The CKM matrix: from the Standard Model to New Physics

KEK, Tsukuba, April 7th 2009

Jérôme Charles

CPT - Marseille

the CKMfitter group

JC, theory, Marseille

Olivier Deschamps, LHCb, Clermont-Ferrand

Sébastien Descotes-Genon, theory, Orsay

Ryosuke Itoh, Belle, Tsukuba

Andreas Jantsch, ATLAS, Munich

Heiko Lacker, ATLAS, Berlin

Andreas Menzel, Atlas, Berlin

Stéphane Monteil, LHCb, Clermont-Ferrand

Valentin Niess, LHCb, Clermont-Ferrand

Jose Ocariz, BaBar, Paris

Stéphane T'Jampens, LHCb, Annecy-le-Vieux

Vincent Tisserand, BaBar, Annecy-le-Vieux

Karim Trabelsi, Belle, Tsukuba

http://ckmfitter.in2p3.fr

The CKMfitter project

Our goal

- combine as many as possible experimental measurements related to quark flavor mixing
- define and understand the theoretical uncertainties, and propose ways to control them
- work within a rigorous frequentist statistical framework taking into account the different error types and possible biases due to theory, low statistics, non linearities, nuisance parameters ...
- test the Standard Model and different New Physics scenarios

Outline

update of the CKM matrix with a few details on current tensions

New Physics in meson mixing

LQCD averages (Sébastien's talk)

2HDM model (Sébastien's talk)

Quark mixing

mixing of the quark flavors because of the weak interaction

$\longrightarrow$ bi-diagonalization *via* the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

this unitary matrix is complex as soon as there are at least three quark generations: this produces CP violation (Kobayashi-Maskawa, Nobel Prize '08)

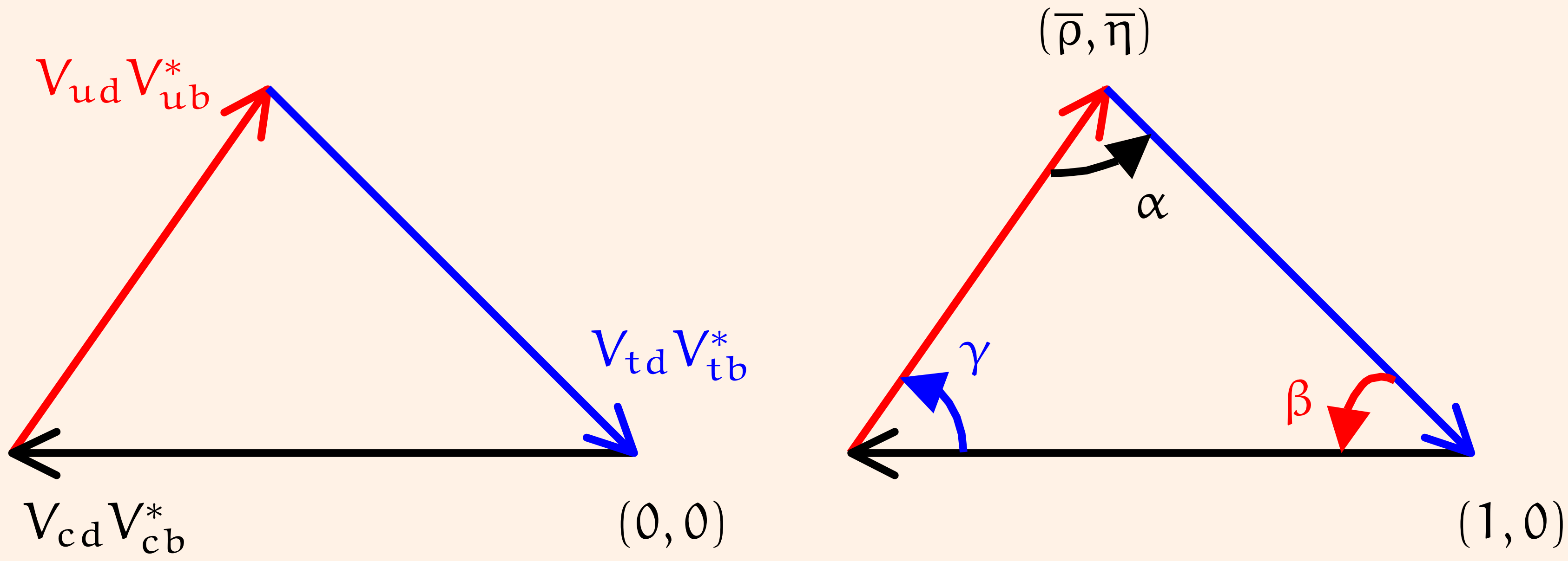
CKM with three generations is predictive, in the sense one can prove the existence of CP-violation from CP-conserving measurements only

Hierarchy and Unitarity Triangle(s)

strong hierarchy of the CKM matrix:

- diagonal couplings $\propto 1$
- 1st $\leftrightarrow$ (resp. 2nd $\leftrightarrow$ 3rd) generation $\propto \lambda \sim 0.22$ (resp. $\propto \lambda^2$)
- 1st $\leftrightarrow$ 3rd generation $\propto \lambda^3$

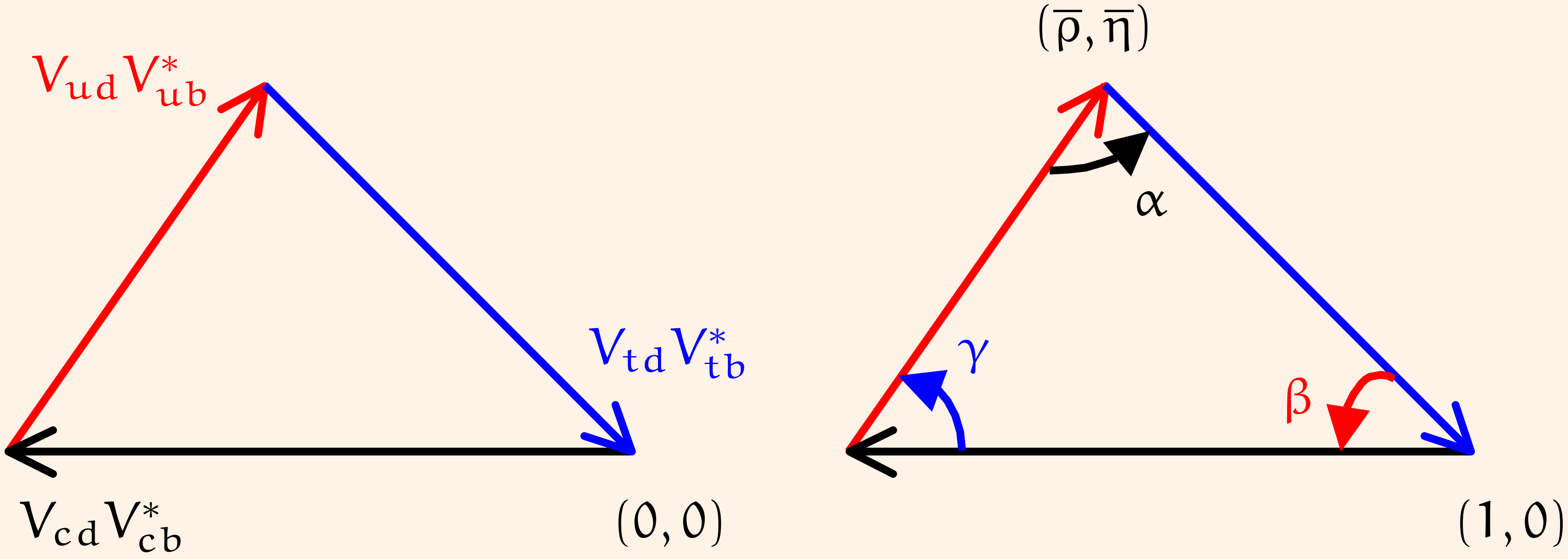
CKM unitarity $\Rightarrow$ six triangles in the complex plane, of which four are quasi flat, two are non flat and quasi degenerate



unitary-exact and phase-convention-independent version of the Wolfenstein parametrization

$$\lambda^2 \equiv \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2} \qquad A^2\lambda^4 \equiv \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2}$$

$$\bar\rho + i\bar\eta \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



Jetlag effect

very often in this talk: β, α, γ convention instead of ϕ_1, ϕ_2, ϕ_3

I apologize !

The statistical framework

we use a standard frequentist approach: likelihood maximization (χ^2 minimization)

where necessary, we treat non gaussian behavior by Monte-Carlo simulation of vir tual experiments

theoretical errors

no model-independent treatment available, due to lack of precise definit ion; we use the Rfit model: a theoretical parameter that has been computed (e.g. B_K) is assumed to lie within a definite range, without any preference inside this range

the best fit will thus be searched by moving uniformly in the theoretical parameter space

references: A. Höcker *et al.*, EPJC 21 (2001); JC *et al.*, EPJ C 41 (2005); http://ckmfitter.in2p3.fr

The global CKM fit

the constraints on the CKM matrix come from the decays of the neutron, the kaon, the B meson and to a lesser extent the D meson

"standard fit": uses all constraints on which we think we have a good theoretical control

$\lvert V_{ud}\rvert, \lvert V_{us}\rvert, \lvert V_{cb}\rvert$	PDG, HFAG and Flavianet WG
ε_K	exp: KTeV/KLOE, theo: OOA
$\lvert V_{ub}\rvert$	OOA
Δm_d	exp: last WA, theo: OOA
Δm_s	dominated by CDF, theo: OOA
β	last WA
α	exp: last $\pi\pi$, $\rho\pi$, $\rho\rho$ WA, theo: SU(2)
γ	exp: last $B \to DK$ WA, theo: GLW/ADS/GGSZ
$B \to \tau\nu$	exp: last WA, theo: OOA

(more details in Sébastien's talk and http://ckmfitter.in2p3.fr)

The global CKM fit: result

Moriond 2009

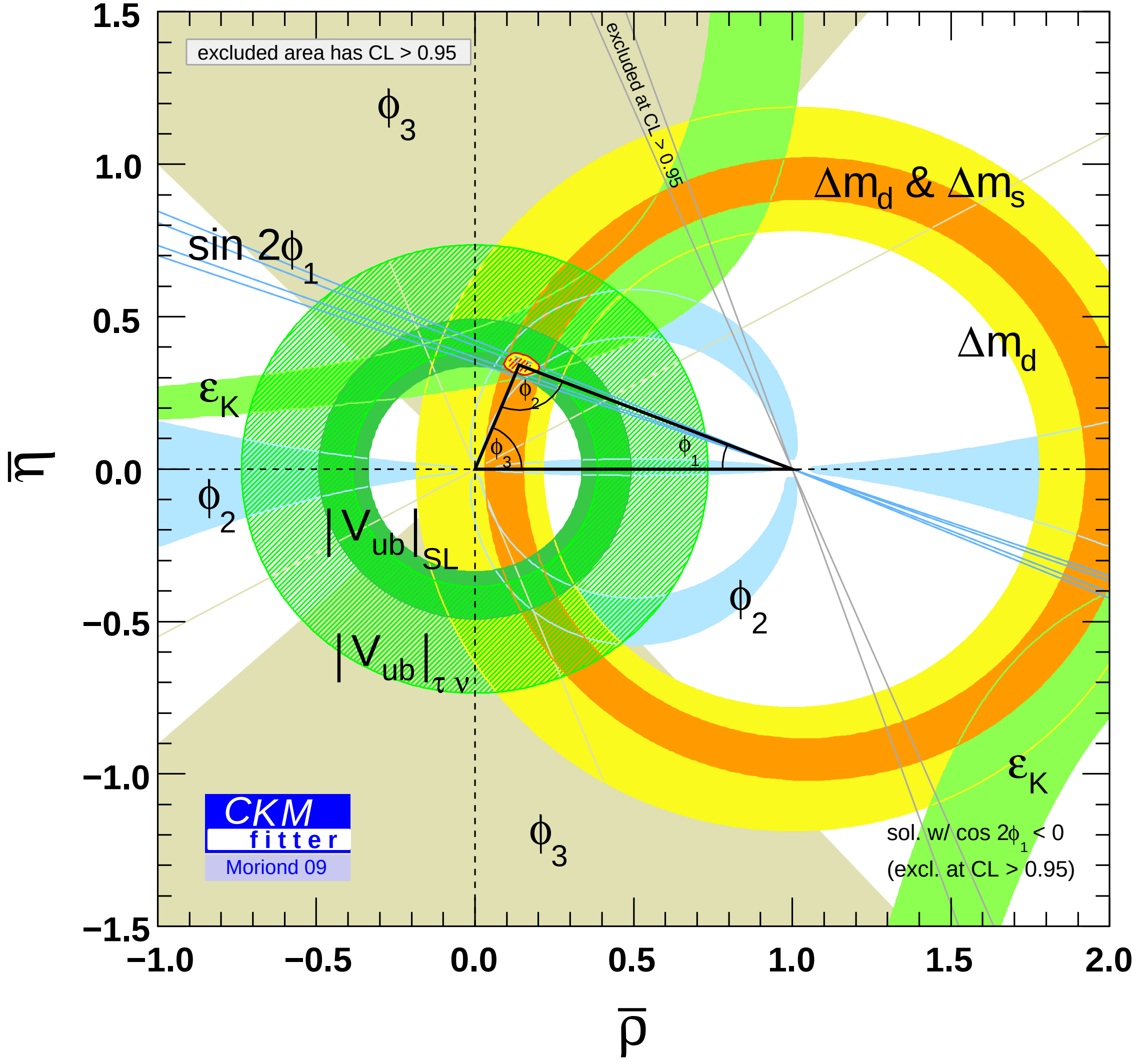
once A and λ have been mainly determined from $|V_{ud}|$, $|V_{us}|$ and $|V_{cb}|$, $(\bar{\rho}, \bar{\eta})$ are constrained by combination of all the observables

$A = 0.8116^{+0.0097}_{-0.0241}$

$\lambda = 0.22521 \pm 0.00082$

$\bar{\rho} = 0.139^{+0.025}_{-0.027}$

$\bar{\eta} = 0.341^{+0.016}_{-0.015}$



More on α

combined isospin analysis of $B \to \pi\pi,\ \rho\pi,\ \rho\rho$ modes

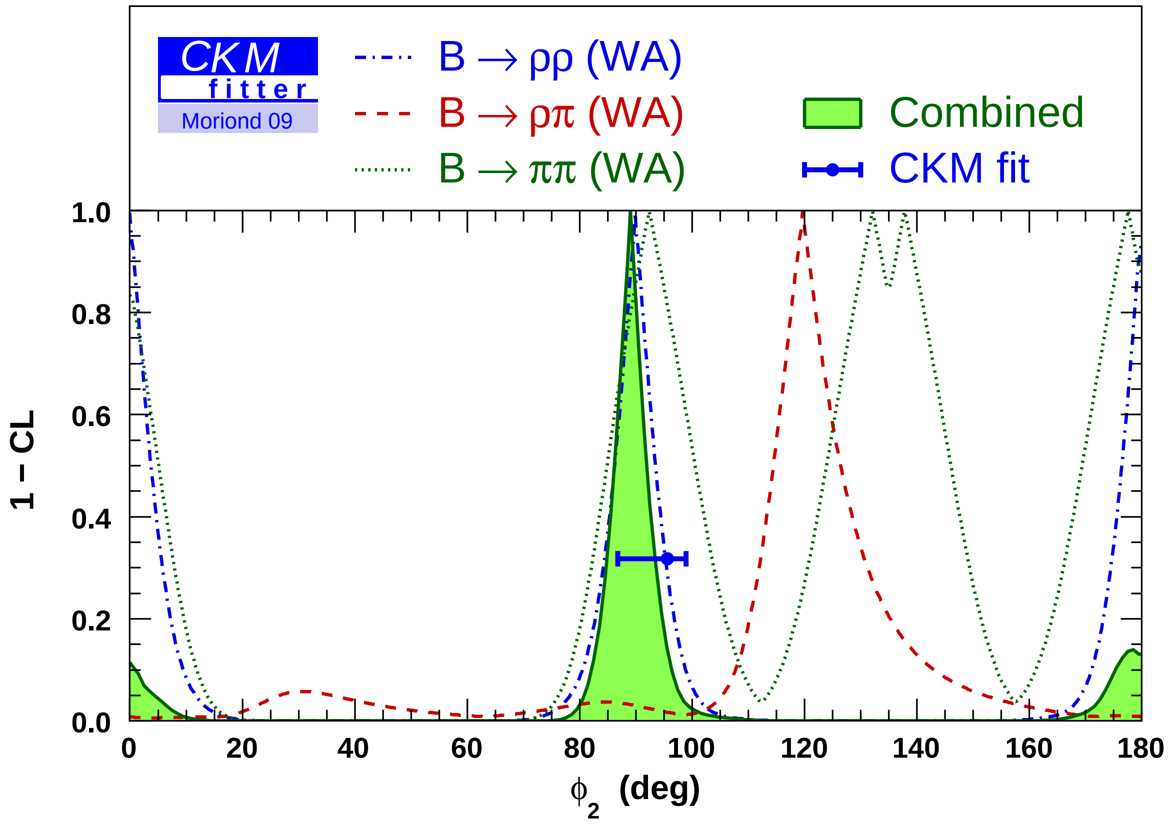
new (final) BaBar measurement of $B \to \rho^+\rho^0$ branching ratio, isospin triangle in $\rho\rho$ does not close and induces a very good constraint on α that dominates the WA

direct measurement

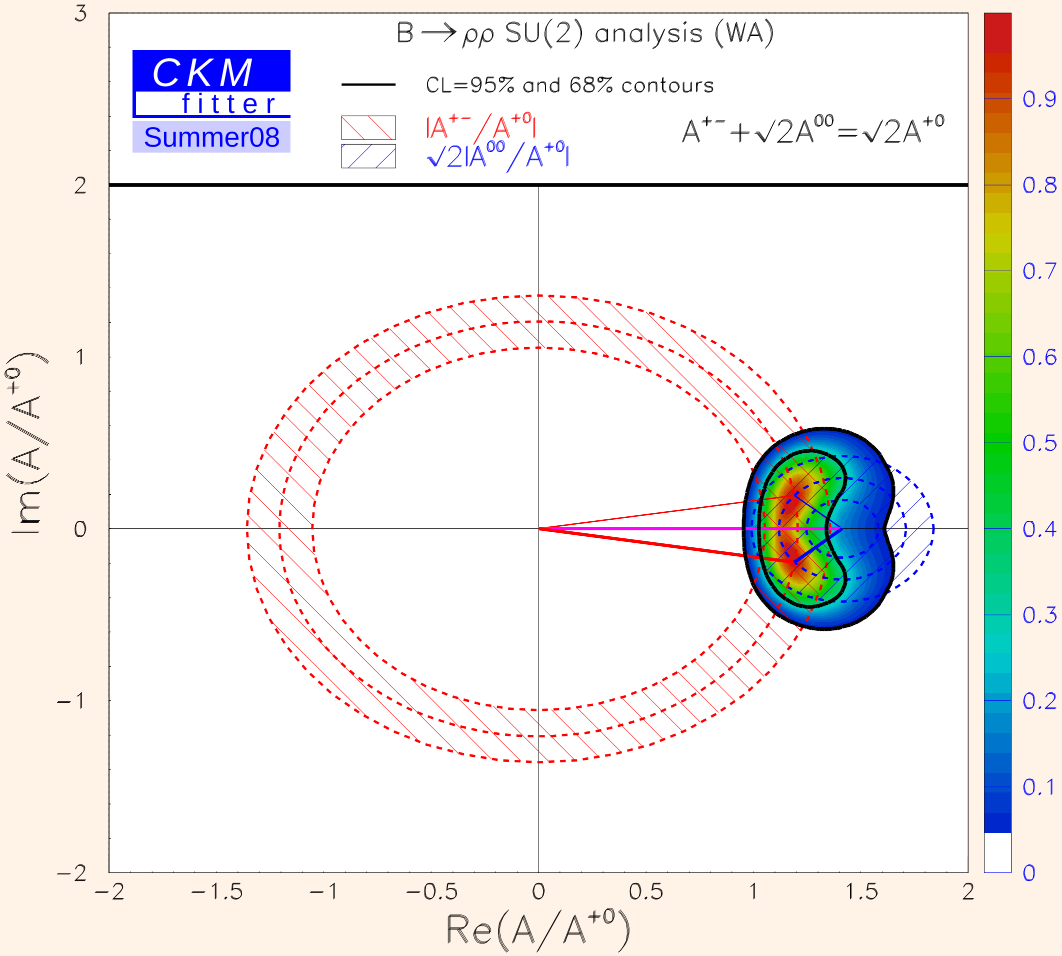
$\alpha = (89.0^{+4.4}_{-4.2})^\circ$

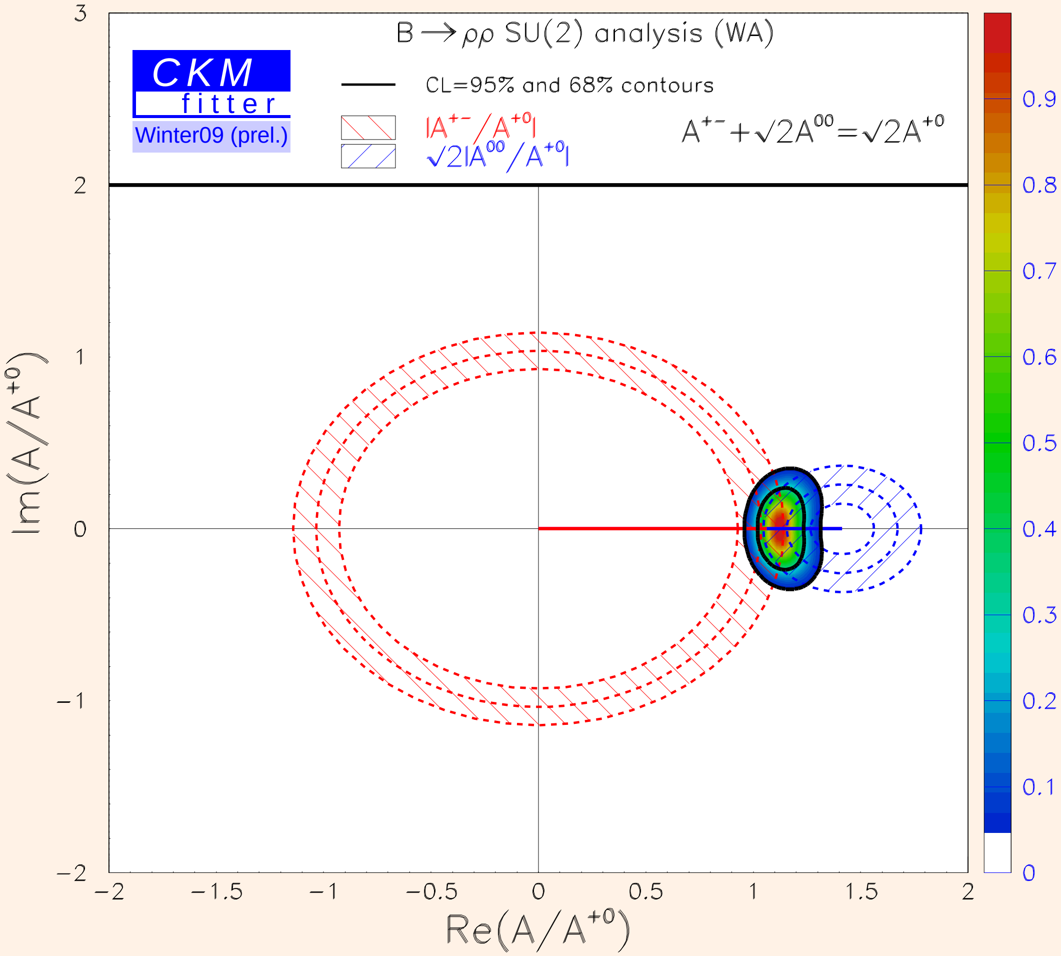
indirect CKM fit

$\alpha = (95.6^{+3.3}_{-8.8})^\circ$



violation of the triangular SU(2) relation





A lucky configuration

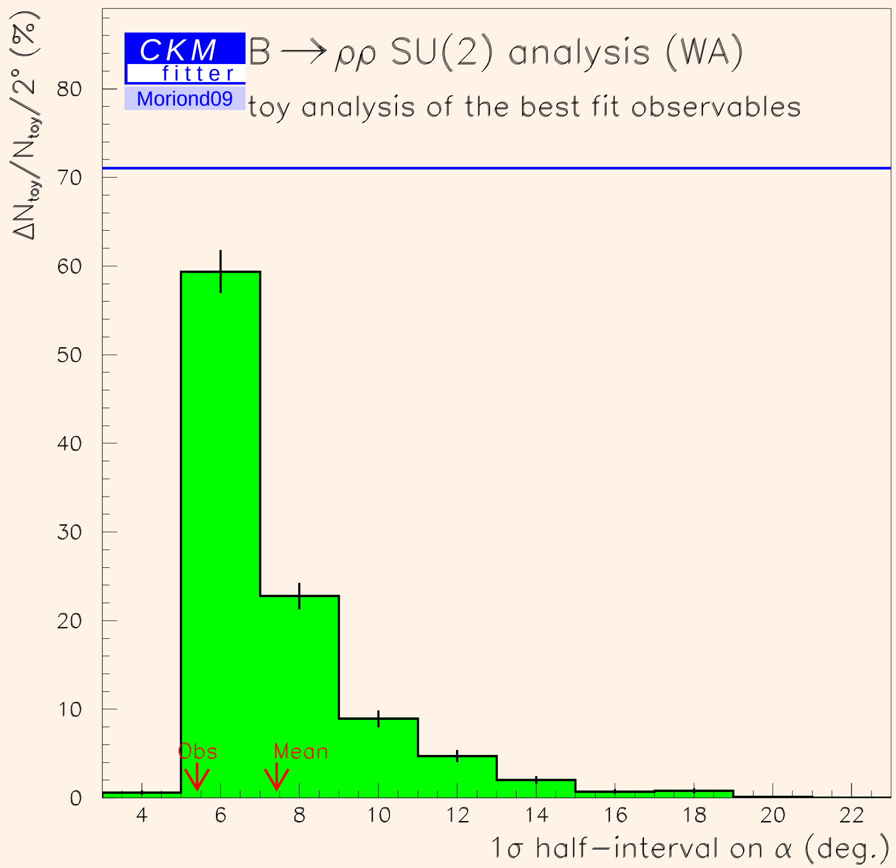
is it consistent to assume SU(2) for the extraction of α when the SU(2) triangular relation is violated ?

a simple toyMC study: take parameters at their best fit values and generate a lot of "measurements"

only 34% of the toyMC triangles close

average toyMC error: 7.5° (to be compared to actual observation 5.4°; 68% of the toys have larger error than actual data)

large asymmetric tail when toy triangles do close: mirror solution reappears



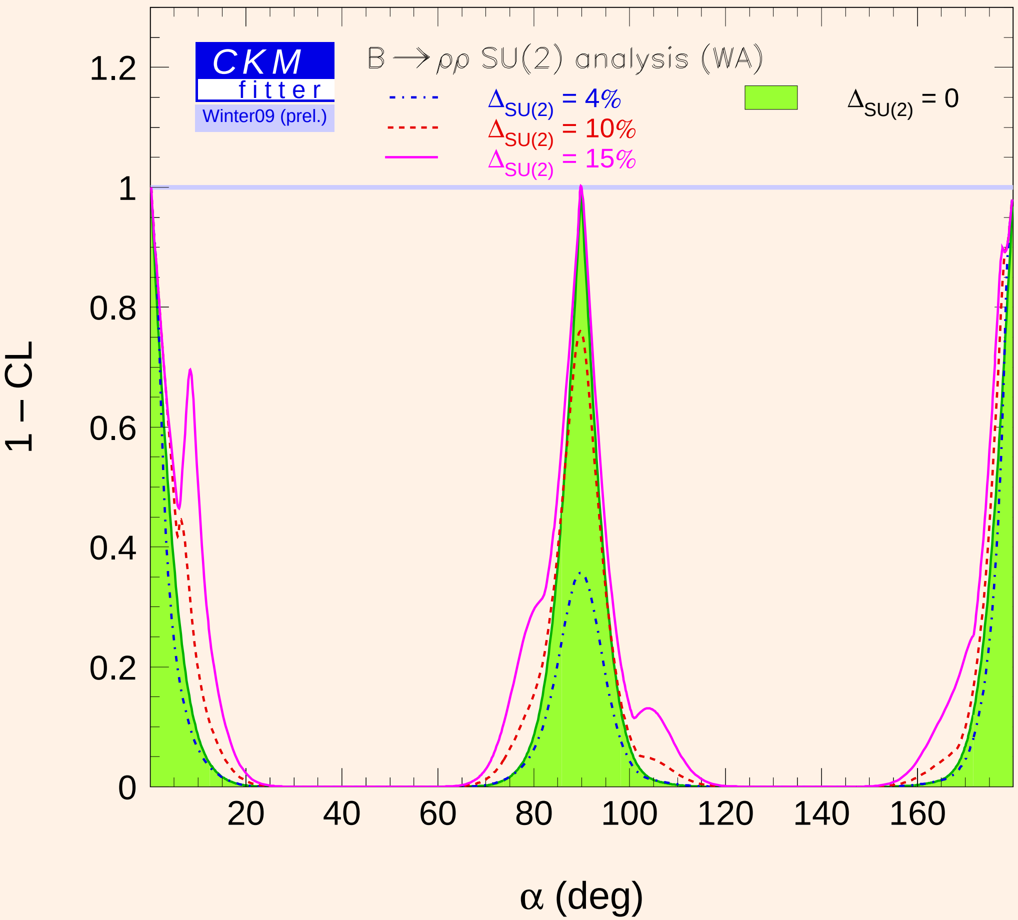
Breaking isospin symmetry

various sources: QCD ($m_u \neq m_d$), QED ($Q_u \neq Q_d$), amount to 1–3° (Zupan CKM'06)

largest effect presumably comes from finite width, $\Gamma_\rho \neq 0$ allows $I = 1$ antisymmetric contribution, $\Gamma_\rho^2/m_\rho^2 \sim \mathcal{O}(\triangle\%)$ (Grossmann et al.)

test 4%, 10% and 15% violation of the triangular relation, with arbitrary additional amplitude

small values of isospin breaking does not really change the pattern; a nice constraint on α in any case



Breaking isospin symmetry

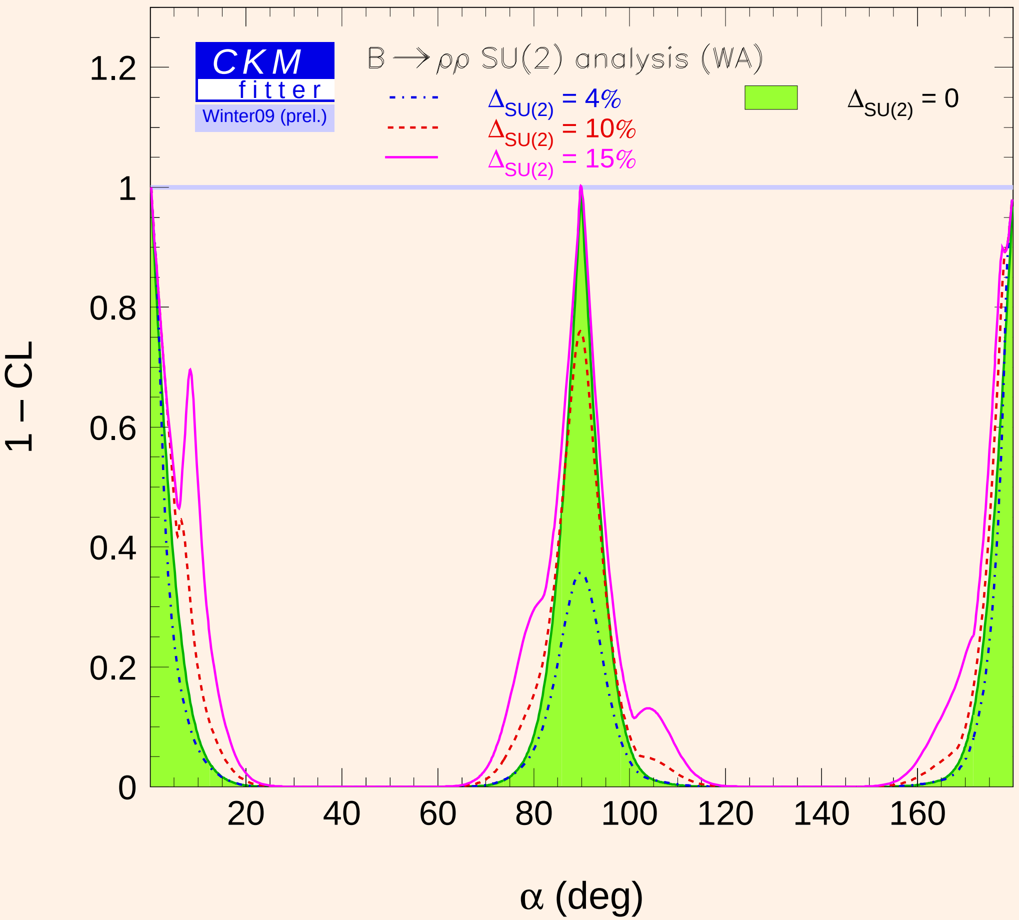
various sources: QCD ($m_u \neq m_d$), QED ($Q_u \neq Q_d$), amount to 1–3° (Zupan CKM'06)

largest effect presumably comes from finite width, $\Gamma_\rho \neq 0$ allows $I = 1$ antisymmetric contribution, $\Gamma_\rho^2/m_\rho^2 \sim \mathcal{O}(\triangle\%)$ (Grossmann et al.)

test 4%, 10% and 15% violation of the triangular relation, with arbitrary additional amplitude

small values of isospin breaking does not really change the pattern; a nice constraint on α in any case

message: yes, we are in a lucky situation: 1) the "true" triangle is presumably close to be flat 2) actual data lead to better constraint than average toy data



More on γ

constraint on γ comes from CP interferences between $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions (GLW, ADS, GGSZ methods): $B \to D^{(*)}K^{(*)}$ exclusive modes

from the theory point of view it is clean: model-dependence only arises in the Dalitz model of the D decay in the GGSZ method

there are non trivial statistical issues due to non-linearities; the error on γ depends on the ratio r_B of interfering amplitudes: when it is small the fitted value of r_B is small and gets biased, which in turn implies that the error on γ is underestimated

in statistical jargon one says that the naive χ^2 treatment leads to the *undercoverage* effect: e.g. the 68% CL interval on γ does not contain the true value at the correct frequency; this is dangerous: one might claim erroneously for a three- or five-σ effect

one corrects for bad coverage by doing a toy MC analysis, which is a computer simulation of many similar experiments; in the absence of *nuisance parameters*, i.e. when all the observables and their distribution can be computed in terms of γ only, there is an exact construction due to Neyman that allows to compute CL intervals with perfect coverage in presence of nuisance parameters, there is no general approach to construct CL intervals with exact coverage: one has to choose between accepting some undercoverage (take your ticket to Stockholm too early) or some overcoverage (miss your flight)

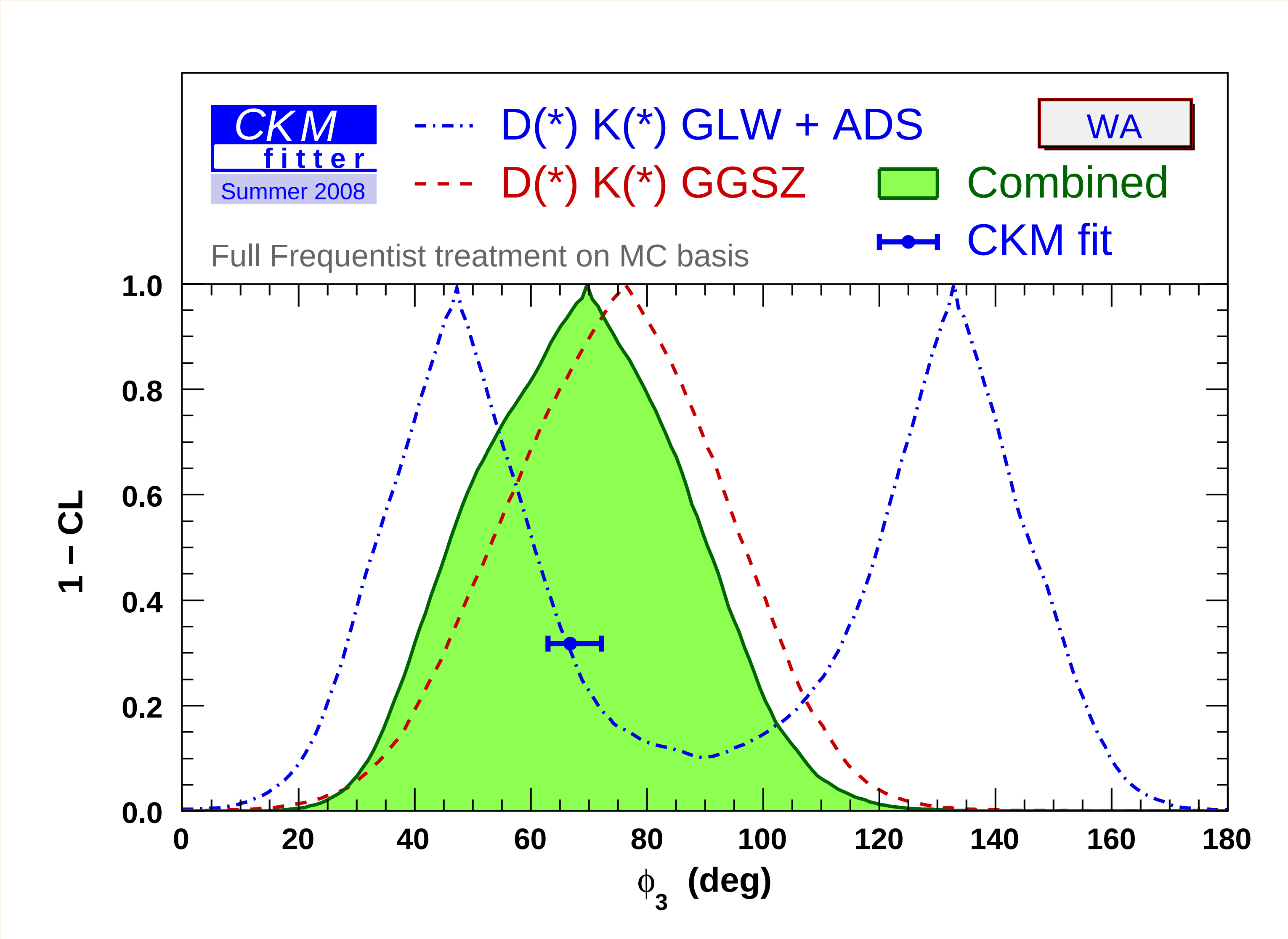
a common and technically simple method amounts to replacing the (unknown) true value of the nuisance parameters by their best fit estimate: *plugin* approach; however we have shown it can be plagued by significant undercoverage (up to 56%(68%CL)/91%(95%CL) for the full GLW/ADS/GGSZ analysis, and small value of the r_B's)

up to now we have used the conservative *supremum* method, that amounts to take the "worst" configuration for the nuisance parameters (largest error on γ); however it is computationally very heavy, and it does overcover

part of the CKMfitter contribution to FJPPL is related to the implementation of a better (and general) approach: we are now close to give the final results, and generalize to other problems than the γ analysis

Combined constraint on γ

$\gamma = (70^{+27}_{-30})^\circ$ (direct) vs. $\gamma = (67.8^{+4.2}_{-3.9})^\circ$ (indirect)



Possible tensions in the global CKM fit

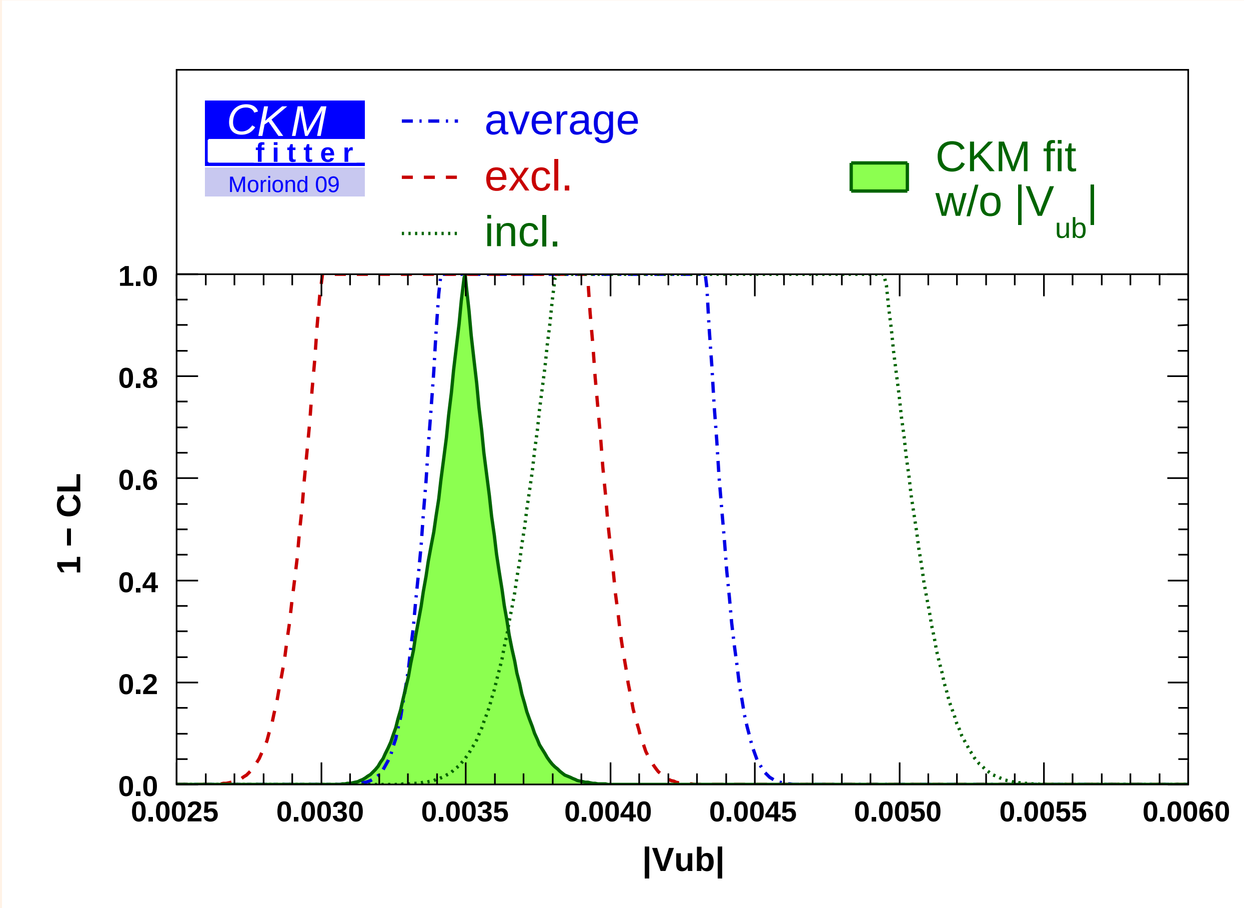
where could be New Physics ?

$|V_{ub}|$ measurement vs. indirect fit

some people reported about a tension between the direct determination of $|V_{ub}|$ (exclusive and inclusive modes) and the indirect fit prediction

it is actually more a *exclusive* vs. *inclusive* tension; $\sin 2\beta$ prefers the smaller $|V_{ub}|$ from exclusive modes, and it is difficult to average consistently exclusive with inclusive values

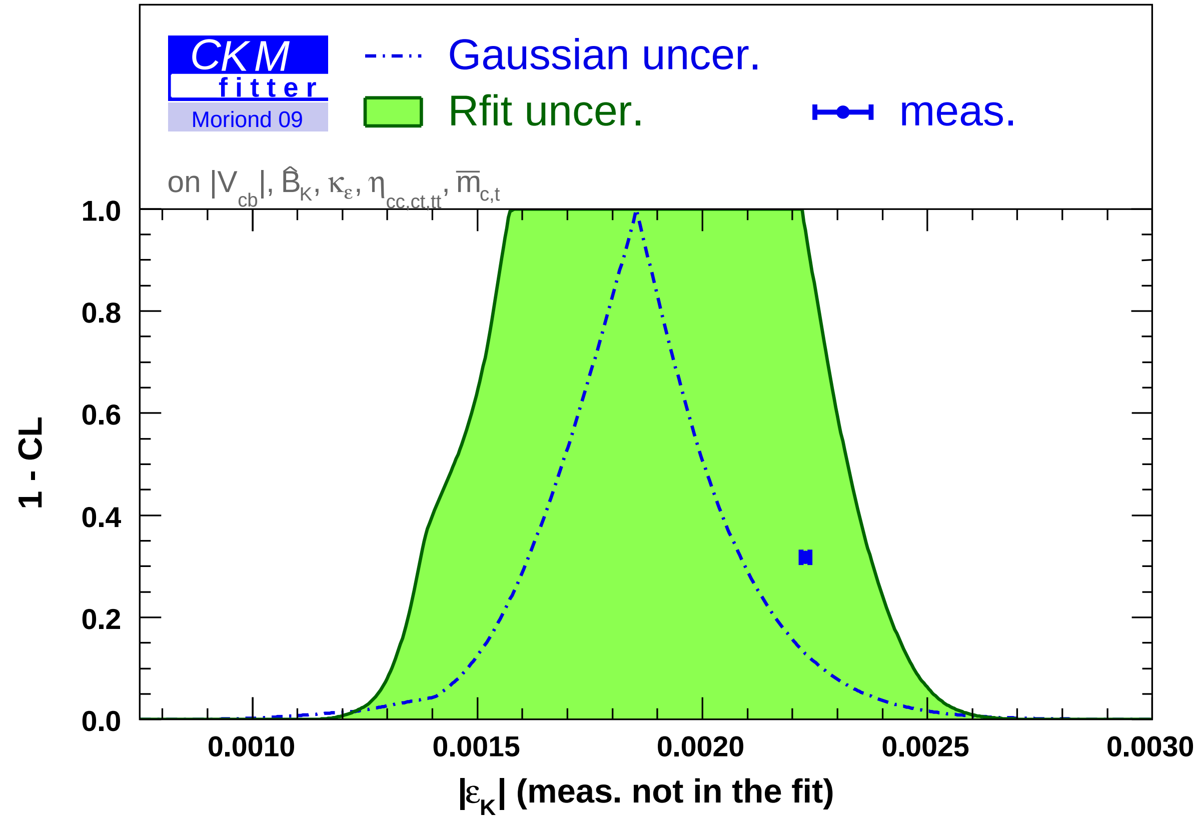
no tension is found unless aggressive treatment of theoretical uncertainties is made



$|\varepsilon_K|$ measurement vs. indirect fit

thanks to the better estimate of B_K (a kind of benchmark for lattice QCD), Soni & Lunghi stressed a tension between the direct measurement of $|\varepsilon_K|$ and the indirect fit prediction; the tension is enhanced by the reminder by Buras & Guadagnoli that one must take into account a contribution from $I = 0$ contribution

with Rfit flat treatment of theoretical uncertainties, and with a conservative average of $|V_{cb,excl}|$ and $|V_{cb,incl}|$ ($|\varepsilon_K| \sim A^4$), one does not see a deviation from the SM here



A new tension in $B \to \tau\nu$

the leptonic decay is the simplest from the theory side ($\Delta B = 1$ weak current matrix element f_B) and is a good test of chirality and possible charged Higgs contribution

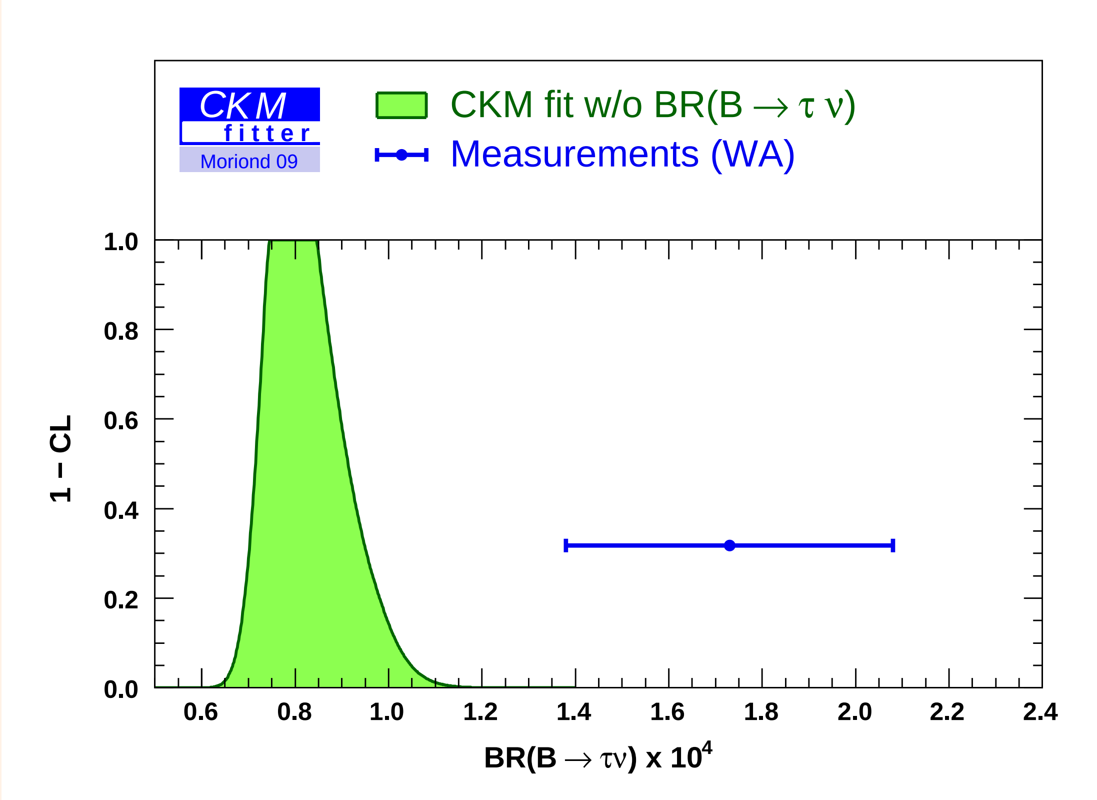
from the global analysis,

$$BR(B \to \tau\nu_\tau) = (0.796^{+0.154}_{-0.093}) \times 10^{-4}$$

Summer 08 experimental update (BaBar/Belle):

$$BR(B \to \tau\nu_\tau) = (1.73 \pm 0.35) \times 10^{-4}$$

a 2.4σ discrepancy

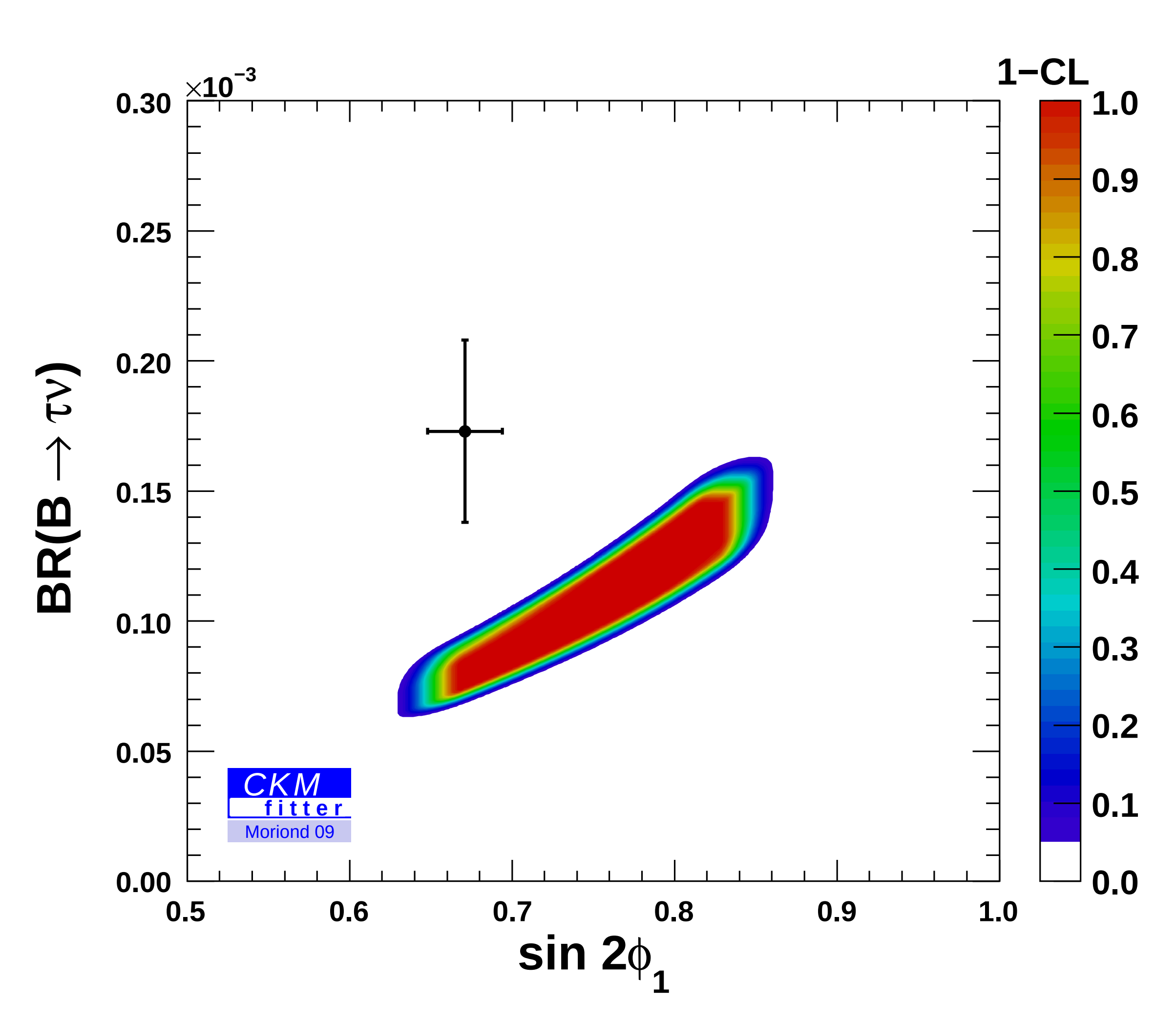


contrary to the naive expectation, the effect is not driven by f_{B_d} nor $|V_{ub}|$: it comes mainly from the CKM angles and B_{B_d} ($\Delta B = 2$ matrix element)

$B \to \tau\nu$: a closer look

$B \to \tau\nu$ vs. $\sin 2\beta$

cross is direct measurement; color levels are indirect fit prediction

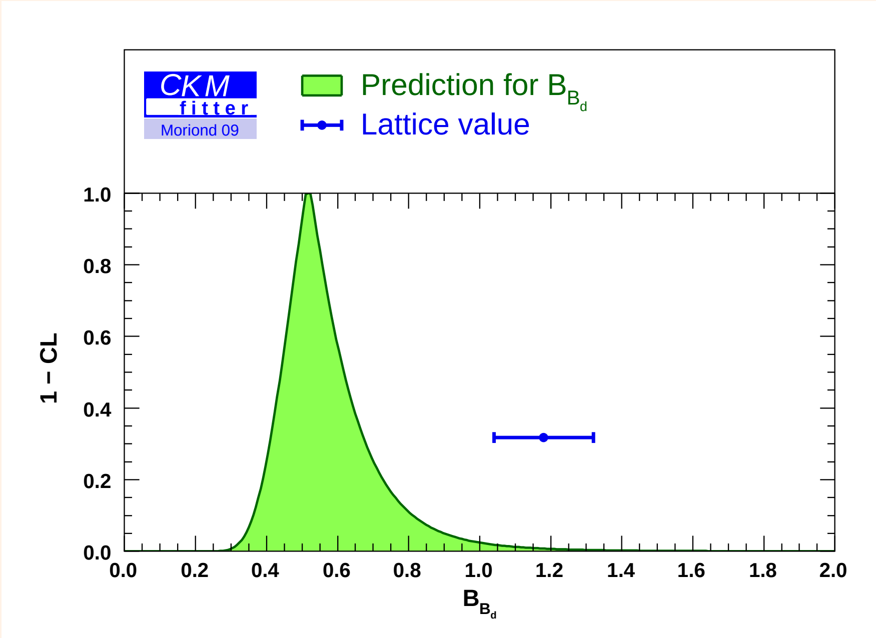


we have found that the shape of the correlation is given by the ratio $\mathrm{BR}(B \to \tau\nu)/\Delta m_d$:

$$\frac{\mathrm{BR}(B \to \tau\nu)}{\Delta m_d} = \frac{3\pi}{4} \frac{m_\tau^2}{m_W^2 S(x_t)} \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \tau_{B^+} \frac{1}{B_{B_d}} \frac{1}{|V_{ud}|^2} \left(\frac{\sin\beta}{\sin\gamma}\right)^2$$

where $B_{B_d} = 1.17 \pm 0.06 \pm 0.08$ is the only source of theoretical uncertainty

alternatively one can take the above formula as a pure experimental prediction for the bag parameter B_{B_d}



here the discrepancy is 2.7σ (taking only Δm_d, α, β, γ as inputs), where the contribution from the theory uncertainty is subdominant

Possible explanations

electromagnetic corrections ? in principle taken into account at the experimental level

lattice: $B_{B_d} \lesssim 1$?

conspiration of statistical fluctuations in $B \to \tau\nu$, α, β and γ ?

New Physics in B-$\overline{B}$ mixing, and thus in β ?

New Physics in $B \to \tau\nu$?

in any case, ideal channel for superB ...

New Physics in $B\overline{B}$ mixing

abstract from a more complete work in collaboration with A. Lenz and U. Nierste

Model-independent parametrization

$$\langle B_q \,|\, \mathcal{H}^{\mathrm{SM+NP}}_{\Delta B=2} \,|\, \bar{B}_q \rangle \equiv \langle B_q \,|\, \mathcal{H}^{\mathrm{SM}}_{\Delta B=2} \,|\, \bar{B}_q \rangle \times \left(\mathrm{Re}(\Delta_q) + i\,\mathrm{Im}(\Delta_q)\right)$$

SM is thus located at $\Delta_d = \Delta_s = 1$; additional notation $2\theta_q \equiv \arg(\Delta_q)$

this cartesian parametrization allows for a simple geometrical interpretation of each individual constraint (Lenz & Nierste 2006)

Model-independent parametrization

$$\langle B_q | \mathcal{H}_{\Delta B=2}^{\mathrm{SM+NP}} | \bar{B}_q \rangle \equiv \langle B_q | \mathcal{H}_{\Delta B=2}^{\mathrm{SM}} | \bar{B}_q \rangle \times (\mathrm{Re}(\Delta_q) + i\,\mathrm{Im}(\Delta_q))$$

SM is thus located at $\Delta_d = \Delta_s = 1$; additional notation $2\theta_q \equiv \arg(\Delta_q)$

this cartesian parametrization allows for a simple geometrical interpretation of each individual constraint (Lenz & Nierste 2006)

Strategy and inputs

- assume that tree-level transitions are 100% SM
- fix SM parameters with $|V_{ud}|$,$|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, γ and $\alpha = \pi - \gamma - \beta_{\mathrm{eff}}((c\bar{c})K)$
- $(\mathrm{Re}(\Delta_d), \mathrm{Im}(\Delta_d))$ are then constrained by Δm_d (circle), by $\phi_d = 2\beta_{\mathrm{eff}} = 2\beta + 2\theta_d$ (straight line) and by $\alpha = \pi - \gamma - \beta_{\mathrm{eff}}((c\bar{c})K)$
- $(\mathrm{Re}(\Delta_s), \mathrm{Im}(\Delta_s))$ are constrained by Δm_s (circle) and by $\phi_s = -2\beta_s + 2\theta_s$
- additional information is brought by the measurement of the semileptonic asymmetries A_{SL}^d, A_{SL}^s (circle) and the width difference $\Delta\Gamma_q = \cos\phi_s\,\Delta\Gamma_q^{\mathrm{SM}}$ (straight line)

NP in mixing modified predictions

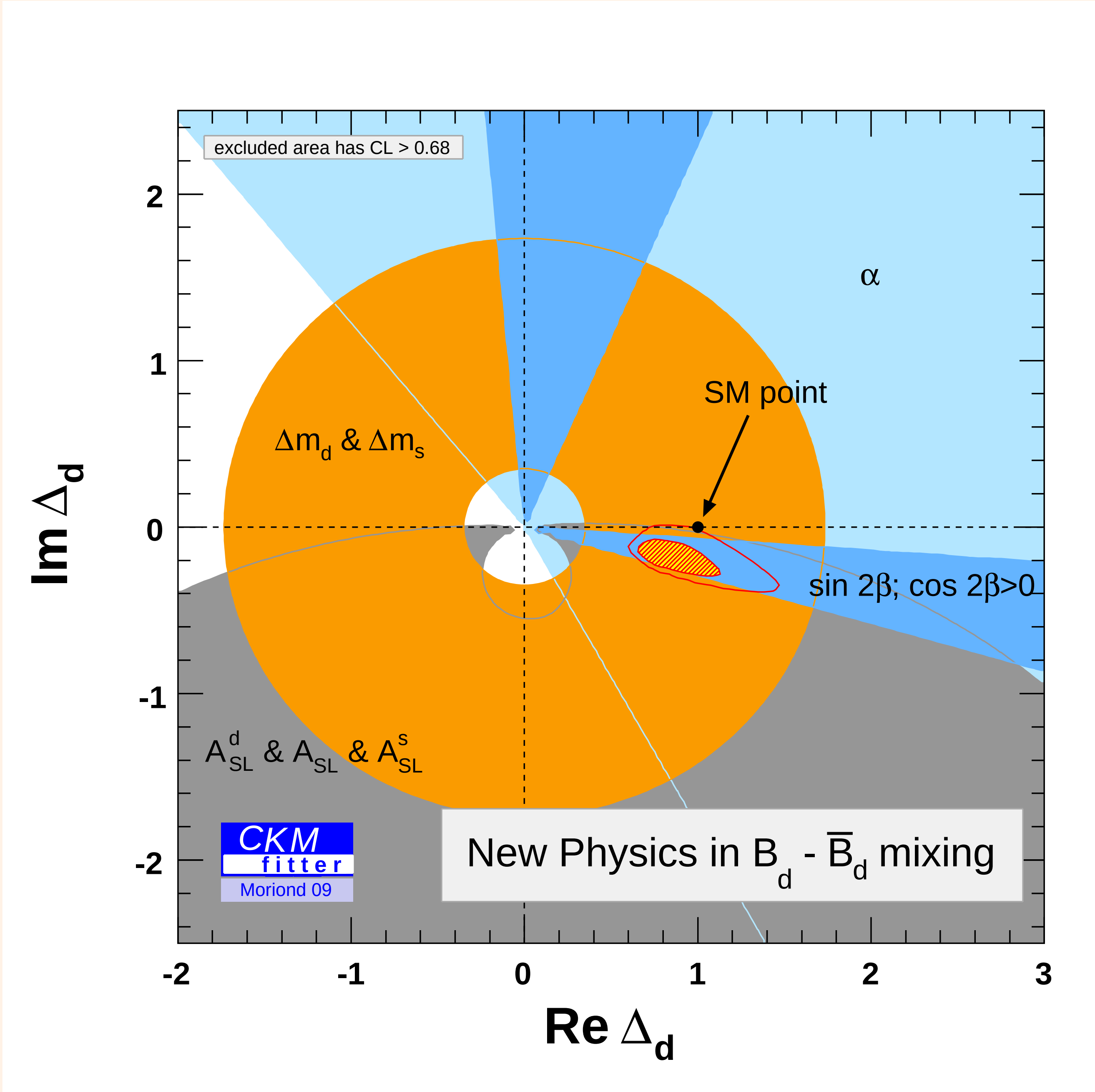
observable	NP prediction
Δm_q	$\Delta m_{q,SM} \times \lvert\Delta_q\rvert$
$2\beta_{c\bar{c}K}$	$2\beta + \mathrm{Arg}(\Delta_d)$
$\phi_{s,\psi\phi}$	$-2\beta_s + \mathrm{Arg}(\Delta_s)$
$2\alpha_{\pi\pi,\rho\pi,\rho\rho}$	$2\alpha - \mathrm{Arg}(\Delta_d)$
$A_{sl,q}$	$\frac{\Gamma 12_{q,SM}}{M12_{q,SM}} \times \frac{\sin(\phi 12_{q,SM}+Arg(\Delta_q))}{\lvert\Delta_q\rvert}$
$\Delta\Gamma_q$	$2\Gamma 12_{q,SM} \times \cos(\phi 12_{q,SM} + Arg(\Delta_q))$

NB: $\Gamma 12$ (in A_{sl} and $\Delta\Gamma$) has a very complicated theoretical expression, taken from Lenz-Nierste 2006; in this quantity theoretical uncertainties play a major rôle and are not completely under control

Result in the $\mathrm{Re}(\Delta_d), \mathrm{Im}(\Delta_d)$ plane

warning: only 68% CL regions are shown because of large errors

no striking evidence for New Physics, but sizable contributions are allowed; the deviation of $\mathrm{Arg}(\Delta_d)$ from 0 is related to the $B \to \tau\nu$ tension: the 2D SM hypothesis $\Delta_d =$ is excluded at 2.1σ, which reduces to 0.6σ if $B \to \tau\nu$ is removed from the inputs

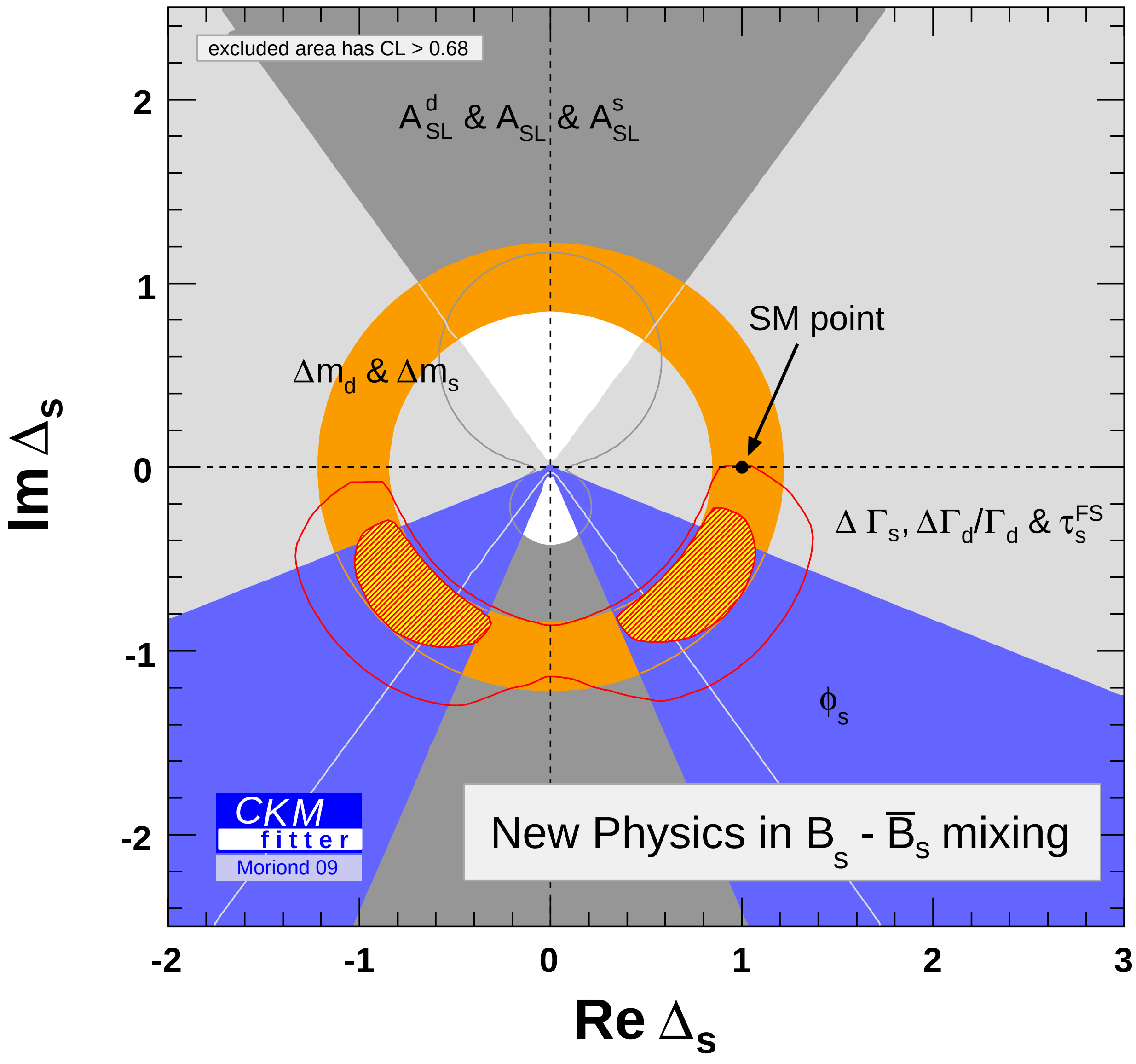


Result in the $\mathrm{Re}(\Delta_s), \mathrm{Im}(\Delta_s)$ plane

warning: only 68% CL regions are shown because of large errors

one sees that the dominant constraints are Δm_s (in agreement with SM) and ϕ_s (small discrepancy)

here the 2D SM hypothesis $\Delta_s = 0$ is excluded at 1.9σ, with the tension almost completely driven by the direct TeVatron measurement of ϕ_s in $B_s \to J/\psi\phi$



Summary

the Standard Model hypothesis within the generic New Physics in mixing scenario is disfavored at about the 2σ level (2.5σ for the 4D hypothesis $\Delta_d = \Delta_s = 1$)

in the B_s system the full combined analysis does not add much information with respect to the bare measurement: the bulk of the effect is contained in ϕ_s; in the B_d system, in contrast, the anomaly related to $B \to \tau\nu$ comes from a specific correlation with the CKM angles, only found in the global fit

we are waiting for new data...

Conclusion

CKM analyses have reached a high level of maturity and establish the KM phase as the dominant source of CP-violation at the electroweak scale and below

nevertheless, there is significant room for non standard contributions to flavour transitions

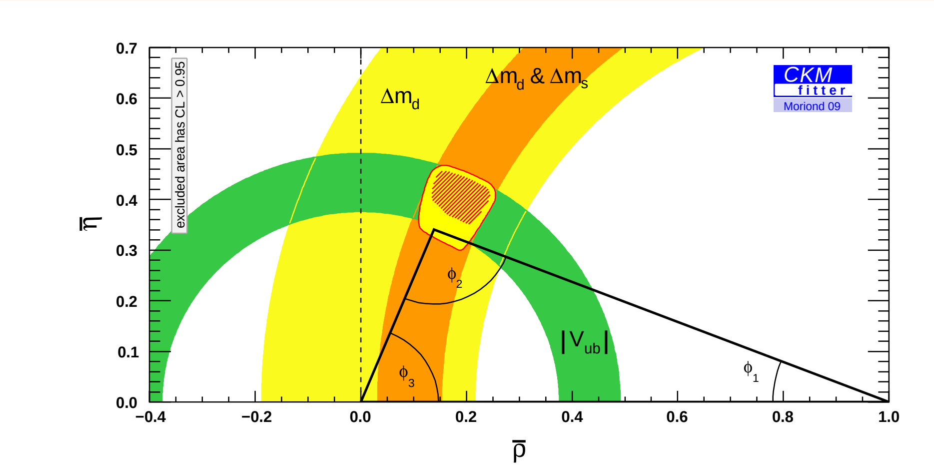
tensions in the B_d system mostly comes from the comparison of the direct measurement of $B \to \tau\nu$ with the fit prediction, thanks to an interesting and non trivial correlation

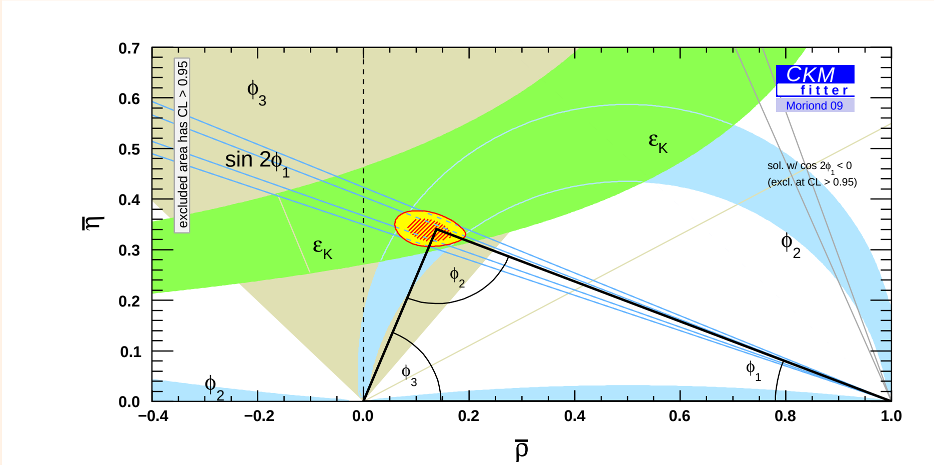
tensions in the B_s system mostly comes from the comparison of the direct measurement of CP-violation in the decay to $J/\psi\phi$ with the SM vanishing value, and is mostly orthogonal to the rest of the global fit

both tensions can well be accomodated in a generic scenario of New Physics contributions to meson mixing, but there are plenty of other viable models

Backup

Testing the CKM paradigm

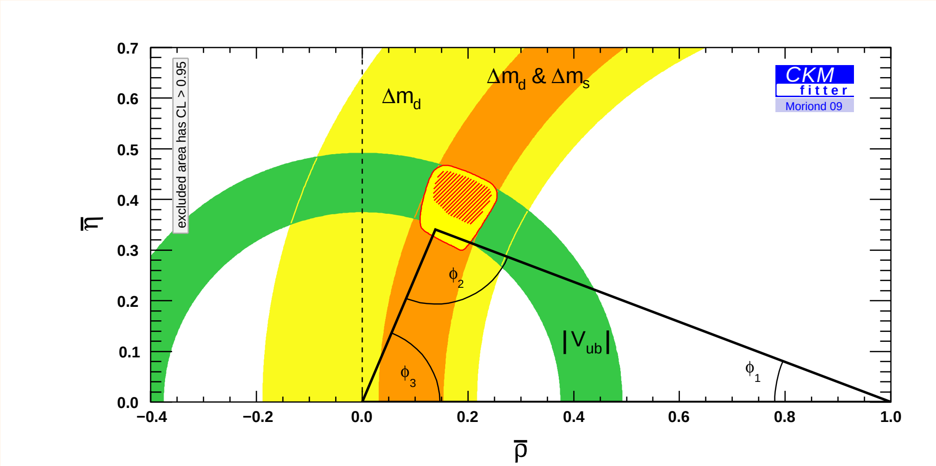


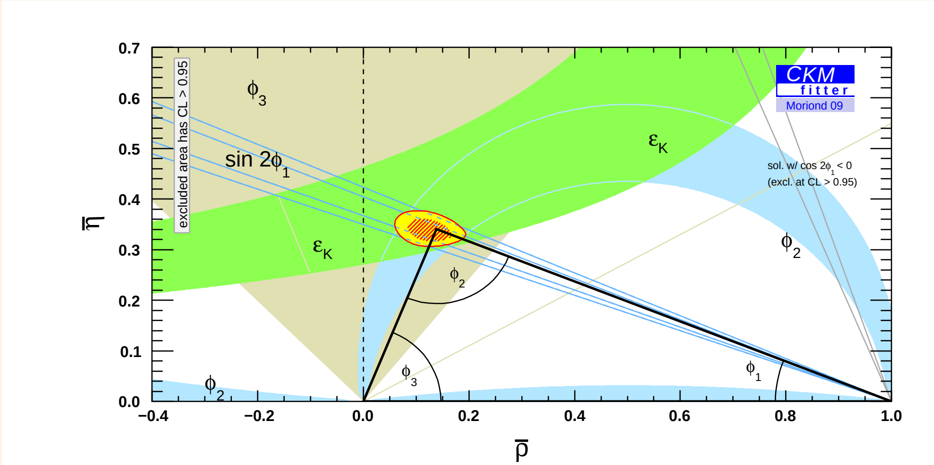


CP-conserving…

…vs. CP-violating

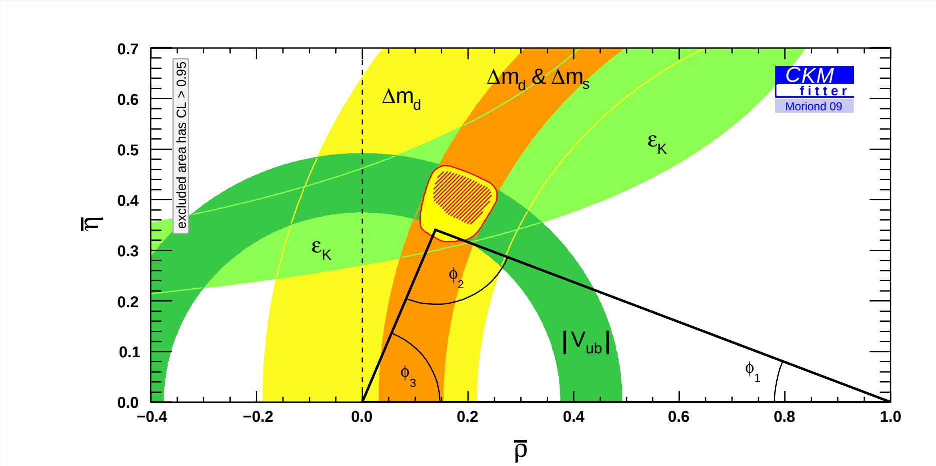
Testing the CKM paradigm

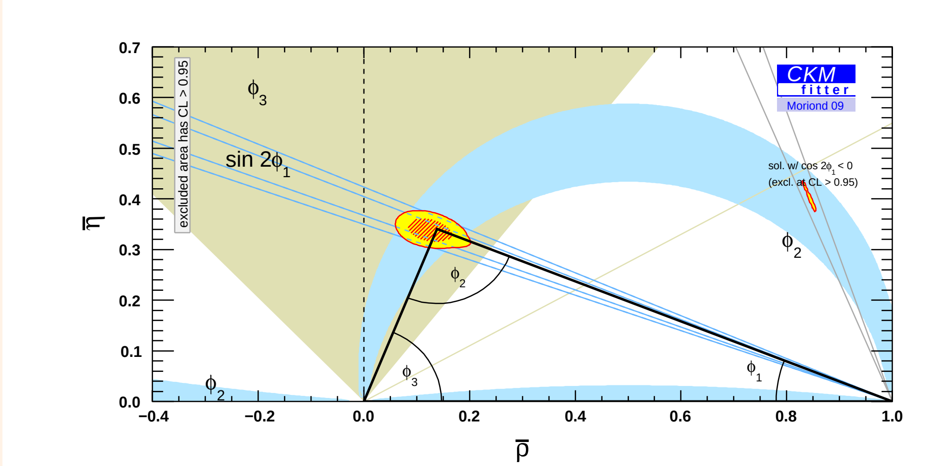




CP-conserving...

...vs. CP-violating

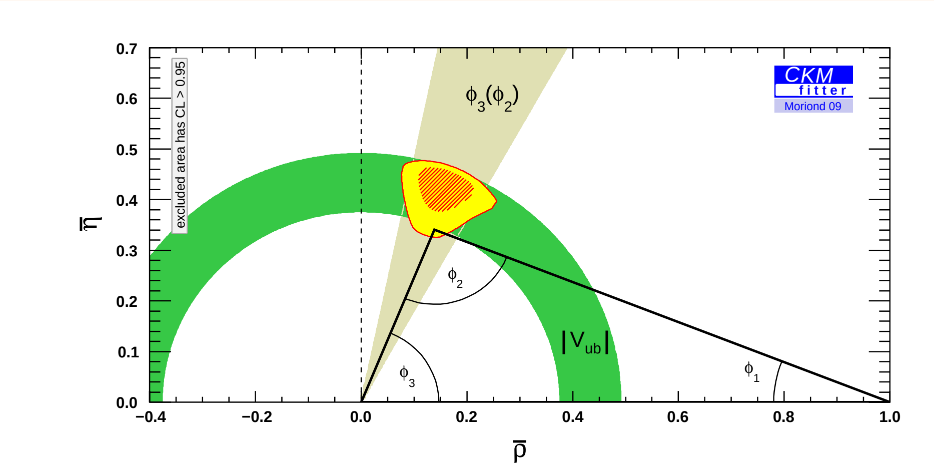


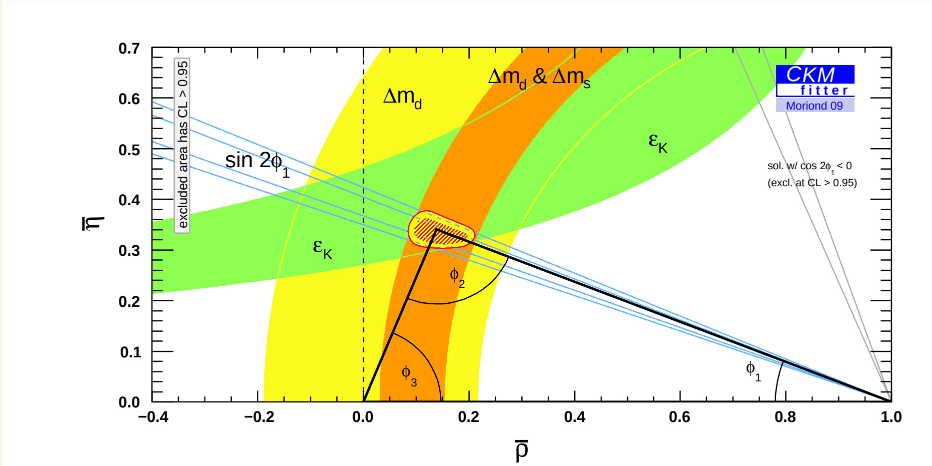


no angles (with theory)...

...vs. angles (without theory)

Testing the CKM paradigm

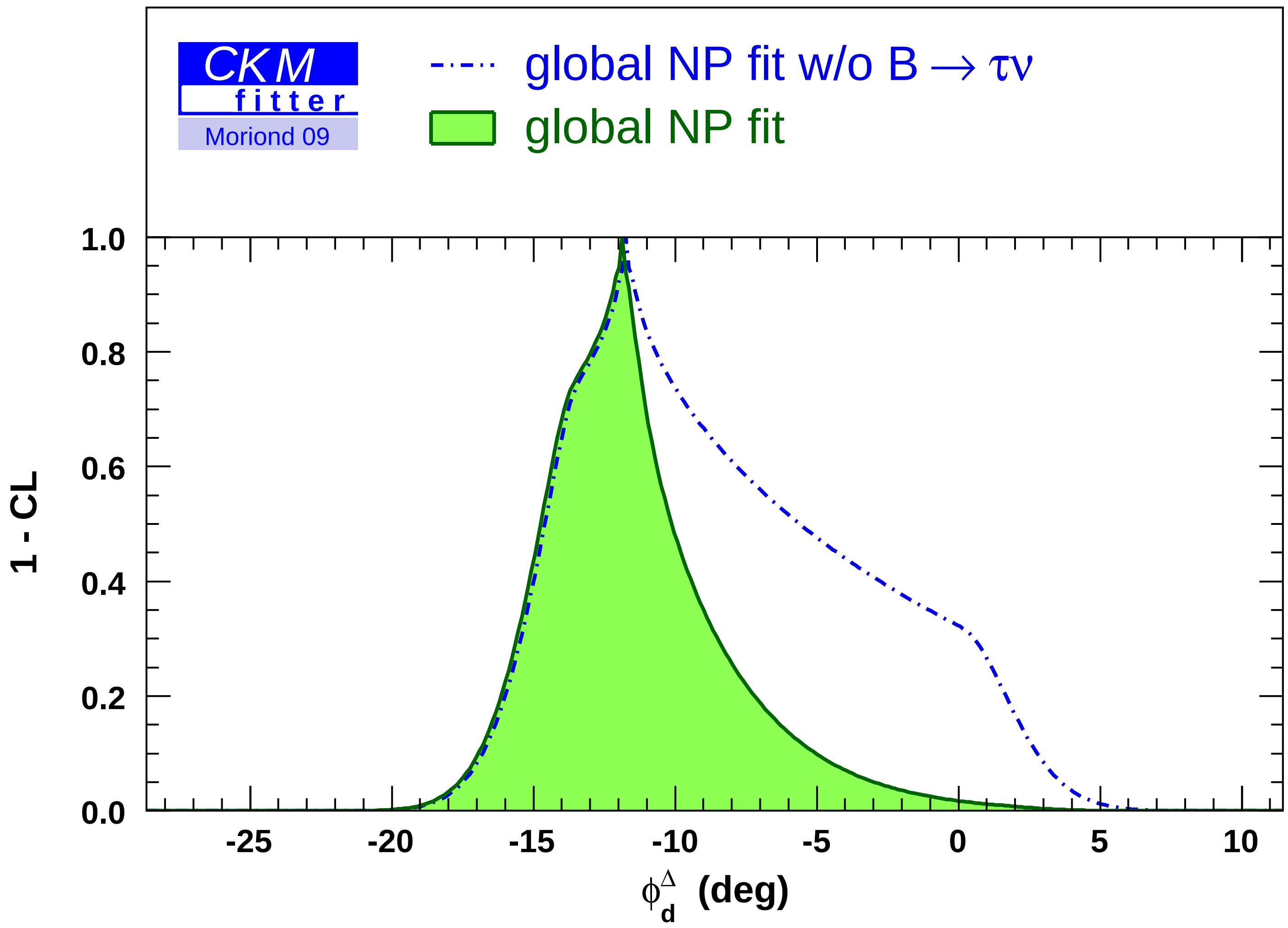




tree…

…vs. loop

1D constraint on Arg(Δ_d)



Frequentist statistics in a nutshell

we want to test the hypothesis $\mathcal{H}_\gamma : \gamma_{\text{True}} = \gamma$ through the construction of a p-value

however the hypothesis $\mathcal{H}_\gamma$ is *composite*: the distribution of the experimental observables under this hypothesis is not completely specified. Instead we (assume to) know $\text{PDF}(\text{data}|\gamma, \mu)$ where μ is the (vector of) nuisance parameter(s)

the validity of $\mathcal{H}_\gamma$ is described by a test statistic $t(\gamma; \text{data})$, small value of which indicates support in favor of $\mathcal{H}_\gamma$. There is full freedom in the choice of t. In our case we take

$$t(\gamma; \text{data}) = \Delta\chi^2(\gamma; \text{data}) \equiv \text{Min}_\mu \chi^2(\gamma, \mu; \text{data}) - \text{Min}_{\gamma,\mu} \chi^2(\gamma, \mu; \text{data})$$

where

$$\chi^2(\gamma, \mu; \text{data}) \equiv -2 \ln \text{PDF}(\text{data}|\gamma, \mu)$$

this choice is motivated by its asymptotic properties in case of Gaussian PDF, see below

then the master formula for the p-value is

$$1 - p_{v,\mu}(\gamma; \text{data}) = \int_0^{t(\gamma;\text{data})} dt \, \text{PDF}_t(t|\gamma, \mu)$$

with

$$\text{PDF}_t(t|p) = \int d\text{exp} \, \delta[t - t(p; \text{exp})] \, \text{PDF}(\text{exp}|p)$$

for any fixed value of μ, and for $\mu_{\text{True}} = \mu$, $p_{v,\mu}(\gamma; \text{data})$ is simply the CDF of $\text{PDF}_t(t|\gamma;\mu)$, thus its distribution under $\mathcal{H}_\gamma$ is flat

$$\mathcal{P}\left[p_{v,\mu}(\gamma; \text{data}) \leq \alpha\right] = \alpha$$

(exact coverage). In this case measuring $p_v < \alpha$ on the real data allows to exclude $\mathcal{H}_\gamma$ at the $1 - \alpha$ confidence level (Type I error)

however the true value of μ is unknown; thus the distribution of $p_{v,\mu}$ is *a priori* not flat if the true value of μ is not the one that is used in $p_{v,\mu}$ (independently of whether $\mathcal{H}_\gamma$ is true or not)

under $\mathcal{H}_\gamma$, if $\mu_{\text{True}} \neq \mu$, one may have

$$\mathcal{P}\left[p_{v,\mu}(\gamma; \text{data}) \leq \alpha\right] > \alpha \quad \text{(undercoverage: small p-values occur too often)}$$

or

$$\mathcal{P}\left[p_{v,\mu}(\gamma; \text{data}) \leq \alpha\right] < \alpha \quad \text{(overcoverage: small p-values occur too rarely)}$$

on the other hand, if $\mathcal{H}_\gamma$ is not true, one will get too many small p-values wrt the case where $\mathcal{H}_\gamma$ is true. Hence, *rejecting $\mathcal{H}_\gamma$ because of small p-value is only correct if the method used for it does not undercover*

conversely, in presence of overcoverage one may miss a discovery, by not rejecting $\mathcal{H}_\gamma$ because of large p-value in order to construct a p-value in terms of γ only, one may choose to replace μ by some estimator $\tilde{\mu}$ based on the real data

$$p_{v,\mu}(\gamma;\mathrm{data}) \to p_v(\gamma;\mathrm{data}) \equiv p_{v,\tilde{\mu}(\mathrm{data})}(\gamma;\mathrm{data})$$

in this case, even if (by chance) $\mu_{\mathrm{True}} = \tilde{\mu}(\mathrm{data})$, the distribution of $p_v(\gamma;\mathrm{data})$ is *a priori* not flat, because it is not a CDF anymore (unless one uses the same value of $\mu = \tilde{\mu}(\mathrm{data})$ in the calculation of the p-value of any experiment exp$\neq$data)

there is no obvious best choice for the estimator $\tilde{\mu}$; what we call the $\hat{\mu}$ method is the choice $\tilde{\mu}(\mathrm{data}) = \hat{\mu}(\gamma;\mathrm{data})$ corresponding to the value of μ that minimizes the χ^2. One could well choose the position of the global minimum $\tilde{\mu} = \hat{\hat{\mu}} = \hat{\mu}(\hat{\hat{\gamma}})$ instead

one may hope that the dependence wrt the specific choice is weak for "good" estimators

the simplest solution to construct a valid (conservative) p-value for $\mathcal{H}_\gamma$ is the supremum method

$$p_v(\gamma;\mathrm{data}) \equiv \sup_\mu p_{v,\mu}(\gamma;\mathrm{data})$$

which guarantees (over)coverage for any true value of μ

supremum method is costly $\sim N_\gamma \times 2N_{min}[\chi^2] \times N_{toys} \times N_{sup}[p_v]$

the big drawback of the supremum method is that one includes in the maximization all possible values of μ, even those that are not supported by the data (recall that the test statistic t has been designed to optimize the information on γ independently of μ); hence overcoverage can be sizable

note that if one works in the full (γ, μ) parameter space, there is no nuisance parameter; hence the master formula gives an exact p-value. Projecting onto the γ subspace necessarily throws up some information, which means *there is no general method to get exact coverage when one is only interested in* γ

asymptotic limit: for small enough errors, and for Gaussian PDF of the data (our assumption in the following), the asymptotic distribution of the statistic $\Delta\chi^2(\gamma; \text{data})$ is a χ^2 with $N_{dof} = \text{dim}(\mu)$, and thus does not depend on γ nor μ; the corresponding CDF is the generalized error function (Prob)

asymptotic limit means that the errors are sufficiently small so that the χ^2 function can be Taylor expanded around its minimum, and becomes quadratic in all directions of the parameter space