Quantum dynamics of continuously monitored many-body systems

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Outline

1. Continuously monitored many-body dynamics
   1-1. quantum statistics
   1-2. quantum criticality
   1-3. full-counting dynamics
   1-4. thermalization

2. Quantum Zeno dynamics
   2-1. Zeno Hall effect
   2-2. retroreflection
Continuously monitored many-body dynamics

Yuto Ashida

Shunsuke Furukawa
Enables observation of the many-body wave function at the single-site resolution.

Heisenberg’s uncertainty relation

The higher the resolution, the greater the backaction will be.

Harvard, MPQ, Strathclyde, Kyoto, ...

How are the many-body dynamics and quantum phase transitions altered by the measurement backaction?

Frontier of QGM requires generalization of quantum-statistical mechanics that incorporates effects of measurement backaction.
Multi-Particle Quantum Dynamics under Continuous Observation
Our model is to continuously monitor atoms in an optical lattice lattice by spatially resolved measurement.

We shine off-resonant laser light on atoms in an optical lattice and measure scattered photons on the screen.

**Measurement operator**

\[
\hat{M}(X) = \sqrt{\gamma} \sum_{m} f(X - md) \hat{n}_m
\]

- **strength of measurement**
- **point spread function**
- **atom-number operator at lattice site** \(m\)
The dynamics of atoms under continuous monitoring is governed by three terms:

\[ d|\psi\rangle = -\frac{i}{\hbar}\hat{H}|\psi\rangle dt \quad \text{unitary evolution for Bose-Hubbard Hamiltonian} \]

\[ -\frac{1}{2} \int dX \left( \hat{M}^\dagger(X)\hat{M}(X) - \langle \hat{M}^\dagger(X)\hat{M}(X) \rangle \right)|\psi\rangle dt \quad \text{non-unitary evolution without photodetection} \]

\[ + \int dX \left( \frac{\hat{M}(X)|\psi\rangle}{\sqrt{\langle \hat{M}^\dagger(X)\hat{M}(X) \rangle}} - |\psi\rangle \right) dN(X; t) \quad \text{jump process upon photodetection} \]

### Continuous-Measurement Limit

1. frequent \( \gamma \gg J \)
   - measurement rate \( J \)
   - hopping rate of atoms

2. weak \( \sigma \gg d \)
   - spatial resolution \( d \)
   - lattice constant

while keeping the ratio \( \gamma / \sigma^2 \) constant.

- The weak measurement \( (\sigma >> d) \) keeps the measurement disturbance small.

- With such low resolution, one can still achieve single-site resolution by means of Bayesian inference (Y. Ashida and MU, PRL 115, 095301 (2015)).

- From the central limit theorem, the marked point process \( dN(X; t) \) is replaced by the Wiener process \( dW(X) \).

\[
dN(X) = \langle \hat{M}^\dagger(X) \hat{M}(X) \rangle dt + \sqrt{\langle \hat{M}^\dagger(X) \hat{M}(X) \rangle} dW(X)
\]
Continuous measurement of distinguishable particles can be modeled by a generalization of the single-particle model due to S. Diosi [Phys. Rev. A 40, 1165 (1989)].

\[
d\hat{\rho} = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho} \right] dt - \frac{\gamma_{\text{tot}} d^2}{4\sigma^2} \left[ \hat{X}_{\text{CM}}, \left[ \hat{X}_{\text{CM}}, \hat{\rho} \right] \right] dt + \sqrt{\frac{\gamma_{\text{tot}} d^2}{2\sigma^2}} \left\{ \hat{X}_{\text{CM}} - \langle \hat{X}_{\text{CM}} \rangle, \hat{\rho} \right\} dW
\]

\[
- \frac{d^2}{4\sigma^2} \sum_{i=1}^{N} \gamma_i \left[ \hat{r}_i, \left[ \hat{r}_i, \hat{\rho} \right] \right] dt + \sum_{i=1}^{N} \sqrt{\frac{\gamma_i d^2}{2\sigma^2}} \left\{ \hat{r}_i - \langle \hat{r}_i \rangle, \hat{\rho} \right\} dW_i
\]

The measurement backaction leads to decoherence of both the COM coordinate and the relative positional coordinates. ← All DOF can be distinguished and hence susceptible to measurement backaction.
Indistinguishable Particles

For indistinguishable particles, the “relative positions” cannot be distinguished.

In fact, the relative-coordinate part is canceled out due to two-particle interference!

\[
d\hat{\rho} = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho} \right] dt - \frac{\gamma_{\text{tot}} d^2}{4\sigma^2} \left[ \hat{X}_{CM}, \left[ \hat{X}_{CM}, \hat{\rho} \right] \right] dt + \sqrt{\frac{\gamma_{\text{tot}} d^2}{2\sigma^2}} \left\{ \hat{X}_{CM} - \langle \hat{X}_{CM} \rangle, \hat{\rho} \right\} dW
\]

center-of-mass decoherence

\[
-\frac{d^2}{4\sigma^2} \sum_{i=1}^{N} \gamma_i \left[ \hat{r}_i, \left[ \hat{r}_i, \hat{\rho} \right] \right] dt + \sum_{i=1}^{N} \sqrt{\frac{\gamma_i d^2}{2\sigma^2}} \left\{ \hat{r}_i - \langle \hat{r}_i \rangle, \hat{\rho} \right\} dW_i
\]

relative positional decoherence

Multi-particle quantum interference suppresses the relative positional decoherence, and only the center-of-mass position decoheres.

Initial condition: two particles at adjacent sites

Let the system evolve freely subject to continuous monitoring.

We compare quantum transport for distinguishable particles: fermions, bosons.
Without measurement: unitary evolution

Under continuous observation

Two particles move away in opposite directions, and the density profile is almost independent of quantum statistics.

Quantum dynamics under measurement backaction
Continuous Observation

- uncorrelated
- diffusive random walk

Measurement backaction → COM+relative positional decoherence

Two particles move away in opposite directions, and the density profile is almost independent of quantum statistics.

Distinguishable Particles

- fermions
- bosons
Indistinguishable Particles

**Distinguishable**
Measurement backaction → COM + relative positional decoherence
- uncorrelated
- diffusive random walk

**Indistinguishable**
COM + relative position decoherence (no relative positional decoherence)
- localized
- strongly correlated
- ballistic transport (inertial regime)

Under strong measurement, the dynamics shows diffusive transport due to strong measurement backaction.

Quantum statistics serves as effective repulsion for fermions and attraction for bosons.

Fermions show faster diffusion and bosons show slower diffusion than distinguishable particles due to multi-particle interference.

Quantum critical phenomena under continuous observation
A single quantum trajectory undergoes several quantum jumps.

**nonunitary dynamics during null measurement outcomes**

Effective non-Hermitian Hamiltonian:

\[
\hat{H}_{\text{eff}} = \hat{H} - \frac{i\gamma}{2} \sum_i \hat{M}_i^\dagger \hat{M}_i
\]

\(\hat{M}_i\): jump operators

**measurement backaction for null measurement outcomes between quantum jumps**

* If the ensemble average is taken over all outcomes, the dynamics is described by the Lindblad eq. → Quantum correlations and criticality would then be washed out.

J. Schachenmayer et al., PRA 89, 011601 (2014).
We consider situations in which the Hermitian part of the Hamiltonian $H$ exhibits quantum criticality.

- **Effective Hamiltonian**: 
  \[ \hat{H}_{\text{eff}} = \hat{H} - \frac{i\gamma}{2} \sum_i \hat{M}_i^\dagger \hat{M}_i \]

- **Complex Eigenvalues**: 
  \[ E_\lambda - \frac{i\Gamma_\lambda}{2} \quad E_\lambda \quad \Gamma_\lambda \]
  - Real part: effective *energy* of the system
  - Imaginary part: *decay rate*

- **Effective Ground State**: 
  \[ |\Psi_{\text{GS}}\rangle \]
  Eigenstate of $\hat{H}_{\text{eff}}$ having the *lowest* eigenenergy $E_\lambda$

In our models, $|\Psi_{\text{GS}}\rangle$ also has the minimal $\Gamma_\lambda$ (the *longest* lifetime).

We consider situations in which the Hermitian part of the Hamiltonian $\hat{H}$ exhibits *quantum criticality*. 
Jump process: two-body loss due to inelastic collisions

\[ \hat{H}_{\text{eff}} = \hat{H}_{BH} - \frac{i\gamma}{2} \sum_i \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i \]

\[ \hat{M}_i = \hat{b}_i^2 \]

The SF-Mott insulator transition point is identified with the one at which the energy gap closes.

The measurement backaction shifts the transition point, so that the Mott lobes expand.

Superfluid-Mott Insulator Transition in Bose-Hubbard Model

- jump process: two-body loss due to inelastic collisions

\[ \hat{H}_{\text{eff}} = \hat{H}_{\text{BH}} - \frac{i\gamma}{2} \sum_i \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i \]

\[ \hat{M}_i = \hat{b}_i^2 \]

The SF-Mott insulator transition point is identified as the one at which an energy gap closes.

Expansion of Mott lobes may be interpreted as being caused by the suppression of the hopping rate due to the continuous quantum Zeno effect.

Quantum Criticality of a 1D Bose Gas

- Jump process: two-body loss
  \[ \to \text{modified Lieb-Liniger model} \]

\[ \hat{H}_{\text{LL}} = \int dx \left[ \hat{\Psi}^\dagger (x) \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \hat{\Psi} (x) + \frac{g}{2} \hat{\Psi}^\dagger (x) \hat{\Psi}^\dagger (x) \hat{\Psi} (x) \hat{\Psi} (x) \right] \]

- Low-energy model = non-Hermitian Tomonaga-Luttinger model

\[ \hat{H}_{\text{eff}} = \frac{\hbar}{2\pi} \int_{-\infty}^{\infty} dx \left[ v_J (\partial_x \hat{\Theta})^2 + \tilde{v}_N e^{-i\gamma} (\partial_x \hat{\Phi})^2 \right] \]

non-Hermiticity: \( \delta \gamma \to 0 \ (\gamma \to 0) \)

Behaviors of two critical exponents against the measurement strength

one-particle correlation: $\langle \hat{\Psi}^\dagger (r) \hat{\Psi} (0) \rangle \propto \left( \frac{1}{r} \right)^{\frac{1}{2K_\phi}}$

density-density correlation: $\langle \hat{\rho}(r) \hat{\rho}(0) \rangle - \rho_0^2 = -\frac{K_\theta}{2\pi^2 r^2} + \text{const.} \times \frac{\cos(2\pi \rho_0 r)}{r^2 K_\theta}$

equal for Tomonaga-Luttinger liquid universality class

Behaviors of two critical exponents against the measurement strength

one-particle correlation: \[ \langle \hat{\Psi}^\dagger(r) \hat{\Psi}(0) \rangle \propto \left( \frac{1}{r} \right)^{2K_\phi} \]
density-density correlation: \[ \langle \hat{\rho}(r) \hat{\rho}(0) \rangle - \rho_0^2 = -\frac{K_\theta}{2\pi^2 r^2} + \text{const.} \times \frac{\cos(2\pi \rho_0 r)}{r^{2K_\theta}} \]
equal for TLL univ. class

Two different critical exponents indicate the emergence of the unique 1D quantum critical universality class caused by measurement backaction.

Full-Counting Many-Particle Dynamics toward establishing statistical mechanics under continuous observation

Y. Ashida and M. Ueda, PRL 120, 185301 (2018)
We now take into account quantum jump events as well.

Full-counting statistics $P_n(t)$: probability of observing $n$ quantum jumps during a time interval $[0,t]$

Garrahan and Lesanovsky, PRL 104, 160601 (2010)
We initially prepare $N$ atoms.
At time $t$, some atoms are lost due to quantum jumps.
Measure the remaining total atom number using QGM.

Full-counting dynamics \( \hat{\rho}^{(n)}(t) \): ensemble of trajectories for the varying number of quantum jumps
General solution for $n$ quantum jumps:

$$
\hat{\rho}^{(n)}(t) = \sum_{\{a_k\}_{k=1}^{n}} \int_0^t dt_n \cdots \int_0^{t_2} dt_1 \prod_{k=1}^{n} \left[ \hat{U}_{\text{eff}}(\Delta t_k) \hat{L}_{a_k} \right] \times \hat{U}_{\text{eff}}(t_1) \hat{\rho}(0) \hat{U}_{\text{eff}}^\dagger(t_1) \prod_{k=1}^{n} \left[ \hat{L}_{a_k}^\dagger \hat{U}_{\text{eff}}^\dagger(\Delta t_k) \right]
$$

$\rightarrow$ ensemble of all possible trajectories with $n$ quantum jumps
Solvable model: fermions under periodic dissipation

site-resolved detection of the atom number

periodic dissipative potential (one-body loss)

half-filled fermions in a superlattice

Lindblad equation (ensemble level)
\[
\frac{d\hat{\rho}(t)}{dt} = -i \left( \hat{H}_{\text{eff}} \hat{\rho} - \hat{\rho} \hat{H}_{\text{eff}}^\dagger \right) + \mathcal{J}[\hat{\rho}]
\]

effective Hamiltonian:
\[
\hat{H}_{\text{eff}} = \hat{H}_{\text{PT}} - 2i\gamma \hat{N}
\]

PT-symmetric Hamiltonian:
\[
\hat{H}_{\text{PT}} = -\sum_{l=0}^{L-1} \left[ (J + (-1)^l i\gamma) \left( \hat{c}_l^\dagger \hat{c}_{l+1} + \hat{c}_{l+1}^\dagger \hat{c}_l \right) + (-1)^l \hbar \hat{c}_l^\dagger \hat{c}_l \right]
\]

jump term:
\[
\mathcal{J}[\hat{\rho}] = 2\gamma \sum_l \left[ 2\hat{c}_l \hat{\rho} \hat{c}_l^\dagger + (-1)^l (\hat{c}_l \hat{\rho} \hat{c}_{l+1}^\dagger + \hat{c}_{l+1} \hat{\rho} \hat{c}_l^\dagger) \right]
\]

Y. Ashida and MU, PRL 120, 185301 (2018)
We calculate the full-counting dynamics under the same condition.

full-counting dynamics: \( C^{(n)}(l, t) = \text{Tr}[\hat{\rho}^{(n)}(t)\hat{c}_l^\dagger \hat{c}_0] \)  
\( n: \) number of jumps
How thermalization and heating proceeds in open quantum many-body systems?

Y. Ashida, K. Saito, and M. Ueda, PRL 121, 170402 (2018)
Three types of thermalization

(1) System in contact with thermal reservoir

(2) Isolated systems

(3) Systems under control and measurement
We consider repeated indirect or continuous measurement.

**Jump processes** \( m = 1, \ldots, M \)

\[
\mathcal{E}_m(\hat{\rho}) = \gamma \delta t \hat{L}_m \hat{\rho} \hat{L}_m^\dagger
\]

**No-count process** \( m = 0 \)

\[
\mathcal{E}_0(\hat{\rho}) \simeq \left( 1 - i \hat{H}_{\text{eff}} \delta t \right) \hat{\rho} \left( 1 + i \hat{H}_{\text{eff}}^\dagger \delta t \right)
\]

\[
\hat{H}_{\text{eff}} = \hat{H} - i \hat{\Gamma} / 2 \quad \hat{\Gamma} = \gamma \sum_m \hat{L}_m^\dagger \hat{L}_m
\]

Each realization of a quantum trajectory is determined by specifying a sequence of measurement outcomes:

\[
\mathcal{T} = (t_1, \ldots, t_n) \quad \mathcal{M} = (m_1, \ldots, m_n)
\]

If the jump operator does not commute with the Hamiltonian, the measurement can heat/cool the system.
Numerical simulations: a local measurement

➢ Hard-core bosons with NN and NNN interactions.
➢ Site number = 18, particle number = 6
➢ Jump operator \( \hat{L}_l = \hat{n}_l \)
➢ Every time an atom is detected, a high-density region emerges which subsequently diffuses away toward an equilibrium value.

The kinetic energy increases stepwise upon quantum jump, with a concomitant decrease of the $k=0$ component.

Matrix-vector product ensemble

\[ \hat{\rho}_M \propto \sum_a [\mathcal{V}_m \cdots \mathcal{V}_1 \rho_{eq}]_a \hat{P}_a \quad (\mathcal{V}_m)_{ab} = |\langle E_a | \hat{L}_m | E_b \rangle|^2 \]

Zeno Hall Effect

Zongping Gong

Sho Higashikawa

PRL 118, 200401 (2017)
A quantum state can be transformed to another with almost unit probability by repeated projective measurements.

In fact, one can construct a series of projective measurements that smoothly connect from $|\psi\rangle$ to $|\varphi\rangle$:

$$\hat{P}_j = |\psi_j\rangle\langle\psi_j|, \quad |\psi_j\rangle = \cos \frac{\pi j}{2N} |\psi\rangle + \sin \frac{\pi j}{2N} |\phi\rangle$$

$$|\psi_0\rangle = |\psi\rangle, \quad |\psi_N\rangle = |\phi\rangle$$

$$P_N = |\langle \phi | \hat{P}_N \ldots \hat{P}_2 \hat{P}_1 |\psi\rangle|^2$$

$$= \prod_{j=1}^{N} |\langle \psi_j | \psi_{j-1} \rangle|^2 = \cos^{2N} \frac{\pi}{2N} = 1 - O(N^{-1})$$

$$\lim_{N \to \infty} P_N = 1$$
von Neumann’s idea can be implemented by using atomic internal states connected via time-dependent Rabi couplings.

Excite the system from $|s\rangle$ to $|e\rangle$, and detect fluorescence photons.

Projective measurement onto the dark state $|D(t)\rangle$, indicated by the absence of fluorescence signals.

Dark state $|D(t)\rangle \propto \Omega_\phi(t)|\psi\rangle - \Omega_\psi(t)|\phi\rangle$

\[
\begin{align*}
\Omega_\psi(t) &= -\Omega \sin \frac{\omega t}{2}, \\
\Omega_\phi(t) &= \Omega \cos \frac{\omega t}{2}
\end{align*}
\]

Perform measurement at discrete times. Von Neumann’s idea is faithfully realized.

M. Porrati and S. Putterman, PRA **36**, 929 (1987)
Can we apply von Neumann's idea to transport a wave packet from one location $|\psi>$ to another $|\phi>$ by measurement?
Appears to be difficult because an intermediate state must be a superposition state between spatially separated states.
At each point in the Brillouin zone, we introduce the same Λ-scheme, but make the Rabi couplings depend on time and the crystal momentum.
The SOC emerges from a nontrivial $k$-dependence of the dark state which can be implemented through synthetic gauge fields.

\[
\psi_\sigma(k, t) = \sqrt{\rho(k)} u_\sigma(k, t) \quad \sigma = \uparrow, \downarrow
\]

**dark-state wave function**

\[
\sum_\sigma \Omega_{k\sigma}(t) u_\sigma(k, t) = 0, \quad \sum_\sigma |u_\sigma(k, t)|^2 = 1
\]
Dynamics of the center of mass of the wave packet

\[ \langle \dot{\mathbf{r}} \rangle = \int \frac{d^2 k}{(2\pi a^{-1})^2} \rho(k) \mathcal{F}(k, t) \]

\( k-t \) Berry curvature, reflecting the geometry of the restricted Hilbert space (nonholonomy)

\[ \mathcal{F}(k, t) = i[\langle \partial_t u(k, t) | \nabla_k u(k, t) \rangle - \langle \nabla_k u(k, t) | \partial_t u(k, t) \rangle] \]

\[ |u(k, t)\rangle = \sum_\sigma u_\sigma(k, t) |k\sigma\rangle \]

cf. Thouless pump  \quad D. J. Thouless, PRB 27, 6083 (1983)

1(space)+1(time) dimensions
The integral of \( F \) is quantized, giving the Chern number.


Here

2(space)+1(time) dimensions
The integral of \( F \) is not quantized.

anomalous Hall effect
In our case, the purely geometrical quantum dynamics is caused by measurement or dissipation.

\[
\langle \dot{\mathbf{r}} \rangle = \int \frac{d^2 \mathbf{k}}{(2\pi a^{-1})^2} \rho(\mathbf{k}) \mathcal{F}(\mathbf{k}, t)
\]

\[
\mathcal{F}(\mathbf{k}, t) = i[\langle \partial_t u(\mathbf{k}, t) | \nabla_\mathbf{k} u(\mathbf{k}, t) \rangle - \langle \nabla_\mathbf{k} u(\mathbf{k}, t) | \partial_t u(\mathbf{k}, t) \rangle]
\]

\[
|u(\mathbf{k}, t)\rangle = \sum_\sigma u_\sigma(\mathbf{k}, t) |k\sigma\rangle
\]

To have a Hall effect, we have only to apply a uniform force \( F \).

\[
\Omega_{k\sigma}(t) = \Omega_{k+Ft/\hbar,\sigma}
\]

\[
\Rightarrow u_\sigma(\mathbf{k}, t) = u_\sigma(\mathbf{k} + F t / \hbar)
\]
The Berry curvature acts as an effective magnetic field, and particles move perpendicularly to the external force due to the nontrivial Berry curvature. \( \Rightarrow \text{anomalous Hall effect} \)
The nonlocal term is absent.

$$\dot{\hat{\rho}} = \frac{i}{\hbar} [\hat{F} \cdot \hat{r}, \hat{\rho}]$$

potential gradient

The position operators commute.

$$[\hat{x}, \hat{y}] = 0 \quad \Rightarrow \quad \langle \hat{k} \rangle = \frac{F}{\hbar}, \quad \langle \hat{r} \rangle = 0$$

Then, a potential gradient alone cannot generate any real-space dynamics!
With the nonlocal term,

\[ \dot{\hat{\rho}}_t = \frac{i}{\hbar} [\hat{F} \cdot \hat{\textbf{r}}, \hat{\rho}_t] + \sum_{\textbf{r}} \kappa^{-1} \hat{D}[\hat{L}_{\textbf{r}}] \hat{\rho}_t \]

potential gradient  nonlocal one-body loss

The accessible Hilbert space is constrained to a single Bloch band (quantum Zeno subspace), and the position operators do not commute due to a nonzero Berry curvature.

Non-commutativity of position operators \( \rightarrow \) transverse motion

\[ [\hat{x}_-, \hat{y}_-] = i \hat{\mathcal{B}} \equiv i \sum_{\textbf{k}} \mathcal{B}_{xy}(\textbf{k}) |\textbf{k}-\rangle \langle \textbf{k}-| \]

\[ \Rightarrow \quad \langle \hat{k} \rangle = \frac{F}{\hbar}, \quad \langle \hat{\textbf{r}} \rangle = -\frac{F}{\hbar} \times e_z \langle \hat{\mathcal{B}} \rangle \]

N. Nagaosa et al., RMP 82, 1539 (2010)
D. Xiao et al., RMP 82, 1959 (2010)
At early times, the wave packet moves perpendicular to the force. After colliding with the boundary, it is retroreflected!

\[ F = \frac{F}{\sqrt{10}} (-1, 3) \]

Z. Gong, S. Higashikawa, and MU, PRL 118, 200401 (2017)
The velocity of the wave packet depends on the sign of the Berry curvature.

Upon collision with the wall, it squashes, and spreads in momentum space.

$$ (i) \quad \dot{\mathbf{r}} = -\frac{F}{\hbar} \times e_z \mathbf{B}_{xy}(\mathbf{k}) $$

$$ (ii) \quad \Delta x \cdot \Delta p_x \geq \frac{\hbar}{2} $$

Z. Gong, S. Higashikawa, and MU, PRL 118, 200401 (2017)
Summary I

I. Quantum critical phenomena under continuous observation

PRA 94, 053615 (2016); 95, 022124 (2017) ; Nat. Commun. 8, 15791 (2017)

- Many-particle transport depends on quantum statistics.
- Mott-insulator regime expands due to the quantum Zeno effect.
- Beyond-TLL universality class emerges in non-Hermitian critical systems

II. Full-counting many-particle dynamics

PRL 120, 185301 (2018)

- Nonequilibrium many-body dynamics at the quantum trajectory level.
- Superluminal propagation of correlations due to the non-Hermiticity in the full-counting dynamics.
- Chiral propagation of correlations near the exceptional point.

III. Quantum thermalization

PRL 121, 170402 (2018)

- Single-trajectory thermalization
- Effective temperature in open many-body quantum systems
- Results reproduced well by the matrix-vectrix-product ensemble
Inspired by von Neumann’s idea of generating quantum dynamics solely by measurement, we predict two consequences of the flat-band anomalous Hall effect:

**Zeno Hall effect**

- Dissipative counterpart of the anomalous Hall effect

**Retroreflection**

- Universal feature of a flat band with a nontrivial Berry curvature

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Z. Gong, S. Higashikawa, and MU, PRL 118, 200401 (2017)