Systematic Beam Study on the Static Robinson Instability at the Photon Factory Storage Ring

The static Robinson instability and coherent synchrotron frequency were systematically studied in the PF 2.5-GeV storage ring [1]. We observed the static Robinson instability at the beam current of 350 mA by gradually decreasing the RF voltage. The observed threshold condition and growth of the beam phase were consistent with the theory of the static Robinson instability. However, we found a remarkable discrepancy between the theory and experiment when we measured the coherent synchrotron frequency. When the operation condition of the RF system was close to the threshold condition of the instability, we found an unexpected narrow peak in the spectrum of the beam.

The *static Robinson instability* [2] is one of the beam instabilities caused by the radio-frequency (RF) accelerating system in a storage ring. The theoretical background is outlined in Fig. 1. Here, the RF accelerating voltage $V_c(\phi)$ excited in RF cavities is expressed as the function of RF phase ϕ. When the beam current is low (Fig. 1(a)), the total RF voltage is equal to the generator-induced voltage $V_g(\phi)$ that is excited by high-power RF sources like klystrons. The particles in each bunch coherently oscillate around the synchronous phase ϕ_s where the energy gain from the RF voltage compensates the radiation loss per turn U_0. In this case, the restoring force ($F(\phi)$ in Fig. 1(a)) of the coherent oscillation (i.e., coherent synchrotron oscillation) is proportional to the slope of the RF voltage at $\phi=\phi_s$. As the beam current becomes larger (Fig. 1(b)), the voltage induced by the beam itself $V_b(\phi)$ also becomes larger. However, because the phase of the beam-induced voltage oscillates synchronously with the beam coherent oscillation, it does not act as the restoring force. As a result, the restoring force becomes weak and the coherent synchrotron frequency decreased. When the beam current reaches a threshold determined by parameters of the RF system (Fig. 1(c)), the restoring force vanishes and the oscillation becomes unstable. This instability is called the static Robinson instability.

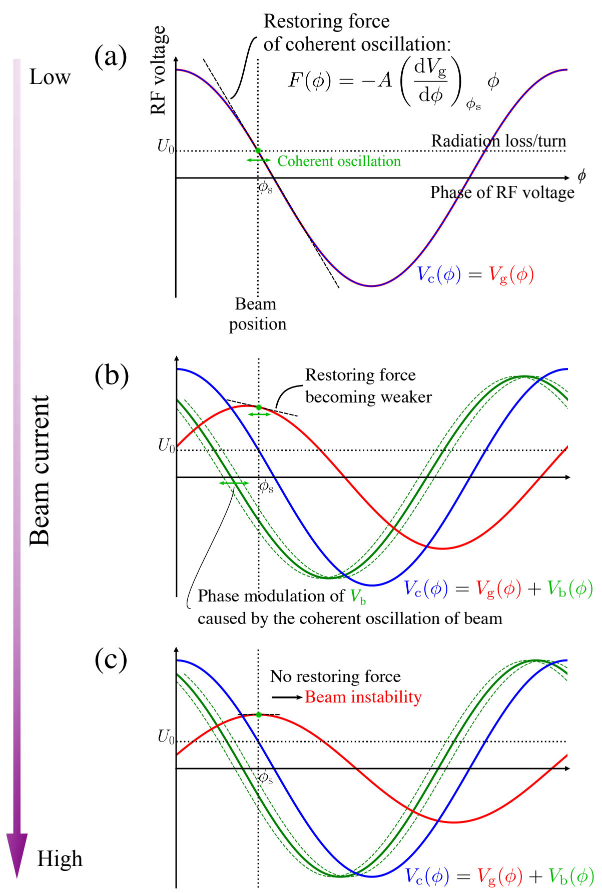

Figure 1: Schematic view that explains the static Robinson instability. The three graphs show the total RF accelerating voltage $V_c(\phi)$ (blue), generator-induced voltage $V_g(\phi)$ (red), and beam-induced voltage $V_b(\phi)$ (green) as the function of RF phase at different beam currents of **(a)** low current limit, **(b)** nominal beam current for user operation of the PF ring, and **(c)** the threshold beam current of the static Robinson instability. The amplitude of $V_c(\phi)$ is fixed in the three cases.

This instability is important for circular accelerators storing high-intensity hadron or lepton beams, such as the synchrotron light sources. Especially, for the RF systems of the next-generation light sources, the static Robinson instability is one of the essential topics in conjunction with the bunch-lengthening harmonic cavities. However, there are only a few reports on experimental studies of the static Robinson instability. Based on such growing interest, we studied the static Robinson instability in the PF 2.5-GeV storage ring.

We conducted two types of experiments. In the first experiment, we stored a beam current of 350 mA at the RF voltage of 1.7 MV that is typical in the PF RF system. After that, we gradually decreased the RF voltage until the static Robinson instability occurred. At the moment the instability occurred, we recorded the in-phase and quadrature-phase components of the beam signal measured at the RF frequency as shown in Fig. 2. The beam signal was obtained from one of the beam position monitors in the ring. Before the RF system was turned off by the fast RF interlock system (RF trip), the beam phase gradually shifted. This indicates that the beam lost the restoring force and the beam phase exponentially grew up. The threshold RF voltage at which the instability occurred and the growth rate of the beam phase before the RF trip were consistent with the theory of the static Robinson instability.

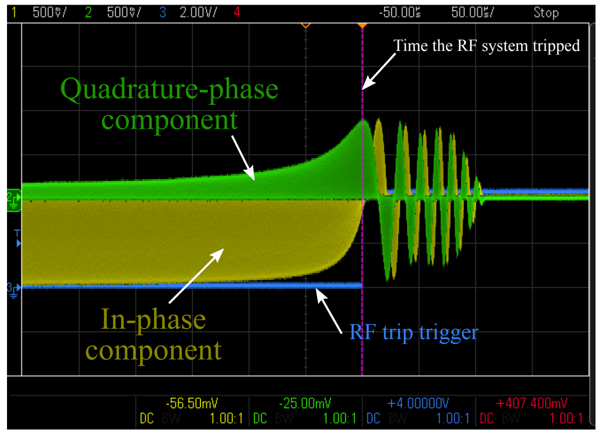

Figure 2: Measured beam signal at the moment the static Robinson instability occurred. The yellow and green curves represent the in-phase and quadrature-phase components of the beam signal observed at the RF frequency. The vertical dashed line indicates the time when the RF system tripped.

In the second experiment, we measured the coherent synchrotron frequency as the function of the beam current or RF voltage. Using a spectrum analyzer having a tracking generator, we measured a frequency response of the coherent oscillation to RF phase modulation. We found a remarkable discrepancy between the theory and experiment when measuring the coherent frequency as the function of the RF voltage at the beam current of 350 mA. Figure 3 shows the spectra of the synchrotron oscillation at the different RF voltages. The RF voltage was decreased from 1.7 MV until the instability occurred at lower than 1.13 MV. The center peak shows the RF frequency and its sidebands indicate the coherent frequency. Focusing on the upper sideband, we can see that there are two different peaks: the broad peak (marked by blue arrows) and narrow peak (marked by red arrows). As decreasing the RF voltage, the frequency of the broad peak decreased, which is roughly consistent with the theory. Simultaneously, the narrow peak at approximately 5.2 kHz grew up. However, this peak could not be expected from the theory. Moreover, the instability seems to have occurred when the lowering broad peak frequency coincided with the narrow peak frequency. Conducting more detailed study on the narrow peak, we confirmed that it was not a phenomenon simply explained by the higher-order modes of the RF cavities or some resonances of the RF feedback loops. Since the narrow peak is possibly an essential phenomenon for the static Robinson instability, we are going to further study it experimentally in other electron storage rings.

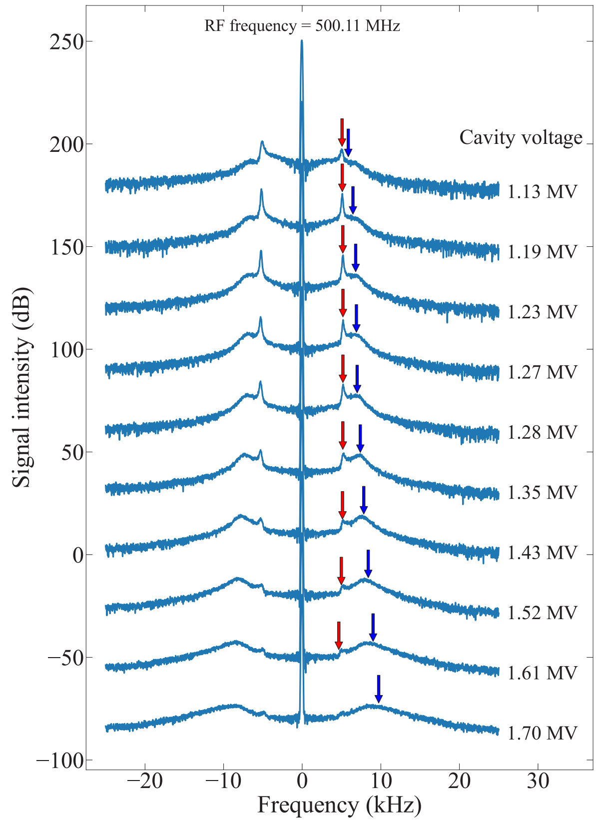

Figure 3: Measured beam spectra around the RF frequency at different RF voltages. Each dataset is offset by 30 dB. The center peak shows the RF frequency, and the upper and lower sidebands represent the frequency of the synchrotron oscillation. In the upper sideband, we marked the "broad peak" and "narrow peak" with blue and red arrows, respectively.

REFERENCES

[1] T. Yamaguchi, S. Sakanaka, N. Yamamoto, D. Naito and T. Takahashi, *Phys. Rev. Accel. Beams* **26**, 044401 (2023).
[2] K. W. Robinson, *CEAL-1010* (MIT, Cambridge, MA, 1964).

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