Broken S_3 Symmetry in Flavor Physics

Hirotaka Sugawara, JSPS Office in Washington DC

Work being done with Toshiaki Kaneko, KEK, Japan

Long time ago、I made a proposal together with S.Pakvasa, to understand flavor physics based on non-Abelian discrete symmetry such as S_3.

S.Pakvasa and H.Sugawara, Phys.Lett.B73,61(1978); B82,105(1979)

Recently, many attempts are being made to understand, at least in the leptonic part of flavor physics based on all kinds of non-Abelian discrete symmetries.

We still do not know:

(1) What is the right symmetry?

(2) How is it broken?

(3) What is the origin of the symmetry?

In this continuing work, we show that :

(1) S_3 symmetry with specific way of its breaking is cosistent with both quark sector and lepton sector flavor dynamics.

(2) The understanding the origin of the symmetry will give a strong restriction on the way gauge symmetry is broken.

ABC of S_3

$S_3 : \{1, (1,2), (2,3), (3,1), (1,2,3), (1,3,2)\}$

wave function $: \psi_1, \psi_2, \psi_3$

singlet $: \dfrac{\psi_1 + \psi_2 + \psi_3}{\sqrt{3}} = \alpha$

doublet $: \begin{pmatrix} \dfrac{\psi_1 - \psi_2}{\sqrt{2}} \\ \dfrac{\psi_1 + \psi_2 - 2\psi_3}{\sqrt{6}} \end{pmatrix} = \begin{pmatrix} \beta \\ \gamma \end{pmatrix}$

Higgs coupling or mass term when Higgs is

S_3 singlet is :

$$\mathrm{a}\overline{\alpha}_{\mathrm{R}}\alpha_L + b(\overline{\beta}_R\beta_L + \overline{\gamma}_R\gamma_L)$$

$$= (\overline{\psi}_1, \overline{\psi}_2, \overline{\psi}_3)(\frac{a}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{b}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix})\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

How can s_3 be broken?

KM - -Arbitrary phase for the mixing matrix

$\rightarrow$ CP violation

mixing matrix $\leftarrow$ mass matrix

Allow all possible phases to the mass matrix

$\rightarrow$ both CP and S_3 are broken

Then

$$M = \begin{pmatrix} ae^{i\delta_{11}} & be^{i\delta_{12}} & be^{i\delta_{13}} \\ be^{i\delta_{21}} & ae^{i\delta_{22}} & be^{i\delta_{23}} \\ be^{i\delta_{31}} & be^{i\delta_{32}} & ae^{i\delta_{33}} \end{pmatrix}$$

simple algebra shows :

$$M^*M = \begin{pmatrix} k & f & \overline{g} \\ \bar{f} & k & h \\ g & \overline{h} & k \end{pmatrix}$$

$k : real$

$\operatorname{Re} f \le k, \operatorname{Re} g \le k, \operatorname{Re} h \le k$

The relation of the parameters to the quark masses:

$$k = \frac{1}{3} s_1$$

$$\left|f\right|^2 + \left|g\right|^2 + \left|h\right|^2 = \frac{1}{3} s_1^2 - s_2 \equiv -A$$

$$fgh + \bar{f}\bar{g}\bar{h} = \frac{2}{27} s_1^3 - \frac{1}{3} s_1 s_2 + s_3 \equiv -B$$

with

$$s_1 = m_1^2 + m_2^2 + m_3^2, s_2 = m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2$$

$$s_3 = m_1^2 m_2^2 m_3^2$$

The phase convention and the number of variables (quark sector)

There are only two phases one can adjust

weak current : $\overline{\psi}^{u}_{L,i}\gamma_{\mu}\psi^{d}_{L,i}$

phase of $\psi^{u}_{L,i}$ and $\psi^{d}_{L,i}$ must be changed simultaneously.

$$k, f, g, h --- 7 \times 2(\text{u and d})$$

$$\text{number of parameters} = 14 - 2 = 12$$

$$\text{number of observables} = 6(\text{masses}) + 4$$

We can put either up-quark or down quark f, h real but not both.

Up quark f,h real case

$$(MM^*)_u = \begin{pmatrix} 1 & 1 & 0.9999 - 0.1305\times10^{-4}i \\ 1 & 1 & 0.9999 \\ 0.9999 + 0.1305\times10^{-4}i & 0.9999 & 1 \end{pmatrix}$$

$$(MM^*)_d = \begin{pmatrix} 1 & 0.9994 + 0.02453i & 0.9935 + 0.09677i \\ 0.9994 - 0.02453i & 1 & 0.9965 + 0.07300i \\ 0.9935 + 0.09677i & 0.9965 - 0.07300i & 1 \end{pmatrix}$$

$$U_u = \begin{pmatrix} 0.6012 + 0.1913i & 0.5173 + 0.03279i & 0.5774 - 0.5025\times10^{-5}i \\ -0.7392 - 0.1913i & 0.2875 - 0.03280i & 0.5774 - 0.2513\times10^{-5}i \\ 0.1380 & -0.8048 & 0.5773 \end{pmatrix}$$

$$U_d = \begin{pmatrix} 0.3763 + 0.3792i & 0.5864 + 0.1932i & 0.5746 + 0.05607i \\ -0.6857 - 0.3918i & 0.1658 - 0.1236i & 0.5760 + 0.04208i \\ 0.3014 & -0.7589 & 0.5772 \end{pmatrix}$$

$Mixing \quad matrix$

$$\begin{pmatrix} 0.9743 & 0.2252 & 0.001293-0.003269i \\ -0.2251-0.1381\times10^{-3}i & 0.9735 & 0.04117 \\ 0.008012-0.003182i & -0.0404-0.7367\times10^{-3}i & 0.9991 \end{pmatrix}$$

Parameter	**Experimental**	**Calculate**
λ (Cabibbo)	0.2252	0.225195
A	0.8116	0.811834
ρ	0.139	0.136225
η	0.341	0.343641

Lepton case

(1)neutrino sector

Assume large Majorana masses of right handed neutrino but zero majorana masses of left handed neutrino

$$M_\nu = (\nu_L^T, (\nu_R^C)^T) C \begin{pmatrix} 0 & M^D \\ (M^D)^T & M^M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix}$$

$$M^M = (M^M)^T$$

$$\begin{pmatrix} 0 & M^D \\ (M^D)^T & M^M \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \lambda \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$M^D V_2 = \lambda V_1, \quad (M^D)^T V_1 = (\lambda - M^M) V_2$$

For small λ, we get,

$$-M^D (M^M)^{-1} (M^D)^T V_1 = \lambda V_1$$

$$V_2 \approx 0$$

$$M^{D} = m_{\nu}\begin{pmatrix} 1 & c & c \\ c & 1 & c \\ c & c & 1 \end{pmatrix}$$

$$M^{M} = a\begin{pmatrix} 1 & \eta e^{i\delta_1} & \eta e^{i\delta_2} \\ \eta e^{i\delta_1} & 1 & \eta e^{i\delta_3} \\ \eta e^{i\delta_2} & \eta e^{i\delta_3} & 1 \end{pmatrix}$$

$$\eta e^{i\delta_i} = (1-\chi)(1+i\delta_i+\cdots)$$

$$\approx (1+i(\delta_i+i\chi))$$

$$\equiv 1+i\delta_i$$

$$\delta_i : complex$$

$$M_\nu = \frac{m_\nu \varepsilon^2}{az}[-\chi\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} + i\begin{pmatrix} -(x_1+x_2) & x_1 & x_2 \\ x_1 & -(x_3+x_1) & x_3 \\ x_2 & x_3 & -(x_2+x_3) \end{pmatrix}]$$

"Real" part and "imaginary" part of the matrix commute with each other.

-→ We can diagonalize this by an orthogonal matrix

$$c = 1 + \varepsilon$$

$$z = 1 + q_1 q_2 q_3 - q_1^2 - q_2^2 - q_3^2$$

$$q_i = \eta e^{i\delta_i}$$

$$x_i = \delta_{i+1} + \delta_{i+2} - \delta_i$$

Here δ's are defined without imaginary part.

(1) Normal hierarchy case : $x_1 x_2 + x_2 x_3 + x_3 x_1 = 0$

$$U_\nu = \left(\frac{1}{N_1}\begin{pmatrix} x_2 + 2x_3 \\ -(2x_2 + x_3 \\ x_2 - x_3 \end{pmatrix}, \quad \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \frac{1}{N_3}\begin{pmatrix} x_2 \\ x_3 \\ -(x_2 + x_3) \end{pmatrix} \right)$$

The first column correspons to an eigen value 3χ, the secod column to eigen value zeroand the third one to $3\chi - 2i(x_1 + x_2 + x_3)$.

We see that $x_2 = 0$ correspond exactly to tri - bimaximal case.

Therefore, x_2 (θ_{13} in ordinary notation)

indicates the deviation from the tri - bimaximal case.

(2) Inverted hierarchy : $x_1 = x_2 = x_3$

$$U_\nu = \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}$$

The first and the second columns correspond to eigen value $3\chi - 3ix_i$ and the third column to zero eigen value.

This does not correspond to the tri-bimaximal solution.

We have to study the charged lepton matrix to get the mixing matrix.

The fact that the neutrino part is by itself gives good experimental fit

means that we must get identity matrix from the charged lepton sector.

Charged lepton family members are all s(3) singlet rather than singlet doublet. All the quarks and neutrinos are singlet-doublet.

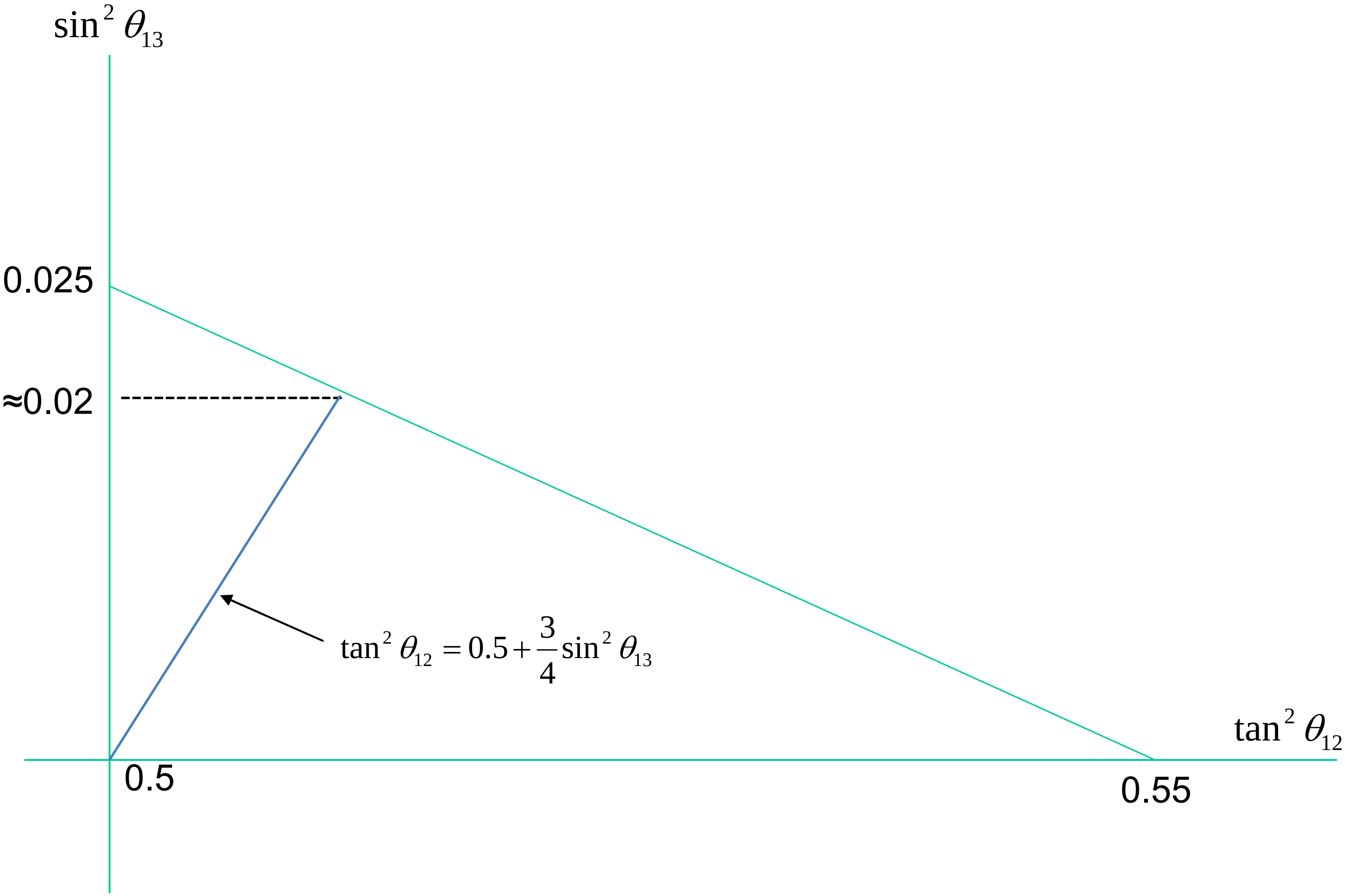
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23} & c_{12}c_{23} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23} & -c_{12}s_{23} - s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

we get for small s_{13} :

$$\sin^2\theta_{13} = \kappa$$

$$\tan^2\theta_{12} = 0.5 + \frac{3}{4}\kappa$$

$$\tan^2\theta_{23} = 1 + 2\sqrt{2\kappa}$$



Kamland data taken from M.C.Gonzalez-Garcia
etal.arXiv:1001.4524v3,page9

The most important question is “Why S(3)?”

There are two kinds of symmetry in nature:

(1)Guage symmetry which is good at high energy(short distance).

(2)Non-gauge symmetry such as baryon number conservation,

CP invariance, parity etc.. These are good only in lower energy region(long distance.

For example,

Baryon number conservation gets vilated when quarks and lepton go into a same multiplet. This makes sense at high energy.

CP is violated when 3^{rd} generation quarks gets into the same multiplet as the other two generations.

S(3) must belong to this category.

One possibility

$E_8 \to SU_3 \times E_6$

$248 \to (1,78) + (8,1) + (3,27) + (\overline{3},\overline{27})$

We use this SU_3 as the flavor group.

This SU_3 will be broken at high energy.

The following S_3 may remain unbroken "except for phases":

$(1,2) = e^{-i\frac{\pi}{2}} e^{i\frac{\pi}{2}T_1}$

$T_1 = \lambda_1 + \frac{1}{3}(I - \sqrt{3}\lambda_8)$

etc.

In fact we are using U_3 rather than SU_3

This will give strong constraints how grand unfied group is

broken down to standard model.

Question :

Is it possible to make a model which is consistent with our observation?

(1) Can charged lepton be singlets?

(2)What sort of Higgs multiplets will lead to mass matrix :

$$M^* M = \begin{pmatrix} k & f & g \\ \overline{f} & k & \overline{h} \\ \overline{g} & h & k \end{pmatrix} \quad ?$$