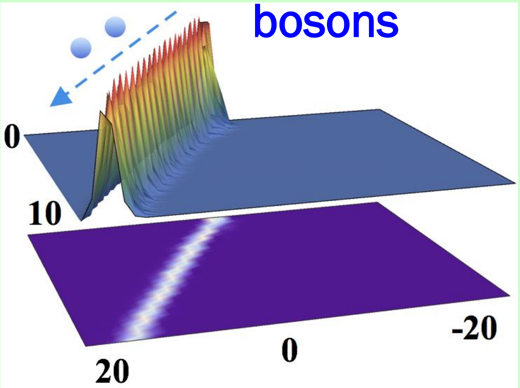
COM
- localized
- strongly correlated
- ballistic transport (inertial regime)

fermions



antibunching

bosons

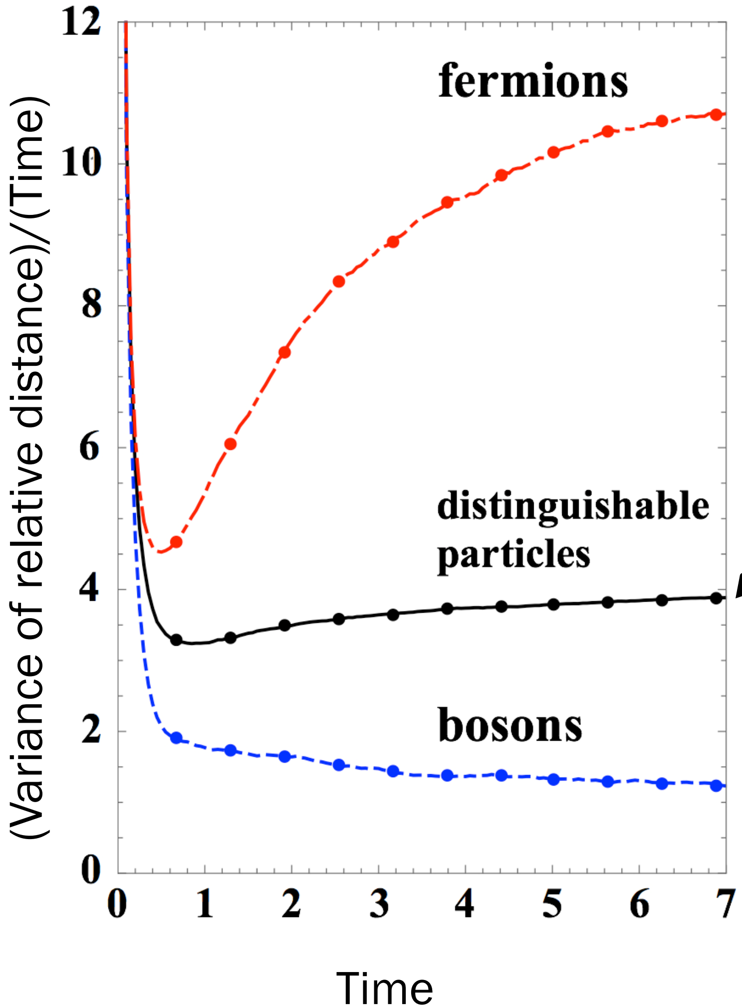


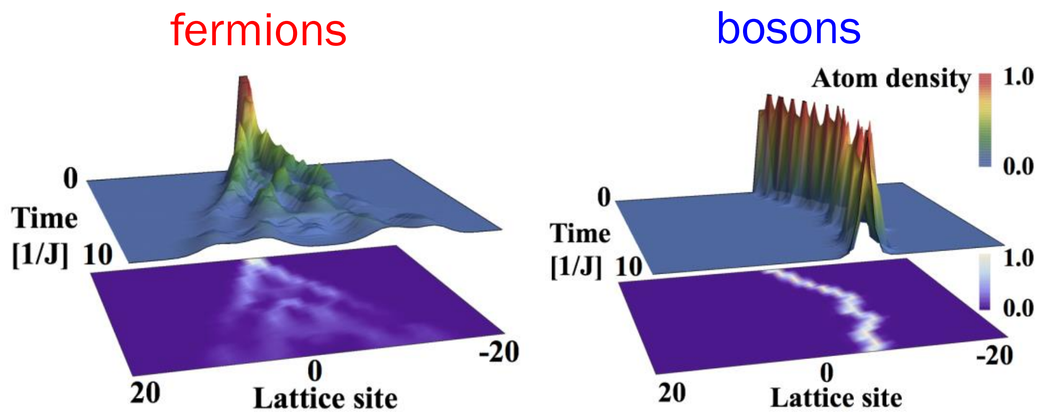
bunching

Y. Ashida and MU, Phys. Rev. A**95**, 022124 (2017)

Strong Measurement: Diffusive Transport

Under strong measurement, the dynamics shows diffusive transport due to strong measurement backaction.





classical diffusion constant: $2D_c = \dfrac{32J^2\sigma^2}{\gamma\hbar^2 d^2}$

- **Quantum statistics** serves as effective repulsion for fermions and attraction for bosons.
- Fermions show faster diffusion and bosons show slower diffusion than distinguishable particles due to multi-particle intereference.

Y. Ashida and MU, Phys. Rev. A**95**, 022124 (2017)

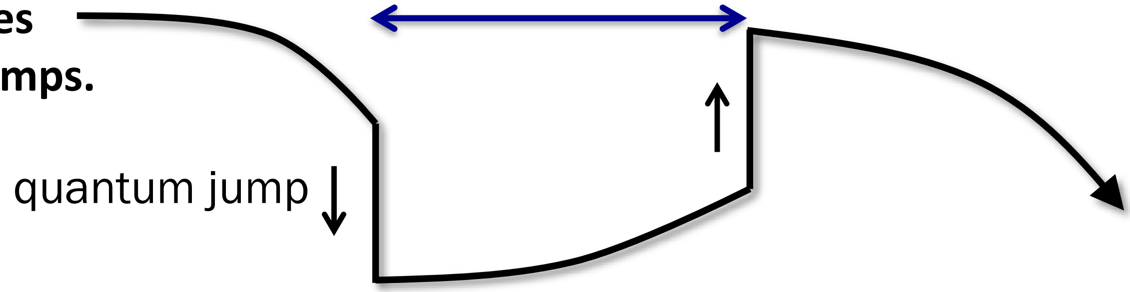
Quantum critical phenomena under continuous observation

Model

— continuously monitored quantum many-body system —

A single quantum trajectory undergoes several quantum jumps.

nonunitary dynamics during null measurement outcomes



effective non-Hermitian Hamiltonian

$$\hat{H}_{\mathrm{eff}} = \hat{H} - \underbrace{\frac{i\gamma}{2}\sum_i \hat{M}_i^\dagger \hat{M}_i}$$

$\hat{M}_i$: jump operators

measurement backaction for null measurement outcomes between quantum jumps

* If the ensemble average is taken over all outcomes, the dynamics is described by the Lindblad eq. $\rightarrow$ Quantum correlations and criticality would then be washed out.

J. Schachenmayer et al., PRA 89, 011601 (2014).
Y. Yanay and E. J. Mueller, PRA 90, 023611 (2014).

Physical Meaning of Non-Hermitian Hamiltonian

- **effective Hamiltonian:** $\hat{H}_{\mathrm{eff}} = \hat{H} - \frac{i\gamma}{2}\sum_i \hat{M}_i^\dagger \hat{M}_i$

- **complex eigenvalues:** $E_\lambda - \frac{i\Gamma_\lambda}{2}$

  E_λ real part: effective **energy** of the system

  Γ_λ imaginary part: **decay rate**

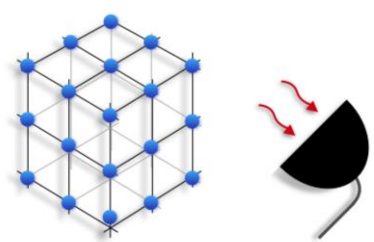
- **effective ground state:** $|\Psi_{\mathrm{GS}}\rangle$ eigenstate of $\hat{H}_{\mathrm{eff}}$ having the **lowest** eigenenergy E_λ

In our models, $|\Psi_{\mathrm{GS}}\rangle$ also has the **minimal** Γ_λ (the **longest** lifetime).

We consider situations in which the Hermitian part of the Hamiltonian $\hat{H}$ exhibits quantum criticality.

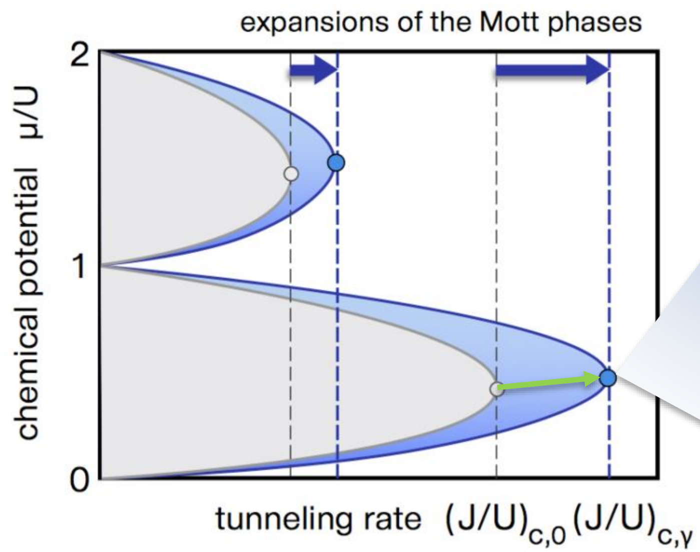
Superfluid-Mott Insulator Transition in Bose-Hubbard Model

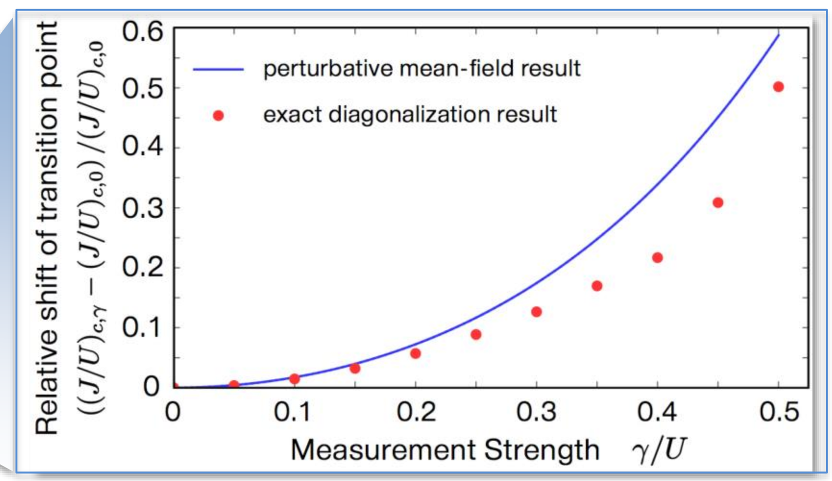
- **jump process: two-body loss due to inelastic collisions**



$$\hat{H}_{\mathrm{eff}} = \hat{H}_{\mathrm{BH}} - \frac{i\gamma}{2}\sum_i \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i \qquad \hat{M}_i = \hat{b}_i^2$$

The SF-Mott insulator transition point is identified with the one at which the energy gap closes.



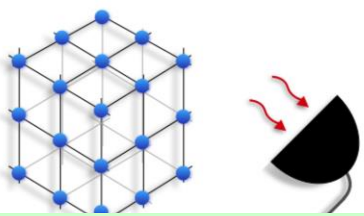


The measurement backaction shifts the transition point, so that the Mott lobes expand.

Y. Ashida, S. Furukawa, and MU, Phys. Rev. A **94**, 053615 (2016)

Superfluid-Mott Insulator Transition in Bose-Hubbard Model

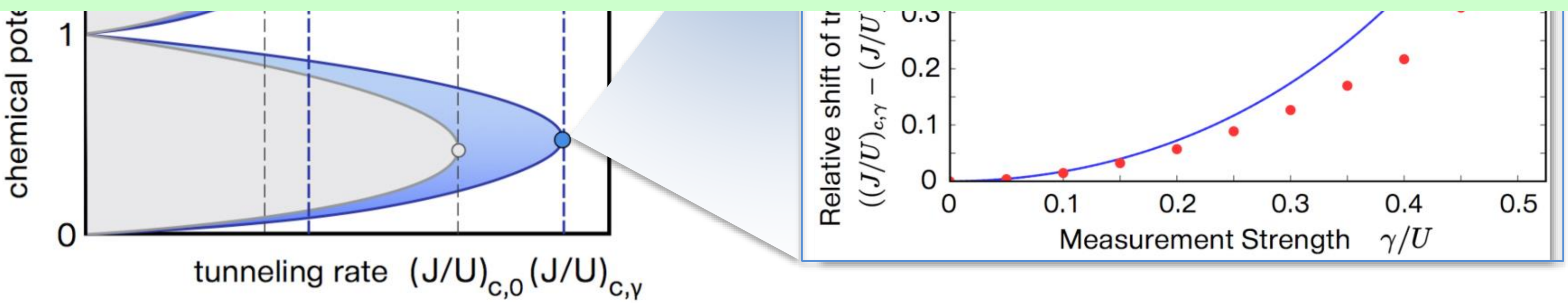
- **jump process: two-body loss due to inelastic collisions**



$$\hat{H}_{\mathrm{eff}} = \hat{H}_{\mathrm{BH}} - \frac{i\gamma}{2}\sum_i \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i \qquad \hat{M}_i = \hat{b}_i^2$$

The SF-Mott insulator transition point is identified

Expansion of Mott lobes may be interpreted as being caused by the suppression of the hopping rate due to the continuous quantum Zeno effect



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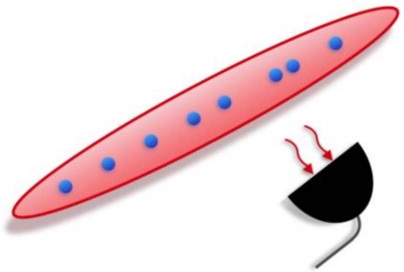
Quantum Criticality of a 1D Bose Gas

- **Jump process: two-body loss**
  $\rightarrow$ **modified Lieb-Liniger model**



two-body loss

$$\hat{H}_{\mathrm{LL}} = \int dx \left[\hat{\Psi}^\dagger(x) \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \hat{\Psi}(x) + \frac{g - i\gamma}{2} \hat{\Psi}^\dagger(x) \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \hat{\Psi}(x) \right]$$

- **Low-energy model = non-Hermitian Tomonaga-Luttinger model**

$$\hat{H}_{\mathrm{eff}} = \frac{\hbar}{2\pi} \int_{-\infty}^{\infty} dx [v_J (\partial_x \hat{\theta})^2 + \tilde{v}_N e^{-i\delta_\gamma} (\partial_x \hat{\phi})^2]$$

non-Hermiticity: $\delta_\gamma \to 0 \;\; (\gamma \to 0)$

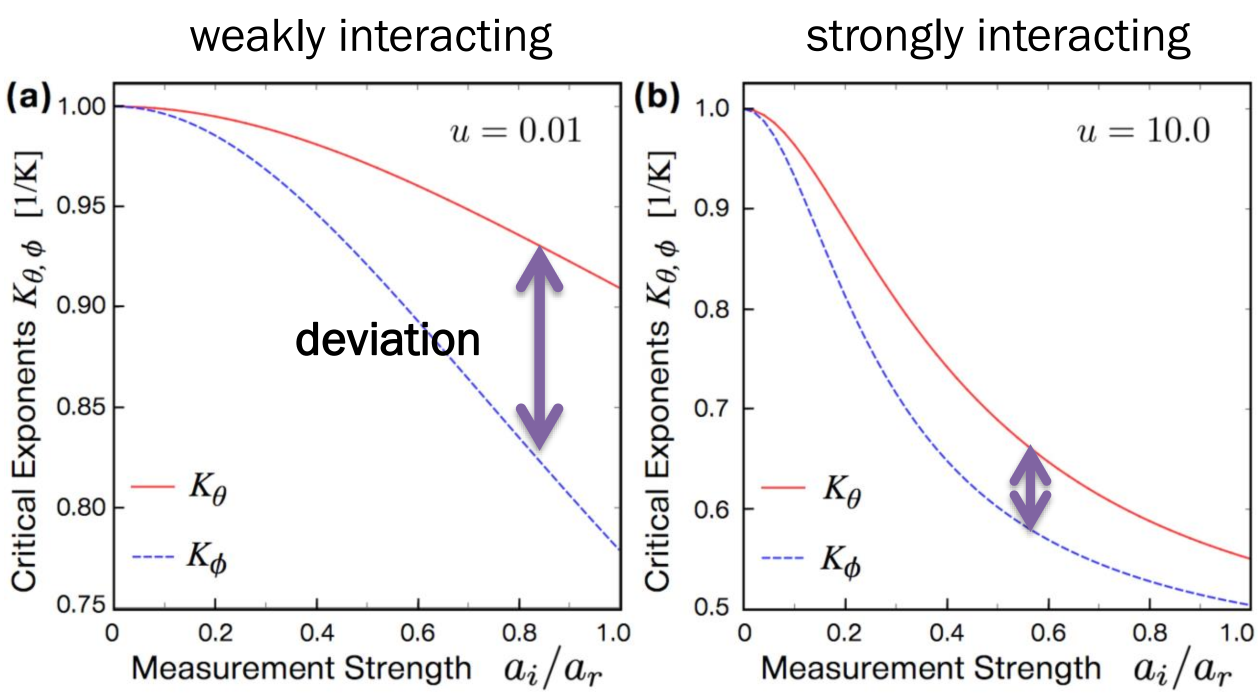
Y. Ashida, S. Furukawa, and MU, Phys. Rev. A **94**, 053615 (2016)

Behaviors of two critical exponents against the measurement strength

one-particle correlation: $\langle \hat{\Psi}^\dagger(r)\hat{\Psi}(0)\rangle \propto \left(\frac{1}{r}\right)^{\frac{1}{2K_\phi}}$

equal for Tomonaga-Luttinger liquid universality class

density-density correlation: $\langle \hat{\rho}(r)\hat{\rho}(0)\rangle - \rho_0^2 = -\frac{K_\theta}{2\pi^2 r^2} + \text{const.} \times \frac{\cos(2\pi\rho_0 r)}{r^{2K_\theta}}$



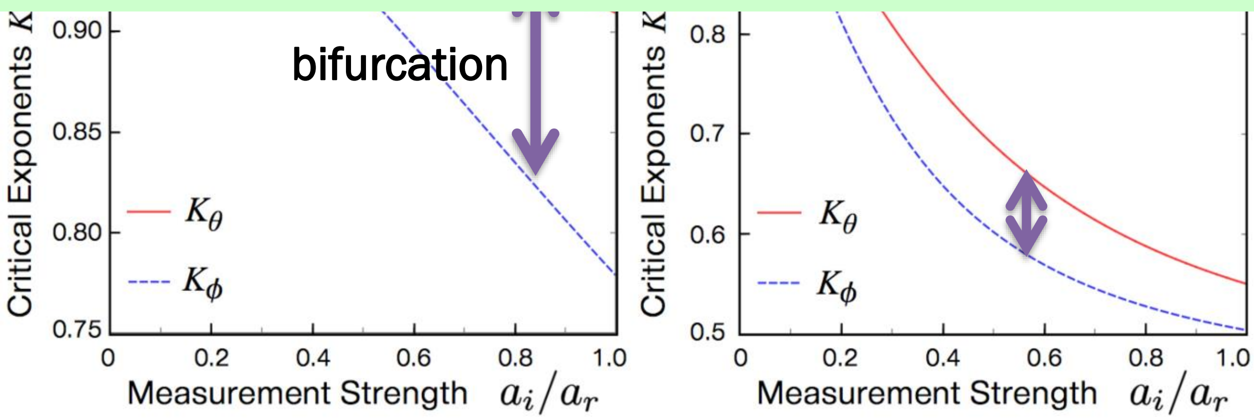
Y. Ashida, S. Furukawa, and MU, Phys. Rev. A **94**, 053615 (2016)

Behaviors of two critical exponents against the measurement strength

one-particle correlation: $\langle\hat{\Psi}^\dagger(r)\hat{\Psi}(0)\rangle \propto \left(\frac{1}{r}\right)^{\frac{1}{2K_\phi}}$ equal for TLL univ. class

density-density correlation: $\langle\hat{\rho}(r)\hat{\rho}(0)\rangle - \rho_0^2 = -\frac{K_\theta}{2\pi^2 r^2} + \text{const.} \times \frac{\cos(2\pi\rho_0 r)}{r^{2K_\theta}}$

Two different critical exponents indicate the emergence of the unique 1D quantum critical universality class caused by measurement backaction.



Y. Ashida, S. Furukawa, and MU, Phys. Rev. A **94**, 053615 (2016)

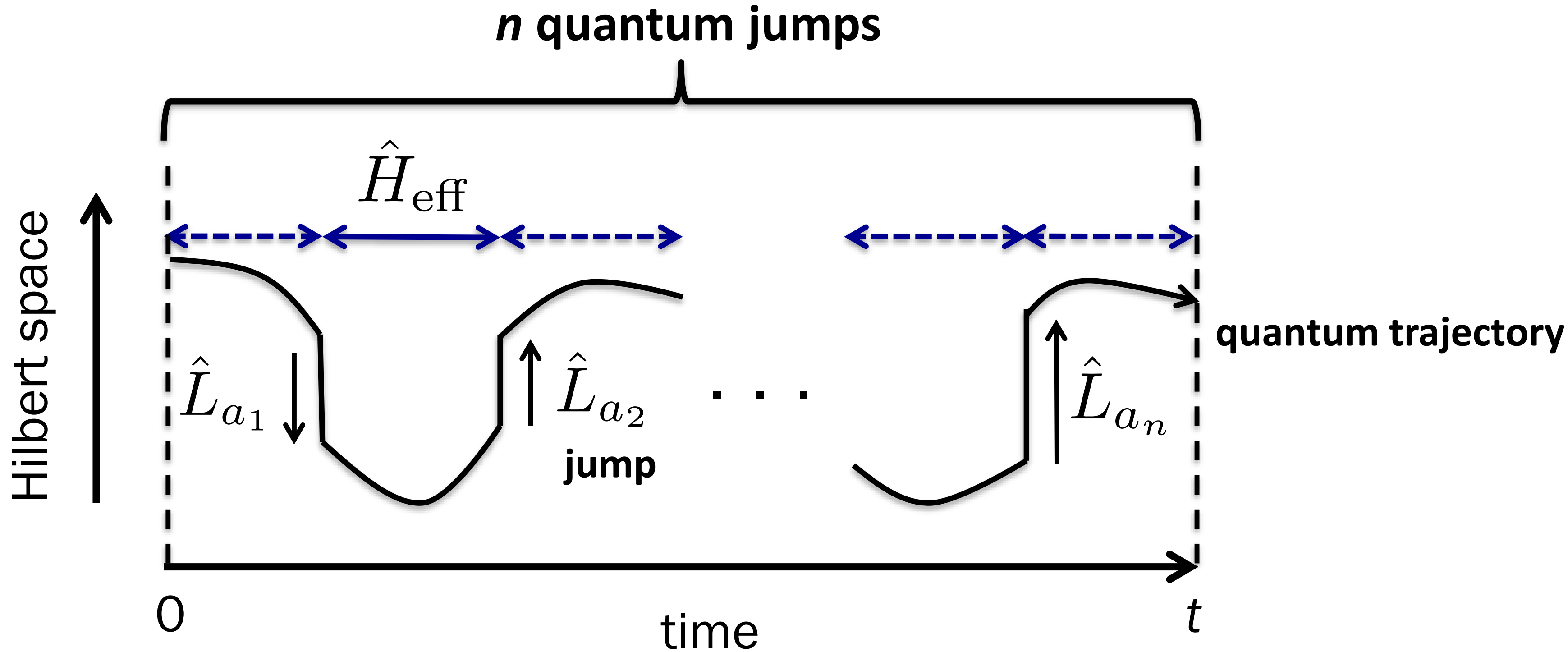
Full-Counting Many-Particle Dynamics

toward establishing statistical mechanics under continuous observation

Y. Ashida and M. Ueda, PRL **120**, 185301 (2018)

Full-counting statistics of quantum jumps

We now take into account quantum jump events as well.



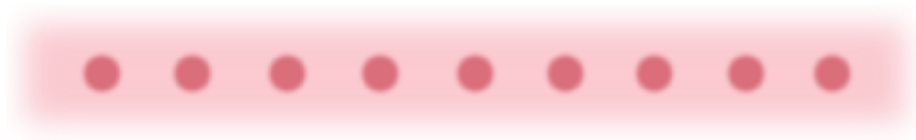
Full-counting statistics $P_n(t)$ **:** probability of observing *n* quantum jumps during a time interval [0,*t*]

Levitov et al., J. Math. Phys. 37, 4845 (1996)
Garrahan and Lesanovsky, PRL 104, 160601 (2010)

Full-counting many-particle dynamics

time

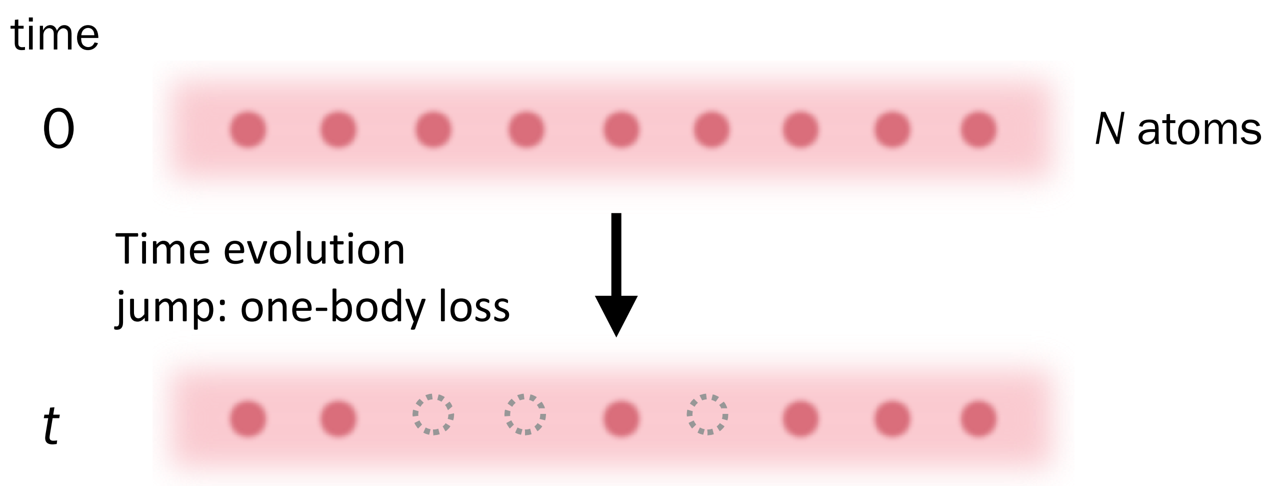
0



N atoms

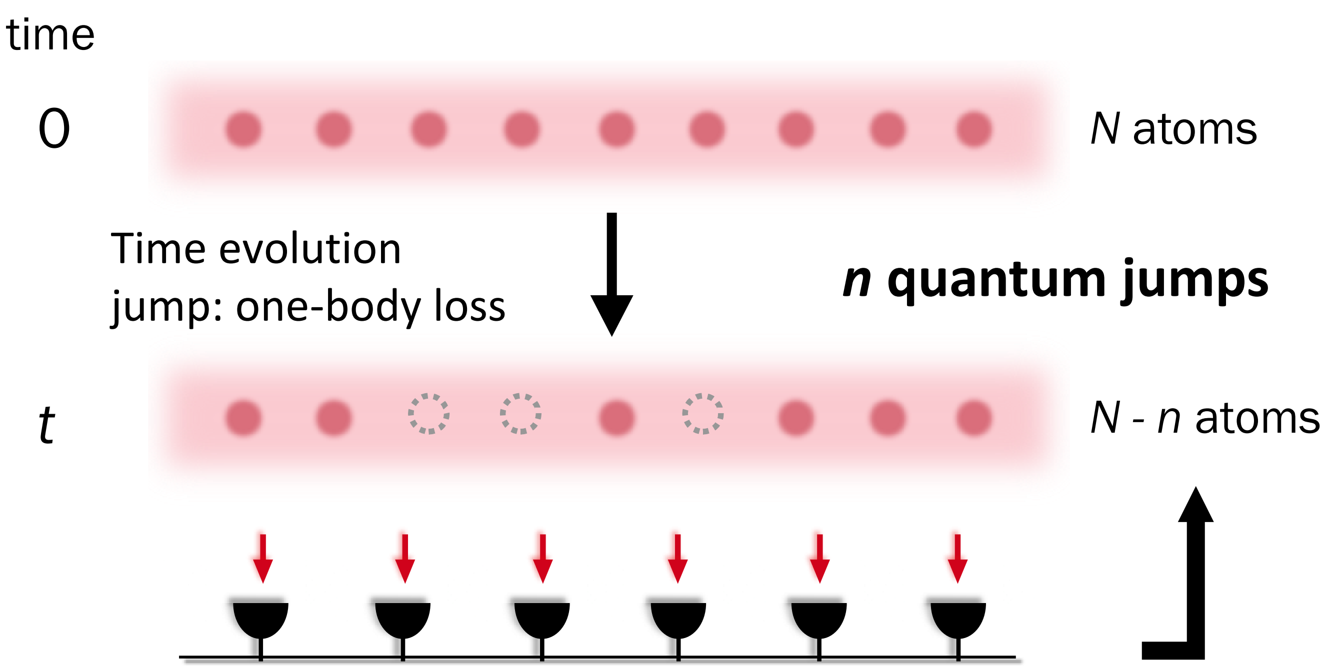
We initially prepare *N* atoms.

Full-counting many-particle dynamics



At time t, some atoms are lost due to quantum jumps.

Full-counting many-particle dynamics



Measure the remaining total atom number using QGM.

Full-counting dynamics $\hat{\rho}^{(n)}(t)$ **:** ensemble of trajectories for the varying number of quantum jumps

General Theory

Solve the Lindblad master equation:

$$\frac{d\hat{\rho}(t)}{dt} = -i\left(\hat{H}_{\text{eff}}\hat{\rho} - \hat{\rho}\hat{H}_{\text{eff}}^{\dagger}\right) + \mathcal{J}[\hat{\rho}]$$

$$\hat{H}_{\text{eff}} = \hat{H} - \frac{i}{2}\sum_a \hat{L}_a^{\dagger}\hat{L}_a \qquad \mathcal{J}[\hat{\rho}] = \sum_a \hat{L}_a\hat{\rho}\hat{L}_a^{\dagger}$$

General solution for *n* quantum jumps:

$$\hat{\mathcal{U}}_{\text{eff}}(t) = e^{-i\hat{H}_{\text{eff}}t}$$

$$\hat{\varrho}^{(n)}(t) = \sum_{\{a_k\}_{k=1}^{n}} \int_0^t dt_n \cdots \int_0^{t_2} dt_1 \prod_{k=1}^{n}\left[\hat{\mathcal{U}}_{\text{eff}}(\Delta t_k)\hat{L}_{a_k}\right] \times \hat{\mathcal{U}}_{\text{eff}}(t_1)\hat{\rho}(0)\hat{\mathcal{U}}_{\text{eff}}^{\dagger}(t_1)\prod_{k=1}^{n}\left[\hat{L}_{a_k}^{\dagger}\hat{\mathcal{U}}_{\text{eff}}^{\dagger}(\Delta t_k)\right]$$

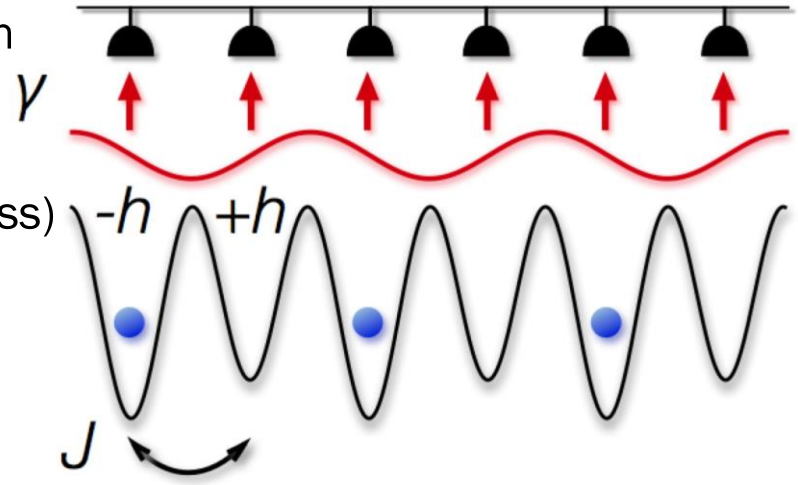
→ ensemble of all possible trajectories with ***n* quantum jumps**

Solvable model: fermions under periodic dissipation

site-resolved detection
of the atom number

periodic dissipative
potential (one-body loss)

half-filled fermions
in a superlattice



Lindblad equation
(ensemble level)

$$\frac{d\hat{\rho}(t)}{dt} = -i\left(\hat{H}_{\mathrm{eff}}\hat{\rho} - \hat{\rho}\hat{H}_{\mathrm{eff}}^{\dagger}\right) + \mathcal{J}[\hat{\rho}]$$

effective Hamiltonian: $\hat{H}_{\mathrm{eff}} = \hat{H}_{\mathrm{PT}} - 2i\gamma\hat{N}$

PT-symmetric Hamiltonian: $$\hat{H}_{\mathrm{PT}} = -\sum_{l=0}^{L-1}\left[\left(J+(-1)^l i\gamma\right)\left(\hat{c}_{l+1}^{\dagger}\hat{c}_l + \hat{c}_l^{\dagger}\hat{c}_{l+1}\right) + (-1)^l h\hat{c}_l^{\dagger}\hat{c}_l\right]$$

jump term: $$\mathcal{J}[\hat{\rho}] = 2\gamma\sum_l[2\hat{c}_l\hat{\rho}\hat{c}_l^{\dagger} + (-1)^l(\hat{c}_l\hat{\rho}\hat{c}_{l+1}^{\dagger} + \hat{c}_{l+1}\hat{\rho}\hat{c}_l^{\dagger})]$$

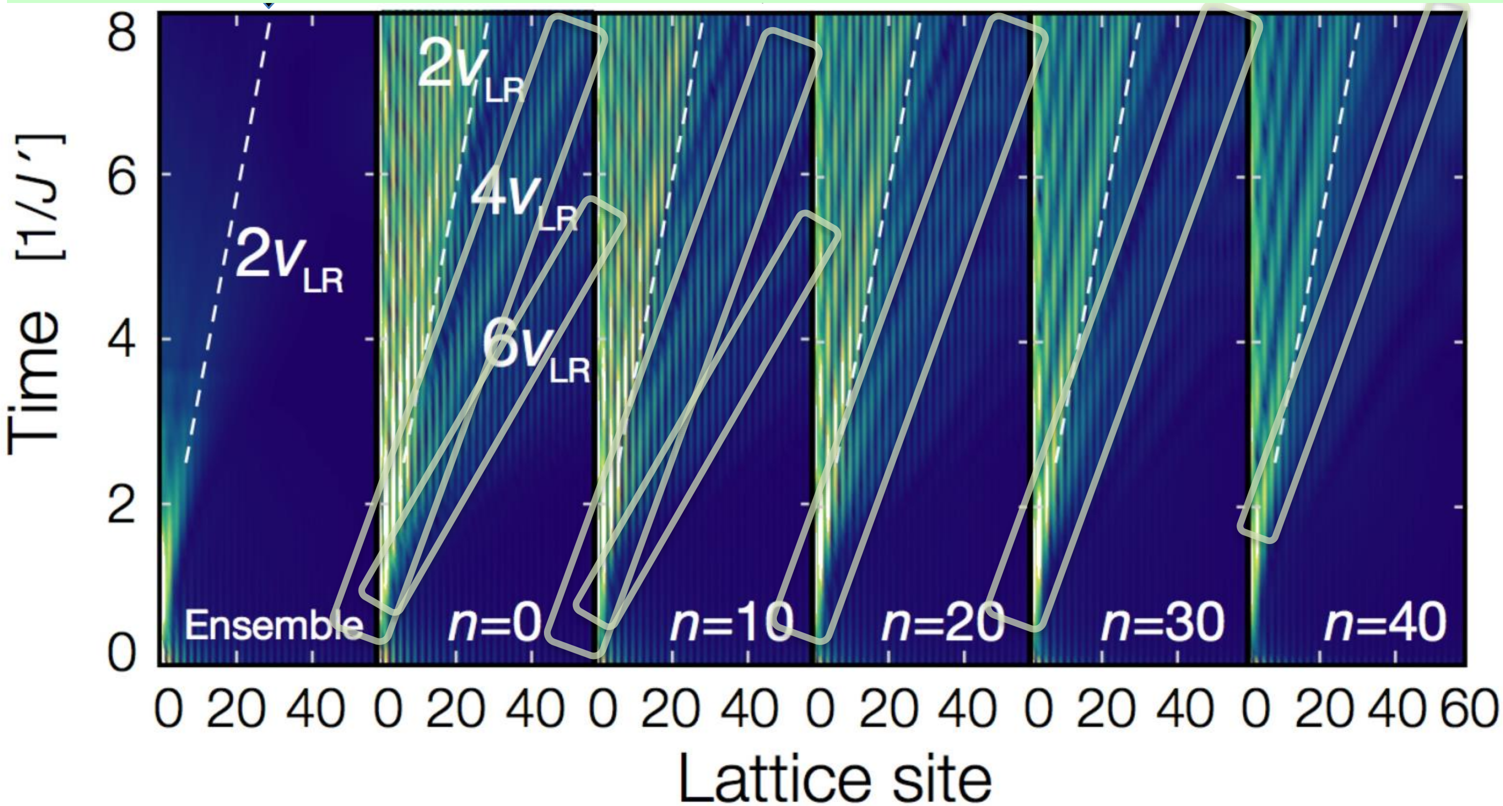
Y. Ashida and MU, PRL **120**, 185301 (2018)

Spreading of correlation: full-counting dynamics

We calculate the full-counting dynamics under the same condition.

full-counting dynamics: $C^{(n)}(l,t) = \mathrm{Tr}[\hat{\rho}^{(n)}(t)\hat{c}_l^\dagger \hat{c}_0]$ n: number of jumps

Faster propagation at integer multiples of LR bound!



How thermalization and heating proceeds in open quantum many-body systems?

Y. Ashida, K. Saito, and M. Ueda, PRL **121**, 170402 (2018)

Three types of thermalization

(1) System in contact with thermal reservoir

(2) Isolated systems

(3) Systems under control and measurement

Formulation of the problem

We consider repeated indirect or continuous measurement.

jump processes m=1, ... , M

$$\mathcal{E}_m(\hat{\rho}) = \gamma \delta t \hat{L}_m \hat{\rho} \hat{L}_m^\dagger$$

no-count process m=0

$$\mathcal{E}_0(\hat{\rho}) \simeq (1 - i\hat{H}_{\rm eff}\delta t)\hat{\rho}(1 + i\hat{H}_{\rm eff}^\dagger \delta t)$$

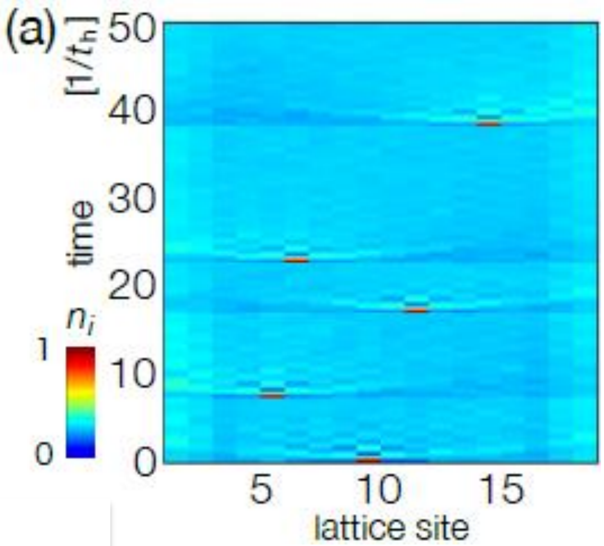
$$\hat{H}_{\rm eff} = \hat{H} - i\hat{\Gamma}/2 \qquad \hat{\Gamma} = \gamma \sum_m \hat{L}_m^\dagger \hat{L}_m$$

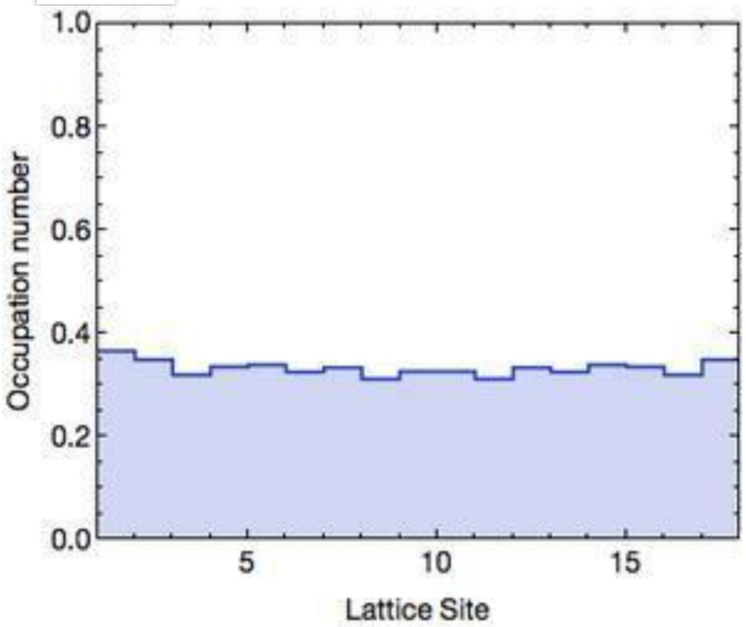
Each realization of a quantum trajectory is determined by specifying a sequence of measurement outcomes:

$$\mathcal{T} = (t_1, \ldots, t_n) \qquad \mathcal{M} = (m_1, \ldots, m_n)$$

If the jump operator does not commute with the Hamiltonian, the measurement can heat/cool the system.

Numerical simulations: a local measurement



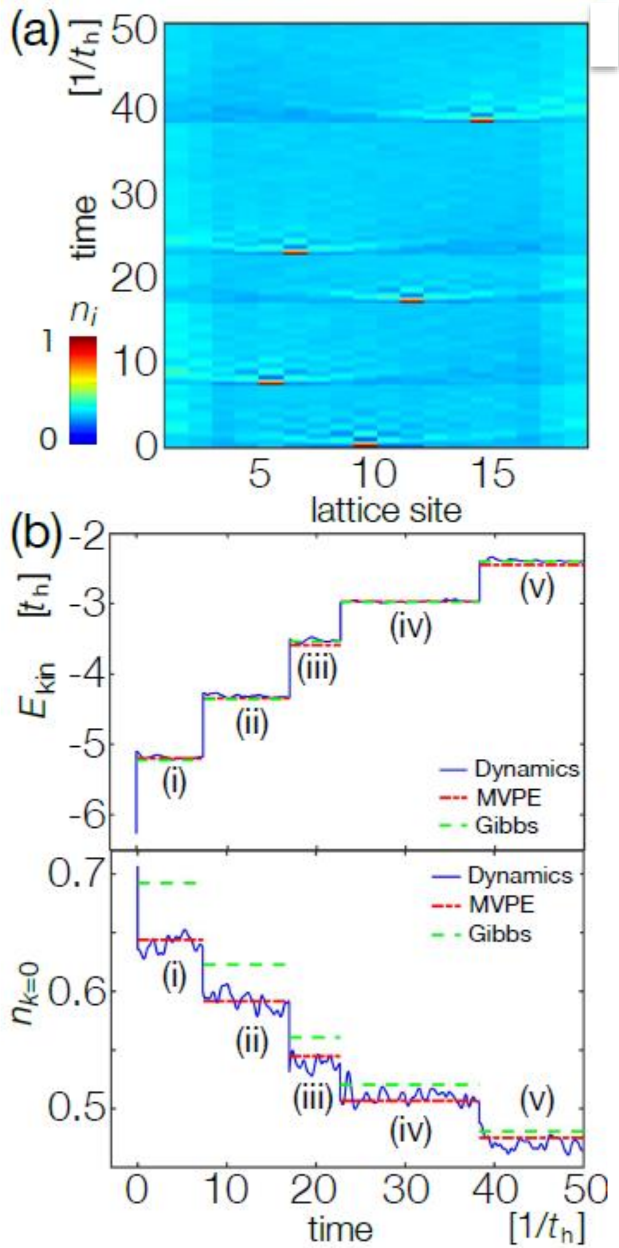


- Hard-core bosons with NN and NNN interactions.
- Site number = 18, particle number = 6
- Jump operator $\hat{L}_l = \hat{n}_l$

- Every time an atom is detected, a high-density region emerges which subsequently diffuses away toward an equilibrium value.

Y. Ashida, et al., PRL **121**, 170402 (2018)

Numerical simulations: a local measurement

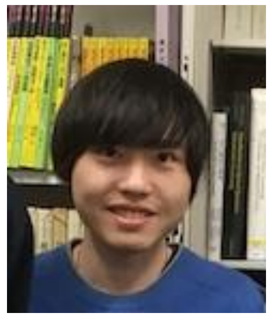


- The kinetic energy increases stepwise upon quantum jump, with a concomitant decrease of the *k=0* component.
- Matrix-vector product ensemble

$$\hat{\rho}_{\mathcal{M}} \propto \sum_a [\mathcal{V}_{m_n} \cdots \mathcal{V}_{m_1} p_{\mathrm{eq}}]_a \hat{P}_a \qquad (\mathcal{V}_m)_{ab} = |\langle E_a|\hat{L}_m|E_b\rangle|^2$$

Y. Ashida, et al., PRL **121**, 170402 (2018)

Zeno Hall Effect



Zongping Gong

Sho Higashikawa

PRL **118**, 200401 (2017)

A seminal idea of QZE – von Neumann

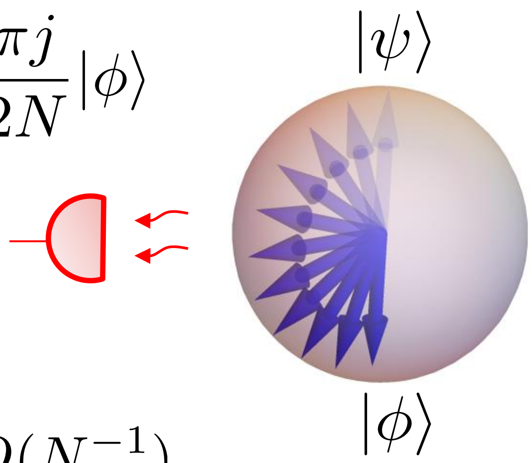
A quantum state can be transformed to another with almost unit probability by repeated projective measurements.

John von Neumann, *Die Mathematische Grundlagen der Quantenmechanik* (1932) p. 366 in English edition

In fact, one can construct a series of projective measurements that smoothly connect from $|\boldsymbol{\psi}\rangle$ to $|\boldsymbol{\varphi}\rangle$:

$$\hat{P}_j = |\psi_j\rangle\langle\psi_j|, \qquad |\psi_j\rangle = \cos\frac{\pi j}{2N}|\psi\rangle + \sin\frac{\pi j}{2N}|\phi\rangle$$

$$|\psi_0\rangle = |\psi\rangle, \qquad |\psi_N\rangle = |\phi\rangle$$

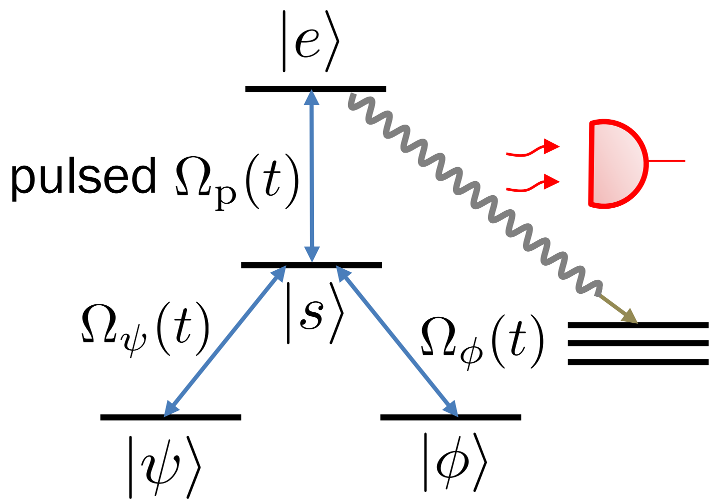


$$\begin{aligned} P_N &= |\langle\phi|\hat{P}_N \ldots \hat{P}_2\hat{P}_1|\psi\rangle|^2 \\ &= \prod_{j=1}^{N} |\langle\psi_j|\psi_{j-1}\rangle|^2 = \cos^{2N}\frac{\pi}{2N} = 1 - O(N^{-1}) \end{aligned}$$

$$\boxed{\lim_{N\to\infty} P_N = 1}$$

Implementation of von Neumann’s idea

von Neumann’s idea can be implemented by using atomic internal states connected via time-dependent Rabi couplings.



Excite the system from |s> to |e>, and detect fluorescence photons.

↓

Projective measurement onto the dark state |*D*(*t*)>, indicated by the absence of fluorescence signals

dark state $|D(t)\rangle \propto \Omega_\phi(t)|\psi\rangle - \Omega_\psi(t)|\phi\rangle$

$$\Omega_\psi(t) = -\Omega \sin\frac{\omega t}{2}, \qquad \Omega_\phi(t) = \Omega \cos\frac{\omega t}{2}$$

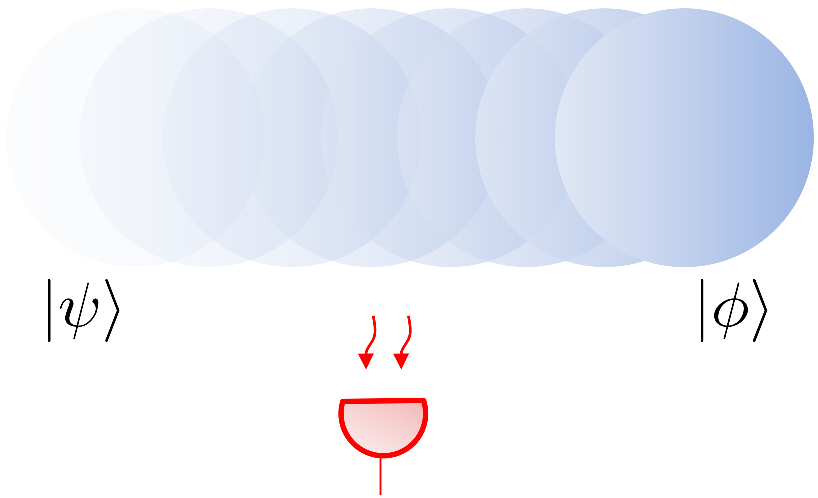
$$|D(t_j)\rangle = |\psi_j\rangle, \qquad t_j = \frac{j\pi}{N\omega}$$

Perform measurement at discrete times. Von Neumann’s idea is faithfully realized.

M. Porrati and S. Putterman, PRA **36**, 929 (1987)

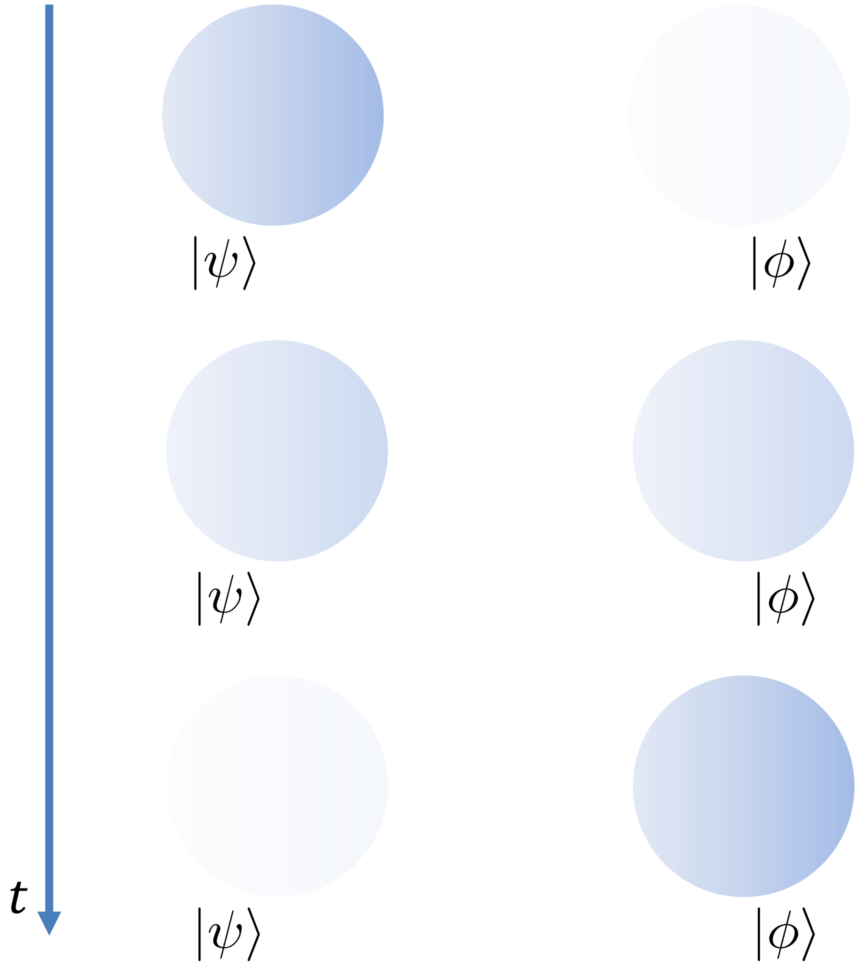
Question addressed in this talk

Can we apply von Neumann's idea to transport a wave packet from one location |ψ> to another |φ> by measurement?



Measurement-based transport of a wave packet

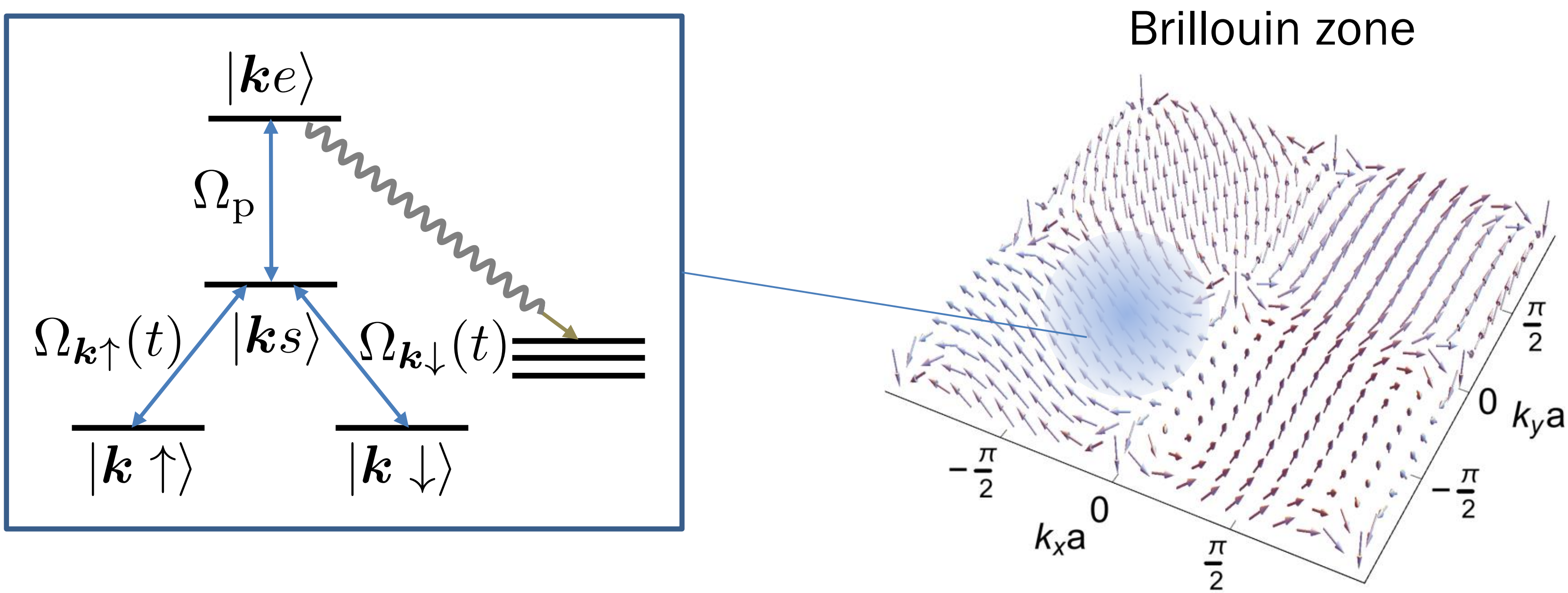
Appears to be difficult because an intermediate state must be a superposition state between spatially separated states.



superposition between spatially separated states

Our idea

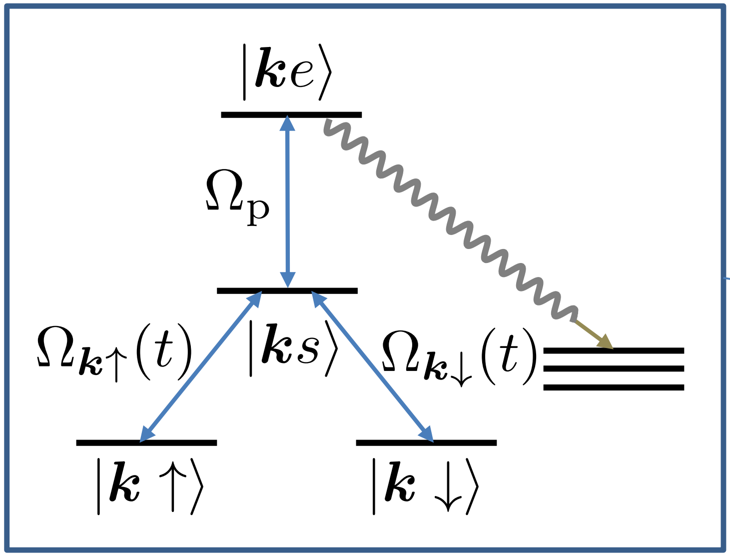
Consider a 2D square lattice with lattice constant *a*.

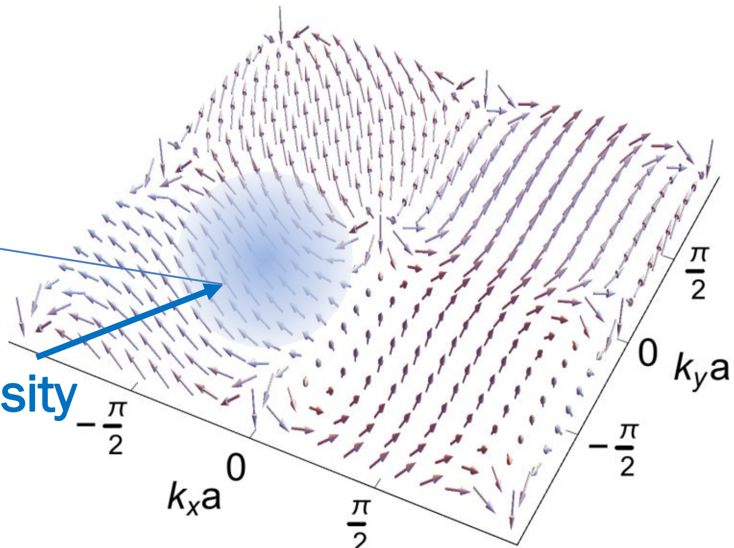


At each point in the Brillouin zone, we introduce the same Λ-scheme, but make the Rabi couplings depend on time and the crystal momentum.

Spin-orbit coupling

The SOC emerges from a nontrivial *k*-dependence of the dark state which can be implemented through synthetic gauge fields.





$$\psi_\sigma(\boldsymbol{k},t) = \sqrt{\rho(\boldsymbol{k})}\underline{u_\sigma(\boldsymbol{k},t)} \quad \sigma = \uparrow,\downarrow$$

dark-state wave function

$$\sum_\sigma \Omega_{\boldsymbol{k}\sigma}(t) u_\sigma(\boldsymbol{k},t) = 0, \quad \sum_\sigma |u_\sigma(\boldsymbol{k},t)|^2 = 1$$

Wave-packet dynamics

Dynamics of the center of mass of the wave packet

$$\langle \dot{\hat{\boldsymbol{r}}} \rangle = \int \frac{d^2\boldsymbol{k}}{(2\pi a^{-1})^2} \rho(\boldsymbol{k}) \boldsymbol{\mathcal{F}}(\boldsymbol{k}, t)$$

k-*t* Berry curvature, reflecting the geometry of the restricted Hilbert space (nonholonomy)

$$\boldsymbol{\mathcal{F}}(\boldsymbol{k}, t) = i[\langle \partial_t u(\boldsymbol{k}, t) | \nabla_{\boldsymbol{k}} u(\boldsymbol{k}, t) \rangle - \langle \nabla_{\boldsymbol{k}} u(\boldsymbol{k}, t) | \partial_t u(\boldsymbol{k}, t) \rangle]$$

$$|u(\boldsymbol{k}, t)\rangle = \sum_\sigma u_\sigma(\boldsymbol{k}, t) |\boldsymbol{k}\sigma\rangle$$

cf. Thouless pump   D. J. Thouless, PRB **27**, 6083 (1983)

1(space)+1(time) dimensions

The integral of *F* is quantized, giving the Chern number.

Exp. S. Nakajima, et al., Nat. Phys. **12**, 296 (2016)

M. Lohse, et al., Nat. Phys. **12**, 350 (2016)

Here

2(space)+1(time) dimensions

The integral of *F* is not quantized.

anomalous Hall effect

Purely geometrical quantum dynamics

In our case, the purely geometrical quantum dynamics is caused by measurement or dissipation.

$$\langle \dot{\hat{\boldsymbol{r}}} \rangle = \int \frac{d^2\boldsymbol{k}}{(2\pi a^{-1})^2} \rho(\boldsymbol{k}) \boldsymbol{\mathcal{F}}(\boldsymbol{k}, t)$$

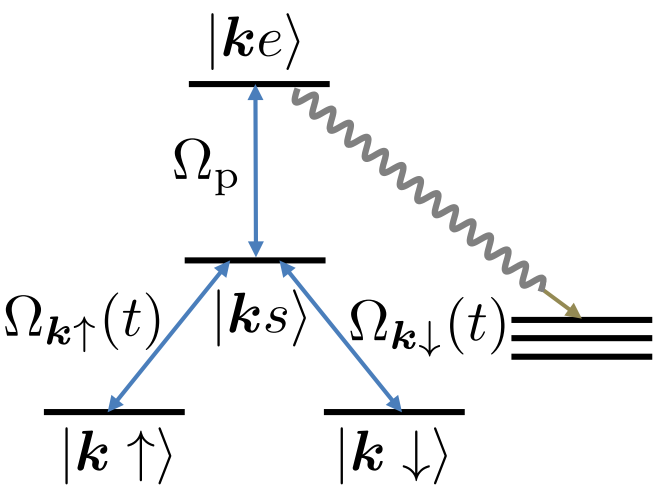
$$\boldsymbol{\mathcal{F}}(\boldsymbol{k}, t) = i[\langle \partial_t u(\boldsymbol{k}, t) | \nabla_{\boldsymbol{k}} u(\boldsymbol{k}, t) \rangle - \langle \nabla_{\boldsymbol{k}} u(\boldsymbol{k}, t) | \partial_t u(\boldsymbol{k}, t) \rangle]$$

$$|u(\boldsymbol{k}, t)\rangle = \textstyle\sum_\sigma u_\sigma(\boldsymbol{k}, t) |\boldsymbol{k}\sigma\rangle$$

To have a Hall effect, we have only to apply a uniform force *F*.

$$\Omega_{\boldsymbol{k}\sigma}(t) = \Omega_{\boldsymbol{k}+\boldsymbol{F}t/\hbar,\sigma}$$

$$\Rightarrow \quad u_\sigma(\boldsymbol{k}, t) = u_\sigma(\boldsymbol{k} + \boldsymbol{F}t/\hbar)$$

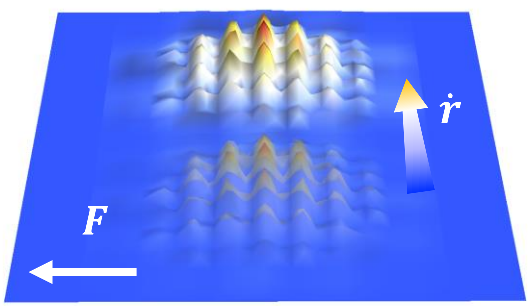


Dynamics of COM of the wave packet

$$\langle \dot{\hat{\boldsymbol{r}}} \rangle = -\frac{\boldsymbol{F}}{\hbar} \times \int \frac{d^2\boldsymbol{k}}{(2\pi a^{-1})^2} \rho(\boldsymbol{k}) \boldsymbol{\mathcal{B}}(\boldsymbol{k} + \boldsymbol{F}t/\hbar)$$

$$\boldsymbol{\mathcal{B}}(\boldsymbol{k}) = i\nabla_{\boldsymbol{k}} \times \langle u(\boldsymbol{k}) | \nabla_{\boldsymbol{k}} u(\boldsymbol{k}) \rangle$$

The Berry curvature acts as an effective magnetic field, and particles move perpendicularly to the external force due to the nontrivial Berry curvature. → anomalous Hall effect



Isolated dynamics

The nonlocal term is absent.

$$\dot{\hat{\rho}}_t = \frac{i}{\hbar}[\boldsymbol{F} \cdot \hat{\boldsymbol{r}}, \hat{\rho}_t]$$

potential gradient

The position operators commute.

$$[\hat{x}, \hat{y}] = 0 \qquad \Rightarrow \quad \langle \dot{\hat{\boldsymbol{k}}} \rangle = \frac{\boldsymbol{F}}{\hbar}, \qquad \langle \dot{\hat{\boldsymbol{r}}} \rangle = \boldsymbol{0}$$

Then, a potential gradient alone cannot generate any real-space dynamics!

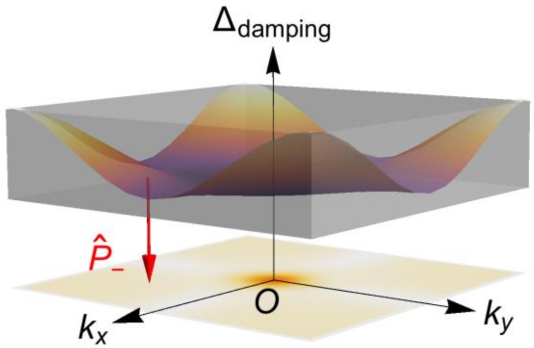
Quantum Zeno dynamics

With the nonlocal term,

$$\dot{\hat{\rho}}_t = \frac{i}{\hbar}[\boldsymbol{F}\cdot\hat{\boldsymbol{r}}, \hat{\rho}_t] + \sum_{\boldsymbol{r}} \kappa^{-1}\mathcal{D}[\hat{L}_{\boldsymbol{r}}]\hat{\rho}_t$$

potential gradient

nonlocal one-body loss



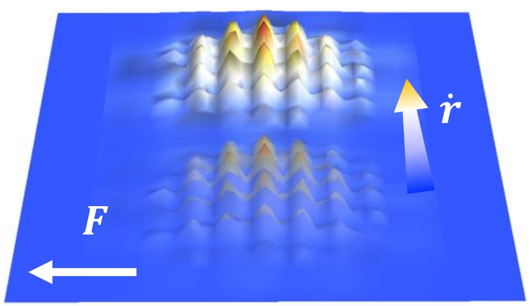
The accessible Hilbert space is constrained to a single Bloch band (quantum Zeno subspace), and the position operators do not commute due to a nonzero Berry curvature.

quantum Zeno subspace

Non-commutativity of position operators → transverse motion

$$[\hat{x}_-, \hat{y}_-] = i\hat{\mathcal{B}} \equiv i\sum_{\boldsymbol{k}} \mathcal{B}_{xy}(\boldsymbol{k})|\boldsymbol{k}-\rangle\langle\boldsymbol{k}-|$$

$$\Rightarrow \quad \langle\dot{\hat{\boldsymbol{k}}}\rangle = \frac{\boldsymbol{F}}{\hbar}, \quad \langle\dot{\hat{\boldsymbol{r}}}\rangle = -\frac{\boldsymbol{F}}{\hbar}\times\boldsymbol{e}_z\langle\hat{\mathcal{B}}\rangle$$



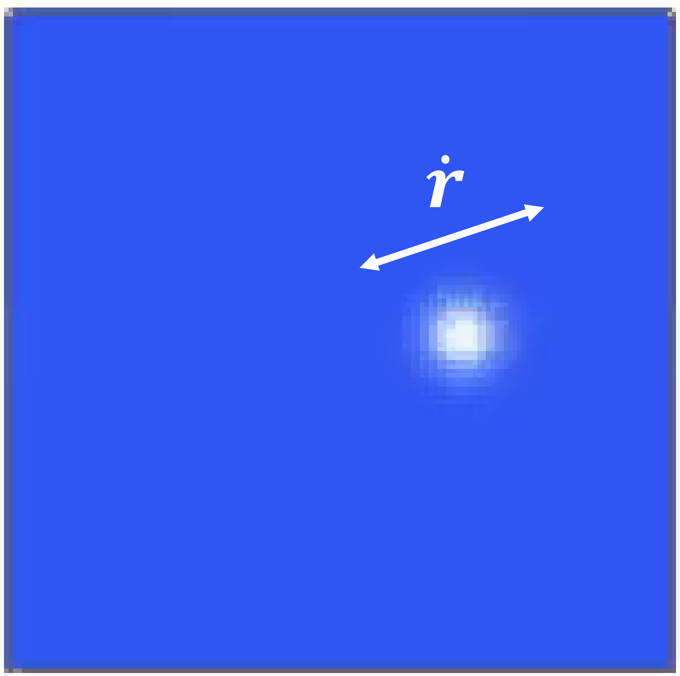
Zeno Hall effect

N. Nagaosa *et al.*, RMP **82**, 1539 (2010)
D. Xiao *et al.*, RMP **82**, 1959 (2010)

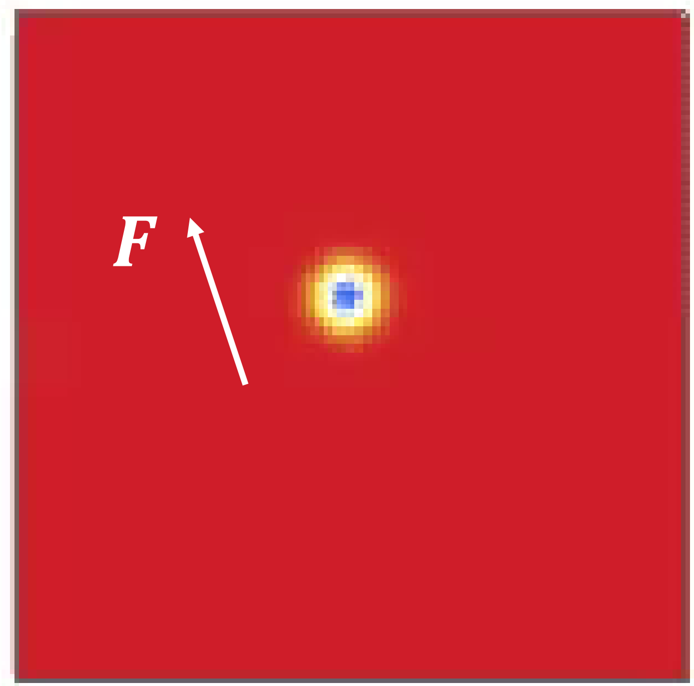
Long-time dynamics

At early times, the wave packet moves perpendicular to the force. After colliding with the boundary, it is retroreflected!

Real space



momentum space



$$\boldsymbol{F} = \frac{F}{\sqrt{10}}(-1, 3)$$

Z. Gong, S. Higashikawa, and MU, PRL **118**, 200401 (2017)

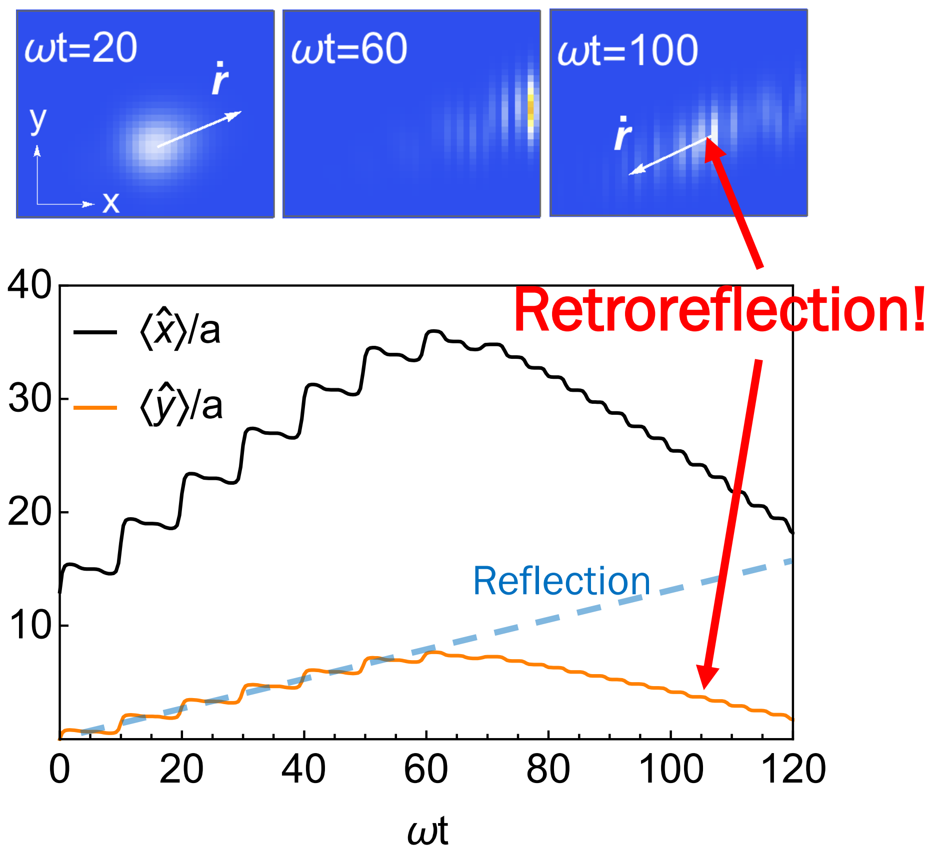
Physical Origin of Retroreflection

The velocity of the wave packet depends on the sign of the Berry curvature.

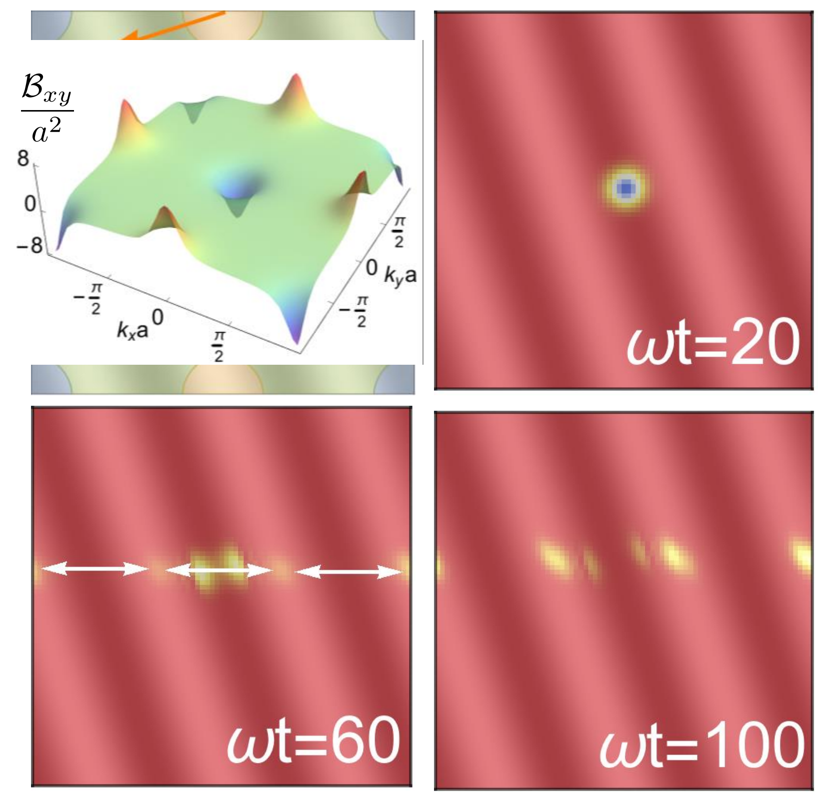
Upon collision with the wall, it squashes, and spreads in momentum space.

$$(i)\ \dot{\boldsymbol{r}} = -\frac{\boldsymbol{F}}{\hbar} \times \boldsymbol{e}_z \mathcal{B}_{xy}(\boldsymbol{k})$$

$$(ii)\ \Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$



Brillouin zone



Z. Gong, S. Higashikawa, and MU, PRL **118**, 200401 (2017)

Summary I

I. Quantum critical phenomena under continuous observation

PRA **94**, 053615 (2016); **95**, 022124 (2017) ; Nat. Commun. 8, 15791 (2017)

- Many-particle transport depends on quantum statistics.
- Mott-insulator regime expands due to the quantum Zeno effect.
- Beyond-TLL universality class emerges in non-Hermitian critical systems

II. Full-counting many-particle dynamics PRL **120**, 185301 (2018)

- Nonequilibrium many-body dynamics at the quantum trajectory level.
- Superluminal propagation of correlations due to the non-Hermiticity in the full-counting dynamics.
- Chiral propagation of correlations near the exceptional point.

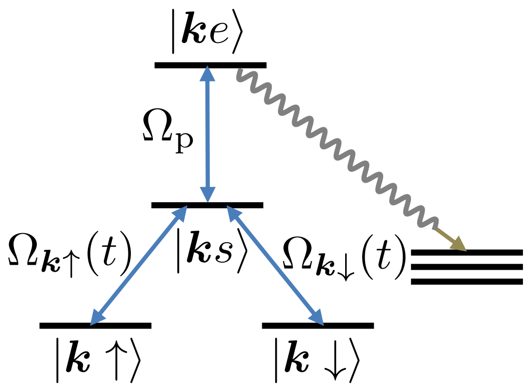
III. Quantum thermalization PRL **121**, 170402 (2018)

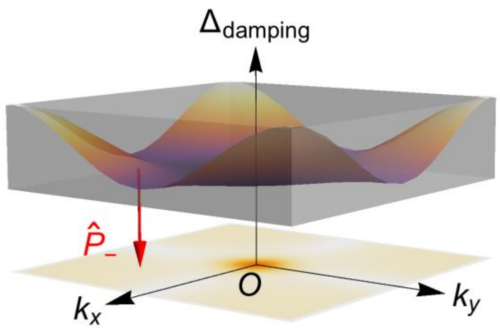
- Sigle-trajectory thermalization
- Effective temperature in open many-body quantum systems
- Results reproduced well by the matrix-vectrix-product ensemble

Summary II

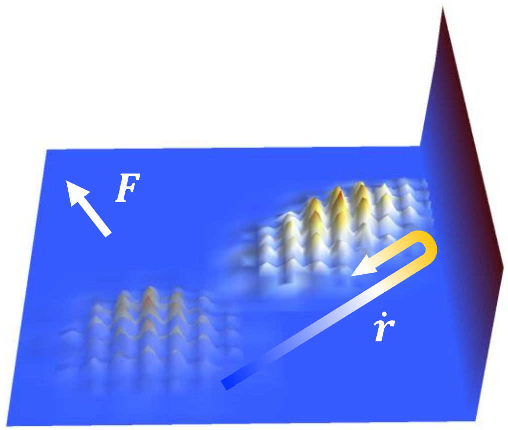
Inspired by von Neumann's idea of generating quantum dynamics solely by measurement, we predict two consequences of the flat-band anomalous Hall effect:

Zeno Hall effect





retroreflection



dissipative counterpart of the anomalous Hall effect

universal feature of a flat band with a nontrivial Berry curvature

Z. Gong, S. Higashikawa, and MU, PRL **118**, 200401 (2017)