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「量子多体系の素核・物性クロスオーバー」
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つくばキャンパス 4号館セミナーホール

物質との超強結合で横電磁場は相転移するか？

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共同研究者・研究費・解説記事

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研究費

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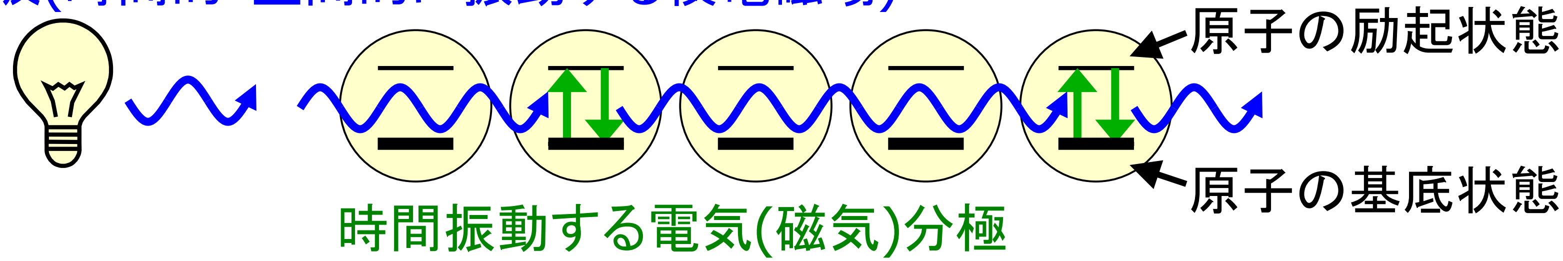
解説記事

- 馬場基彰, 日本物理学会誌 **73**(8), 540-541 (2018)
- 馬場基彰, パリティ **32**(11), 35-40 (2017)
- 馬場基彰, 固体物理 **52**(9), 459-476 (2017)

概要：物質との超強結合で横電磁場は相転移するか？

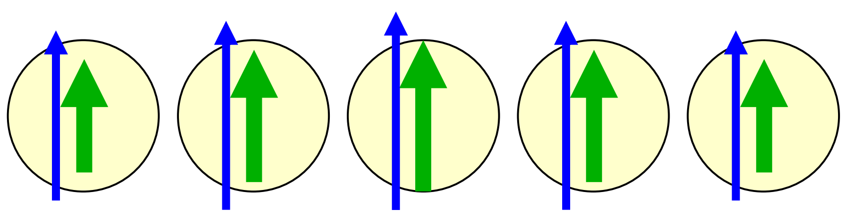
非平衡下

電磁波(時間的・空間的に振動する横電磁場)



- 物質との相互作用 ＝ 電気(磁気)分極との振幅のやり取り(屈折率の変化)
- 超強結合 ＝ 1光子レベルでの振幅のやり取りのレート $\gtrsim$ 原子遷移の振動数

熱平衡下 ($T < T_c$)



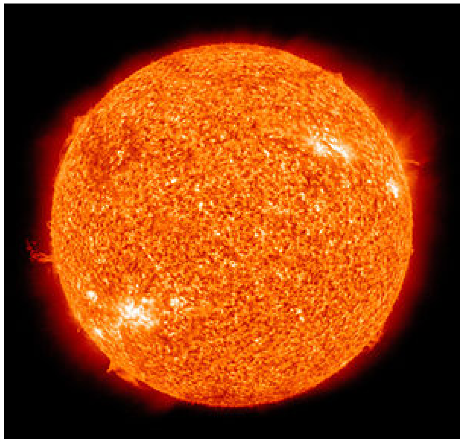
横電磁場と分極が超強結合する系では，熱平衡下で
静的な電場(磁場)と静的な電気(磁気)分極が自発的に現れる可能性がある

このような相転移が起こる物質を実験グループと共に探索している

Optical Science & Technology

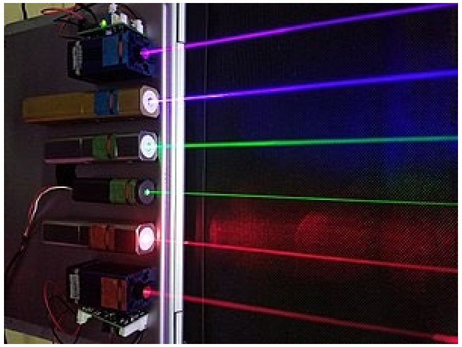
Images by Wikipedia

Photo-emission processes




Thermal radiation


Spontaneous emission



Stimulated emission

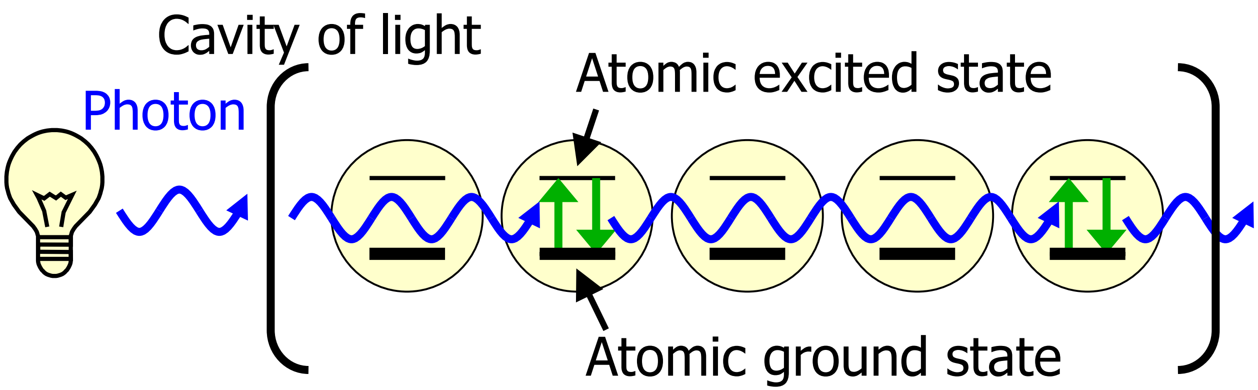
Displays

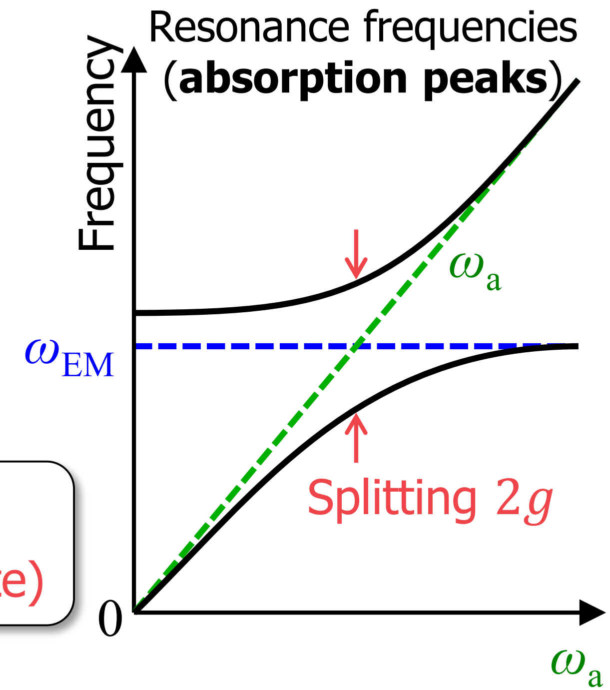
Communications

Photovoltaics

- **Thermal radiation** and **photoelectric effect** contribute the development of quantum theory.
- A variety of devices has been developed.

Light-matter dynamics (non-equilibrium)





Interaction strength $g \propto \sqrt{N}$
(signle-photon-level amplitude exchange rate)

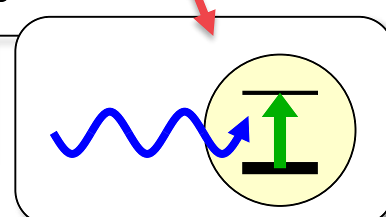
Dicke Hamiltonian

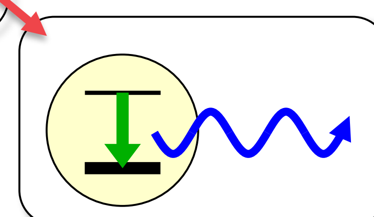
$$\hat{H}_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{EM}}\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}\left(\hat{\sigma}_j^{\dagger}\hat{a} + \hat{a}^{\dagger}\hat{\sigma}_j\right)$$

$\hat{a}$ annihilates a photon
$\hat{\sigma}_j$ lowers atom j

Energy of photons

Energy of atomic excitation





Even in **thermal equilibrium**, **ultra-strong** g makes a dramatic phenomenon

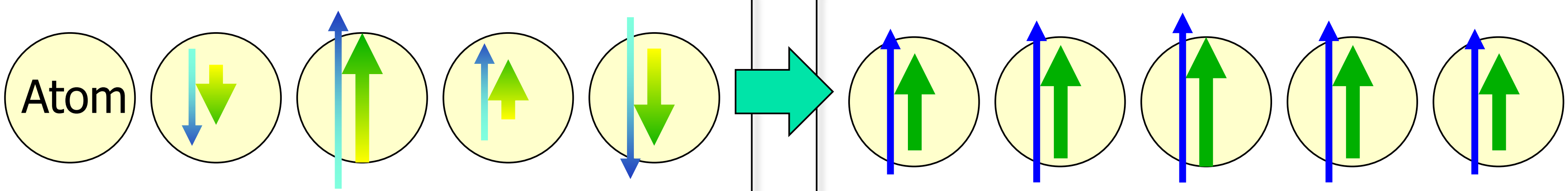
Phase transition of transverse EM field

Super-radiant phase transition (**SRPT**) proposed in 1973

K. Hepp and E. H. Lieb, Ann. Phys. **76**, 360 (1973)

SRPT might appear in **special materials** with ultra-strong interaction

High temperature | **Low temperature**



Thermal motion of charges & **thermal** radiation

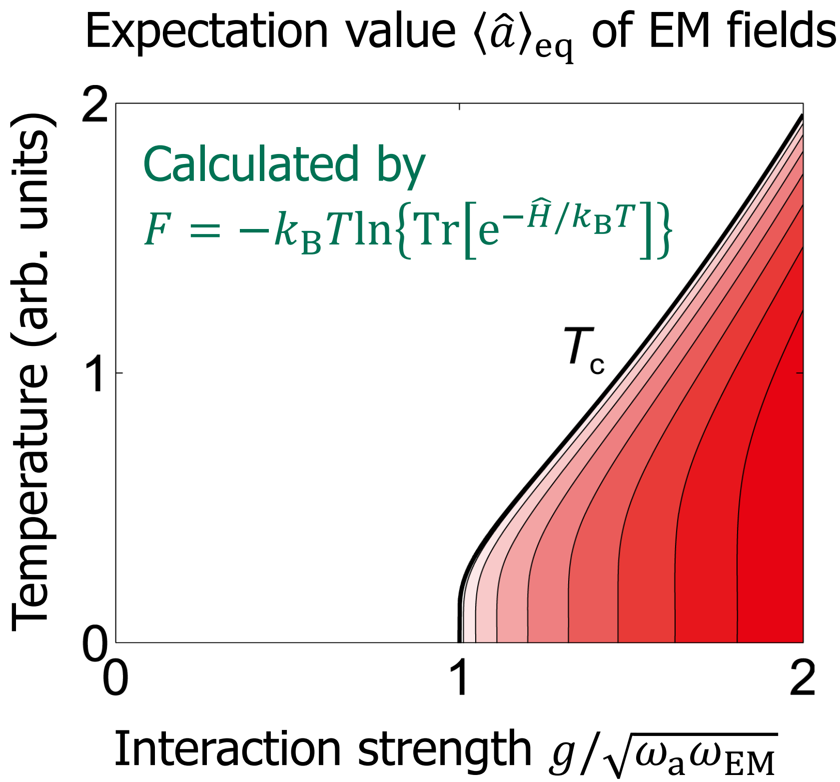
Spontaneous appearance of **static** electric displacement field $\boldsymbol{D}$ & electric polarization $\boldsymbol{P}$

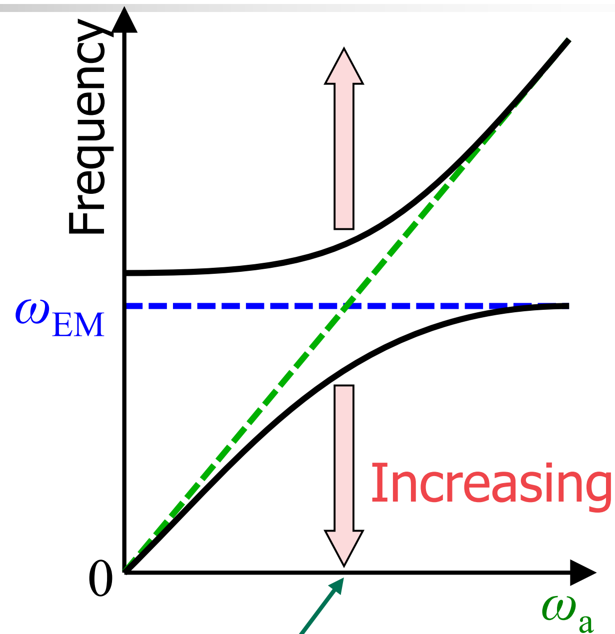
$$\hat{H}_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{EM}}\hat{a}^{\dagger}\hat{a} + \sum_{j=1}^{N}\frac{\hbar\omega_{\mathrm{a}}}{2}\hat{\sigma}_j^{z} + \frac{\hbar g}{\sqrt{N}}\sum_{j=1}^{N}\left(\hat{\sigma}_j^{\dagger}\hat{a} + \hat{a}^{\dagger}\hat{\sigma}_j\right)$$

Energy **cost** for appearance of $\boldsymbol{D}$ and $\boldsymbol{P}$ < Energy **benefit** by interaction

→ **SRPT** $(\langle\hat{a}\rangle_{\mathrm{eq}} \neq 0)$

Phase diagram (thermal equilibrium)



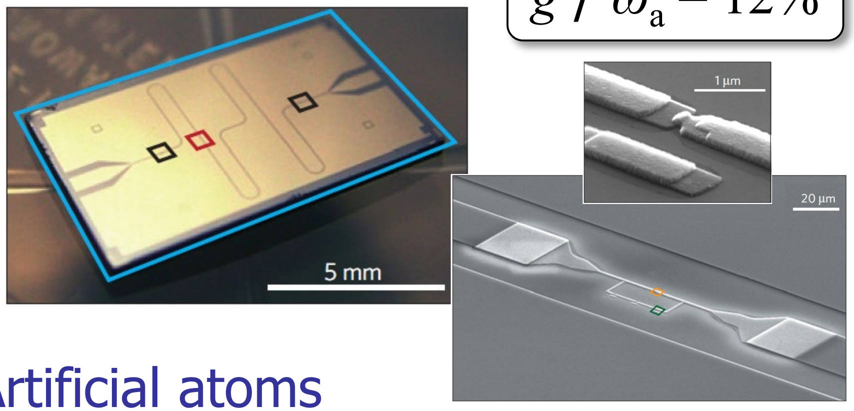


At $g \to \sqrt{\omega_a \omega_{EM}}$, $\omega \to 0$, i.e., **instability** of normal ground state ($\langle \hat{a} \rangle = 0$) ⇒ SRPT

- **Ultra-strong** interaction $g > \sqrt{\omega_a \omega_{EM}}$ is required
  - Quite large compared with typical materials in optical science & technology
- g is determined mainly by **materials** (not enhanced by radiation power)
- Since 2009, ultra-strong g has been implemented experimentally

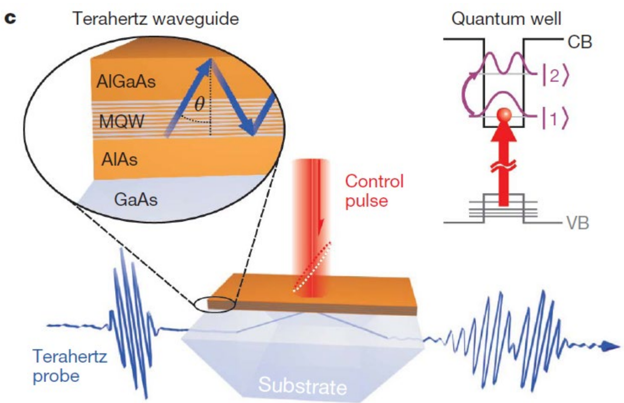
Materials showing ultra-strong interaction 1

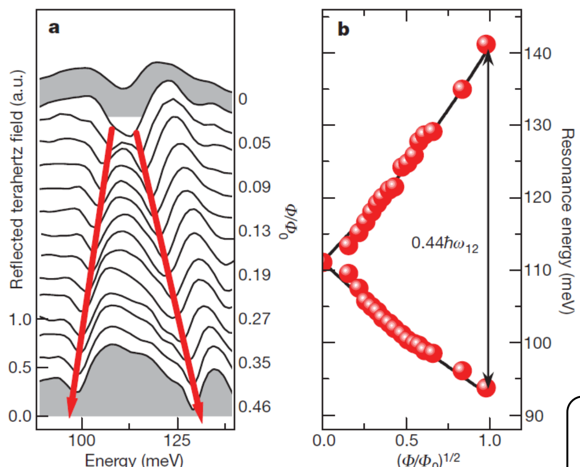
$g / \omega_a = 12\%$



Artificial atoms
in superconducting circuits (microwave)

T. Niemczyk, et al., Nature Phys. **6**, 772 (2010)

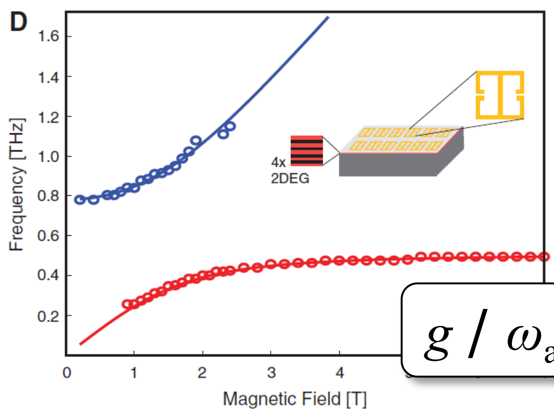




$g / \omega_a = 22\%$

Inter-subband transition in QWs (THz)

G. Gunter, et al., Nature **458**, 178 (2009)



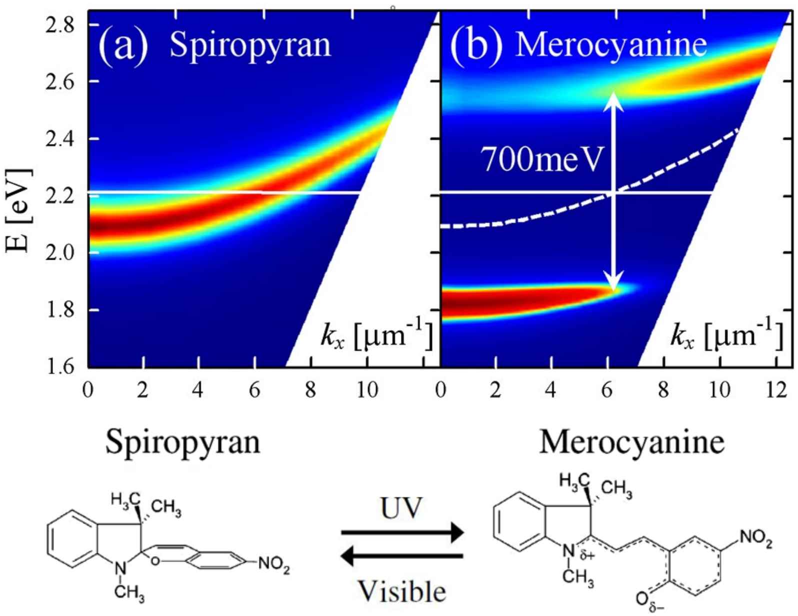
$g / \omega_a = 60\%$

Cyclotron transition of 2DEG (THz)

G. Scalari, et al., Science **335**, 1323 (2012)

Materials showing ultra-strong interaction 2

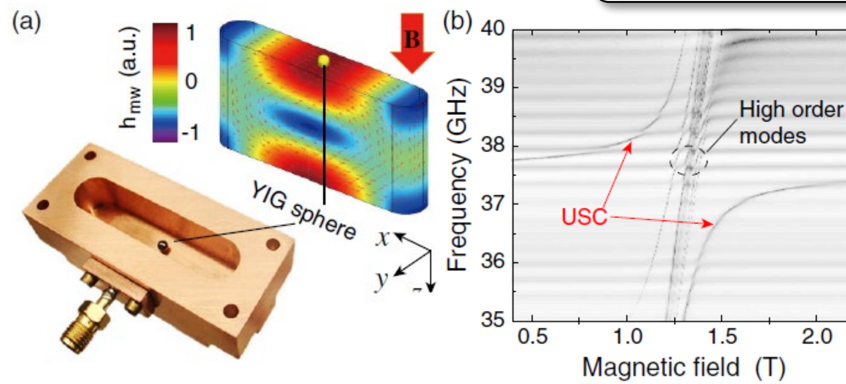
$g / \omega_a = 16\%$



Dye molecules (visible)

T. Schwartz, et al., PRL **106**, 196405 (2011)

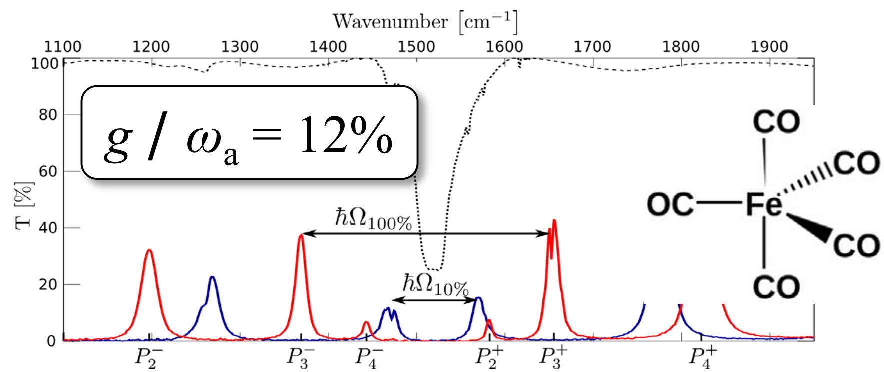
$g / \omega_a = 7\%$



Magnons in YIG sphere (microwave)

X. Zhang, et al., PRL **113**, 156401 (2014)

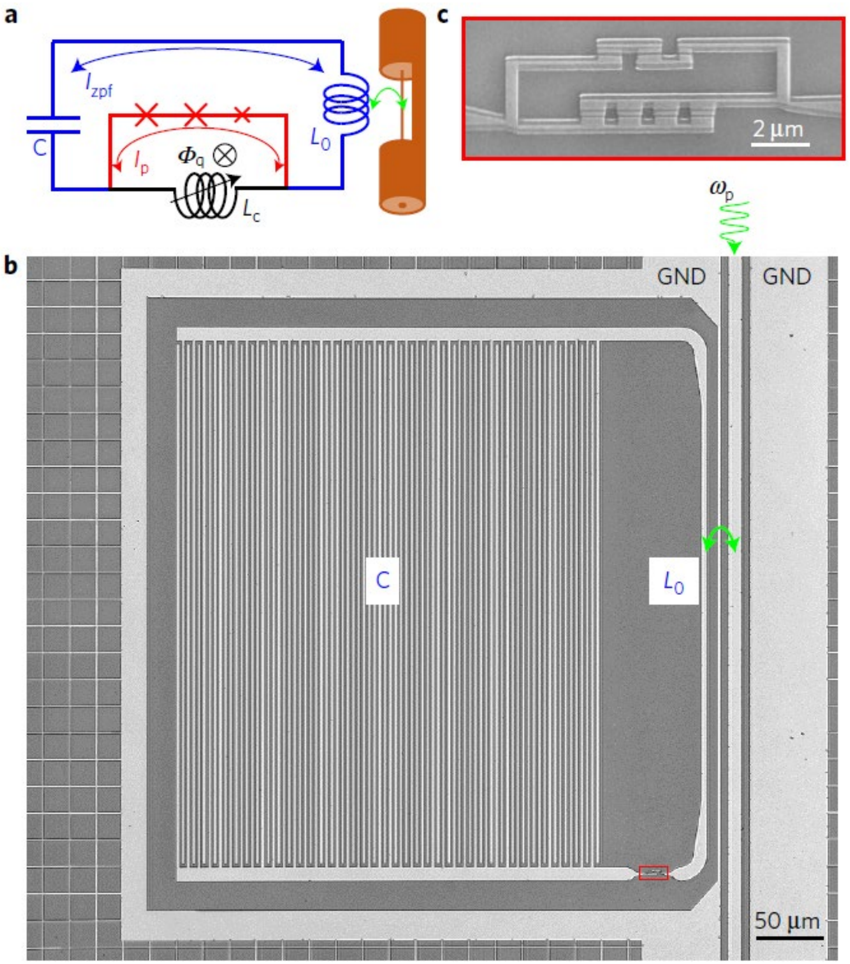
$g / \omega_a = 12\%$



Molecular vibration (infra-red)

J. George, et al., PRL **117**, 153601 (2016)

Recent progress: Superconducting circuit



$$g / \omega_a = 134\%$$

LC circuit (micro-wave) &
Superconducting flux qubit

F. Yoshihara, et al., Nat. Phys. **13**, 44 (2017)

Experimental techniques for **ultra-strong** interactions are highly developed

However, SRPT is **NOT** yet realized since the first proposal in 1973

History of SRPT

- 1973 Proposed theoretically
  K. Hepp and E. H. Lieb, Ann. Phys. (N.Y.) **76**, 360 (1973)
- 1975 A no-go theorem of **charge**-mediated SRPT
  K. Rzążewski, *et al.*, Phys. Rev. Lett. **35**, 432 (1975)
- 2009 Ultra-strong interactions started to be implemented
- 2010 A **non-equilibrium analogue** of SRPT was demonstrated
  K. Baumann, et al., Nature **464**, 1301 (2010)
- 2016 Proposal of **thermal-equilibrium analogue** of SRPT in a superconducting circuit
  M. Bamba, K. Inomata, and Y. Nakamura, Phys. Rev. Lett. **117**, 173601 (2016)
- 2018 A step toward another **thermal-equilibrium analogue** in experiment with **magnetic** material $ErFeO_3$
  X. Li, M. Bamba, N. Yuan, Q. Zhang, Y. Zhao, M. Xiang, K. Xu, Z. Jin, W. Ren, G. Ma, S. Cao, D. Turchinovich, and J. Kono, Science **361**, 794 (2018)

No-go theorem of charge-mediated SRPT

Hamiltonian of EM fields and charges

Maxwell equations

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r},t) = \rho(\boldsymbol{r},t)/\epsilon_0$$
$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r},t) = 0$$
$$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\dot{\boldsymbol{B}}(\boldsymbol{r},t)$$
$$\nabla \times \boldsymbol{B}(\boldsymbol{r},t) = \mu_0 \boldsymbol{J}(\boldsymbol{r},t) + \dot{\boldsymbol{E}}(\boldsymbol{r},t)/c^2$$

Newton's equation (Lorentz force)

$$m_j \ddot{\boldsymbol{r}}_j(t) = q_j \boldsymbol{E}(\boldsymbol{r}_j,t) + q_j \dot{\boldsymbol{r}}_j(t) \times \boldsymbol{B}(\boldsymbol{r}_j,t)$$

Charge density: $\rho(\boldsymbol{r}) = \sum_j q_j \delta(\boldsymbol{r} - \boldsymbol{r}_j)$

Current denstiy: $\boldsymbol{J}(\boldsymbol{r}) = \sum_j q_j \dot{\boldsymbol{r}}_j \delta(\boldsymbol{r} - \boldsymbol{r}_j)$

↓ In Coulomb gauge ($\nabla \cdot \boldsymbol{A} = 0$)

Minimal-coupling Hamiltonian (velocity form)

$$\hat{H}_{\mathrm{min}} = \int \mathrm{d}\boldsymbol{r} \left\{ \frac{\varepsilon_0 \hat{\boldsymbol{E}}_\perp(\boldsymbol{r})^2}{2} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right\} + \sum_j \frac{\left[\hat{\boldsymbol{p}}_j - q_j \hat{\boldsymbol{A}}(\hat{\boldsymbol{r}}_j)\right]^2}{2m_j} + V(\{\hat{\boldsymbol{r}}_j\})$$

↕ Unitary transform (in long-wavelength approximation)

Hamiltonian in length form (electric dipole "gauge")

$$\hat{H}_{\mathrm{len}} = \int \mathrm{d}\boldsymbol{r} \left\{ \frac{\left[\hat{\boldsymbol{D}}(\boldsymbol{r}) - \hat{\boldsymbol{P}}(\boldsymbol{r})\right]^2}{2\varepsilon_0} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right\} + \sum_j \frac{\hat{\boldsymbol{p}}_j^{\,2}}{2m_j} + V(\{\hat{\boldsymbol{r}}_j\})$$

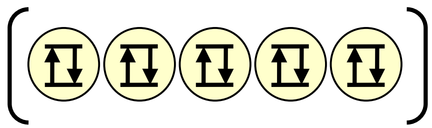
A no-go theorem of charge-mediated SRPT

Hamiltonian from Maxwell eqs. & Lorentz force

$$\hat{H} = \int \mathrm{d}\boldsymbol{r} \left\{ \frac{\left[\hat{\boldsymbol{D}}(\boldsymbol{r}) - \hat{\boldsymbol{P}}(\boldsymbol{r})\right]^2}{2\varepsilon_0} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right\} + \sum_j \frac{\hat{\boldsymbol{p}}_j{}^2}{2m_j} + V(\{\hat{\boldsymbol{r}}_j\})$$

Electric energy | Magnetic energy | Kinetic energy | Coulomb energy

For two-level atoms in a cavity



$$\hat{S}_x \equiv \sum_{j=1}^{N} \frac{\hat{\sigma}_j^\dagger + \hat{\sigma}_j}{2}$$

$$\hat{H} \approx \hbar\omega_{\mathrm{EM}} \hat{a}^\dagger \hat{a} + \sum_{j=1}^{N} \frac{\hbar\omega_{\mathrm{a}}}{2} \hat{\sigma}_j^z + \frac{4\hbar g^2}{N\omega_{\mathrm{EM}}} \hat{S}_x{}^2 + \frac{2\hbar g}{\sqrt{N}} \hat{S}_x (\hat{a}^\dagger + \hat{a})$$

$\hat{\boldsymbol{P}} \cdot \hat{\boldsymbol{D}}$

Does **NOT** show SRPT

P^2 term (additional **energy cost**)

Neglecting P^2 term

Dicke Hamiltonian

$$\hat{H}'_{\mathrm{Dicke}} = \hbar\omega_{\mathrm{EM}} \hat{a}^\dagger \hat{a} + \sum_{j=1}^{N} \frac{\hbar\omega_{\mathrm{a}}}{2} \hat{\sigma}_j^z + \frac{\hbar g}{\sqrt{N}} \sum_{j=1}^{N} (\hat{\sigma}_j^\dagger + \hat{\sigma}_j)(\hat{a}^\dagger + \hat{a})$$

P^2 terms was **NOT** considered at the first proposal in 1973

Another no-go theorem (classical theory)

$$\hat{H}_{\rm len} = \int \mathrm{d}\boldsymbol{r} \left\{ \frac{[\hat{\boldsymbol{D}}(\boldsymbol{r}) - \hat{\boldsymbol{P}}(\boldsymbol{r})]^2}{2\varepsilon_0} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right\} + \sum_j \frac{\hat{\boldsymbol{p}}_j{}^2}{2m_j} + V(\{\hat{\boldsymbol{r}}_j\})$$

$$\hat{H}_{\rm len} = \int \mathrm{d}\boldsymbol{r} \left\{ \frac{\hat{\boldsymbol{D}}(\boldsymbol{r})^2 + \hat{\boldsymbol{P}}(\boldsymbol{r})^2}{2\varepsilon_0} - \frac{\hat{\boldsymbol{D}}(\boldsymbol{r}) \cdot \hat{\boldsymbol{P}}(\boldsymbol{r})}{\varepsilon_0} + \frac{\hat{\boldsymbol{B}}(\boldsymbol{r})^2}{2\mu_0} \right\} + \sum_j \frac{\hat{\boldsymbol{p}}_j{}^2}{2m_j} + V(\{\hat{\boldsymbol{r}}_j\})$$

"Energy cost < Interaction energy" **cannot** be obtained

J. M. Knight, Y. Aharonov, and G. T. C. Hsieh, PRA **17**, 1454 (1978).

- More rigorous **quantum** analyses were also performed
  I. Bialynicki-Birula and K. Rzążewski, PRA **19**, 301 (1979);
  K. Gawędzki and K. Rzążnewski, PRA **23**, 2134 (1981).

- Counter examples are still being discussed
  T. Grieβer, A. Vukics, and P. Domokos, PRA **94**, 033815 (2016);
  G. Mazza and A. Georges, arXiv:1804.08534 [cond-mat.str-el].

- **Charge**-mediated light-matter interactions hardly give the SRPT

Motivation and Recent Progresses

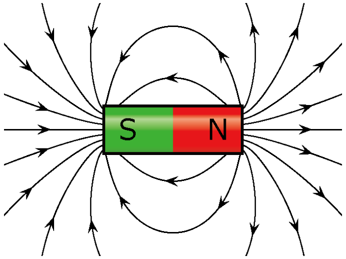
Motivation and strategy in SRPT study

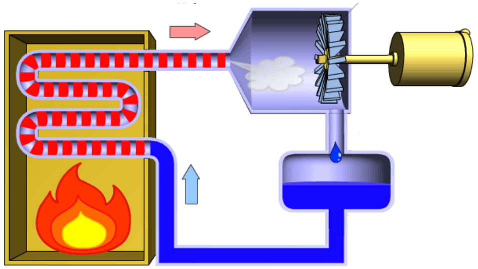
Optical Science & Technology

Non-equilibrium dynamics
of light and matters

Condensed matters & Thermodynamics

Thermal equilibrium of matters





Motivation

- Introduction of **heat** & **phase transitions** into optical science
- Realization of SRPT enhances the potential of **optical technology** and advances the **non-equilibrium** statistical physics

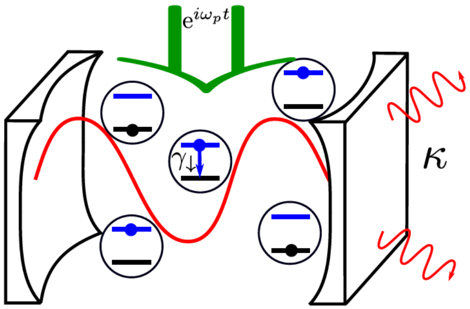
Strategy

- Experimental demonstration of **analogues** in a variety of systems
- Interaction mediated by **spins** or something except charges

Recent theoretical progresses on SRPT

- Quantum chaos, entanglement entropy, critical exponent, etc. in SRPT
  C. Emary and T. Brandes, PRL **90**, 044101 (2003); C. Emary and T. Brandes, PRE **67**, 066203 (2003); N. Lambert, C. Emary, and T. Brandes, PRL **92**, 073602 (2004); J. Larson and E. K. Irish, J. Phys. A Math. Theor. **50**, 174002 (2017); etc.

- Analyses of **non-equilibrium** SRPT (in cold atoms)
  e.g., P. Kirton, *et al.*, Adv. Quantum Technol. 1800043 (2018);
  H. J. Carmichael, Phys. Rev. X 5, 031028 (2015).



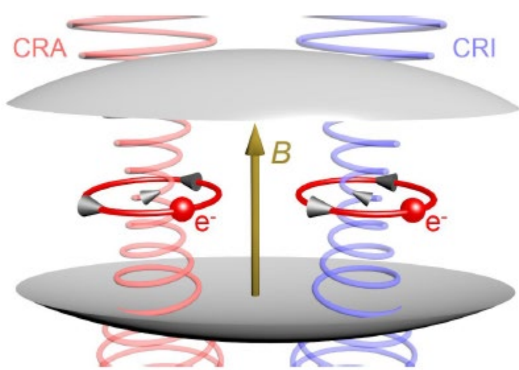
- Proposals of **charge**-mediated **thermal** SRPT (controversial)
  T. Grießer, A. Vukics, and P. Domokos, PRA **94**, 033815 (2016);
  G. Mazza and A. Georges, arXiv:1804.08534 [cond-mat.str-el].

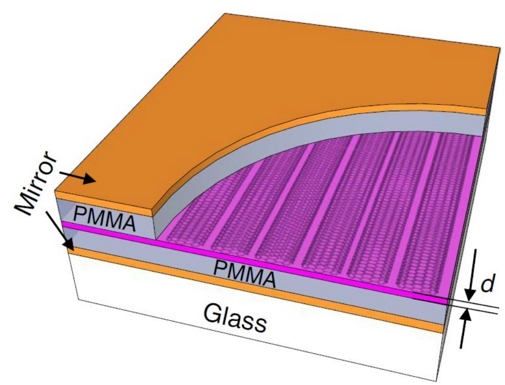
- Reminding the importance of **spin** for **thermal** SRPT
  M. Bamba and T. Ogawa, PRA **90**, 063825 (2014)
  - Originally, J. M. Knight, Y. Aharonov, and G. T. C. Hsieh, PRA **17**, 1454 (1978)

- Superconducting **circuit** showing a **thermal** "SRPT"
  M. Bamba, K. Inomata and Y. Nakamura, PRL **117**, 173601 (2016)

Recent experimental progresses for SRPT

- Signature (?) of **charge**-mediated SRPT (2DEG cyclotron resonance)
  J. Keller, G. Scalari, F. Appugliese, C. Maissen, J. Haase, M. Failla, M. Myronov, D. R. Leadley, J. Lloyd-Hughes, P. Nataf, and J. Faist, arXiv:1708.07773 [cond-mat.mes-hall]
- Quantitative evaluation of **additional energy cost** of EM fields
  X. Li, M. Bamba, Q. Zhang, S. Fallahi, G. C. Gardner, W. Gao, M. Lou, K. Yoshioka, M. J. Manfra, and J. Kono, Nature Photonics **12**, 324 (2018)
- Samples embedding carbon nanotubes with **easily-tunable** g
  W. Gao, X. Li, M. Bamba and J. Kono, Nature Photonics **12**, 362 (2018)
- A step toward "SRPT" in **magnetic** material $ErFeO_3$
  X. Li, M. Bamba, N. Yuan, Q. Zhang, Y. Zhao, M. Xiang, K. Xu, Z. Jin, W. Ren, G. Ma, S. Cao, D. Turchinovich, and J. Kono, Science **361**, 794 (2018)



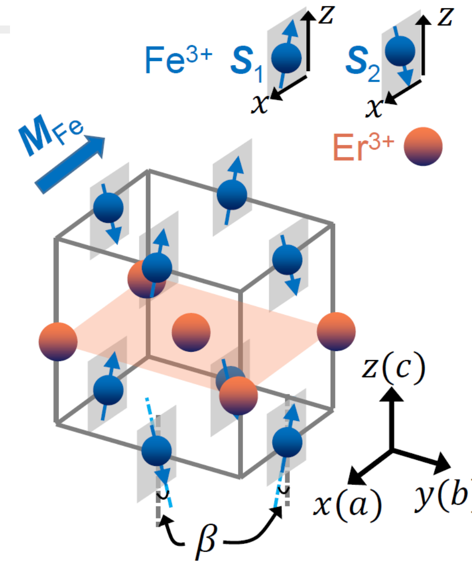


Cooperative interaction in magnetic material $ErFeO_3$

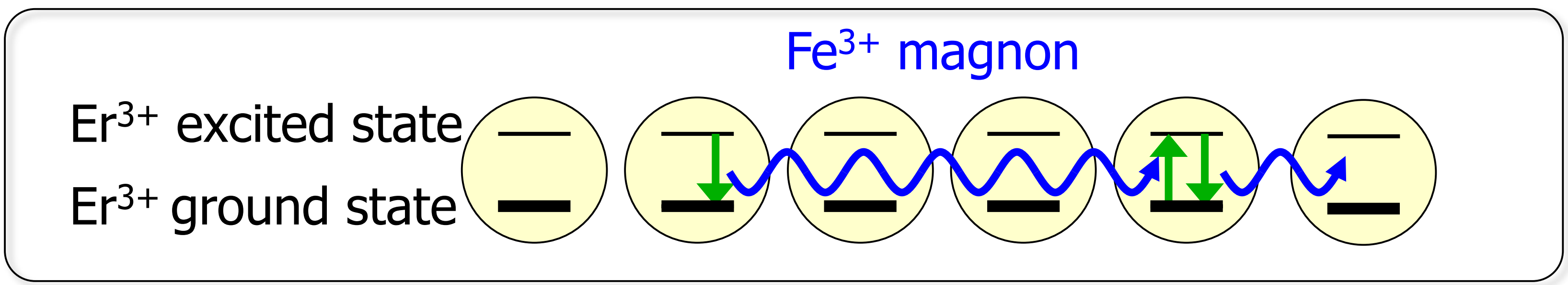
X. Li, M. Bamba, N. Yuan, Q. Zhang, Y. Zhao, M. Xiang, K. Xu, Z. Jin, W. Ren, G. Ma, S. Cao, D. Turchinovich, and J. Kono, Science **361**, 794 (2018)

A step toward "SRPT" in magnet

- **Spin** degree of freedom may cause a SRPT, which is hardly obtained only by **charge**-mediated interactions
  - **Design** of artificial structures of **magnetic** materials is a potential strategy

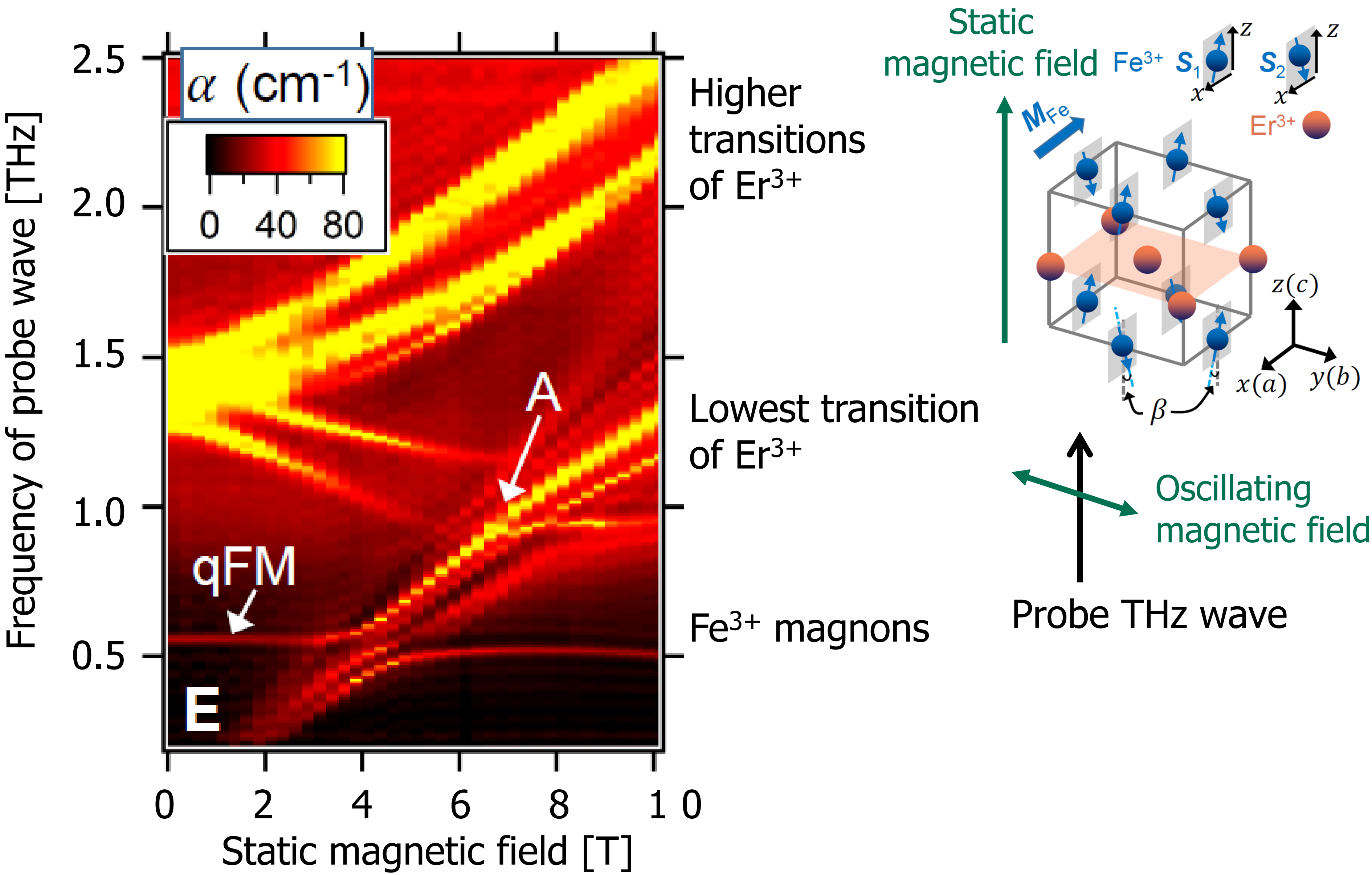


- $ErFeO_3$ shows a phase transition at T = 4.5 K, where Er^{3+} spins are ordered antiferromagnetically
  - Originating from **short**-range Er–Er interactions? (standard picture)

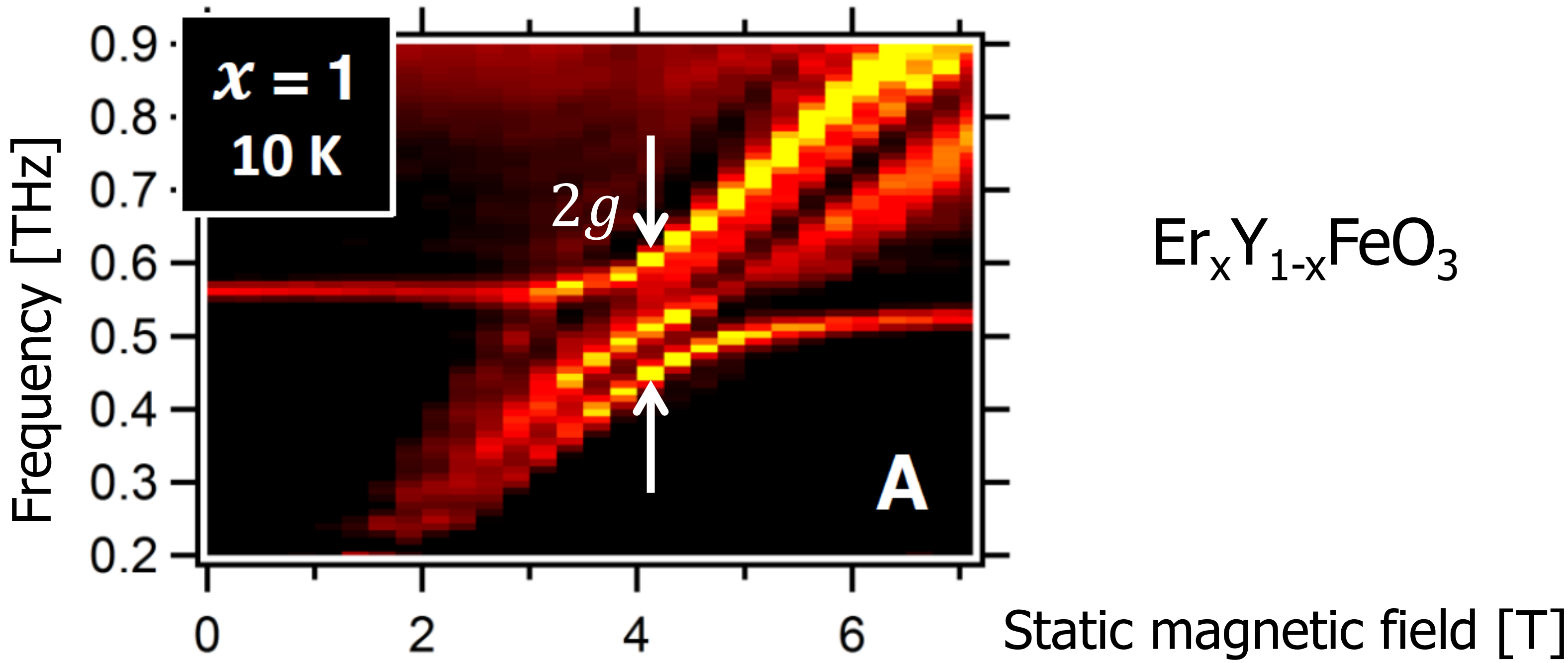


- This picture works ($g \propto \sqrt{N}$) for the interaction between Er^{3+} ensemble and Fe^{3+} magnons
  - A step toward "SRPT" where **photons** are replaced by **magnons**

Absorption spectra



Evidence of cooperative interaction



$Er_xY_{1-x}FeO_3$

Splitting $2g \propto$ (Excitable Er^{3+} density)$^{1/2}$

- Cooperative interaction mediated by **magnons** instead of **photons** long discussed in quantum optics
  X. Li, M. Bamba, N. Yuan, Q. Zhang, Y. Zhao, M. Xiang, K. Xu, Z. Jin, W. Ren, G. Ma, S. Cao, D. Turchinovich and J. Kono, Science **361**(6404), 794–797 (2018)

Theoretical analysis

- Reduction of a spin model of $ErFeO_3$ into Dicke model ($g \propto x^{1/2}$)
- Fe^{3+} – Er^{3+} exchange strength J was estimated from experimental g

Spin model of $ErFeO_3$

$$\hat{H}_{\mathrm{ErFeO_3}} = \hat{H}_{\mathrm{Er}} + \hat{H}_{\mathrm{Fe}} + \hat{H}_{\mathrm{Fe-Er}}, \qquad \hat{H}_{\mathrm{Er}} = -\sum_{i=1}^{N} \hat{\boldsymbol{\mu}}_i \cdot \boldsymbol{B}_{\mathrm{stat}}$$

$$\hat{H}_{\mathrm{Fe}} = J_{\mathrm{Fe}} \sum_{\mathrm{n.n.}} \hat{\boldsymbol{S}}_i^A \cdot \hat{\boldsymbol{S}}_{i'}^B - D_{\mathrm{Fe}} \sum_{\mathrm{n.n.}} \left(\hat{S}_{i,z}^A \hat{S}_{i',x}^B - \hat{S}_{i',z}^B \hat{S}_{i,x}^A \right)$$
$$- \textstyle\sum_{i=1}^{N} \left(A_x {\hat{S}_{i,x}^{A}}{}^2 + A_z {\hat{S}_{i,z}^{A}}{}^2 + A_{xz} \hat{S}_{i,x}^A \hat{S}_{i,z}^A \right) - \textstyle\sum_{i=1}^{N} \left(A_x {\hat{S}_{i,x}^{B}}{}^2 + A_z {\hat{S}_{i,z}^{B}}{}^2 - A_{xz} \hat{S}_{i,x}^B \hat{S}_{i,z}^B \right)$$

$$\hat{H}_{\mathrm{Er-Fe}} = \sum_{i=1}^{N} \left[J_A \hat{\boldsymbol{\sigma}}_i \cdot \hat{\boldsymbol{S}}_i^A + J_B \hat{\boldsymbol{\sigma}}_i \cdot \hat{\boldsymbol{S}}_i^B + \boldsymbol{D}_A \cdot \left(\hat{\boldsymbol{\sigma}}_i \times \hat{\boldsymbol{S}}_i^A \right) + \boldsymbol{D}_B \cdot \left(\hat{\boldsymbol{\sigma}}_i \times \hat{\boldsymbol{S}}_i^B \right) \right]$$

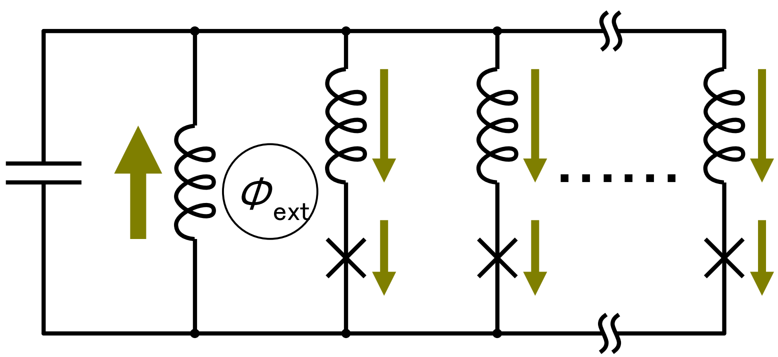
Dicke model

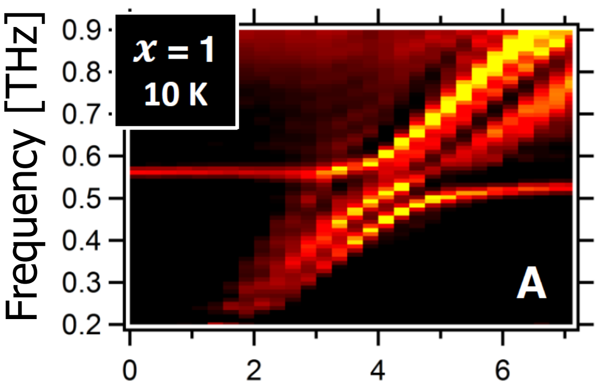
$$\hat{H}_{\mathrm{ErFeO_3}} \approx \hbar\omega_{\mathrm{magnon}} \hat{a}^\dagger \hat{a} + \sum_{i=1}^{N} \frac{\hbar\omega_{\mathrm{Er}}}{2} \hat{\sigma}_i^z + \frac{\hbar g}{\sqrt{N}} \left(\hat{a}^\dagger + \hat{a} \right) \sum_{i=1}^{N} \left(\hat{\sigma}_i^\dagger + \hat{\sigma}_i \right)$$

Analogy with SRPT is under investigation ($2g > \sqrt{\omega_{\mathrm{FM}}\omega_{\mathrm{Er}}}$)

Summary

- The SRPT introduces **heat** & **phase transition** into **optical science**
- Its non-equilibrium analogue was demonstrated in cold atoms
- Thermal SRPT is **not** yet realized (**charge**-mediated SRPT is hard)
- Thermal SRPT analogue was predicted in superconducting **circuit**
- Thermal **spin**-mediated SRPT (analogue) is now being explored





Remaining problems

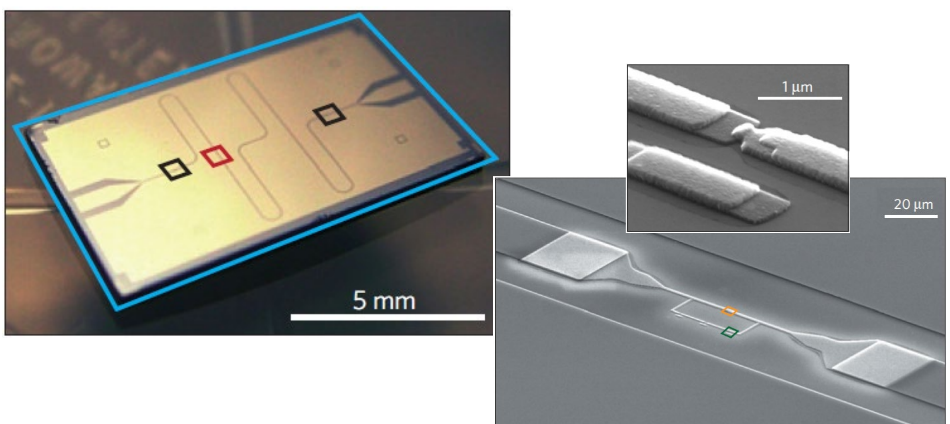
- Experimental distinguishability from magnetic phase transitions
- Magnetic transition dipoles are basically small, etc.

Thermal-equilibrium analogue in a superconducting circuit

M. Bamba, K. Inomata, and Y. Nakamura, Phys. Rev. Lett. **117**, 173601 (2016)

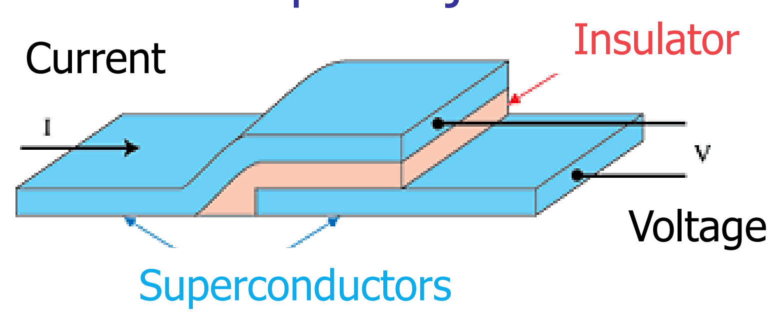
Superconducting circuits

- **Superconducting** current (or charges) in circuits is interpreted as “electromagnetic (EM) fields” and “atoms”
- Experimental techniques are highly developed together with the development of quantum computers
  - Quantum computers of D-wave, Google, IBM, etc. consist of superconducting circuits
- Advantage: We can **design** interaction forms of “EM fields” and “atoms”



T. Niemczyk, et al., Nature Phys. **6**, 772 (2010)

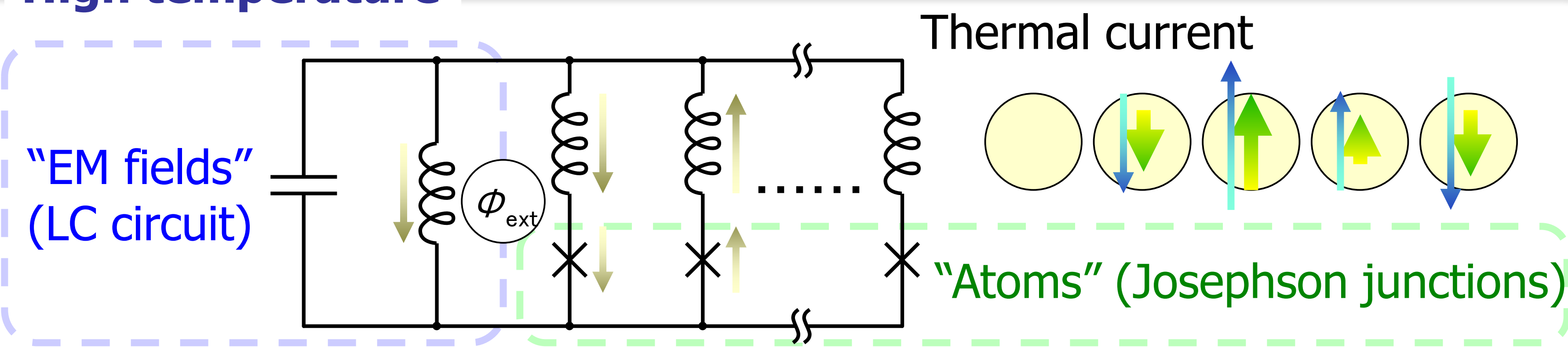
“Atom” Josephson junctions



http://www.sei.co.jp/super/about/feature.html

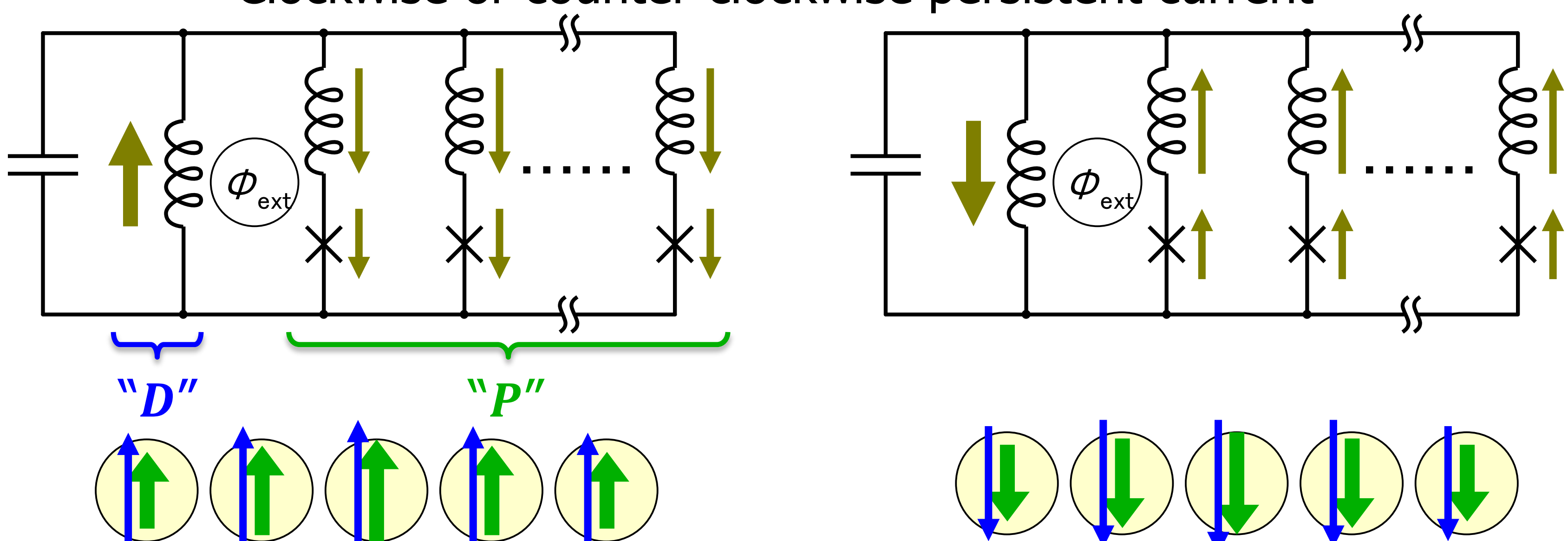
Circuit showing "SRPT"

High temperature



Low temperature

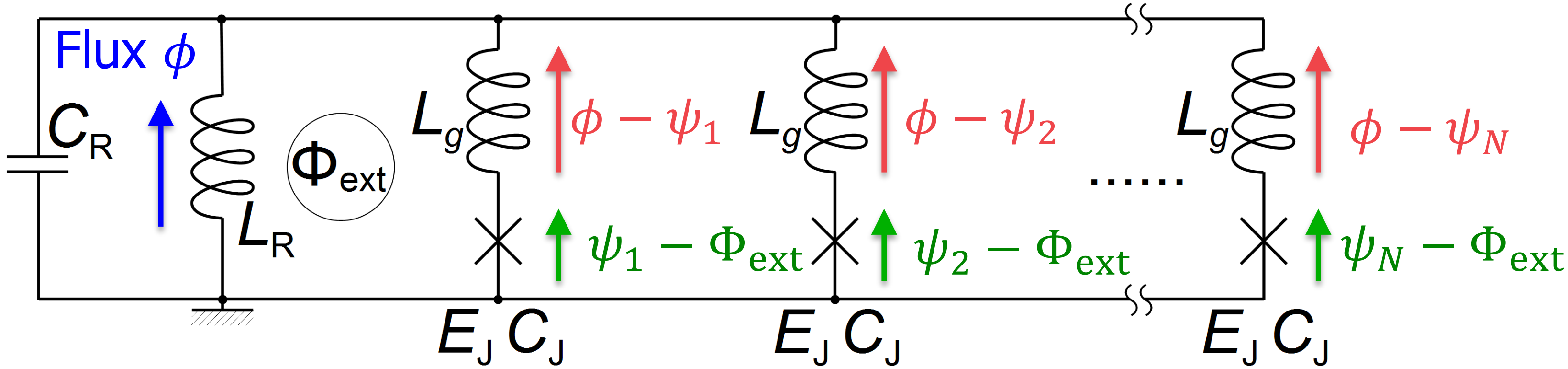
Clockwise or counter-clockwise persistent current



Superconducting circuit showing “SRPT”

M. Bamba, K. Inomata, and Y. Nakamura, PRL **117**, 173601 (2016)

Current $I = \phi/L_{\mathrm{R}}$



Charge | Flux | Flux difference | Charge | Effective flux

$$\hat{H} = \frac{\hat{q}^2}{2C_{\mathrm{R}}} + \frac{\hat{\phi}^2}{2L_{\mathrm{R}}} + \sum_{j=1}^{N}\left[\frac{\left(\hat{\phi} - \hat{\psi}_j\right)^2}{2L_g} + \frac{\hat{\rho}_j{}^2}{2C_{\mathrm{J}}} \underline{+} E_{\mathrm{J}} \cos\frac{2\pi\hat{\psi}_j}{\Phi_0}\right]$$

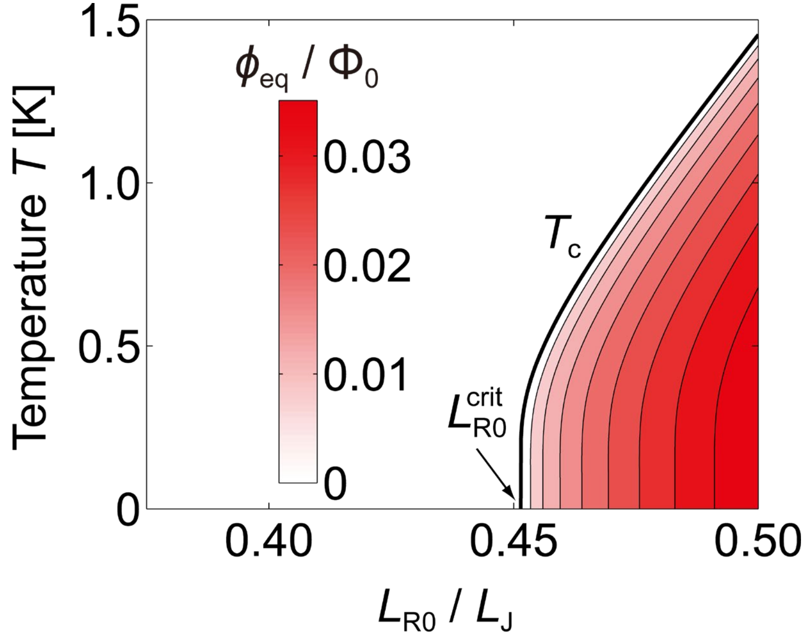
External magnetic flux $\Phi_{\mathrm{ext}} = \Phi_0/2$ flips the sign ($\Phi_0 = h/2e$)

“Photonic” variables $[\hat{\phi}, \hat{q}] = \mathrm{i}\hbar$

“Atomic” variables $[\hat{\psi}_j, \hat{\rho}_{j'}] = \mathrm{i}\hbar\delta_{j,j'}$

Phase diagram of superconducting current

Flux amplitude $\phi_{\rm eq} = L_R\, I_{\rm eq}$ is calculated through $Z(T) = \mathrm{Tr}\left[e^{-\hat{H}/k_{\rm B}T}\right]$ in thermodynamic limit ($N \to \infty$)

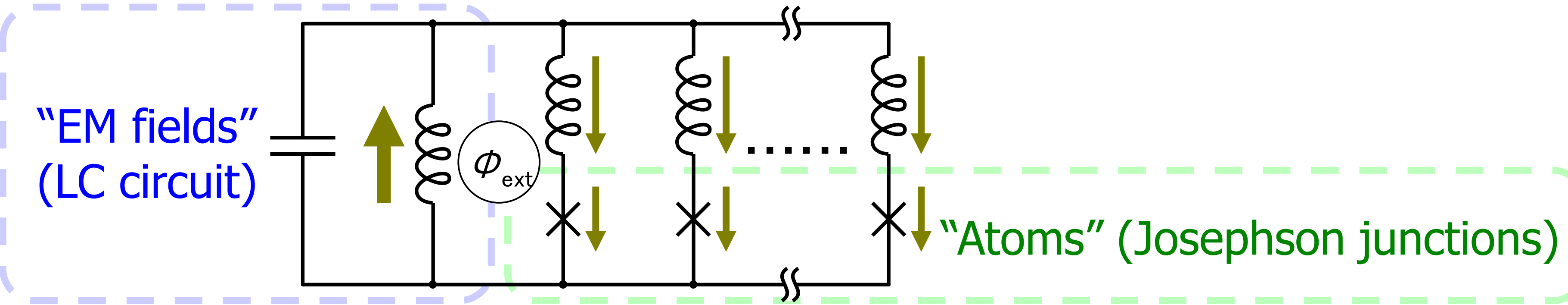


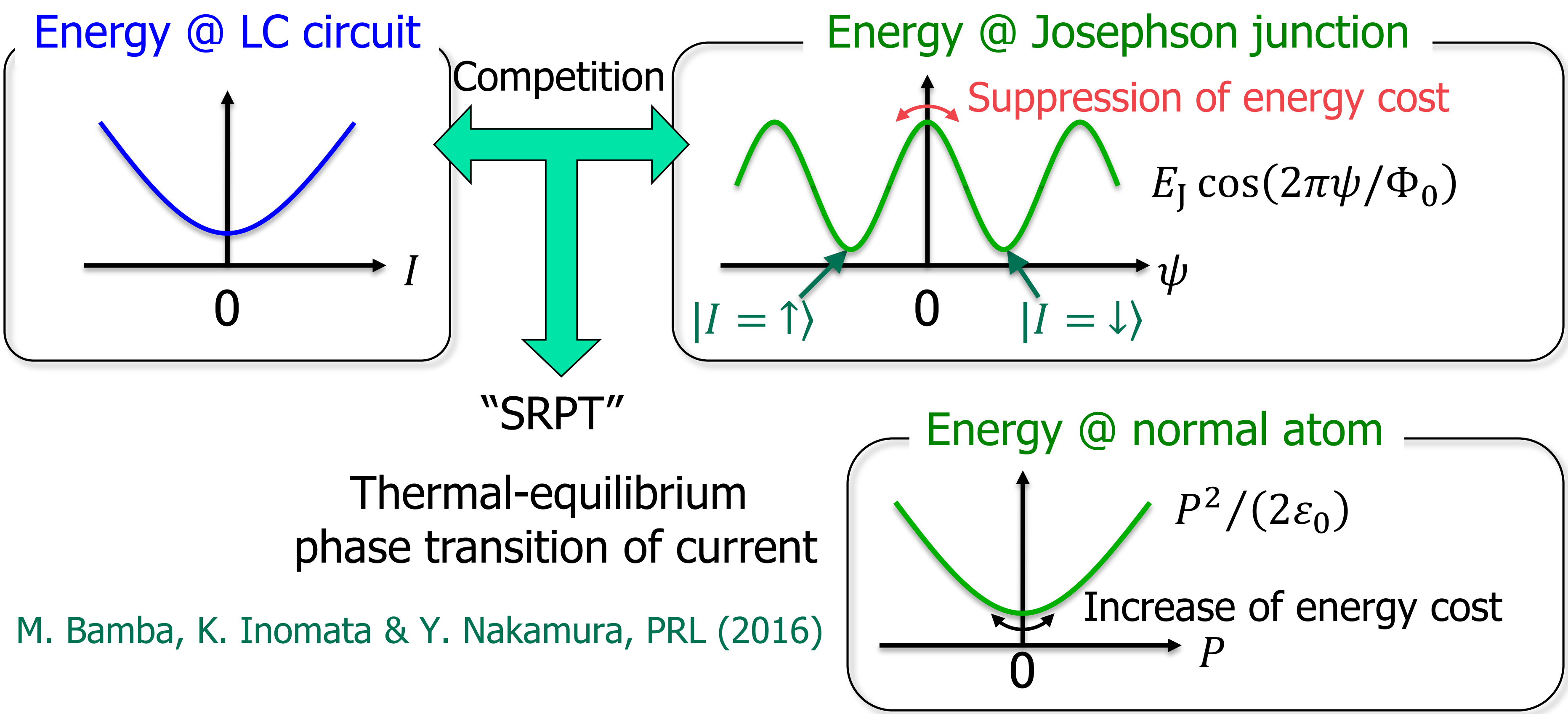
Parameters

$$L_{\rm J} = 0.75\ \mathrm{nH}$$
$$L_g = 0.6 L_{\rm J} = 0.45\ \mathrm{nH}$$
$$C_{\rm J} = 24\ \mathrm{fF}$$
$$C_{\rm R0} = 2\ \mathrm{fF} = C_{\rm R}/N$$
$$L_{\rm R0} = N L_{\rm R}$$

- A second-order phase transition appears
- The circuit Hamiltonian is certainly reduced to the Dicke model
- The "photonic" flux $\phi_{\rm eq} \neq 0$ get an amplitude spontaneously (persistent current appears)

Why we can get "SRPT" ?





"SRPT"

Thermal-equilibrium
phase transition of current

M. Bamba, K. Inomata & Y. Nakamura, PRL (2016)