PROPHECY4F:

a PROPer description for the Higgs dECaY into 4 fermions

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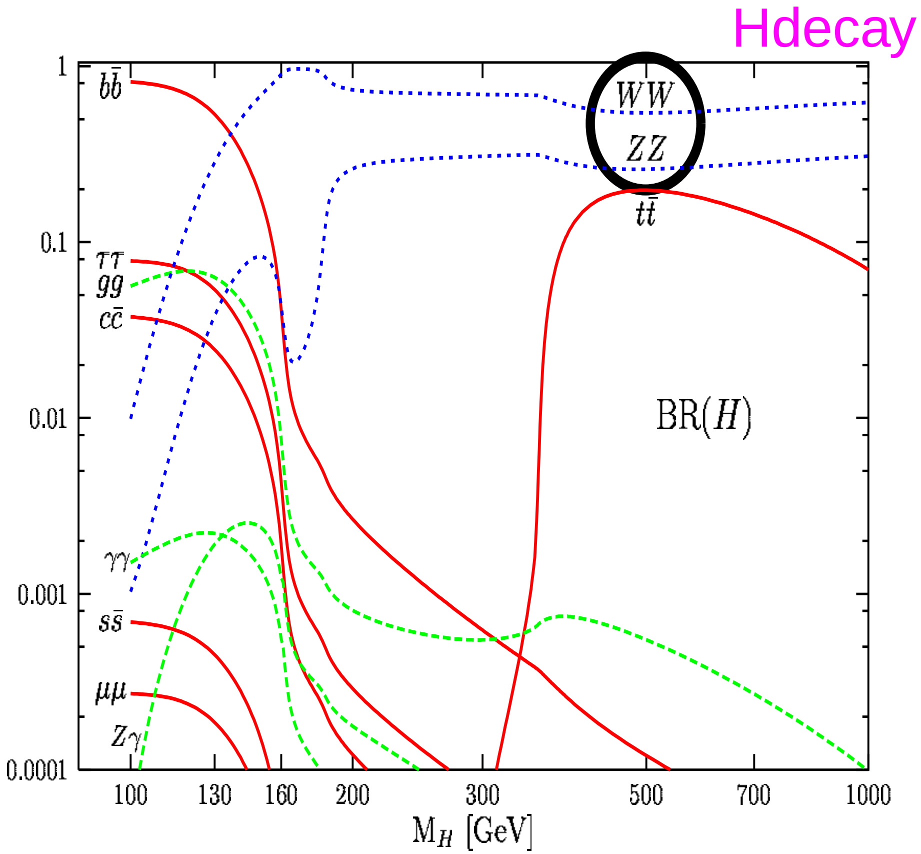
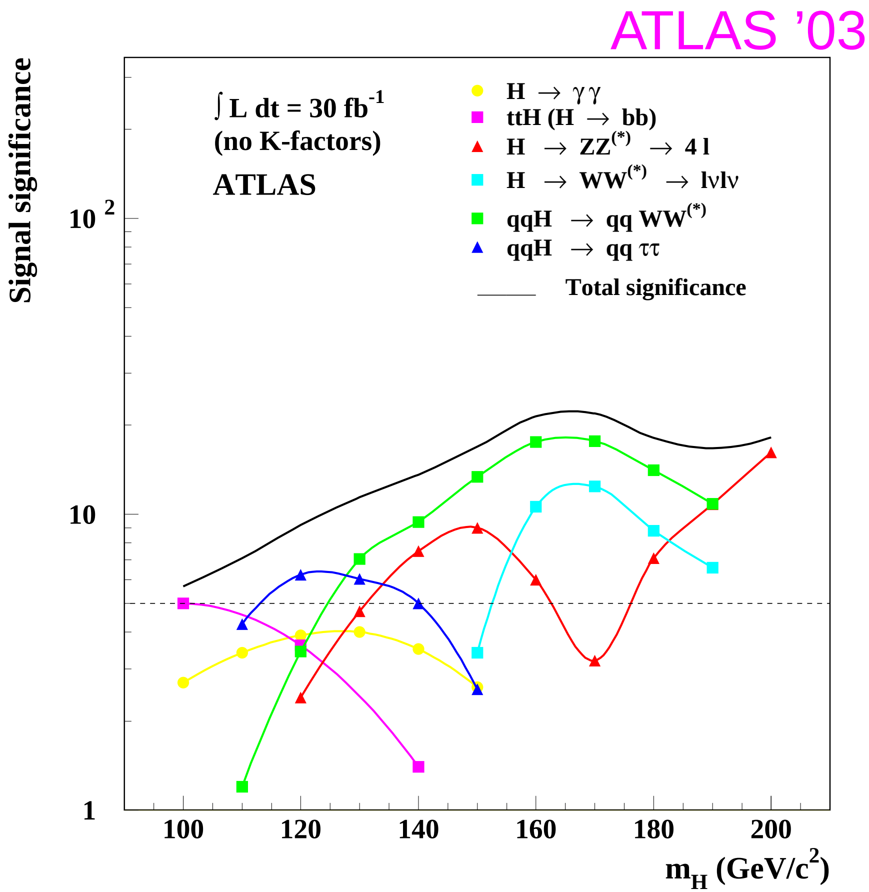
Contents

- Relevance of $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f$
- Calculation of EW and QCD corrections
- Numerical results

$\mathbf{H \to WW^{(*)}/ZZ^{(*)}}$





LHC: – most important Higgs decay channels for $M_{\mathrm{H}} \gtrsim 125\,\mathrm{GeV}$

– most precise determination of M_{H} via $\mathrm{H}{\to}\mathrm{ZZ}{\to}4l$ for $M_{\mathrm{H}} \gtrsim 130\,\mathrm{GeV}$

ILC: – measurements of branching ratios etc. at per-cent level

– full reconstruction of $\mathrm{H} \to \mathrm{WW}$ in semileptonic / hadronic final states

$\mathbf{H \to WW^{(*)}/ZZ^{(*)}}$

Previous work on partical decay widths:

above $\mathrm{WW/ZZ}$ threshold : $\mathcal{O}(\alpha)$ corrections for stable $\mathrm{W/Z}$

Kniehl '91; Bardin, Vilenski, Khristova '91

below threshold (and in transition region): tree-level predictions

(e.g. by HDecay: Djouadi, Kalinowski, Spira '97)

$\rightarrow$ Needed:

- corrections for off-shell W's,Z's
- distributions in fermion momenta
  - ◇ kinematical reconstruction of W's,Z's,H (influenced by corrections, especially γ radiation)
  - ◇ use information of 4f to verify Higgs spin and CP properties

Nelson '88; Soni, Xu '93; Chang et al.'93;
Skjold, Osland '93; Barger et al.'93;
Arens, Sehgal '94; Buszello et al.'02; Choi et al.'03

$\Rightarrow$ MC generator for $\mathrm{H \to WW/ZZ \to 4}f$ with corrections

Recent developments:

- Carloni-Calami et al.: QED corrections to $\mathrm{H \to ZZ \to 4}l$ (in progress)
- PROPHECY4F: complete EW and QCD NLO corrections to $\mathrm{H \to WW/ZZ \to 4}f$

PROPHECY4F

Monte Carlo Generator for all channels $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f$

- EW and QCD NLO corrections
- beyond NLO: final-state radiation via structure functions, heavy-Higgs effects
- still to be done: unweighted events, interface to parton shower, anomalous Higgs couplings

Employs

- multi-channel Monte Carlo integration (checked by Vegas)
  Berends, Kleiss, Pittau '94; Kleiss, Pittau '94
- dipole subtraction method for soft/collinear singularities (generalized to non-collinear safe observables, checked by phase-space slicing)
  Catani, Seymour '96; Dittmaier '99; A.B., Dittmaier, Roth '05

NLO calculation relies heavily on

- new techniques for stable evaluation of tensor integrals Denner, Dittmaier '05
- complex-mass scheme for treatment of resonances of unstable particles
  Denner, Dittmaier, Roth, Wieders '05

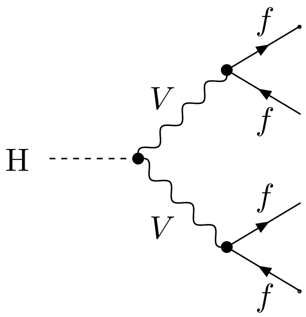
Further Details of PROPHECY4F

- G_μ-scheme includes some higher-order corrections already at tree level (electromagnetic coupling constant α is derived from Fermi constant)
- External fermion masses neglected whenever possible (everywhere but in mass-singular logarithms)
- Improved Born approximation (IBA) for partial decay widths with most important corrections (Coulomb singularity, leading effects for $m_t, M_H \gg M_W$, one fitting constant)
- Narrow-width approximation (NWA) for partial decay widths (applicable above threshold)

$\mathbf{H \to WW/ZZ \to 4}\boldsymbol{f}$

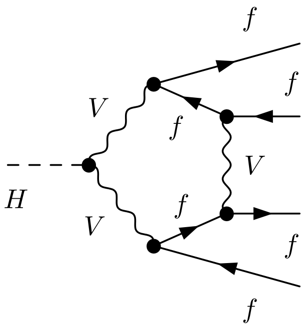
Some typical diagrams

Tree level:

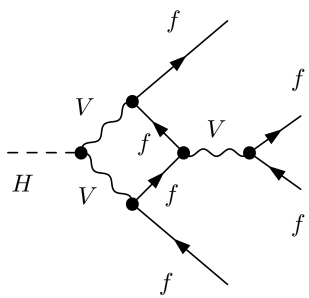


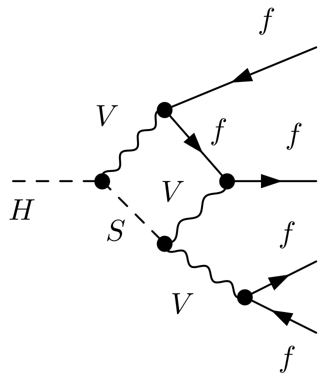
electroweak $\mathcal{O}(\alpha)$ corrections

pentagon

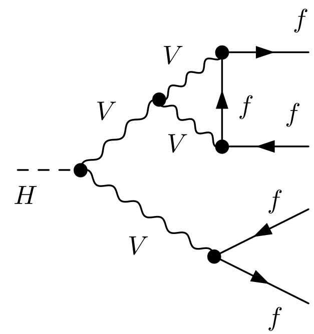


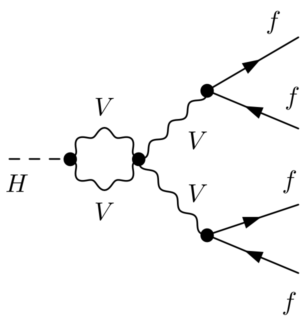
box



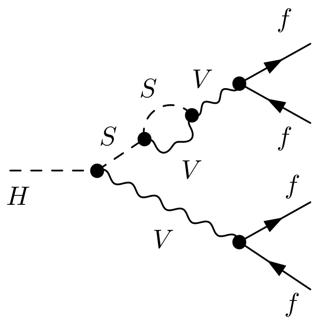


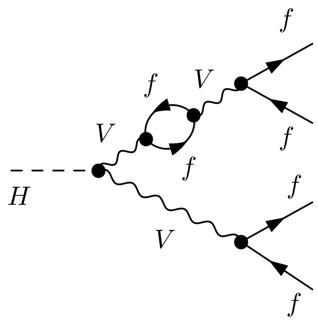
vertex





self energy





diagrams, #=$\mathcal{O}(200 - 400)$

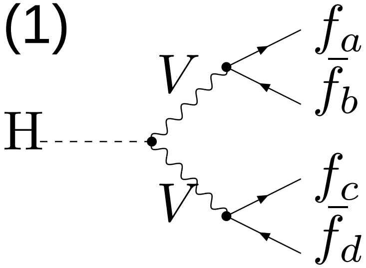
+ real corrections

$\mathbf{H \to WW/ZZ \to 4}\boldsymbol{f}$

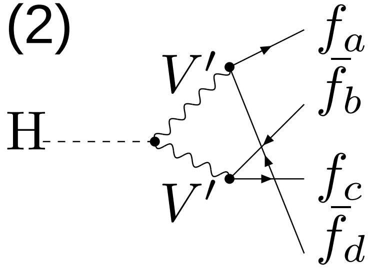
QCD corrections to semileptonic or hadronic final states:

Possible tree-level diagrams:

(1)



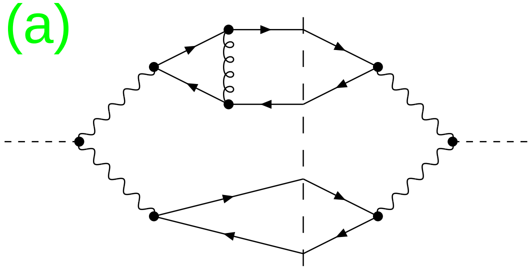
(2)



diagrams (2) only for
$q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ final states
(q' = weak-isospin partner of q)

Classification of QCD corrections into four categories: (typical diagrams)

(a)

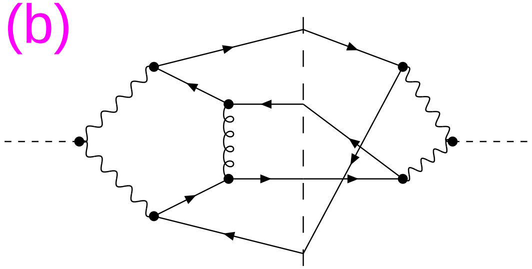


(a) = correction to W/Z decays

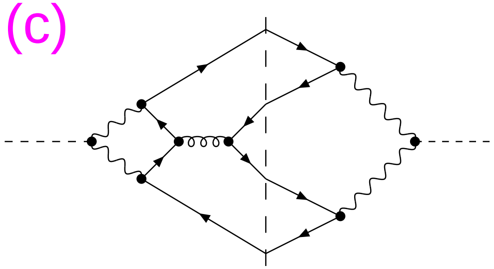
(b,c,d) = corrections to interferences

(only for $q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ final states)

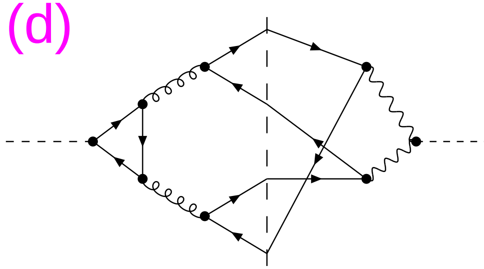
(b)



(c)



(d)



Virtual corrections: technical challenges

Reduction of tensor integrals:
appearance of small Gram determinants in standard Passarino-Veltman reduction (at phase-space boundary, but not only) $\rightarrow$ numerical instabilities
two different approaches to circumvent this problem

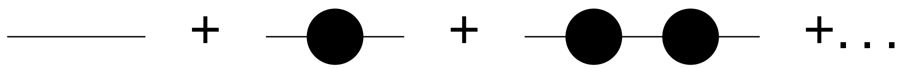
Same methods used as calculation of $\mathcal{O}(\alpha)$ corrections to $\mathrm{e}^+\mathrm{e}^- \rightarrow 4f$:
Denner, Dittmaier '05 Denner, Dittmaier, Roth, Wieders '05

- 5-point integrals are reduced to 4-point integrals without inverse Gram determinant
- 3-/4-point integrals:
  special treatment of phase-space points with small Gram determinant
  2 alternatives:
  - ◇ semi-numerical method + analytical special cases
    $\rightarrow$ avoids inverse Gram determinant
  - ◇ expansion in small Gram and other kinematical determinants

Virtual corrections: conceptual issues

Unstable particles require Dyson summation for description of resonances

$P(p^2) = \frac{i}{p^2-m^2} \sum_{n=0}^{\infty} \left(\frac{-\Sigma(p^2)}{p^2-m^2} \right)^n = \frac{i}{p^2-m^2+\Sigma(p^2)}$ (propagator of scalar particle)



mixes different orders of perturbation theory, potentially violates gauge invariance

tree level: various schemes (e.g. naive fixed width, complex-mass scheme) $\surd$

proposed solution at one loop:

- naive fixed width: breaks gauge invariance only mildly(?), spoils resonance structure
- pole expansion Aeppli et al. '93, '94; Stuart '91; Beenakker et al. '98; Denner et al. '00
  not relyable in threshold region
- effective field theory approach Beneke et al. '04
  combination of expansions suitable for different phase-space regions
- complex-mass scheme at one loop Denner, Dittmaier, Roth, Wieders '05
  valid everywhere in phase space

Complex-mass scheme at $\mathcal{O}(\alpha)$

Denner, Dittmaier, Roth, Wieders '05

Basic idea: mass^2 = location of propagator pole in complex p^2 plane

$$M^2_{\mathrm{W/Z}} \to \mu^2_{\mathrm{W/Z}} = M^2_{W/Z} - \mathrm{i} M_{\mathrm{W/Z}} \Gamma$$

everywhere in Feynman rules, also in loop integrals

modified renormalization conditions, e.g. $\hat{\Sigma}_T^{\mathrm{W/Z}}(\mu^2_{\mathrm{W/Z}}) = 0$ (on-shell scheme)

bare Lagrangian unchanged

Virtues

- gauge invariant, unitarity cancellations preserved
- valid everywhere in phase-space
- no matching of different expansions required
- perturbative calculations as usual, easy to implement

Drawbacks

- loop integrals with complex masses
- unitarity-violating spurious terms of $\mathcal{O}(\alpha^2)$

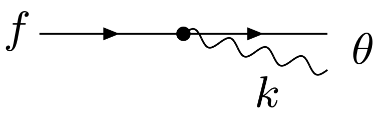
Phase-space integration

Multi-channel Monte Carlo integration with adaptive optimization

Berends, Kleiss, Pittau '94 — Kleiss, Pittau '94

- mappings for propagators (e.g. resonances) $\rightarrow$ integrand is flattened
- "coherent" combination of different mappings (channels)
- adaptive optimization finds most important channels
- below threshold: additional channels for non-resonant propagators

Virtual and real corrections have soft and collinear singularities



$k, \theta \rightarrow 0$

$\sim \log(\frac{s}{\lambda^2}), \quad \log(\frac{s}{m_f^2}), \quad \log(\frac{s}{\lambda^2}) \cdot \log(\frac{s}{m_f^2})$

λ: photon/gluon mass
m_f: fermion mass

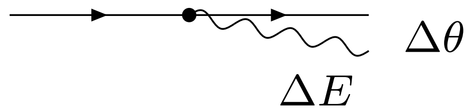
soft, collinear, soft & collinear

(ϵ^{-1},ϵ^{-1},ϵ^{-2} in dimensional regularization)

matching of singularities between virtual and real corrections:

$\rightarrow$ dipole subtraction, phase-space slicing

Phase-space slicing



$$\int \mathrm{d}\phi_{4f\gamma}\,|\mathcal{M}_\mathrm{real}|^2 = \int_{\substack{E_\gamma<\Delta E\\ \text{or}\,\theta(\gamma,f_i)<\Delta\theta}} \mathrm{d}\phi^\mathrm{sing}_{4f\gamma}\,|\mathcal{M}_\mathrm{real}|^2 \quad + \quad \underbrace{\int \mathrm{d}\phi^\mathrm{finite}_{4f\gamma}\,|\mathcal{M}_\mathrm{real}|^2}_{\text{finite}}$$

$\Downarrow$ analytical integration over $\mathrm{d}\phi_\gamma$
in soft/collinear approximation

$$\underbrace{\int \mathrm{d}\phi_{4f}\,|\mathcal{M}_\mathrm{virt}|^2 + |\mathcal{M}^\mathrm{sing}_\mathrm{real}|^2}_{\text{finite (due to KLN theorem)}}$$

Virtue

- concept/implementation very simple

Drawback

- large CPU time in case of small ΔE,$\Delta\theta$
- accuracy depends on ΔE, $\Delta\theta$ (plateau in (ΔE,$\Delta\theta$)-space has to be found)
- dependence of partial width on ΔE,$\Delta\theta$ cancels only numerically

Dipole subtraction method

Basic idea: subtract and re-add the quantity $|M_{\rm sub}|^2$

$$|\mathcal{M}_{\rm sub}|^2 \sim |\mathcal{M}_{\rm real}|^2 \quad \text{for} \quad \underset{\text{soft}}{k \to 0} \quad \text{or} \quad \underset{\text{collinear}}{p_i k \to 0}$$

k: photon/gluon momentum
p_i: fermion momentum

$$\underbrace{\int \mathrm{d}\phi_{4f\gamma/g} \left(|\mathcal{M}_{\rm real}|^2 - |\mathcal{M}_{\rm sub}|^2\right)}_{\text{finite}} + \underbrace{\int \mathrm{d}\phi_{4f} |\mathcal{M}_{\rm virt}|^2 + \int \overbrace{\mathrm{d}\phi_{4f\gamma/g}}^{\mathrm{d}\phi_{4f} \otimes \mathrm{d}\phi_{\gamma/g}} \cdot \overbrace{|\mathcal{M}_{\rm sub}|^2}^{g(p_i,p_j,k)|\mathcal{M}_{\rm Born}|^2}}_{\text{finite}}$$

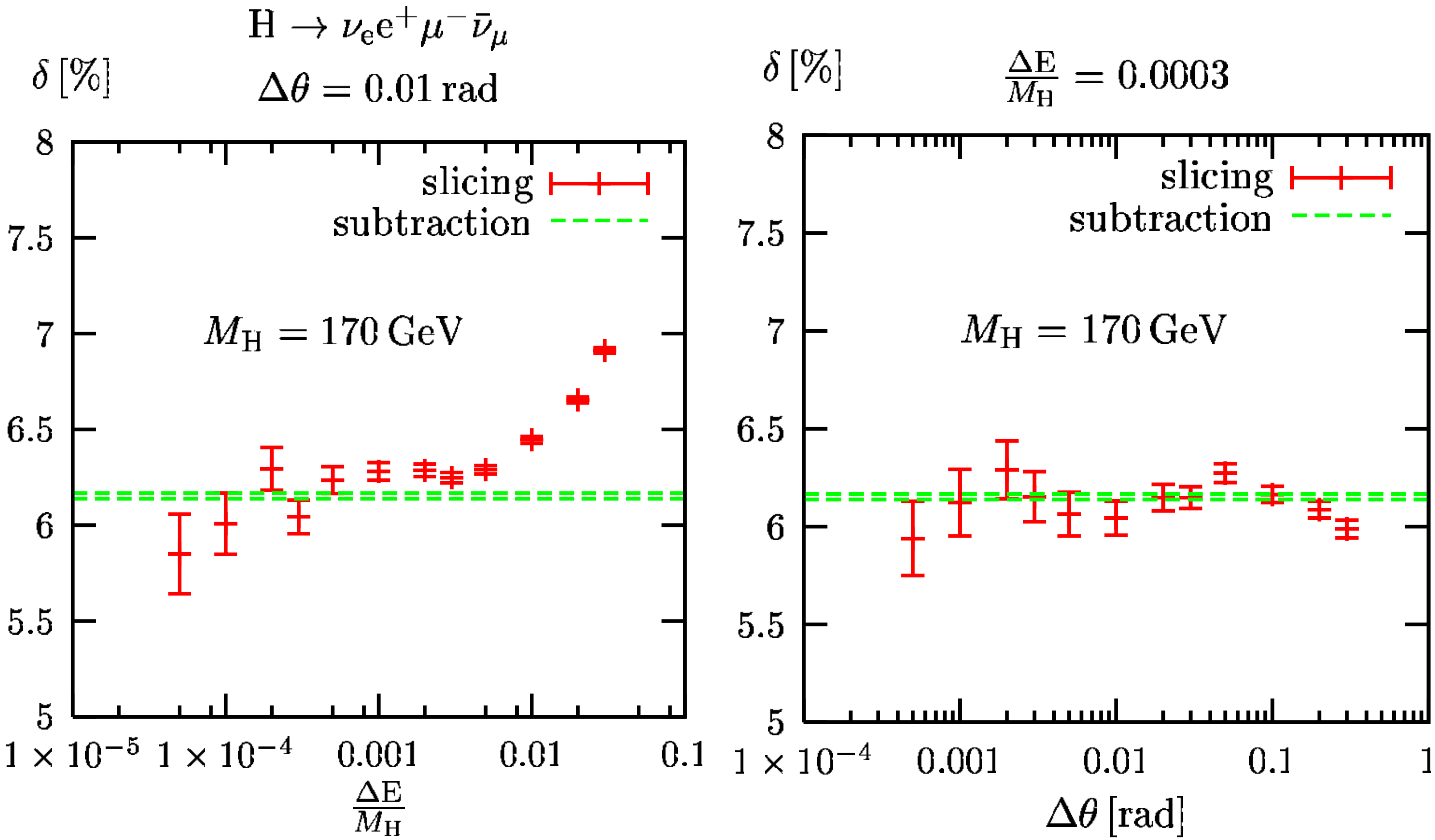
no regulators needed ($m_f, \lambda = 0$)

- $g(p_i, p_j, k)$ universal $\to$ can be integrated once and for all

  Catani, Seymour '96; Dittmaier '99

- fermion i and j are called emitter and spectator
- no soft/collinear approximation, no cut-off parameter dependence

Phase-space slicing – Subtraction



good agreement between subtraction and slicing method

$5 \cdot 10^7$ events

Non-collinear safe observables

$$\int \mathrm{d}\phi_{4f\gamma} \left(|\mathcal{M}_{\mathrm{real}}|^2 \underbrace{\Theta(\phi_{4f\gamma})} - |\mathcal{M}_{\mathrm{sub}}|^2 \underbrace{\Theta(\tilde{\phi}_{4f})} \right)$$

cuts/histogram bins/jet algorithm

subtraction method:
$\Theta(\tilde{\phi}_{4f})$ defined through
mapping from $\Theta(\phi_{4f\gamma})$

inclusive* (collinear-safe) observables: Θ depends only on $(p_i + k)$

$\rightarrow$ all $\log m_f$ terms cancel

exclusive* (non-collinear-safe) observables: $\Theta(\phi_{4f\gamma}) \neq \Theta(\tilde{\phi}_{4f})$ for $p_i k \rightarrow 0$

$\rightarrow$ $\log m_f$ terms do not cancel

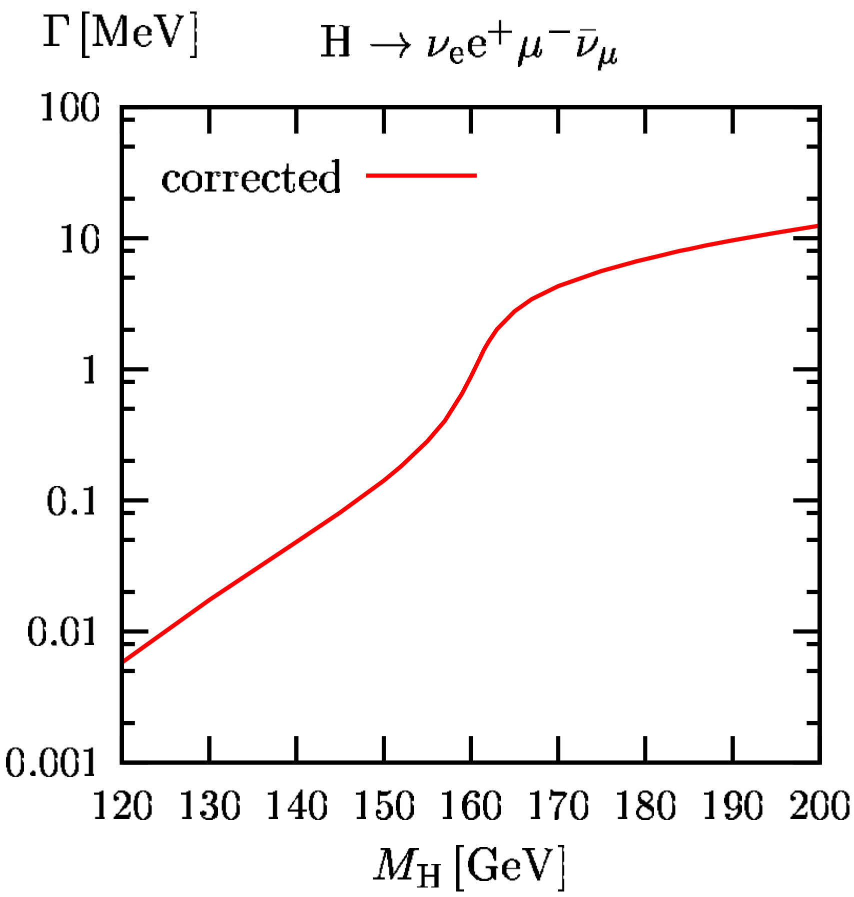
- original formulation of dipole subtraction method only for inclusive observables,
  generalization straightforward for phase-space slicing, but more involved for dipole subtraction method A.B., Dittmaier, Roth '05
- part of the integration over $\mathrm{d}\phi_\gamma$ has to be done numerically
- inclusive observable can be defined by photon recombination ($p_i + k = \tilde{p}_i$ for $p_i k \rightarrow 0$)

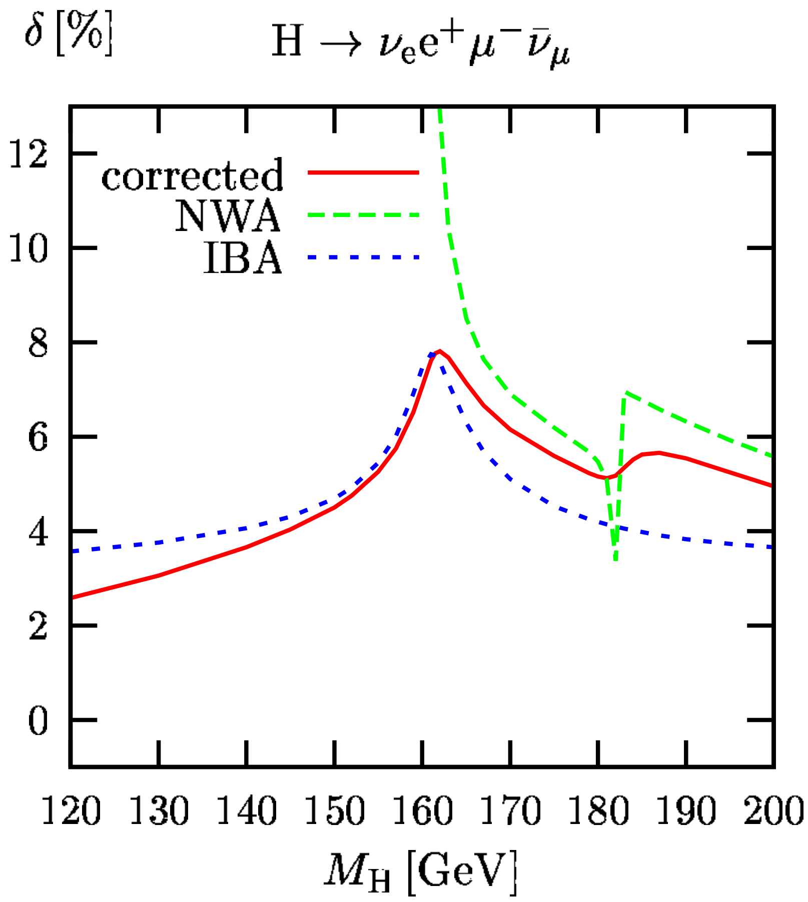
*w.r.t. final state radiation

Consistency checks

- UV: independence of reference mass μ in dimensional regularization
- Soft IR: independence of photon mass (regulator for soft singularities)
- Collinear IR: independence of external fermion masses (needed to describe collinear singularities) in inclusive case
- Gauge independence: calculation in ’t Hooft–Feynman and background-field gauge Denner, Dittmaier, Weiglein ’94
- Different methods for tensor reduction (loop integrals) Denner, Dittmaier ’05
- Real corrections: squared matrix elements compared with MADGRAPH Stelzer, Long ’94
- Different methods for combining soft and collinear singularities: dipole subtraction and phase-space slicing
- Last but not least: two completely independent calculations

Partial decay width for $\mathrm{H} \to \mathrm{WW} \to \nu_e e^+ \mu^- \bar{\nu}_\mu$





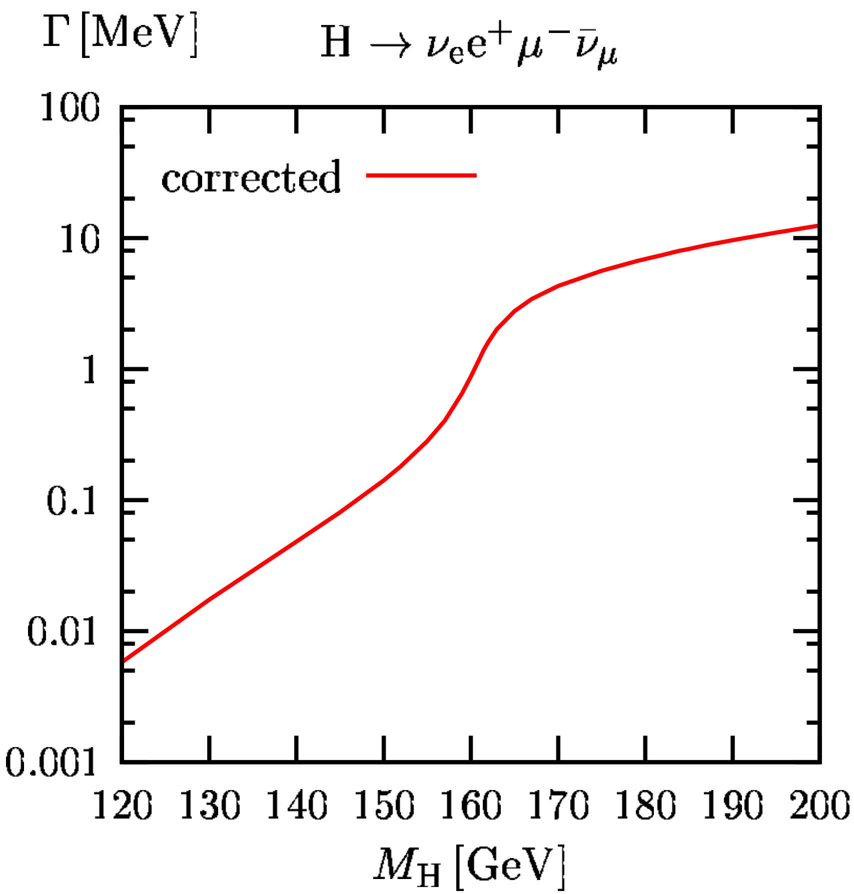
$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{Born}}} - 1$$

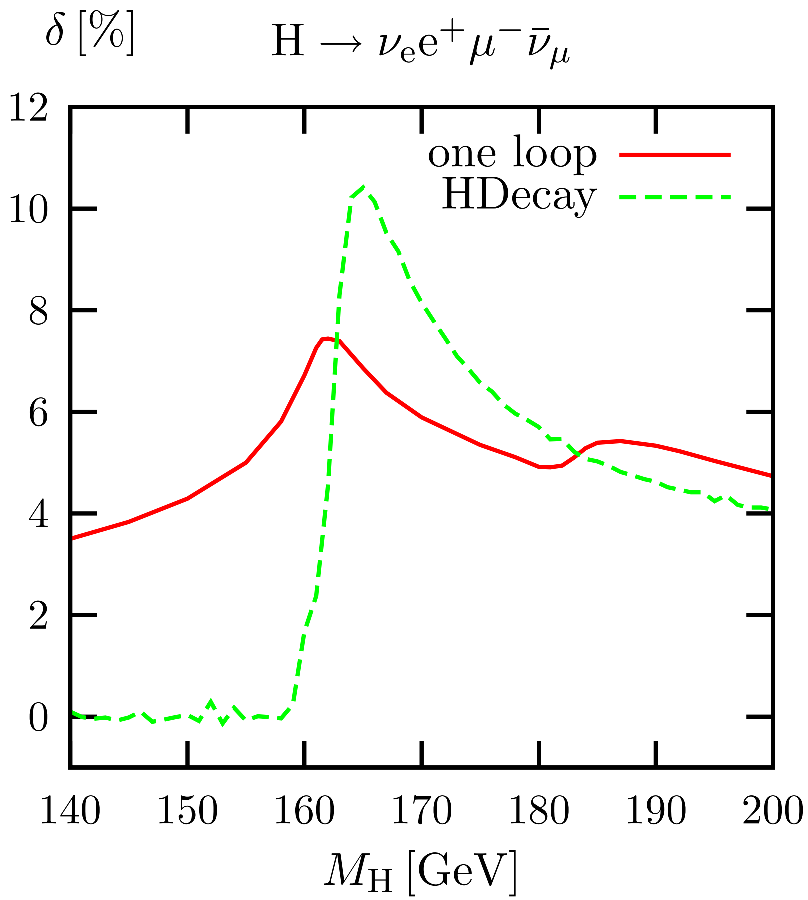
A.B., Denner, Dittmaier, Weber '06

↑ Coulomb singularity for $M_\mathrm{H} \sim 2M_\mathrm{W}$

↑ threshold effect in loops for $M_\mathrm{H} \sim 2M_\mathrm{Z}$

Partial decay width for $\mathrm{H} \to \mathrm{WW} \to \nu_e e^+ \mu^- \bar{\nu}_\mu$





$$\delta = \frac{\Gamma}{\Gamma_\mathrm{Born}} - 1$$

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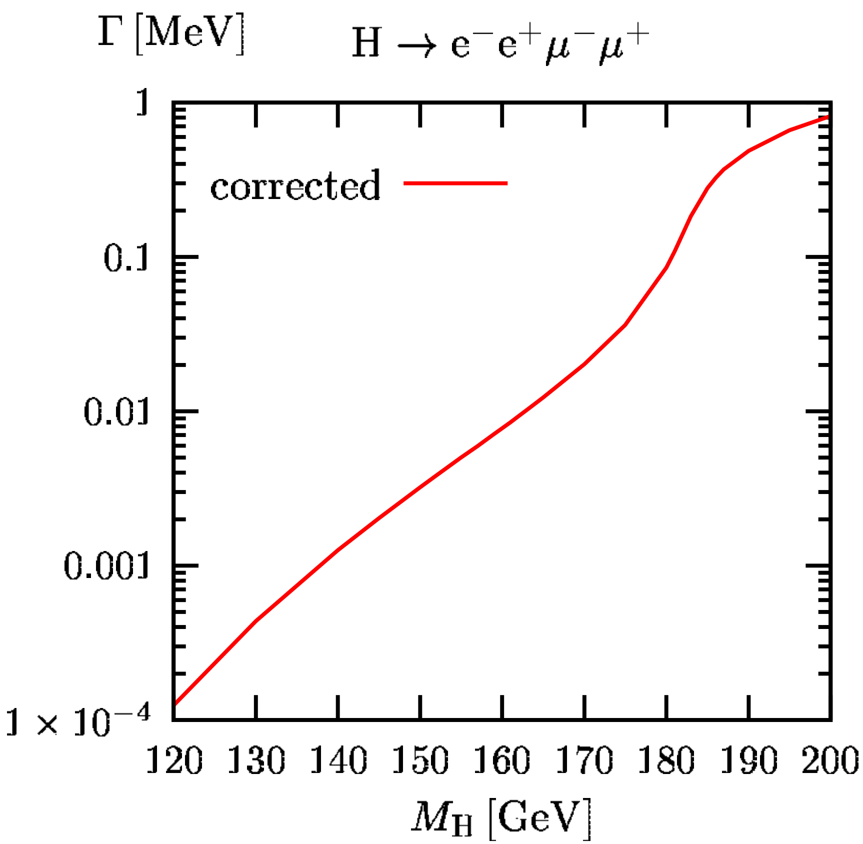
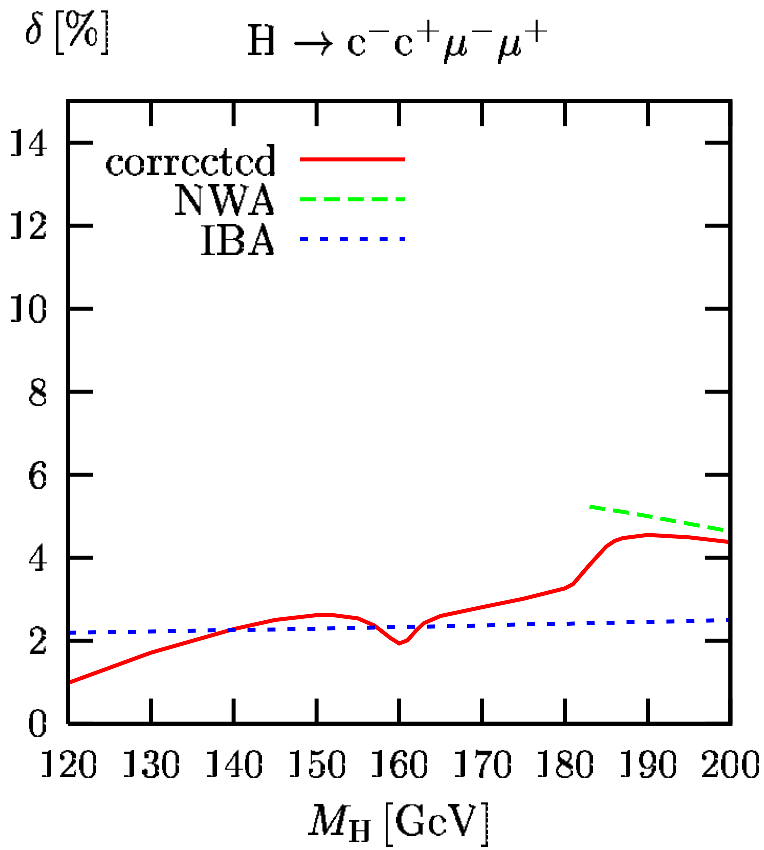
$\uparrow$ Coulomb singularity for $M_\mathrm{H} \sim 2M_\mathrm{W}$

$\uparrow$ threshold effect in loops for $M_\mathrm{H} \sim 2M_\mathrm{Z}$

peak structure in HDECAY is an artefact of the on-shell approximation above threshold

Partial decay width for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$





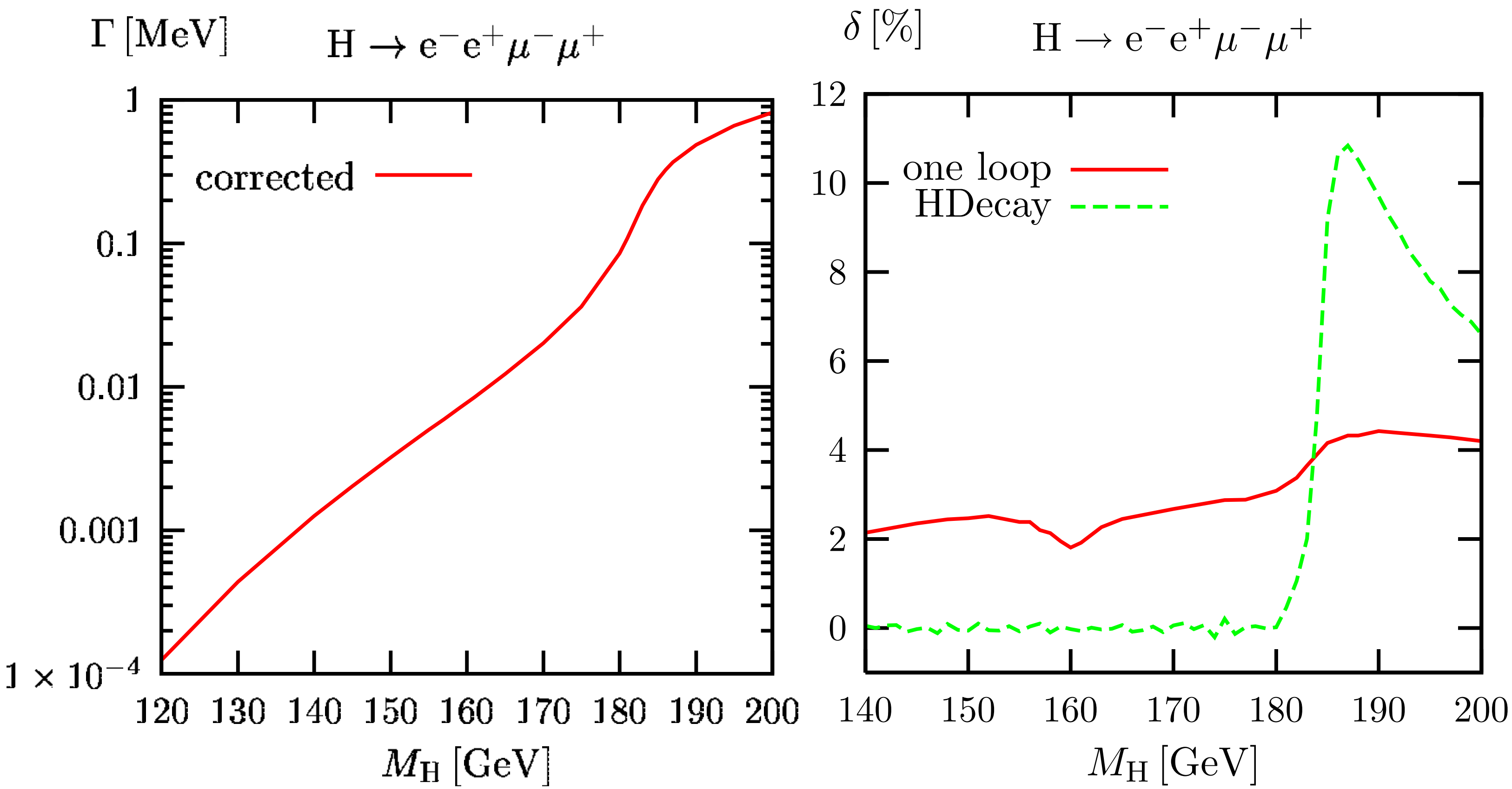
$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{Born}}} - 1$$

A.B., Denner, Dittmaier, Weber '06

$\uparrow$ threshold effects in loops for $M_{\mathrm{H}} \sim 2M_{\mathrm{W}}$

$\uparrow$ kinematical threshold at $M_{\mathrm{H}} \sim 2M_{\mathrm{Z}}$

Partial decay width for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e^-e^+}\mu^-\mu^+$



$$\delta = \frac{\Gamma}{\Gamma_\mathrm{Born}} - 1$$

A.B., Denner, Dittmaier, Weber '06

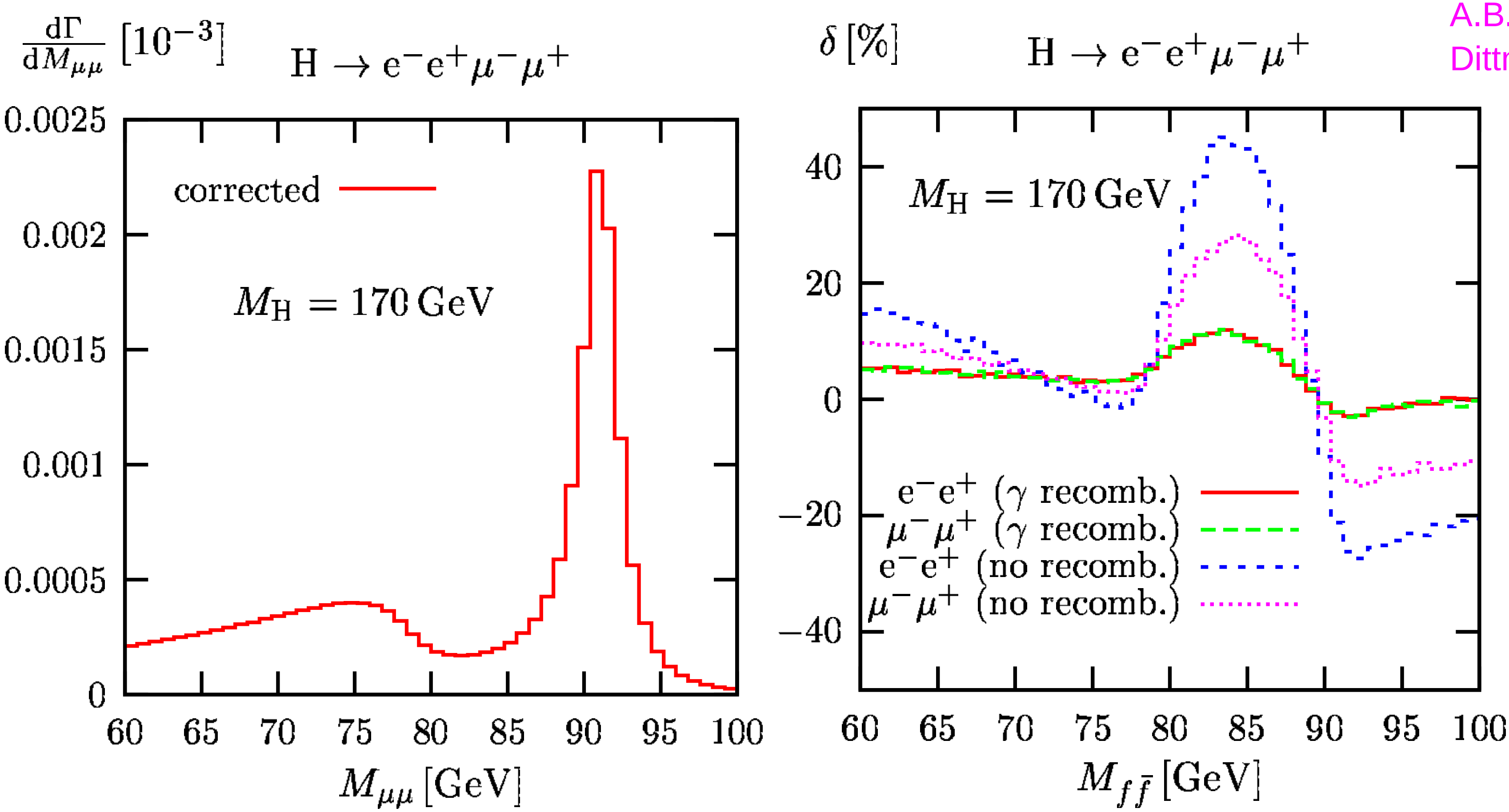
threshold effects in loops for $M_\mathrm{H} \sim 2M_\mathrm{W}$

kinematical threshold at $M_\mathrm{H} \sim 2M_\mathrm{Z}$

peak structure in HDECAY is an artefact of the on-shell approximation above threshold

Z-invariant-mass distribution in $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$

A.B., Denner,
Dittmaier, Weber '06

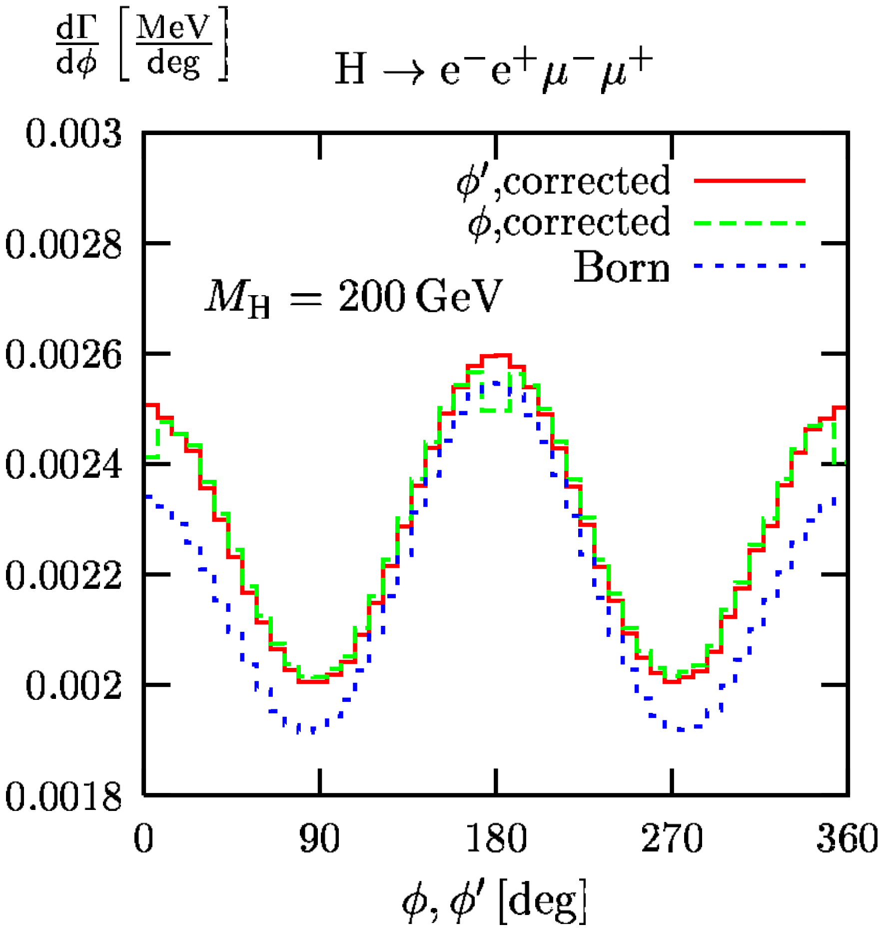


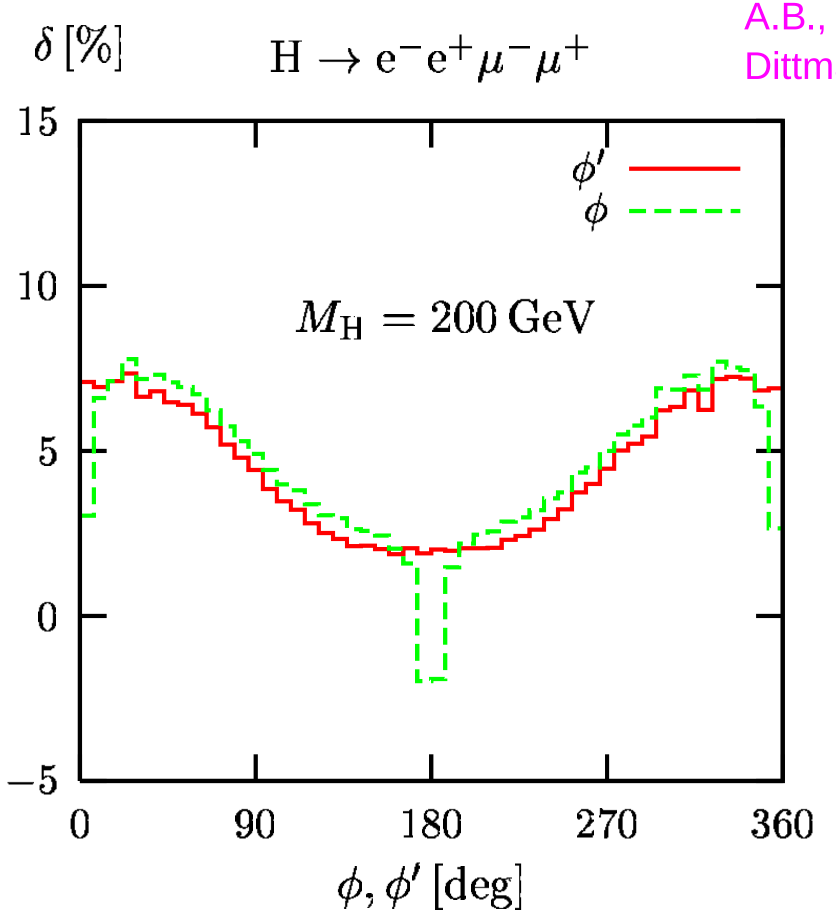
γ recombination if $M_{\mathrm{e}^+\gamma/\mu^-\gamma} < 5\,\mathrm{GeV}$

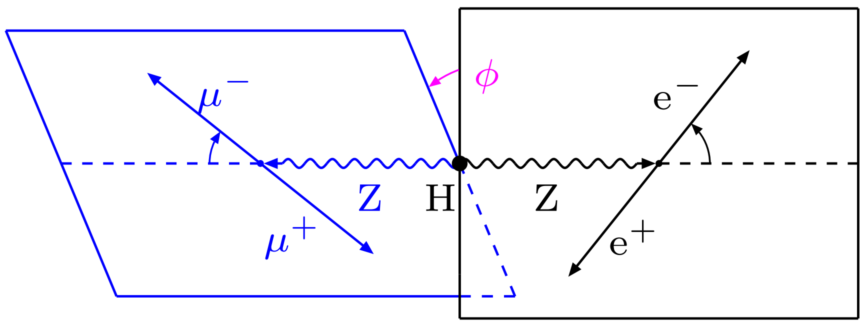
Large corrections from photon radiation in Z reconstruction

Angle between decay planes in $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$

A.B., Denner, Dittmaier, Weber '06





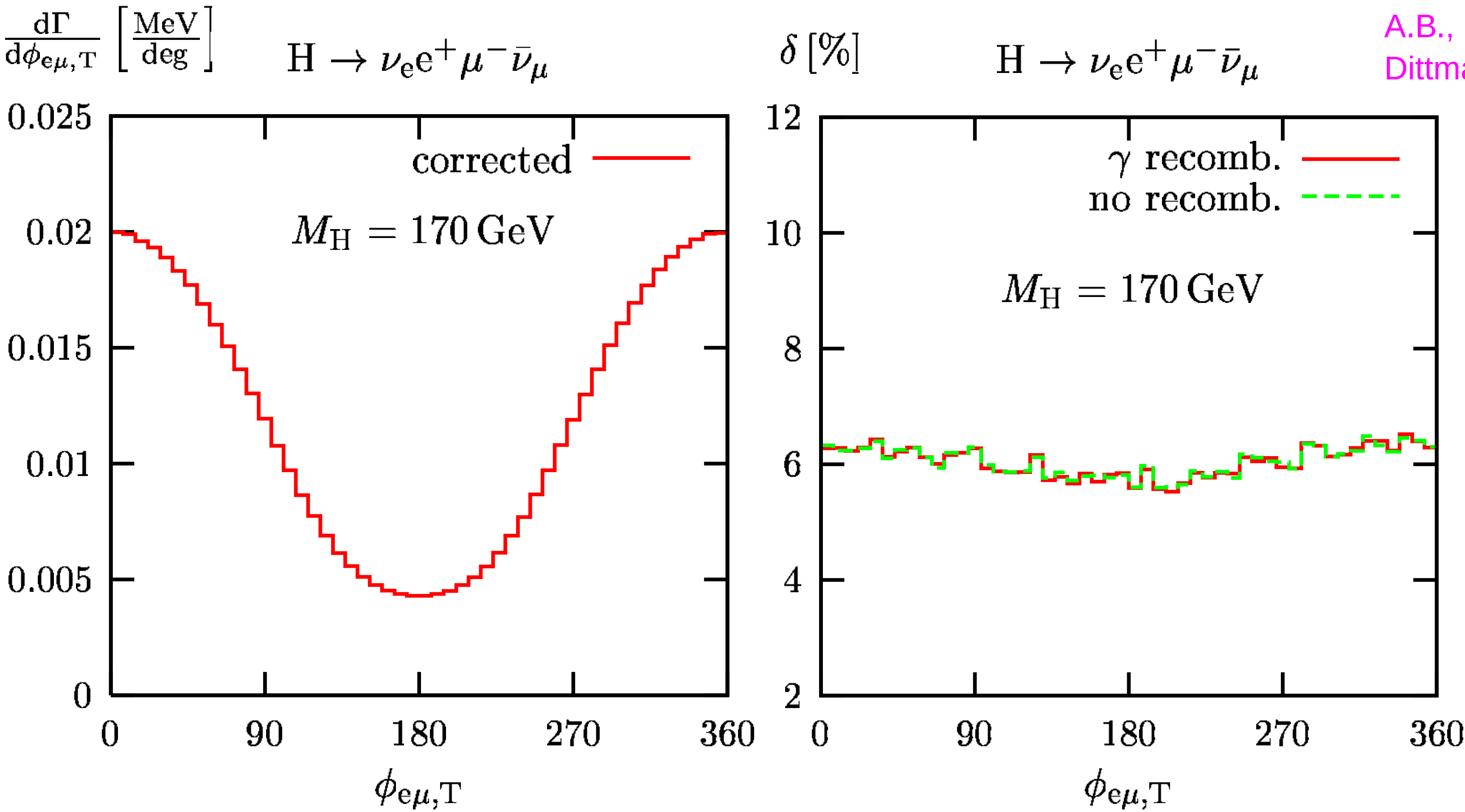


$$\cos\phi = \frac{\left(\mathbf{p}_{\mathrm{e}^-\mathrm{e}^+} \times \mathbf{p}_{\mathrm{e}^-}\right)\left(-\mathbf{p}_{\mu^-\mu^+} \times \mathbf{p}_{\mu^-}\right)}{\left|\mathbf{p}_{\mathrm{e}^-\mathrm{e}^+} \times \mathbf{p}_{\mathrm{e}^-}\right|\left|-\mathbf{p}_{\mu^-\mu^+} \times \mathbf{p}_{\mu^-}\right|}$$

$$\cos\phi' = \frac{\left(\mathbf{p}_{\mathrm{e}^-\mathrm{e}^+} \times \mathbf{p}_{\mathrm{e}^-}\right)\left(\mathbf{p}_{\mathrm{e}^-\mathrm{e}^+} \times \mathbf{p}_{\mu^-}\right)}{\left|\mathbf{p}_{\mathrm{e}^-\mathrm{e}^+} \times \mathbf{p}_{\mathrm{e}^-}\right|\left|\mathbf{p}_{\mathrm{e}^-\mathrm{e}^+} \times \mathbf{p}_{\mu^-}\right|}$$

Transverse angle between e^+ and μ^- in $\mathrm{H} \to \mathrm{WW} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu$

A.B., Denner, Dittmaier, Weber '06

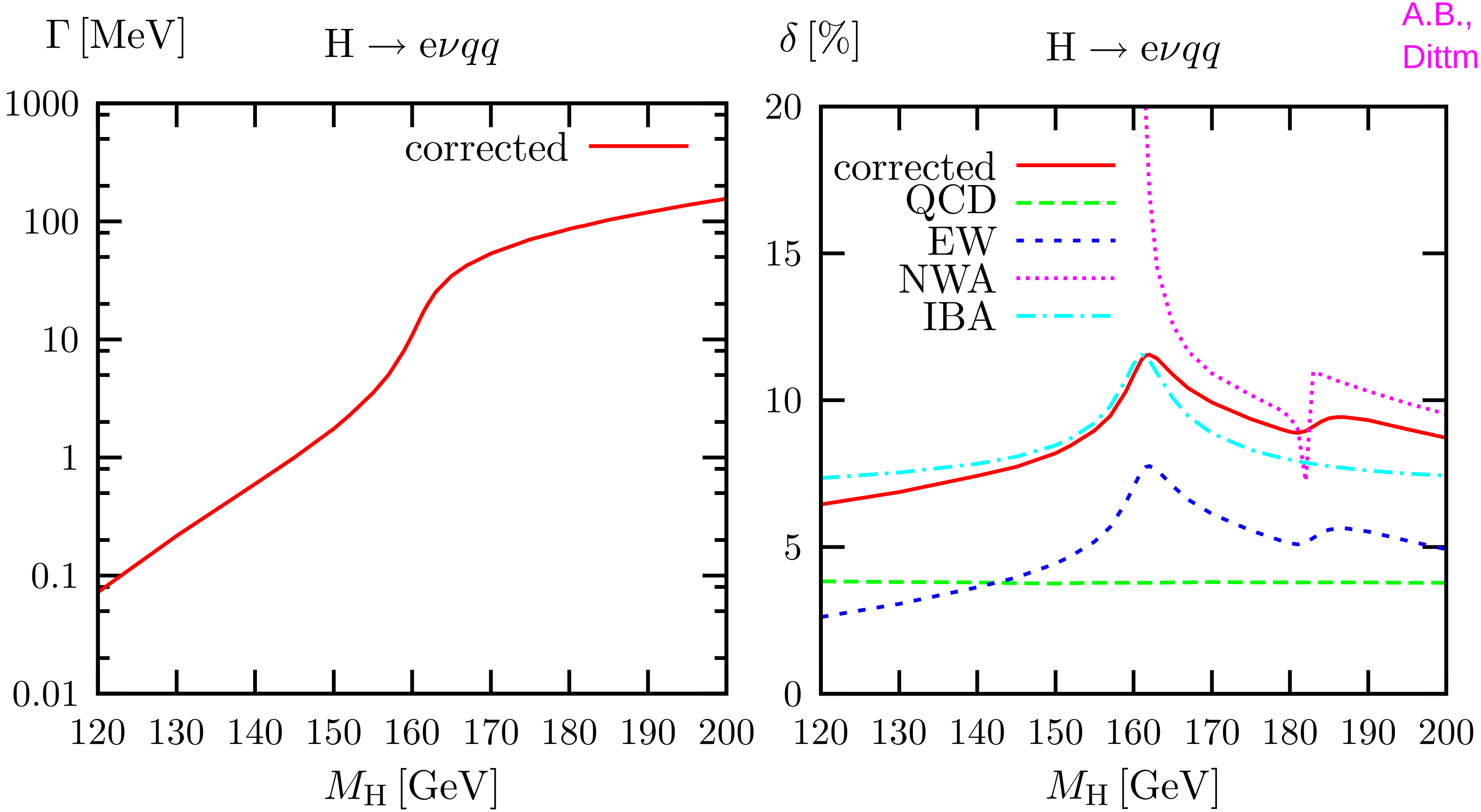


$\phi_{\mathrm{e}\mu,\mathrm{T}}$ is used for background reduction

e.g. Davatz, Dittmar, Giolo-Nicollerat '06

Partial decay width for $\mathrm{H} \to \mathrm{WW} \to \mathrm{e}\nu qq$

A.B., Denner, Dittmaier, Weber '06

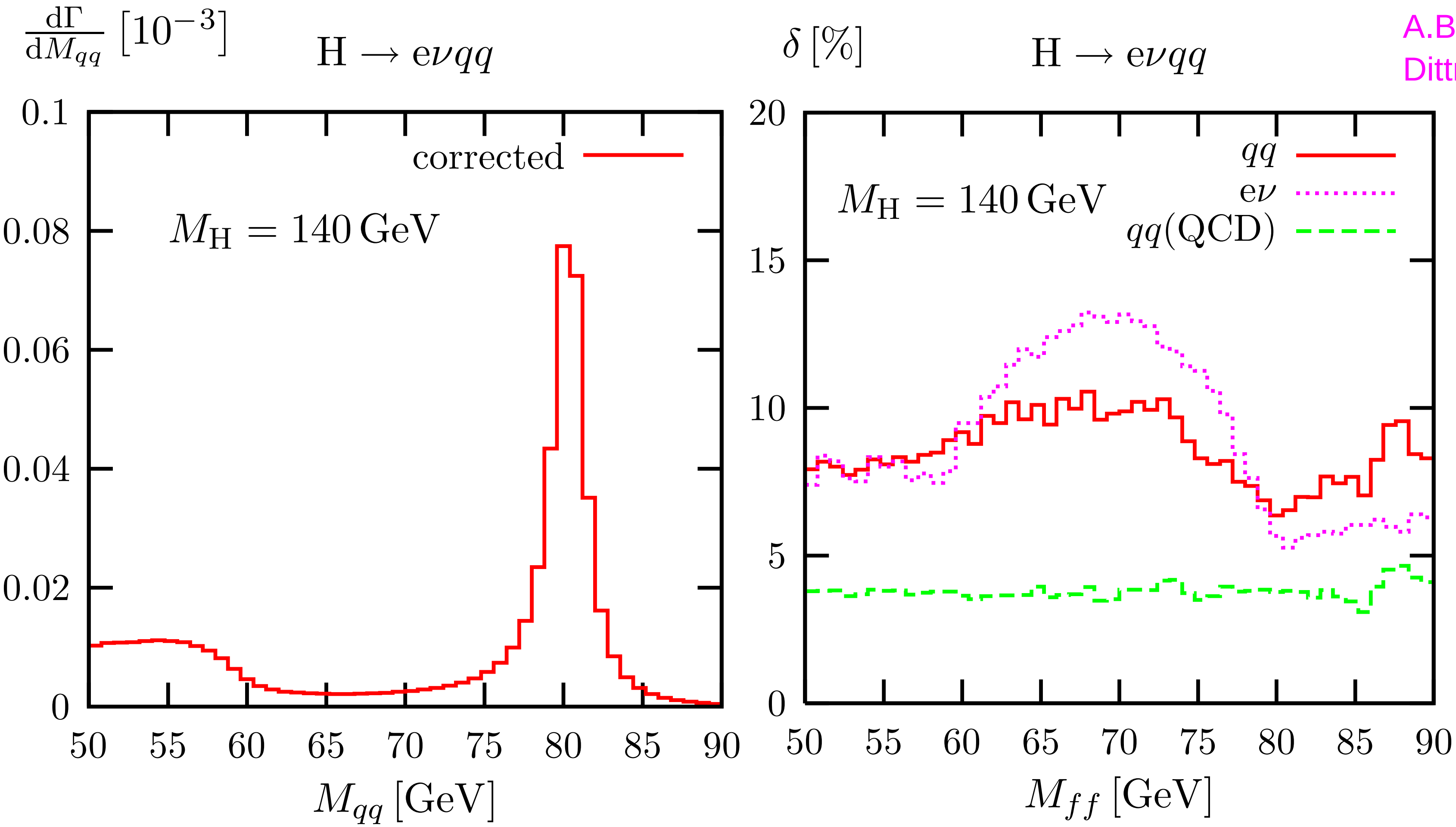


EW corrections: very similar to leptonic final states

QCD corrections: $\approx \frac{\alpha_s}{\pi} = 3.8\%$

W-invariant-mass distribution in $\mathrm{H} \to \mathrm{WW} \to \mathrm{e}\nu qq$

A.B., Denner, Dittmaier, Weber '06



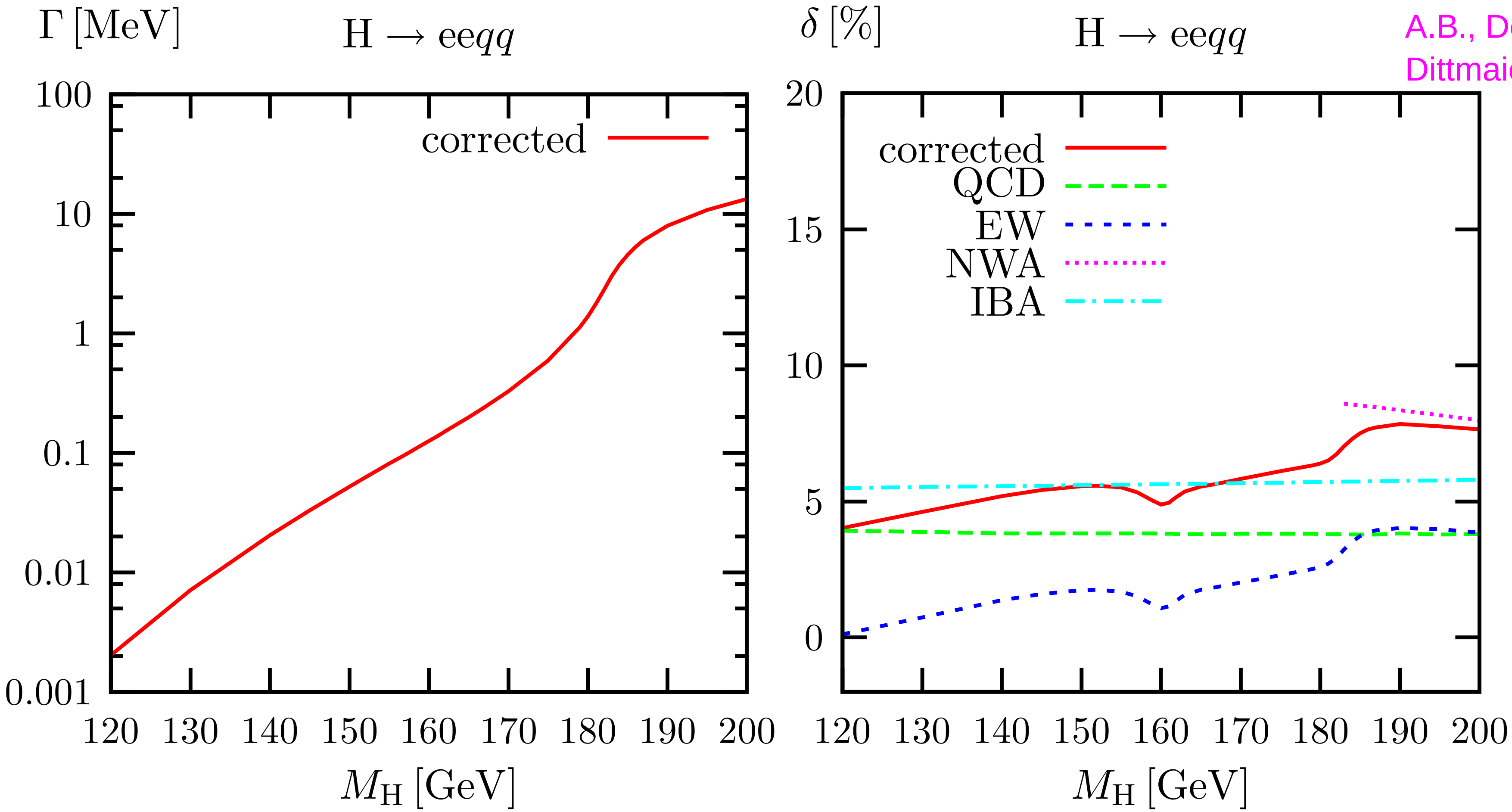
γ recombination if $M_{\mathrm{e}+\gamma} < 5\,\mathrm{GeV}$

$q\bar{q}(g)$ always recombined to two jets

EW corrections similar to leptonic case

QCD corrections are flat since hadronic part is treated inclusively

Partial decay width for H $\longrightarrow$ ZZ $\longrightarrow$ eeqq



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EW corrections: very similar to leptonic final states

QCD corrections: $\approx \frac{\alpha_s}{\pi} = 3.8\%$

Angle between decay planes in $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{ee}qq$

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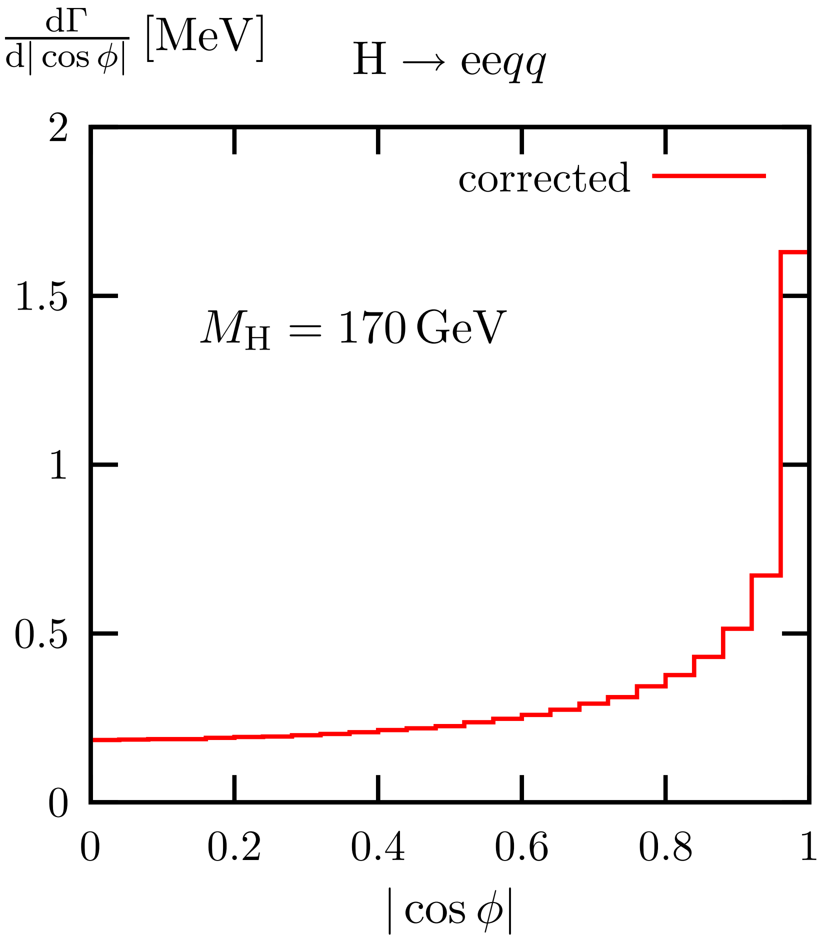
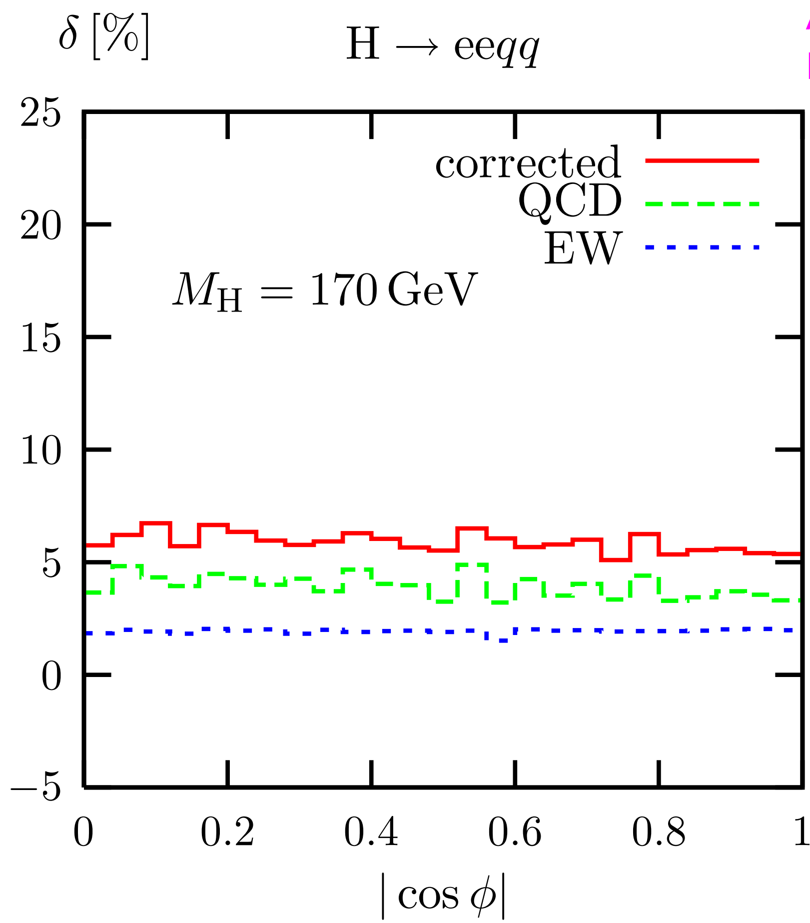
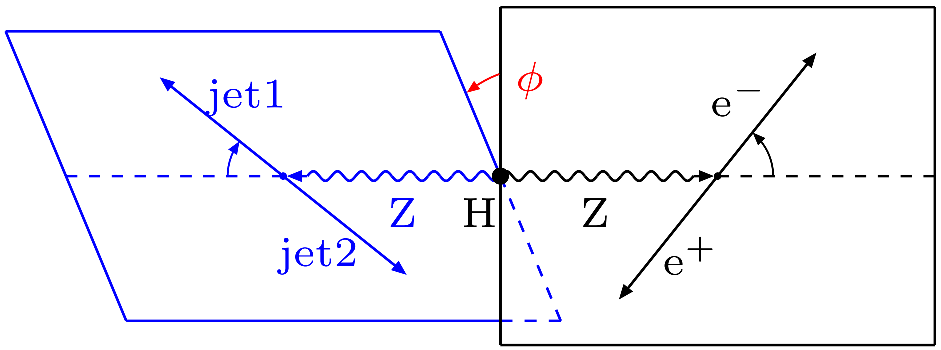


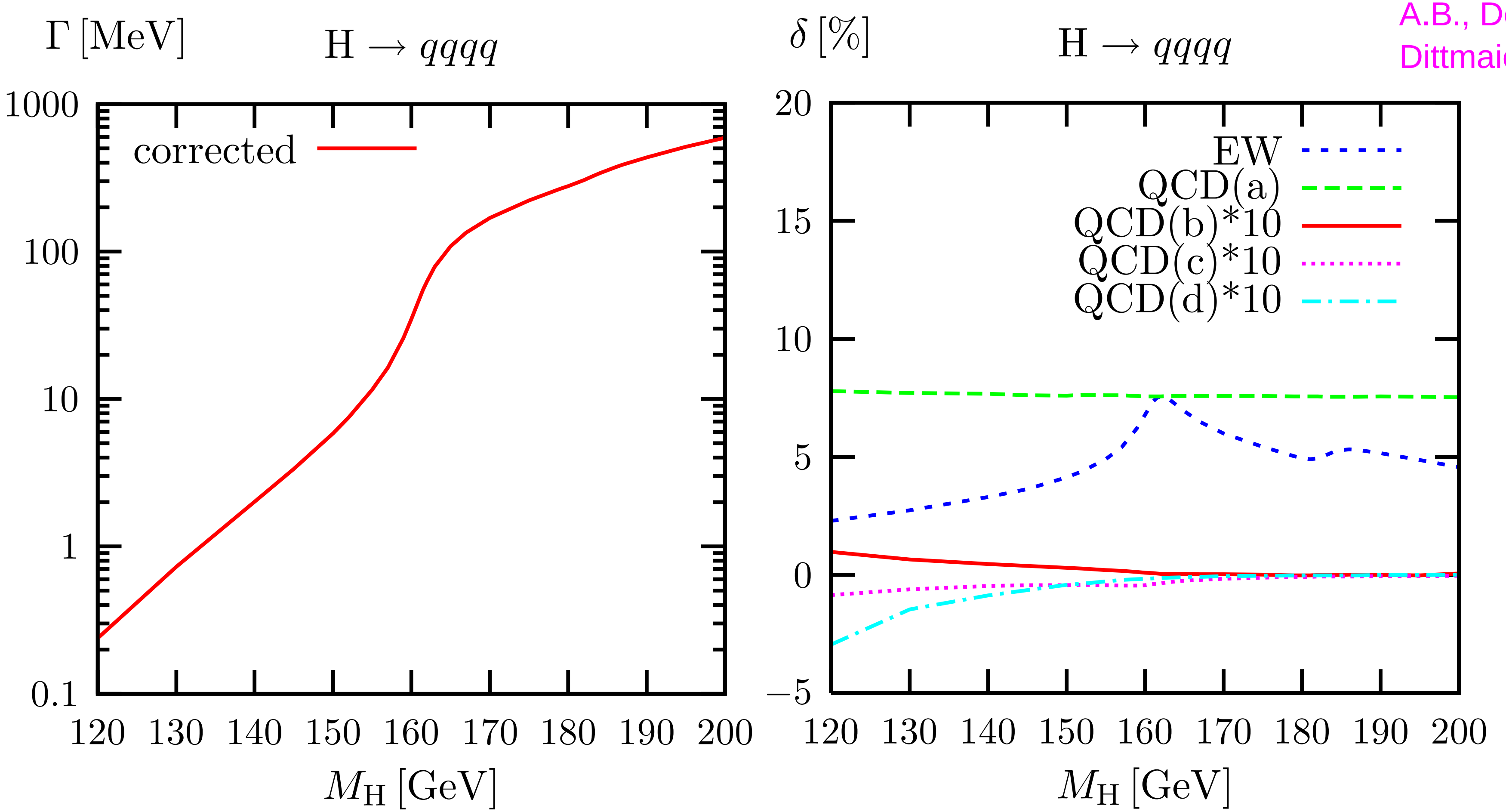




$$\cos\phi = \frac{(\mathbf{p}_{\mathrm{2jets}} \times \mathbf{p}_{\mathrm{e}^-})(\mathbf{p}_{\mathrm{jet1}} \times \mathbf{p}_{\mathrm{jet2}})}{|\mathbf{p}_{\mathrm{2jets}} \times \mathbf{p}_{\mathrm{e}^-}||\mathbf{p}_{\mathrm{jet1}} \times \mathbf{p}_{\mathrm{jet2}}|}$$

Partial decay width for $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to qqqq$

A.B., Denner, Dittmaier, Weber '06



EW corrections: very similar to leptonic final states

QCD corrections: $\approx \frac{2\alpha_s}{\pi} = 7.6\%$

only type (a) significant (corrections to W/Z decays)

Summary

$\mathrm{H} \to \mathrm{WW/ZZ} \to 4f$ is important Higgs decay channel at LHC and ILC for discovery and measurement of Higgs properties

PROPHECY4F: a generator for $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f$ including

- complete $\mathcal{O}(\alpha)$ EW and $\mathcal{O}(\alpha_s)$ QCD corrections by applying
  - complex-mass scheme for W and Z resonances
  - seminumerical or expansion methods for tensor reduction (loop integrals)
- universal corrections beyond $\mathcal{O}(\alpha)$ (final-state radiation, large-M_H effects)

Results obtained with PROPHECY4F:

- partial decay widths: EW corrections of $\mathcal{O}(8\%)$ for $M_\mathrm{H} \lesssim 500\,\mathrm{GeV}$
- angular distributions: EW corrections of $\mathcal{O}(5-10\%)$ distort shapes
- invariant-mass distributions of W's and Z's: EW corrections of some 10% distort shapes (depend on inclusiveness of γ radiation)
- QCD corrections can be associated with W/Z decay (interference effects negligible)