The CKM matrix: from the Standard Model to New Physics (II)

Sébastien Descotes-Genon

Laboratoire de Physique Théorique
CNRS & Université Paris-Sud 11, 91405 Orsay, France

KEK, Tsukuba, Japan
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An impressive global agreement

The Kobayashi Maskawa mechanism describes CP violation in flavour physics

Worth looking at other observables/sectors to find discrepancies

And maybe find new physics?
1. Looking for discrepancies: radiative decays
2. Looking for discrepancies: charm physics
3. A potential loophole: hadronic inputs
4. Testing a simple model of NP with flavour
Radiative decays
Introduction

$b \rightarrow D \gamma$ with $D = d, s$

- access to $|V_{t(d,s)}|$ within SM
- cross-check of neutral $B$ mixing (box/penguin)
- loop processes very sensitive to NP

Inclusive: $B \rightarrow X_s \gamma$

- accurately measured
- computed perturbatively up to (N)NLO [Misiak et al.]
- constrains $|V_{ts}|$ only

Exclusive: $B \rightarrow K^* \gamma$ and $B \rightarrow (\rho, \omega) \gamma$

- Measurements available from Babar & Belle
- Hadronic effects difficult to estimate theoretically
- Naively: $A(B \rightarrow V \gamma) \propto V_{tD} V_{tb}^* C_7 T^{B \rightarrow V}$
  with magnetic operator $Q_7 = \frac{e}{8\pi^2} m_B \bar{D}_\mu \sigma^{\mu\nu} (1 + \gamma_5) F_{\mu\nu} b + \ldots$
$B \to \rho \gamma$ and $B \to K^* \gamma$

$b \to d, s \gamma$: loop processes, give access to $|V_{td(s)}|$, complement $\Delta m_{d,s}$

Early days: focus on magnetic op. $Q_7 = (e/8\pi^2)m_b \bar{D}\sigma^{\mu\nu}(1 + \gamma_5)F_{\mu\nu} b$

and assume short-distance dominance
\( B \rightarrow \rho \gamma \) and \( B \rightarrow K^* \gamma \)

\( b \rightarrow d, s \gamma \) : loop processes, give access to \(| V_{td,s}|\), complement \( \Delta m_{d,s} \)

Early days : focus on magnetic op. \( Q_7 = (e/8\pi^2)m_b \bar{D}\sigma^{\mu\nu}(1 + \gamma_5)F_{\mu\nu} b \)

and assume short-distance dominance

\[
R_{\rho/\omega} = \frac{\bar{B}(\rho^\pm\gamma) + \frac{\tau_B^\pm}{\tau_{B^0}} \left[ \bar{B}(\rho^0\gamma) + \bar{B}(\omega\gamma) \right]}{\bar{B}(K^*\pm\gamma) + \frac{\tau_B^\pm}{\tau_{B^0}} \left[ \bar{B}(K^*0\gamma) \right]}
\]

\[
= \left| \frac{V_{td}}{V_{ts}} \right|^2 \left( \frac{1 - m_{\rho}^2/m_B^2}{1 - m_{K^*}^2/m_B^2} \right)^3 \frac{1}{\xi^2} \left[ 1 + \Delta R \right]
\]

- \( \xi \) ratio of form factors
- \( \Delta R \) estimated as \( \Delta R = 0.1 \pm 0.1 \)

Ali, Lunghi, Parkhomenko 02,04,06

Many open questions : dependence of \( \Delta R \) on CKM matrix ?

isospin breaking ? weak annihilation (tree for \((\rho,\omega)\gamma\) ?

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A more sophisticated analysis

For each final state, estimate all contributions, expressed as factor to the leading amplitude (magnetic operator $Q_7$)

$$\bar{A} \equiv \frac{G_F}{\sqrt{2}} \left( \lambda_U^D a_7^U(V) + \lambda_c^D a_7^C(V) \right) \langle V_\gamma | Q_7 | \bar{B} \rangle \quad \lambda_U^D = V_{UD}^* V_{Ub}$$

$$a_7^U(V) = a_7^{U,\text{QCDF}}(V) + a_7^{U,\text{ann}}(V) + a_7^{U,\text{soft}}(V) + \ldots$$

- QCDF : QCD factorisation for LO in $1/m_b$ up to $O(\alpha_s)$
  
  Bosch and Buchalla 02

- ann,soft : $1/m_b$-suppressed terms from light-cone sum rules
  
  Ball, Jones, Zwicky 06
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- each decay described individually
- $u$ and $c$ internal loops (short and long dist), not “buried” into $\Delta R$
- other operators than $Q_7$ taken into account
Several series of input parameters involved in the analysis

- **Hadronic parameters (nonperturbative)**
  - Decay constants $f_V$
  - Form factors $T^{B \to V}(0)$
  - Moments of distribution amplitudes $a^k_V$, $\lambda_B$

- **QCD parameters**
  - Quark masses $m_{c,b,t}$
  - Strong coupling constant $\alpha_s(\mu)$
  - Factorisation scale $\mu_{mb} = 4.2 \pm 1$ GeV (vertex) and $\mu_{had} = \sqrt{\Lambda_h \mu_{mb}}$ (hard-spectator)

- **Wilson coefficients**
  - NLO for $C_7$ and $C_8$ (magnetic operators)
  - LO for the others
\[ \mathcal{B}(\bar{B} \rightarrow V\gamma) = \frac{T_B}{c_V^2} \frac{G_F^2 \alpha m_B^3 m_b^2}{32 \pi^4} \left( 1 - \frac{m_V^2}{m_B^2} \right)^3 \left[ T_{1B \rightarrow V}(0) \right]^2 \]

\[ \times \left\{ \left| \sum_U \lambda^{(D)}_U a^U_{7L}(V) \right|^2 + \left| \sum_U \lambda^{(D)}_U a^U_{7R}(V) \right|^2 \right\} \]

Isospin factors \( c_{\rho^\pm, K^*, \phi} = 1 \) and \( c_{\rho^0, \omega} = \sqrt{2} \)

Output

- CP-averaged branching ratios
- CP asymmetries
- \( B_{u,d} \) decays into \( K^{*-} \gamma, K^{*0} \gamma, \rho \gamma, \rho^0 \gamma, \omega \gamma \)
- \( B_s \) decays into \( \bar{K}^{*0} \gamma, \phi \gamma \)
From 2007 to 2008

- $|V_{ud}|$, $|V_{us}|$, $|V_{cb}|$
- HFAG Winter 2007

$$K^*-\gamma \quad 40.3 \pm 2.6$$
$$K^{*0}-\gamma \quad 40.1 \pm 2.0$$
$$\rho^+\gamma \quad 0.88^{+0.28}_{-0.26}$$
$$\rho^0\gamma \quad 0.93^{+0.19}_{-0.18}$$
$$\omega\gamma \quad 0.46^{+0.20}_{-0.17}$$
From 2007 to 2008

- $|V_{ud}|$, $|V_{us}|$, $|V_{cb}|$
- HFAG Winter 2008

$K^{*-\gamma} = 45.7 \pm 1.9$
$K^{*0\gamma} = 44.0 \pm 1.5$
$\rho^+ \gamma = 0.98 \pm 0.25$
$\rho^0 \gamma = 0.86 \pm 0.15$
$\omega \gamma = 0.44 \pm 0.17$

- Belle 0804.4770 and 0712.2659

$B(B_s \to \phi \gamma) = 57 \pm 22$
$C(B^- \to \rho^- \gamma) = 0.11 \pm 0.32 \pm 0.09$
Can be turned into a constraint on $|V_{td}/V_{ts}|$

Test of the Standard Model in loops

Excellent agreement between box ($\Delta M$) and penguin ($B \rightarrow V\gamma$)

Can be extended to $B \rightarrow V\ell^+\ell^-$

- More observables (e.g., zero of $A_{fb}$)
- Richer potential to find new physics
- In progress (just now and here !)
Charm physics

Assuming $V_{cs} = 0.97334(23)$

CKM Unitarity

$K_{fast}$  $K_{at~rest}$

$D \to K e^+ \nu_e$
Charm as a test of CKM

Charm sector

- favourite place to test lattice QCD \([m_c \sim \Lambda_{QCD}]\)
- null tests from the SM [GIM mechanism]
- access to NP large couplings to second family

\[
\begin{align*}
|V_{cd}| & = 0.2308 \pm 0.011 \, (5 \%) \\
& \quad \text{CLEO-c on } D \to \pi \ell \nu \\
|V_{cs}| & = 0.97 \pm 0.09 \pm 0.07 \, (12 \%) \\
& \quad \text{CLEO-c on } D \to K \ell \nu
\end{align*}
\]
Charm as a test of CKM

Charm sector
- favourite place to test lattice QCD \([m_c \sim \Lambda_{QCD}]\)
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Current constraints on \(|V_{cd}|\)
- DIS of neutrinos on nucleons \(|V_{cd}| = 0.2308 \pm 0.011(5\%)\)
- CLEO-c on \(D \rightarrow \pi \ell \nu\)
\(|V_{cd}| = 0.222 \pm 0.008(stat) \pm 0.003(syst) \pm 0.023(latt)(3.8\%exp + 10\%th)\)

Current constraints on \(|V_{cs}|\)
- Charm tagged W decays \(|V_{cs}| = 0.97 \pm 0.09 \pm 0.07(12\%)\)
- CLEO-c on \(D \rightarrow K \ell \nu\)
\(|V_{cs}| = 1.018 \pm 0.010(stat) \pm 0.008(syst) \pm 0.106(latt)(1.3\%exp + 10\%th)\)
Charm as a test of CKM: semileptonic decays

- $K$ and nucleon: $V_{ud} \simeq V_{cs}$ and $V_{cd} \simeq V_{us}$ only at first non trivial order in $\lambda$ (need $b$-input to fix higher orders)
- $B$ alone: rather constraining
- Indirect (combination of the two above): already quite well determined
- Unitarity constraint:
  $$|V_{cd}|^2 + |V_{cs}|^2 \leq 1$$

**Direct**
- $|V_{cd}|$ from $\nu N$ scattering
- $|V_{cs}|$ from CLEO-c $D \rightarrow K\ell\nu +$ lattice

**I. Shipsey CLEO-c Aspen workshop**
The trouble with $|V_{cs}|$ (2008 !)

- $D_s \rightarrow \ell \nu$
  (CLEO-c, Belle . . .)

- Unquenched
  (staggered) lattice
  $f_{D_s} = 241 \pm 3$ MeV
  (HPQCD+UKQCD)

- Combined :
  $|V_{cs}| = 1.076 \pm 0.041$

- compared to fit value
  (68% CL)
  $|V_{cs}| = 0.9735^{+0.00020}_{-0.00022}$

$f_{D_s}$ supposedly ideal for lattice (charm strange)
and far worse than $f_K/f_\pi$ (light quarks) !?

Uncontrolled systematics in full (unquenched) QCD simulations ??

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The trouble with $|V_{cs}|$ ? (2009)

- $D_s \rightarrow \mu \nu$ (CLEO-c) and $D_s \rightarrow \tau \nu$ (CLEO, Babar, Belle)
- Own lattice QCD average $f_{Ds} = 246.3 \pm 1.2 \pm 5.3$
- Combined: $|V_{cs}| = 1.032 \pm 0.049$
- compared to fit value (68% CL) $|V_{cs}| = 0.9735^{+0.0002}_{-0.0002}$

Agreement between theory and experiment ?
What has happened?

Theoretical predictions more accurate than direct measurements, in marginal agreement

<table>
<thead>
<tr>
<th></th>
<th>$D_s \rightarrow \mu\nu$</th>
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<tbody>
<tr>
<td><strong>CKMfitter</strong></td>
<td>$5.17 \pm 0.28 \cdot 10^{-3}$</td>
<td>$5.05 \pm 0.27 \cdot 10^{-2}$</td>
</tr>
<tr>
<td><strong>CLEOc 09</strong></td>
<td>$5.65 \pm 0.48 \cdot 10^{-3}$</td>
<td>$5.62 \pm 0.44 \cdot 10^{-2}$</td>
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<tr>
<td><strong>Babar</strong></td>
<td>$6.38 \pm 0.92 \cdot 10^{-3}$</td>
<td>–</td>
</tr>
<tr>
<td><strong>Belle</strong></td>
<td>$6.74 \pm 1.09 \cdot 10^{-3}$</td>
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A few possible explanations

- Experimental issue
  - CLEO-c lower than before and disagreeing with Babar and Belle
  - BES could achieve 0.7% accuracy on these BRs

- Issue of $f_{D_s}$ from lattice, though safer than other hadronic quantities

- New physics in $\Delta F = 1$ processes
How large a disagreement?

Disagreement depends on which value you believe more...

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</tr>
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![Graph showing $1-CL$ vs. $|V_{cs}|$ from the global fit.](image)

CLEO-c + LQCD (FNAL-MILC)

$|V_{cs}|$ from the global fit

Sébastien Descotes-Genon (LPT-Orsay)  CKM: from SM to NP (II)
How large a disagreement?

Disagreement depends on which value you believe more...

\[ f_{Ds} \quad \text{Mean} \quad \text{Stat} \quad \text{Syst} \]

FNAL-MILC07* 254 8 11
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How large a disagreement?

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\[ f_{Ds} \]

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Moriond 2009

CLEO-c + LQCD (Average)

|V_{cs}| from the global fit

Sébastien Descotes-Genon (LPT-Orsay)

CKM : from SM to NP (II)
Hadronic inputs and lattice
The situation

- Several hadronic inputs required as inputs for CKMfitter
  
  \[ f_B, f_{B_s}, B_B, B_{B_s}, B_K \ldots \]

- Lattice QCD the almost only tool able to estimate them with uncertainties that can be checked and improved

- But many collaborations with different methods of simulations, results, and estimations of errors

Need to perform an average

- Up to now: use reviews from the lattice community (e.g., Tantalo CKM 06)

- Problem: averages often performed in a rather personal manner

- Recently, we attempted to perform our own averages
Sources of uncertainties

Euclidean, finite, discrete box
\[
\langle Q \rangle = \int [dA] \hat{Q}[A] (\text{det} S_f[A])^{N_f} \exp(-S_{YM}[A])
\]

observable = statistical average over gauge configurations weighted according to gauge and fermion actions

Statistical

- Size of the ensemble of gauge configurations
- Part of errors listed below (when scaling with size of gauge config)

Systematics

- Fermion action : \( N_f = 2 \), staggered fermions
- Continuum limit/discretisation error \( a \to 0 \)
- Finite volume effects \( L \to \infty \)
- Quark mass extrapolation (chiral limit and heavy quark limit)
List the results

Include
- only unquenched results with 2 or 2+1 dynamical fermions (sea quarks)
- papers and proceedings (but not preliminary studies)
- split error estimates into stat and syst

Potential problems
- include staggered fermions, though status unclear (still QCD ?)
- proceedings not always followed by peer-reviewed papers
- preliminary studies superseded by other papers
- correlations among observables rarely given (e.g., neutral meson mixing with $f_B, B_B, f_B\sqrt{f_B}$...)

Sébastien Descotes-Genon (LPT-Orsay)
A simple example

Let us take reported values for dynamical simulations of $f_{B_s}$

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How to combine them?
Rfit scheme

Each of these results is expressed in the Rfit scheme

- $\chi^2$ with flat bottom (syst) and parabolic walls (stat)
- can be represented as CL assuming $\chi^2$ distribution

$$x = 0^{+0.5}_{-0.7} \pm 0.3$$
Four different ways of performing the average

For our averages, several methods have been considered

- “pure Gaussian”: all errors in quadrature + adding $\chi^2$
- “splitted Gaussian”:
  - separate statistical and systematic errors
  - adding $\chi^2$ separately for syst and stat
  - two errors interpreted as statistical and systematic uncertainties of the combination
- “naive Rfit”: take Rfit $\chi^2$ and add them
- “educated Rfit”:
  - add $\chi^2$ with only the statistical errors
  - theoretical uncertainty of the combination = the one of the most precise method

⇒ Illustration on one quantity $f_{Bs}$ (4 different lattice values)
$f_{B_s} = 236 \pm 9.5 \text{ MeV}$

All errors combined in quadrature + sum of $\chi^2$
\[ f_{B_s} = 236 \pm 2.9 \pm 8.7 \text{ MeV} \]

Separate Gaussian treatment of stat and syst errors + sum of \( \chi^2 \)
Naive Rfit

\[ f_{B_s} = 232^{+9}_{-5} \pm 2.5 \text{ MeV} \]

Sum of Rfit $\chi^2$
Educated Rfit

\[ f_{B_s} = 232 \pm 3 \pm 11 \text{ MeV} \]

Sum of \( \chi^2 \) for stat + min syst uncertainty among values
Final choice

- "Educated Rfit"
- If several Rfit errors, combined linearly (requires to go back to the papers, as it is often combined in quadrature)

Conservative, algorithmic procedure with an internal logic for systematics

- the present state of art cannot allow us to reach a better theoretical accuracy than the best of all estimates
- this best estimate should not be penalized by less precise methods (opposed to combined syst = dispersion of central values)
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Used in global fit (and other analyses such as NP)

- currently: meson decay constants
- currently: bag parameters for $K, B$ and $B_s$ mixing
- soon: $K_{\ell 3}$ form factors
- maybe: exclusive semileptonic $b \to c$ and $b \to u$ decays
From 2007 to 2008

- Lattice-related obs: $\Delta m_d$, $\Delta m_s$, $\epsilon_K$ (also $V_{ub}$ through $B \rightarrow \tau \nu$)
- Impact of change of lattice values (our own averages in 2008)
Beyond the Standard Model: 2HDM
Two Higgs doublet models

- In SM, Higgs mechanism from a single complex doublet for EWSB
- Not imposed by symmetry... one can take $\phi_1$ and $\phi_2$ of opposite hypercharge

Different 2HDM (two-Higgs doublet models)

- type I: $\phi_1$ coupling to both up- and down-type, $\phi_2$ to none
- type II: $\phi_1$ coupling to up-type, $\phi_2$ to down-type (and leptons)
- type III: $\phi_1$ and $\phi_2$ coupling both to both types of quarks

Focus on 2HDM(II): looks like SM with 2 Yukawa matrices $y^d,u$

\[ \mathcal{L}_{II,Y} = -\bar{Q}L\phi_1 y^d D_R - \bar{Q}L\phi_2 y^d U_R - \bar{L}_L\phi_2 y^e E_R + h.c. \]

(SM would be $\phi_2 = i\sigma_2\phi_1^*$).
Electroweak symmetry breaking

EWSB occurs through

\[
|\langle 0|\phi_1|0\rangle| = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix} \quad |\langle 0|\phi_2|0\rangle| = \begin{pmatrix} v_2/\sqrt{2} \\ 0 \end{pmatrix} \quad v_1^2 + v_2^2 = v^2
\]

Charged states: \( \phi^+ \) (new \( W^+ \) polarisation) and \( H^+ \) (charged Higgs)

\[
\begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi^+ \\ H^+ \end{pmatrix}
\]
Electroweak symmetry breaking

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\[ |\langle 0 | \phi_1 | 0 \rangle| = \begin{pmatrix} 0 \\ v_1 / \sqrt{2} \end{pmatrix} \quad |\langle 0 | \phi_2 | 0 \rangle| = \begin{pmatrix} v_2 / \sqrt{2} \\ 0 \end{pmatrix} \quad v_1^2 + v_2^2 = v^2 \]

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\]

The charged Higgs \( H^+ \) has the couplings

\[
\mathcal{L}_{H^+} = \frac{gH^+}{\sqrt{2}} \sum_{ij} \left[ \tan \beta \frac{m_{dj}}{M_W} \bar{u}_{Li} V_{ij} d_{Rj} + \cot \beta \frac{m_{ui}}{M_W} \bar{u}_{Rj} V_{ij} d_{Lj} + \tan \beta \frac{m_{li}}{M_W} \bar{\nu}_{Lj} \ell_{Rj} \right]
\]

CKM matrix \( V_{ij} \) from reexpression of quarks in mass eigenstates
2HDM(II) vs SM

Simple and predictive extension of the Standard Model (embedded in susy models)

- SM-like Yukawa terms for the quark sector
- CKM matrix only source of flavour-changing interactions
- No flavour-changing neutral currents at tree level
- New flavour-changing charged interactions: exchange of a charged Higgs rather than $W$, $S - P$ rather than $V - A$

After EWSB, 5 scalars:

- $H^\pm$ (charged), $A$ (pseudoscalar), $h^0$ and $H^0$ (scalar)

Additional parameters:

- masses of $H^\pm$, $H^0$ and $A$
- ratio of vacuum expectation values $\tan \beta = v_2/v_1$
- angle describing the mixing between $h^0$ and $H^0$
2HDM(II) vs SM

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- CKM matrix only source of flavour-changing interactions
- No flavour-changing neutral currents at tree level
- New flavour-changing charged interactions: exchange of a charged Higgs rather than $W, S - P$ rather than $V - A$

After EWSB, 5 scalars:
- $H^\pm$ (charged), $A$ (pseudoscalar), $h^0$ and $H^0$ (scalar)

Additional parameters:
- masses of $H^\pm, H$ and $A$,
- ratio of vacuum expectation values $\tan \beta = v_2/v_1$
- angle describing the mixing between $h^0$ and $H^0$
Potential problems with leptonic decays ($B \rightarrow \tau \nu$, $D_s \rightarrow \ell \nu \ldots$)

- Restrict analysis to $\Delta F = 1$ or electroweak processes receiving $H^+$ contributions
- Parameters: CKM matrix, $M_{H^+}$, $\tan \beta$ (none from neutral Higgses)

CKM matrix inputs

- Need some inputs to fix CKM matrix
- Take inputs where charged Higgs contribution suppressed because proportional to $\frac{m_{\text{light}} m_{\text{heavy}}}{M_{H^+}^2}$ or $\frac{m_{\text{light}}^2}{M_{H^+}^2}$
- Selects $|V_{ud}|$, $|V_{ub}|$, $|V_{cb}|$, $\gamma$ (from combination of $\alpha$ and $\beta$)
Observables and deviations from SM

Observables chosen for the analysis with SM predictions

Orange band : SM pred with two standard deviations
Points : measurements with exp one standard deviation
Leptonic decays

For any meson $M$, SM leptonic decay rate [rad corr only for $M = K, \pi$]

$$B[M \to \ell\nu_{\ell}]_{\text{SM}} = \frac{G_F^2 m_M m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 |V_{qu}q_d|^2 f_M^2 \tau_M (1 + \delta_{EM}^{M\ell2})$$
Leptonic decays

For any meson $M$, SM leptonic decay rate [rad corr only for $M = K, \pi$]

$$B[M \to \ell \nu_\ell]_{\text{SM}} = \frac{G_F^2 m_M m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 |V_{q_u q_d}|^2 f_M^2 T_M (1 + \delta_{EM}^{M\ell2})$$

Charged Higgs contributions:

$$B[M \to l \nu] = B[M \to l \nu]_{\text{SM}} (1 + r_H)^2$$

$$r_H = \left(\frac{m_{q_u} - m_{q_d} \tan^2 \beta}{m_{q_u} + m_{q_d}}\right) \left(\frac{m_M}{m_{H^+}}\right)^2.$$
Leptonic decays

For any meson $M$, SM leptonic decay rate [rad corr only for $M = K, \pi$]

$$B[M \to \ell \nu_{\ell}]_{\text{SM}} = \frac{G_F^2 m_M m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_M^2}\right)^2 |V_{q_u q_d}|^2 f_M^2 \tau_M (1 + \delta_{EM}^{M\ell\ell})$$

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$$r_H = \left(\frac{m_{q_u} - m_{q_d} \tan^2 \beta}{m_{q_u} + m_{q_d}}\right) \left(\frac{m_M}{m_{H^+}}\right)^2.$$ 

If perfect agreement SM-data, two distinct solutions in 2HDM(II)

- decoupling: $r_H = 0$ ($m_{H^+} \to \infty$, tan $\beta$ small)
- fine-tuned: $r_H = -2$ (linear correlation between $m_{H^+}$ and large tan $\beta$, depends on meson mass)
The 4 leptonic decays: $B \rightarrow \tau \nu$
The 4 leptonic decays: $D \rightarrow \mu \nu$
The 4 leptonic decays: $D_s \to \ell \nu$
The 4 leptonic decays: $K \rightarrow \ell \nu / \pi \rightarrow \ell \nu$
The 4 leptonic decays combined
Semileptonic decays help to remove fined-tuned solutions at 95% CL

\[ \mathcal{B}[B \to D\tau\nu] / \mathcal{B}[B \to D\ell\nu] \]

and

\[ \mathcal{B}[K^0 \to \pi\mu\nu] / \mathcal{B}[K^0 \to \pi\ell\nu] \]

- Easier to study experimentally than purely leptonic decays
- Sensitive to scalar currents through helicity suppressed contribution (scalar form factor)
- Form factors constrained from lattice QCD (including scalar)
- Uses of effective theories (HQET and chiral pert theory)
Further observables and combined fit

- Further observables added: $R_b = \Gamma[Z \rightarrow b\bar{b}]/\Gamma[Z \rightarrow \text{hadrons}]$ and $b \rightarrow s\gamma$
- Charged Higgs contrib = shift in (perturbative) coefficients describing decays in SM

![Graph showing excluded area with CL > 0.95](image)
Further observables and combined fit

\[ \chi^2_{\text{min}} \approx 11 \] obtained for \( m_{H^+} > 600 \) GeV

At high \( H^+ \) mass and any \( \tan \beta \) (decoupling limit), charged Higgs contributions negligible

Large \( \chi^2_{\text{min}} \) value is due to the tension between

- large measured \( \mathcal{B}[B \rightarrow \tau \nu] \) favouring fine-tuned sol at low \( m_{H^+} \)
- other observables agree with SM and select large \( m_{H^+} \)
A limit on $m_{H^+} > 323$ GeV at 95% CL is obtained

- Large $\tan\beta$ (> 30), $B \to \tau\nu$ competes with $b \to s\gamma$ and sharpens its exclusion limit
- At small $\tan\beta$ (< 1), most stringent constraint from the $Z \to b\bar{b}$ partial width

and no constraint on $\tan\beta$ at 95%CL
Conclusions

\[ V = \]

\[
\begin{array}{ccc}
\text{u} & \text{d} & \text{s} \\
\nu_e & e^- & \ell^- \\
\nu & K & B \\
\pi & D & D \\
\pi & B^0 & B^0 \\
\text{t} & B_s & B_s \\
\text{W} & t & b \\
\end{array}
\]
Conclusions (1)

SM very robust
- Kobayashi-Maskawa mechanism at work very efficiently for CP violation
- Fits also additional more involved constraints, such as $B \to V\gamma$ (loop processes)
- Potential tensions from charm sector

Issue of hadronic inputs
- Lattice main conveyor of results from various collaborations
- What to do with systematic uncertainties?
- How to combine them
New physics in $\Delta F = 1$ processes

- Renewed interest due to troubles with leptonic decays ($B \rightarrow \tau \nu$, $D_s \rightarrow \ell \nu$)

- Explicit example: Two Higgs Doublet Models with Charged Higgs contributions

- 2HDM(II) does not fare better than SM: favours slightly the decoupling limit

- More observables (mixing, $B_s \rightarrow \mu \mu$) to be considered, as well as other 2HDM models

Thank you for your attention!
Backup
\[ \langle V \gamma | Q_i | B \rangle = \epsilon^* \cdot \left[ T_{1}^{B \rightarrow V}(0) T_i \right] + \int d\xi \, du \, T_i^{II}(\xi, u) \phi_B(\xi) \phi_{2;V}(u) \cdot \left\{ 1 + O \left( \frac{1}{m_b} \right) \right\} \]

- \( \epsilon_{\mu} \) photon’s polarisation
- \( Q_i \) from effective Hamiltonian for \( b \rightarrow (s, d') \)
- \( T_{1}^{B \rightarrow V} \) form factor \( \phi_B, \phi_{2;V} \) leading-twist distrib. ampl.
\[ \langle V\gamma|Q_i|B\rangle = \epsilon^* \cdot \left[ T_{1}^{B\rightarrow V}(0) T_i^I + \int d\xi \, du \, T_i^{\perp\perp}(\xi, u) \phi_B(\xi) \phi_2; V(u) \right] \cdot \left\{ 1 + O \left( \frac{1}{m_b} \right) \right\} \]

- \( \epsilon_\mu \) photon's polarisation
- \( Q_i \) from effective Hamiltonian for \( b \to (s, d) \)
- \( T_{1}^{B\rightarrow V} \) form factor \( \phi_B, \phi_2; V \) leading-twist distrib. ampl.

Up to \( 1/m_B \) corrections, separation (factorisation) of
- Long distances: form factors, distribution amplitudes
- Short distances: hard-scattering kernels \( T_i^I \) and \( T_i^{\perp\perp} \).
All expressed as factor to leading $Q_7 = \frac{e}{8\pi^2} m_b \bar{D}\sigma^{\mu\nu} (1 + \gamma_5) F_{\mu\nu} b + \ldots$

\[
\bar{A}_{L(R)} \equiv \frac{G_F}{\sqrt{2}} \left( \lambda_u^D a_7^{uL(R)}(V) + \lambda_c^D a_7^{cL(R)}(V) \right) \langle V \gamma_{L(R)} | Q_7^{L(R)} | \bar{B} \rangle
\]

- $\lambda_u^D = V_{UD}^* V_{Ub}$
- $\lambda_t^D + \lambda_c^D + \lambda_u^D = 0$
- $L$ and $R$ polarisation of the photon
- $a_{7L}^{c,u}(V) = C_7 + O(\alpha_s, 1/m_b)$
- $a_{7R}^{c,u}(V) = O(\alpha_s, 1/m_b)$
Beyond QCDF (Ball et al.)

All expressed as factor to leading $Q_7 = \frac{e}{8\pi^2} m_b \bar{D}_\sigma^{\mu\nu} (1 + \gamma_5) F_{\mu\nu} b + \ldots$

$$\bar{A}_{L(R)} \equiv G_F \sqrt{2} \left( \lambda_U^D a_{7L(R)}^u (V) + \lambda_c^D a_{7L(R)}^c (V) \right) \langle V \gamma_{L(R)} | Q_7^{L(R)} | \bar{B} \rangle$$

- $\lambda_U^D = V_{UD}^* V_{Ub}$
- $\lambda_t^D + \lambda_c^D + \lambda_U^D = 0$
- $L$ and $R$ polarisation of the photon
- $a_{7L}^c (V) = C_7 + O(\alpha_s, 1/m_b)$
- $a_{7R}^c (V) = O(\alpha_s, 1/m_b)$

$$a_7^U (V) = a_{7; QCDF}^U (V) + a_{7; ann}^U (V) + a_{7; soft}^U (V) + \ldots$$

- QCDF : $O(\alpha_s)$ terms, but leading $1/m_b$
- ann,soft : $1/m_b$-suppressed terms large or for isospin/CP asym
B → V_γ : QCD factorisation part

Bosch and Buchalla : explicit formulae for $a_{7L}^{U,QCDF}$, complete to $O(\alpha_s)$

- LO contribution to $T_1$ from $Q_7$
- NLO from 4-quark $Q_1...6$ and chromomagnetic $Q_8$

$T_1$ (vertex)  $T_2$ (hard-scattering)
$B \rightarrow V \gamma$: weak annihilation and soft gluons

Weak annihilation
- Short-distance part estimated within QCDF
- Long-distance contribution: photon emission from $B$ meson through LCSR

Soft-gluon emission from a quark loop
- for light-quark loop: fairly complicated LCSR
- for heavy-quark loop: $1/m_Q$ expansion, then LCSR
$b \rightarrow s \gamma$ in 2HDM(II)

\[ \mathcal{R}_{b \rightarrow s \gamma} = \frac{\mathcal{B}[\bar{B} \rightarrow X_s \gamma]}{\mathcal{B}[\bar{B} \rightarrow X_c \ell \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{EM}}{\pi C} (P + N) \]

- **P** QCD perturbative corrections (leading contribution to the $1/m_b$ expansion, up to NNLO)
- **N** non-perturbative corrections (corresponding to higher orders in the $1/m_b$ expansion, starting at $1/m_b^2$)
- **C** phase space difference between charmed semileptonic decays and $\bar{B} \rightarrow X_s \gamma$

Charged Higgs estimated from SusyBSG package, fitting $A$ and $B$

\[ P + N = (C_{7,SM}^{\text{eff},(0)} + B \Delta C_{7,H+}^{\text{eff},(0)})^2 + A, \]

where $\Delta C_{7,H+}$ known function of $m_{H^+}$ and $\tan \beta$
$Z \to b\bar{b}$ in 2HDM(II)

\[ R_b = \frac{\Gamma[Z \to b\bar{b}]}{\Gamma[Z \to \text{hadrons}]} \]

\[
\frac{1}{R_b} = 1 + \frac{S_b}{[\left(\bar{g}_b^L - \bar{g}_b^R\right)^2(1 - 6\mu_b) + (\bar{g}_b^L + \bar{g}_b^R)^2]^2 C_b^{QCD} C_b^{QED}}
\]

Charged Higgs contribution redefine coupling constants:

\[
\bar{g}_b^L = \bar{g}_b^{L,SM} + \frac{2^{3/4} G_F^{3/2} m_W^3}{8\pi^2 \cos \theta_W} \left( \frac{m_t}{m_W \tan \beta} \right)^2 F_z \left[ \frac{m_t}{m_{H^+}} \right]
\]

\[
\bar{g}_b^R = \bar{g}_b^{R,SM} - \frac{2^{3/4} G_F^{3/2} m_W^3}{8\pi^2 \cos \theta_W} \left( \frac{m_b}{m_W \tan \beta} \right)^2 F_z \left[ \frac{m_t}{m_{H^+}} \right]
\]

where the function $F_z$ is

\[
F_z[x] = \frac{x^2 - x - x \ln(x)}{(x - 1)^2}.
\]

Prediction of $\bar{g}_b^L$ and $\bar{g}_b^R$ driven by $\sin^2 \theta_{W}^{\text{eff}}$, mostly insensitive to $R_b$. 

Sébastien Descotes-Genon (LPT-Orsay)  CKM : from SM to NP (II)