Chiral Baryons

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1. Quantum field theory is a necessity
2. Relativistic Mean Field Approximation
3. Baryon wave functions
4. Θ^+ width

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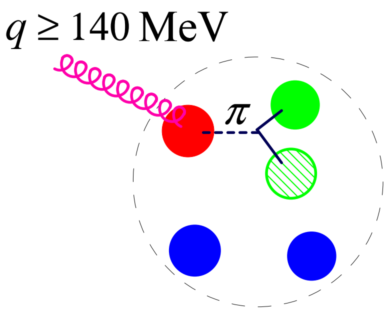


Uncertainty principle at work: When one attempts to measure the quark position in the nucleon to an accuracy better than the pion Compton wave length of $1\,\mathrm{fm}$ one produces a pion, i.e. a new $Q\bar{Q}$ pair. Hence, the quantum-mechanical description of baryons with a fixed number of quarks, is senseless.

The statement "nucleons are made of three quarks" has a limited accuracy.
For some observables the accuracy is very poor, for example,

"Spin crisis": 0.3 ± 0.1 of the nucleon spin is carried by the three valence quarks

Nucleon σ-term: only $\frac{1}{4}$ of it is carried by the three valence quarks,

$$\sigma = \frac{m_u + m_d}{2} < N|\bar{u}u + \bar{d}d|N > \quad = \quad 67 \pm 6\,\mathrm{MeV},$$

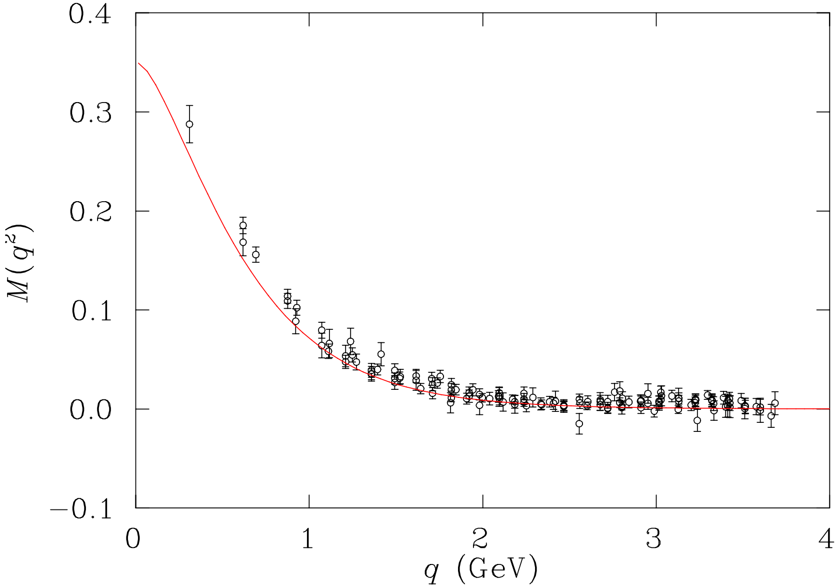
$$\text{From valence quarks}: \frac{4\,\mathrm{MeV} + 7\,\mathrm{MeV}}{2} \times (\leq 3) \quad \leq \quad 17.5\,\mathrm{MeV}.$$

Both paradoxes are explained by additional $\bar{Q}Q$ pairs in baryons.

One needs means to describe baryons as $QQQ + QQQQ\bar{Q} + QQQQQ\bar{Q}\bar{Q} + \ldots$ states at low virtuality.

As a result of the **spontaneous chiral symmetry breaking** nearly massless u, d, s quarks obtain a dynamical mass $M(p)$ and hence necessarily have to interact strongly (!) with the pseudoscalar fields:

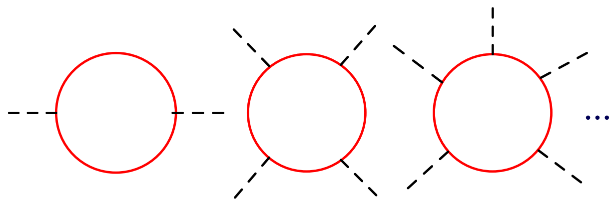
$$\mathcal{L}_{\rm eff} = \bar{q}\,\left[i\partial\!\!\!/ - M\exp(i\,\gamma_5\,\pi^A\lambda^A/F_\pi)\right]\,q, \qquad \pi^A = \pi, K, \eta, \qquad g_{\pi qq}(0) = \frac{M(0)}{F_\pi} \simeq 4$$



Dynamical quark mass $M(p)$ from a lattice simulation [Brower et al. (2003)]. Solid curve: obtained from instantons [DD and Petrov (1986)]. No real solution of the mass-shell equation $p^2 + M^2(p^2) = 0 \Longrightarrow$ quarks are not observable , only their bound states.

Models with massive quarks and (confining) gluon interaction, $\bar{q}\,[i\partial\!\!\!/ + A\!\!\!/ - M]\,q$, contradict chiral symmetry, i.e. invariance under

$$q \rightarrow \exp(i\gamma_5\alpha^A\lambda^A)q, \qquad \bar{q} \rightarrow \bar{q}\exp(i\gamma_5\alpha^A\lambda^A), \qquad e^{i\pi} \rightarrow e^{i\alpha}e^{i\pi}e^{i\alpha}.$$



Pseudoscalar mesons are themselves bound states of constituent quarks: they propagate and interact via virtual quark-antiquark pairs. The sum of all diagrams with any number of external legs is called the effective chiral lagrangian.

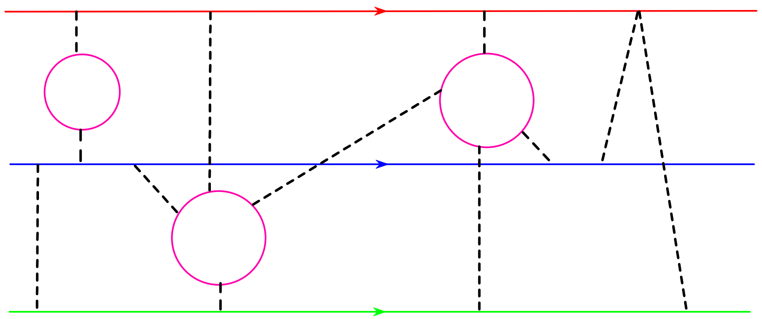
Integrating out quarks one obtains the **effective chiral action** [D.D. and Eides (1983), Dhar, Shankar and Wadia (1985)]

$$
\begin{aligned}
S[\pi(x)] &= \int d^4x \left\{ \frac{F_\pi^2}{4} \mathrm{Tr}\,(\partial_\mu U^\dagger \partial_\mu U) + \frac{N_c}{192\pi^2} \mathrm{Tr}\,[\partial_\mu U^\dagger \partial_\nu U]^2 + \text{infinite series} \right\} \\
&+ \frac{iN_c}{240\pi^2} \int d^5x\, \epsilon^{\alpha\beta\gamma\delta\epsilon}\, \mathrm{Tr}\,(U^\dagger \partial_\alpha U \partial_\beta U^\dagger \partial_\gamma U \partial_\delta U^\dagger \partial_\epsilon U) + \ldots, \\
U &= \exp(i\pi^A \lambda^A).
\end{aligned}
$$

In contrast, the Skyrme model

- neglects all higher derivative terms [it is like replacing e^{-x} by $1-x$]
- adds the Wess–Zumino term by hand

No wonder the agreement of the Skyrme model with data is only qualitative. One has to use the **full** chiral lagrangian to describe baryons.



Quarks in the nucleon (solid lines), interacting via pion fields (dash lines).

Large N_c logic:

If a physical quantity is stable in the limit $N_c \to \infty$, one must be able to extract it from physics surviving at $N_c \to \infty$.

At arbitrary N_c baryons are made of N_c quarks sharing one orbital. Baryon masses: $\sim N_c$; baryon sizes: stable in N_c [Witten(1979)]. Hence, one has to be able to understand quark wave functions in a baryon from physics that survives at $N_c \to \infty$.

When there are many participants one usually exploits the mean field method [Thomas–Fermi approximation to large atoms; shell model for heavy nuclei]. There cannot be any mean colour field – that would break colour symmetry. Only the colour-neutral meson field can play the role of the mean field!

Relativistic Mean Field Approximation **or** the Chiral Quark Soliton Model

[DD and Petrov + Pobylitsa (1986)]

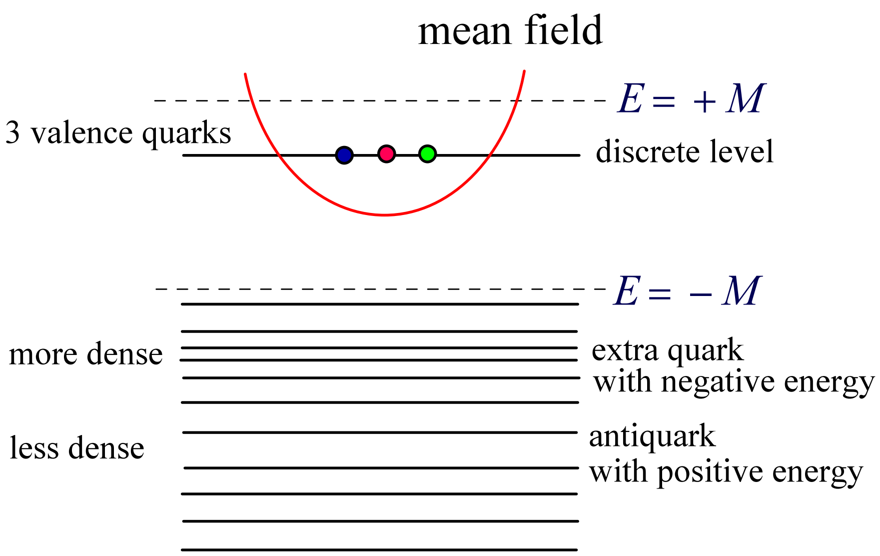


Figure 1: A schematic view of baryons in the Relativistic Mean Field Approximation. There are three "valence" quarks at a discrete energy level created by the mean field, and the negative-energy Dirac continuum distorted by the mean field, as compared to the free one.

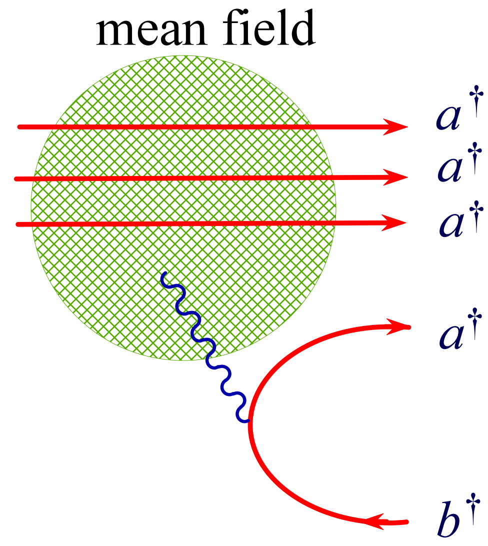


Figure 2: Equivalent view of baryons, where the polarized Dirac sea is presented as $Q\bar{Q}$ pairs. Their wave function is given by the quark Green function in the background mean field, at equal times [Petrov and Polyakov (2003)].

$$\text{Baryon mass} = N_c\ (E_{\rm lev}[\pi(x)] + E_{\rm sea}[\pi(x)])$$

$$\text{Self}-\text{consistent } \pi(x) = \text{ the one minimizing the baryon mass}$$

$$E_{\rm sea}[\pi(x)] = \frac{F_\pi^2}{4}\int d^3x\,{\rm Tr}\,(\partial_i U^\dagger \partial_i U) + \int d^3x\,{\rm Tr}\,[\partial_i U^\dagger \partial_j U]^2 + \ldots, \qquad U = \exp(i\pi^a\tau^a).$$

If the mean field is spatially large, then $E_{\rm sea} \gg E_{\rm lev}$, and it becomes similar to the Skyrme model.

If the chiral field is weak, then $E_{\rm sea} \ll E_{\rm lev} \approx M$, there are few antiquarks, and it becomes the non-relativistic constituent quark model.

The truth is in between: neither the Skyrme model nor the non-relativistic quark model are adequate for describing the nucleon quantitatively.

The only input to get all baryon properties $M(0) = 345\,{\rm MeV}$, $F_\pi = 93\,{\rm MeV}$ (in fact, even these numbers are obtained from $\Lambda_{\rm QCD}$ in e.g. the instanton model of the QCD vacuum.)

For the hedgehog Ansatz for the chiral field, $\pi^a = \mathbf{n}^a \sin P(r), \; \mathbf{n}^a = x^a/r$ where $P(r)$ is the profile function of the self-consistent field, one has to solve the following static Dirac equation:

$$\psi_{\rm lev}(\mathbf{x}) = \begin{pmatrix} \epsilon^{ji} h(r) \\ -i\epsilon^{jk} \, (\sigma\cdot\mathbf{n})^i_k \, j(r) \end{pmatrix}, \qquad \begin{cases} h' + h\, M \sin P - j(M\cos P + E_{\rm lev}) = 0, \\ j' + 2j/r - j\, M\sin P - h(M\cos P - E_{\rm lev}) = 0. \end{cases}$$

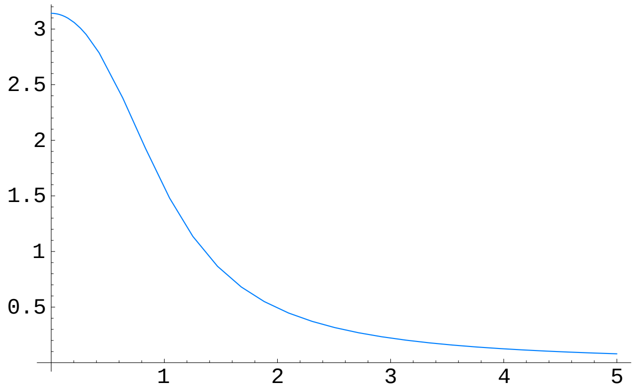


Figure 3: The space profile of the self-consistent chiral field $P(r)$ in light baryons. One unit on the horizontal axis is $r_0 = 0.8/M = 0.46\,\mathrm{fm}$.

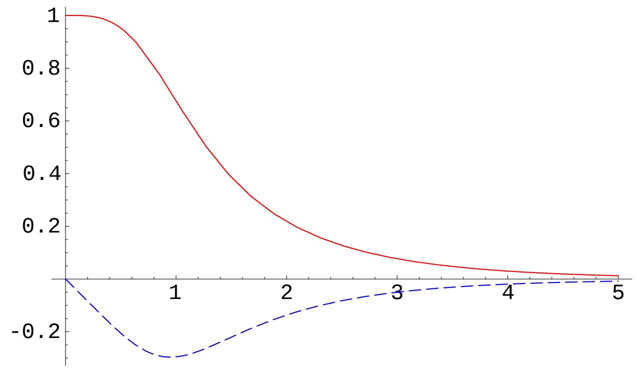


Figure 4: Bound-state quark upper s-wave component $h(r)$ (red) and the lower p-wave component $j(r)$ (blue) in light baryons.

$M(0) = 345\,\mathrm{MeV} \Longrightarrow E_{\rm lev} = 200\,\mathrm{MeV}$. The three "valence" quarks are tightly bound and hence relativistic!

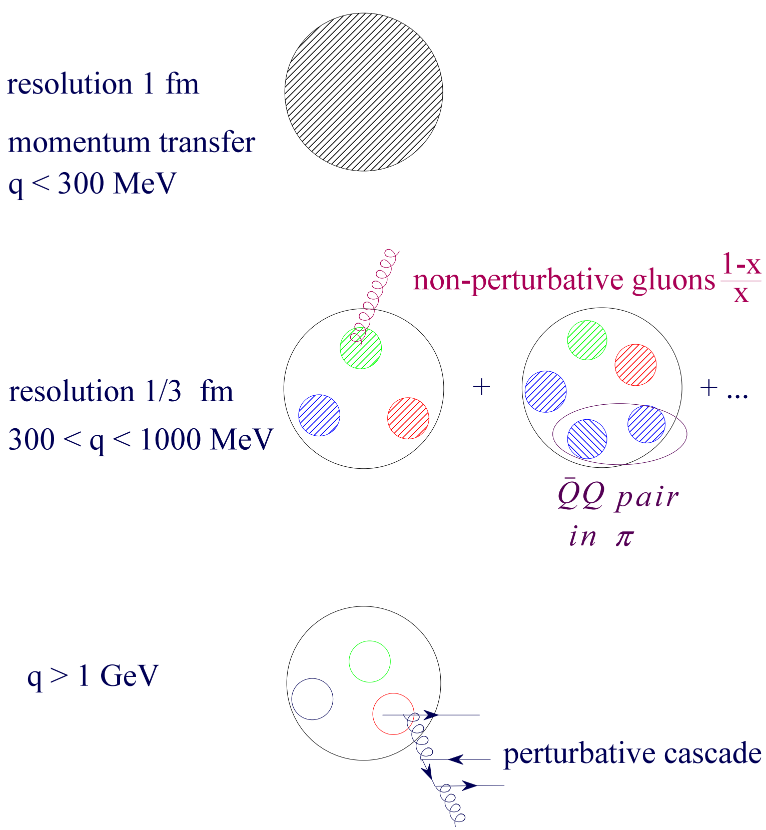
In the Relativistic Mean Field Approximation **all** properties of **all** baryons from the $\left(\mathbf{8}, \frac{1}{2}^+\right)$, $\left(\mathbf{10}, \frac{3}{2}^+\right)$ and $\left(\overline{\mathbf{10}}, \frac{1}{2}^+\right)$ multiplets follow from the shape of the self-consistent pion field, including masses, magnetic moments, formfactors, parton distributions at low virtuality, etc.

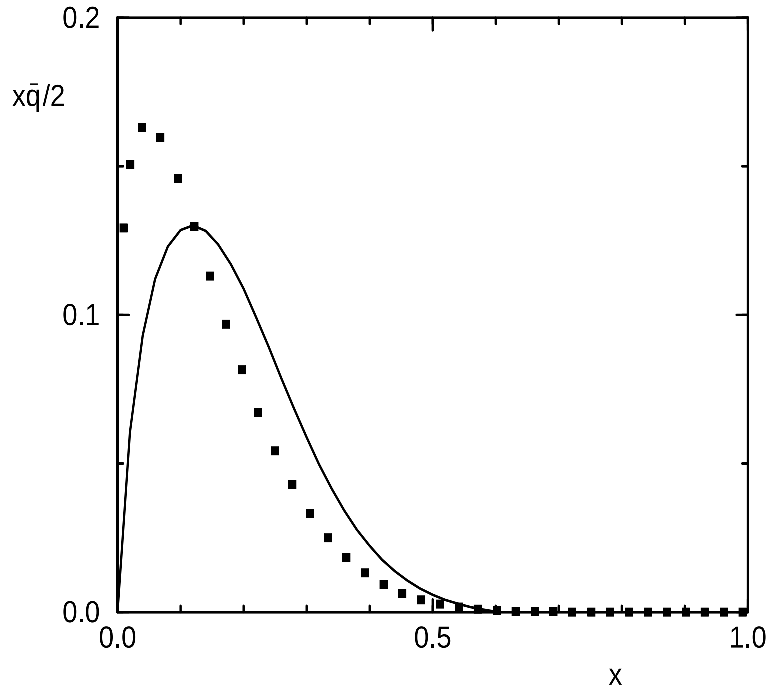
Error sources:

- Precise form of $M(p)$ is presently not known – only certain integrals relating $M(p)$ to F_π, $<\bar{q}q>$
- Corrections from fluctuations about the mean field $\sim 1/N_c$, actually $\sim 1/(2\pi N_c) \approx 6\%$
- Residual color Coulomb interactions, estimated as small by DD, Jaenicke and Polyakov (1992)

On the whole, all baryon properties computed so far are within 15% from the data, with no adjusted or fitting parameters!

Nucleon under a microscope with increasing resolution:





Antiquark distribution at low virtuality.

Parton distributions at low virtuality were studied by St.Petersburg–Bochum groups and by Wakamatsu and Yoshiki (1989–).

Baryon wave functions

The Dirac sea is presented by the coherent exponent of the quark $a^\dagger$ and antiquark $b^\dagger$ creation operators:

$$\text{coherent exponent} = \exp\left(\int (d\mathbf{p})(d\mathbf{p}')\, a^\dagger(\mathbf{p})\, W(\mathbf{p},\mathbf{p}')\, b^\dagger(\mathbf{p}')\right) |0>,$$

where $W(\mathbf{p_1},\mathbf{p_2})$ is the (calculated) quark Green function at equal times in the background chiral field.

Baryon wave function of N_c "valence" quarks and arbitrary number of $Q\bar{Q}$ pairs:

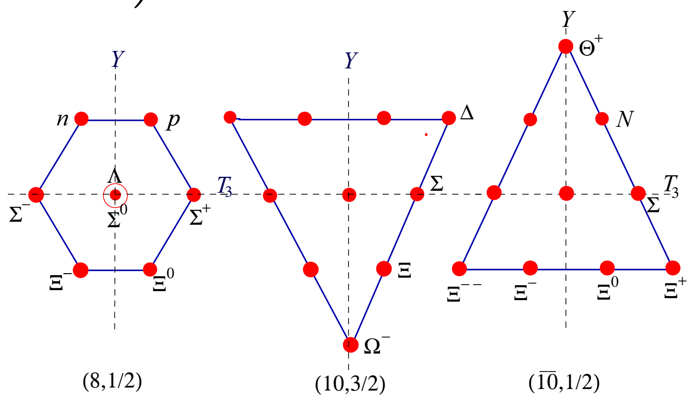
$$\begin{aligned} B[a^\dagger, b^\dagger] &= \prod_{\text{color}=1}^{N_c} \int (d\mathbf{p})\, F(\mathbf{p})\, a^\dagger(\mathbf{p}) \\ &\cdot \exp\left(\int (d\mathbf{p})(d\mathbf{p}')\, a^\dagger(\mathbf{p})\, W(\mathbf{p},\mathbf{p}')\, b^\dagger(\mathbf{p}')\right) |0> . \end{aligned}$$

The mean chiral field is degenerate in overall rotations in ordinary and flavor spaces; hence one has to **project** it to a given flavor and spin baryon state:

$$\Psi_k^B = \int dR\, B_k^*(R)\, \epsilon^{\alpha_1\alpha_2\alpha_3} \prod_{n=1}^{3} \int (d\mathbf{p_n})\, R_{j_n}^{f_n}\, F^{j_n\sigma_n}(\mathbf{p_n})\, a^\dagger_{\alpha_n f_n \sigma_n}(\mathbf{p_n})$$

$$\cdot \exp\left(\int (d\mathbf{p})(d\mathbf{p}')\, a^\dagger_{\alpha f\sigma}(\mathbf{p})\, R_j^f\, W^{j\sigma}_{j'\sigma'}(\mathbf{p},\mathbf{p}')\, R^{\dagger\, j'}_{f'}\, b^{\dagger\,\alpha f'\sigma'}(\mathbf{p}')\right) |0> \;\; .$$

This is the generating functional **for all** $qqq,\;\; qqqq\bar{q},\;\; qqqqq\bar{q}\bar{q}$ quark wave functions in the $\left(\mathbf{8},\frac{1}{2}^+\right)$, $\left(\mathbf{10},\frac{3}{2}^+\right)$ and $\left(\overline{\mathbf{10}},\frac{1}{2}^+\right)$ baryons in the Relativistic Mean Field Approximation.

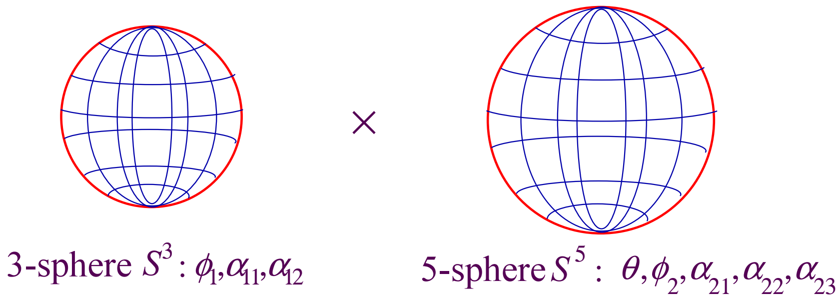


R_j^f is an $SU(3)$ matrix parameterized by 8 "Euler angles". $B_k^*(R)$ is a given baryon's rotational wave function depending on those angles.

Explicit parametrization of the $SU(3)$ rotation matrix:

$$R = \begin{pmatrix} e^{i\alpha_{23}} \cos\theta & 0 & e^{i\alpha_{23}} \sin\theta \\ -e^{i\alpha_{22}} \sin\theta \sin\phi_2 & e^{-i\alpha_{21}-i\alpha_{23}} \cos\phi_2 & e^{i\alpha_{22}} \cos\theta \sin\phi_2 \\ -e^{i\alpha_{21}} \sin\theta \cos\phi_2 & -e^{-i\alpha_{22}-i\alpha_{23}} \sin\phi_2 & e^{i\alpha_{21}} \cos\theta \cos\phi_2 \end{pmatrix}$$
$$\times \begin{pmatrix} e^{-i\alpha_{11}} \cos\phi_1 & e^{i\alpha_{12}} \sin\phi_1 & 0 \\ -e^{-i\alpha_{12}} \sin\phi_1 & e^{i\alpha_{11}} \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$SU(3)$-invariant integration measure is a product of measures over the S^3 and S^5 spheres.



$$R^3_3 = e^{i\alpha_{21}} \cos\phi_2 \cos\theta$$

At $N_c = 3$ the Θ^+ wave function is centered at the latitude 54^o

At $N_c \to \infty$ the Θ^+ wave function is peaked near the "North pole".

Examples of the baryons' (conjugate) rotational wave functions $B^*(R)$:

$$\text{proton, spin projection } k: \qquad p_k^*(R) = \sqrt{8}\,\epsilon_{kl} R_1^{\dagger\,l} R_3^3,$$

$$\text{neutron, spin projection } k: \qquad n_k^*(R) = \sqrt{8}\,\epsilon_{kl} R_2^{\dagger\,l} R_3^3,$$

$$\Delta^{++}, \text{ spin projection } +\frac{3}{2}: \qquad \Delta_{\uparrow\uparrow}^{++\,*}(R) = \sqrt{10}\, R_1^{\dagger\,2} R_1^{\dagger\,2} R_1^{\dagger\,2},$$

$$\Delta^0, \text{ spin projection } +\frac{1}{2}: \qquad \Delta_{\uparrow}^{0\,*}(R) = \sqrt{10}\, R_2^{\dagger\,2}(2R_1^{\dagger\,2} R_2^{\dagger\,1} + R_2^{\dagger\,2} R_1^{\dagger\,1}),$$

$$\Theta^+, \text{ spin projection } k: \qquad \Theta_k^*(R) = \sqrt{30}\, R_3^3 R_3^3 R_k^3, \qquad \left(\times \left(R_3^3\right)^{N_c-3}\right)$$

$$\text{neutron}^* \text{ from } \overline{10}, \text{ spin projection } k: \qquad n_{\overline{10},k}^*(R) = \sqrt{10}\, R_3^3 (2R_3^1 R_k^3 + R_3^3 R_k^1).$$

Normalized in such a way that for any spin projection

$$\int dR\, B_{\text{spin}}^*(R)\, B^{\text{spin}}(R) = 1, \qquad \int dR = 1;$$

$$\int dR\, R_j^f\, R_{f'}^{\dagger\, j'} = \frac{1}{3}\, \delta_{f'}^f\, \delta_j^{j'}, \quad \text{etc.}$$

Baryon wave functions in terms of quarks

If the coherent exponent with $Q\bar{Q}$ pairs is ignored, one gets the 3-quark Fock component of the octet and decuplet baryons. It depends on the quark "coordinates" $\mathbf{r}, \alpha, f, \sigma$ and on the baryon spin projection k.

For example, the neutron $3Q$ wave function is

$$(|n>_k)^{f_1 f_2 f_3, \sigma_1 \sigma_2 \sigma_3}(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}) = \epsilon^{f_1 f_2}\, \epsilon^{\sigma_1 \sigma_2}\, \delta_2^{f_3}\, \delta_k^{\sigma_3}\, h(r_1) h(r_2) h(r_3)$$
$$+ \text{permutations of } 1,2,3 \qquad (\otimes \quad \epsilon^{\alpha_1 \alpha_2 \alpha_3})\,.$$

It is better known in the form

$$|n\uparrow> \;=\; 2\,d\!\uparrow\!(r_1) d\!\uparrow\!(r_2) u\!\downarrow\!(r_3) - d\!\uparrow\!(r_1) u\!\uparrow\!(r_2) d\!\downarrow\!(r_3) - u\!\uparrow\!(r_1) d\!\downarrow\!(r_2) d\!\uparrow\!(r_3)$$
$$+ \quad \text{permutations of } r_1, r_2, r_3,$$

which is the well-known non-relativistic $SU(6)$ wave function of the nucleon!

There are relativistic corrections to the $SU(6)$-symmetric formulae, arising from i) exact treatment of the discrete level, ii) additional $Q\bar{Q}$ pairs. Both effects are not small.

The $5Q$ component of a baryon is obtained when one expands the coherent exponent to the linear order and then projects it onto the concrete baryon in question. For example, the $5Q$ component of the **neutron** has the wave function

$$
\begin{aligned}
&(|n>_k)^{f_1 f_2 f_3 f_4, \sigma_1\sigma_2\sigma_3\sigma_4}_{f_5,\sigma_5}(\mathbf{r_1},\mathbf{r_2},\mathbf{r_3},\mathbf{r_4},\mathbf{r_5}) = h(r_1)h(r_2)h(r_3)\, W^{j_4\sigma_4}_{j_5\sigma_5}(\mathbf{r_4},\mathbf{r_5}) \\
\cdot\quad & \left\{\epsilon^{f_1 f_2}\epsilon_{j_1 j_2}\left[\delta^{f_3}_2\delta^{f_4}_{f_5}\left(4\delta^{j_5}_{j_4}\delta^{k'}_{j_3} - \delta^{j_5}_{j_3}\delta^{k'}_{j_4}\right) + \delta^{f_4}_2\delta^{f_3}_{f_5}\left(4\delta^{j_5}_{j_3}\delta^{k'}_{j_4} - \delta^{j_5}_{j_4}\delta^{k'}_{j_3}\right)\right]\right. \\
+\quad & \left.\epsilon^{f_1 f_4}\epsilon_{j_1 j_4}\left[\delta^{f_2}_2\delta^{f_3}_{f_5}\left(4\delta^{j_5}_{j_3}\delta^{k'}_{j_2} - \delta^{j_5}_{j_2}\delta^{k'}_{j_3}\right) + \delta^{f_3}_2\delta^{f_2}_{f_5}\left(4\delta^{j_5}_{j_2}\delta^{k'}_{j_3} - \delta^{j_5}_{j_3}\delta^{k'}_{j_2}\right)\right]\right\}\epsilon_{k'k}\epsilon^{j_1\sigma_1}\epsilon^{j_2\sigma_2}\epsilon^{j_3\sigma_3} \\
+\quad & \text{permutations of } (1,2,3)
\end{aligned}
$$

Indices 1-3 refer to quarks at the discrete level, 4 refers to the quark in the additional pair, and 5 refers to the antiquark in the pair. $\delta^{f_3}_{f_5} \sim s\bar{s} + u\bar{u} + d\bar{d}$.

The $Q\bar{Q}$ pair wave function W is a combination of four partial waves corresponding to i) pseudoscalar, ii) scalar, iii) vector and iv) axial "mesons" in baryons. The partial waves depend separately on the coordinates $\mathbf{r_{4,5}}$ measured from the baryon center of mass. The pair wave function $W^{j_4\sigma_4}_{j_5\sigma_5}(\mathbf{r_4},\mathbf{r_5})$ is given in: hep-ph/0408219, hep-ph/0505201.

Exotic baryons from the $\left(\overline{10}, \frac{1}{2}^+\right)$

Projecting the three quarks from the discreet level on the Θ^+ rotational function gives an **identical zero**, in accordance with the fact that the Θ^+ cannot be made of 3 quarks.

The non-zero projection is achieved when one expands the coherent exponent at least to the linear order. One gets the $5Q$ component of the Θ wave function:

$$|\Theta_k^+>^{f_1 f_2 f_3 f_4, \sigma_1\sigma_2\sigma_3\sigma_4}_{f_5,\sigma_5}(\mathbf{r_1}\ldots\mathbf{r_5}) = \epsilon^{f_1 f_2}\epsilon^{f_3 f_4}\delta^3_{f_5}\,\epsilon^{\sigma_1\sigma_2}$$

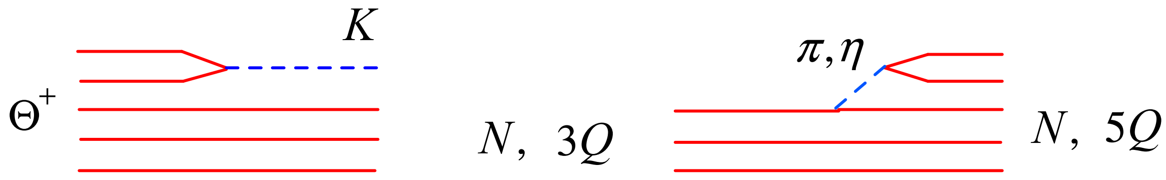
$$\cdot\, h(r_1)h(r_2)h(r_3)\, W^{\sigma_3\sigma_4}_{k\,\sigma_5}(\mathbf{r_4},\mathbf{r_5}) + \text{permutations of } 1,2,3 \qquad (1)$$

$\Theta^+ = uudd\bar{s}$.

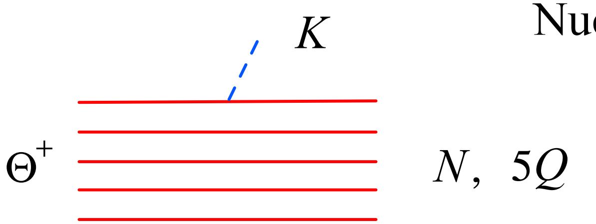
The structure $\epsilon^{f_1 f_2}\epsilon^{\sigma_1\sigma_2}$ clearly shows that there is a pair of ud quarks in the spin and isospin zero combination, exactly as in the nucleon. However, it **does not mean** that there are prominent scalar isoscalar diquarks either in the nucleon or in the Θ: that would require their spatial correlation which, as we see, is absent in the mean field approximation.

The spatial structure (but not the spin-flavor one) of the leading $5Q$ of the Θ^+ is similar to that of the non-leading $5Q$ component of the nucleon.

Θ^+ decay



Nucleon is a mixture of 3Q, 5Q... states



Both processes contribute in the decay

In the Infinite Momentum Frame (IMF) only the second diagram survives, as vector and axial currents with a finite momentum transfer do not create or annihilate quarks with infinite momenta. The baryon matrix elements are thus non-zero only between Fock components with **equal number** of quarks and antiquarks.

Normalization of the $3Q$ and $5Q$ components

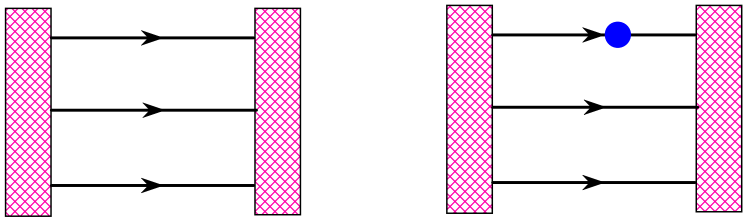


Figure 5: Graphs showing the normalization of a 3-quark component of a baryon (left) and the matrix element of a local operator denoted by a circle (right).

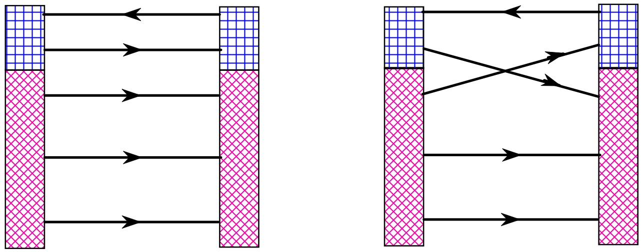


Figure 6: Direct (left) and exchange (right) contributions to the normalization of the 5-quark component of a baryon. The upper rectangles denote $Q\bar{Q}$ pairs.

$$\mathcal{N}_N^{(3)} = 1, \qquad \mathcal{N}_N^{(5)} \approx 0.4\,!$$

Momentum carried by antiquarks in the nucleon at low virtuality, roughly,

$$\frac{0 \cdot 1 + \frac{1}{5} \cdot 0.4}{1 + 0.4} \approx 6\%$$

Baryon matrix elements related to the $5Q$ components

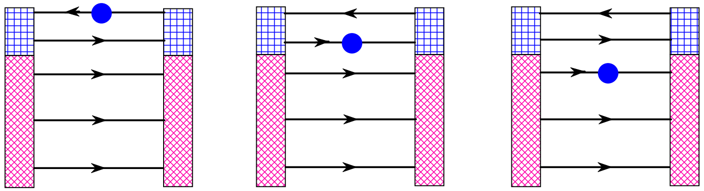


Figure 7: Direct contributions to the matrix element of an operator computed by DD and Petrov (2005).

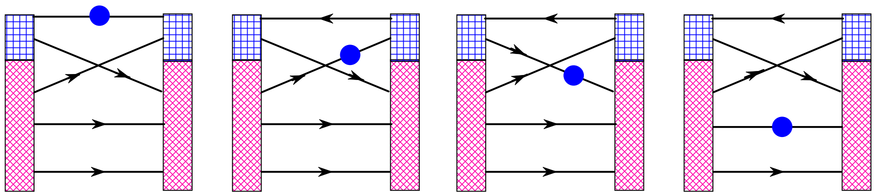


Figure 8: Exchange contributions to the matrix element in the 5-quark component of a baryon computed by Lorcé (2006).

Observables

$$g_A(N)^{\rm exper} = 1.27$$

$$g_A^{(3)}(N) = \frac{A^{(3)}(N)}{\mathcal{N}^{(3)}(N)} = \frac{5}{3} = 1.67$$ in the non-relativistic limit

$$g_A^{(5)}(N) = \frac{A^{(3)}(N)+A^{(5)}(N)+\ldots}{\mathcal{N}^{(3)}(N)+\mathcal{N}^{(5)}(N)+\ldots} = 1.44$$ (1.32 when all $\bar{Q}Q$ pairs are summed up)

$$g_A^{(5)}(\Theta \to KN) = \frac{A^{(5)}(\Theta\to KN)+\ldots}{\sqrt{N_\Theta^{(5)}+\ldots}\sqrt{\mathcal{N}^{(3)}(N)+\mathcal{N}^{(5)}(N)+\ldots}} = 0.15 \Longrightarrow \Gamma_\Theta = 2.3\,{\rm MeV}$$!!

Why Klebanov *et al.* didn't get a narrow Θ^+ from the Skyrme model?

Callan and Klebanov (1989) "bound-state approach" assumes $N_c \to \infty$ meaning the Θ^+ rotational wave function is "sitting at the North pole". Then it is not a rotation but, rather, a small oscillation of the Kaon field about the nucleon soliton, that can be studied, e.g. in the Skyrme model.

Our [DD, Petrov, Polyakov (1997)] eqn. for the width:

$$\Gamma_\Theta = \frac{3|\mathbf{p}|^3}{2\pi(M_N+M_\Theta)^2}\cdot\frac{1}{5}\cdot\left(G_0 - G_1 - \frac{1}{2}G_2\right)^2.$$

Generalization to arbitrary N_c [Praszalowicz (2004)]:

$$\Gamma_\Theta = \frac{3|\mathbf{p}|^3}{2\pi(M_N+M_\Theta)^2}\cdot\frac{3(N_c+1)}{(N_c+3)(N_c+7)}\cdot\left(G_0 - \frac{N_c+1}{4}G_1 - \frac{1}{2}G_2\right)^2.$$

According to Callan–Klebanov, take first the limit $N_c \to \infty$ and then put $N_c = 3$:

$$\Gamma_\Theta = \frac{3|\mathbf{p}|^3}{2\pi(M_N+M_\Theta)^2}\cdot 1\cdot\left(G_0 - \frac{3}{4}G_1 - \frac{1}{2}G_2\right)^2.$$

Our numbers:

$$G_0 \approx 14, \quad G_1 \approx 9, \quad G_2 \approx 2 \quad \Rightarrow \quad \Gamma_\Theta \approx 2.3\,\mathrm{MeV}, \qquad \Gamma_\Theta^{\mathrm{CK}} \approx 41\,\mathrm{MeV}.$$

In addition, the (unrealistic) Skyrme model gives somewhat different numbers for the constants:

$$G_0^{\rm Sk} \approx 17, \quad G_1^{\rm Sk} \approx 5, \quad G_2^{\rm Sk} = 0 \quad \Rightarrow \quad \Gamma_\Theta^{\rm CK} \approx 233\,{\rm MeV!!}$$

The Clebsch–Gordan factor

$$\frac{3(N_c+1)}{(N_c+3)(N_c+7)} = \frac{3}{N_c}\left(1 - \frac{9}{N_c} + \ldots\right) = \frac{1}{5}$$

is approaching its asymptotic value very slowly. At $N_c = 3$ it is still very far from the asymptotics, the rotation is at the latitude of London, and cannot be approximated by a "North pole"!

Conclusions

1. Ordinary baryons are **not** made of 3 quarks only but have a substantial component with the additional $\bar{Q}Q$ pairs. For some observables, their effect is 20% but for some other they change the naive result by a factor 3-4.

2. We have presented a compact and universal technique, how to write explicitly the $3Q, 5Q, 7Q\ldots$ wave functions of the octet, decuplet and antidecuplet baryons, based on the Relativistic Mean Field Approximation.

3. The standard $SU(6)$ wave functions are easily reproduced for the octet and decuplet baryons, if one assumes the non-relativistic limit. However, there are relativistic corrections to the $3Q$ wave function. In particular, 1/3 of the time the nucleon is made of 5 quarks.

4. The exotic Θ^+ width is proportional to the number of additional $\bar{Q}Q$ pairs in $nucleons$ and is thus naturally suppressed as compared to the expected widths of baryons with the dominant $3Q$ component. $\Gamma_\Theta \sim 2\,\mathrm{MeV}$, and could be even less.

5. A broad field of applications: distribution amplitudes, exclusive processes, parton distributions for a fixed number of quarks, etc. Also: all kind of transition amplitudes between various baryons at low energies.

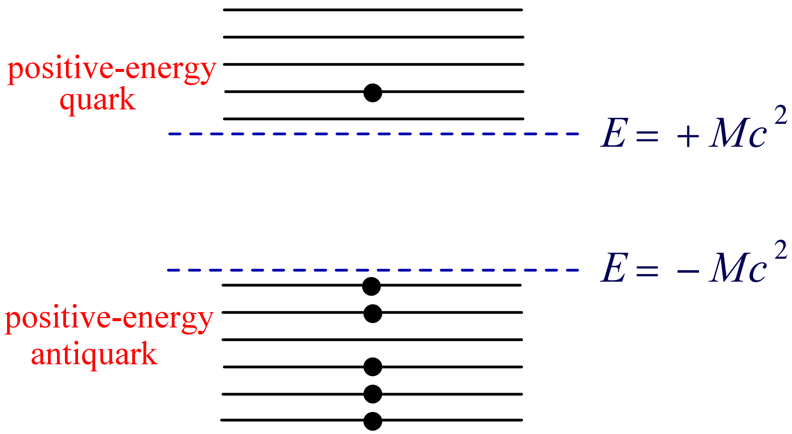


Figure 9: Vector, axial, tensor mesons are particle-hole excitations of the vacuum. They are made of a quark with positive energy and an antiquark with positive energy, hence their mass is roughly $2M$.

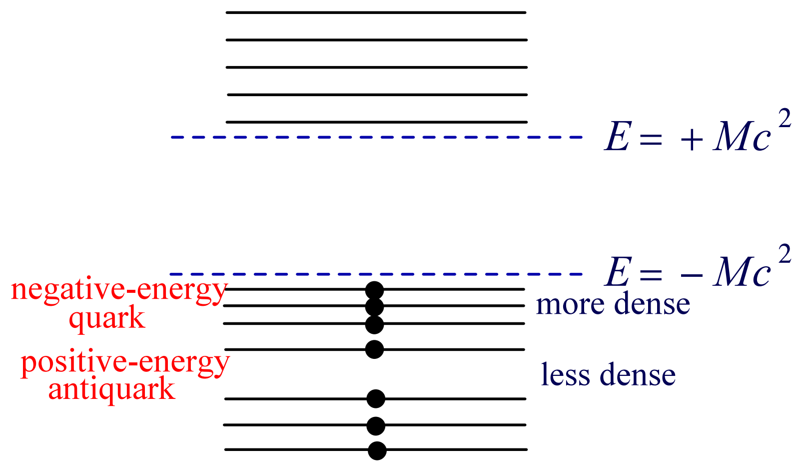


Figure 10: Pseudoscalar mesons are *not* particle-hole excitations but a collective re-arrangement of the vacuum. They are made of an antiquark with positive energy and a quark with *negative* energy, hence their mass is roughly zero, $(M - M) = 0$.