Neutrino diffraction: Theory and implications

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Abstract

Interference of the neutrino in decay of high-energy pion is studied and in this seminar the following four topics will be discussed.

topics

1. Derivation of a diffraction component in the neutrino flux that depends on the absolute value of the neutrino mass.
2. Implicatios of the neutrino diffraction to LSND anomaly and two neutrino experiment.
3. Implications of the neutrino diffraction to the absolute neutrino flux and total cross section.(**T2K**)
4. On neutrino's orbit(trajectory)

Refferences:

Ishikawa and Tobita,

1."Neutrino diffraction induced by light-cone singularity and small neutrino mass", arxiv:1106.4968[hep-ph](Derivation of diffraction term),

2."Resolving LSND anomaly by neutrino diffraction",arxiv:1109.3105[hep-ph](LSND and TWN),

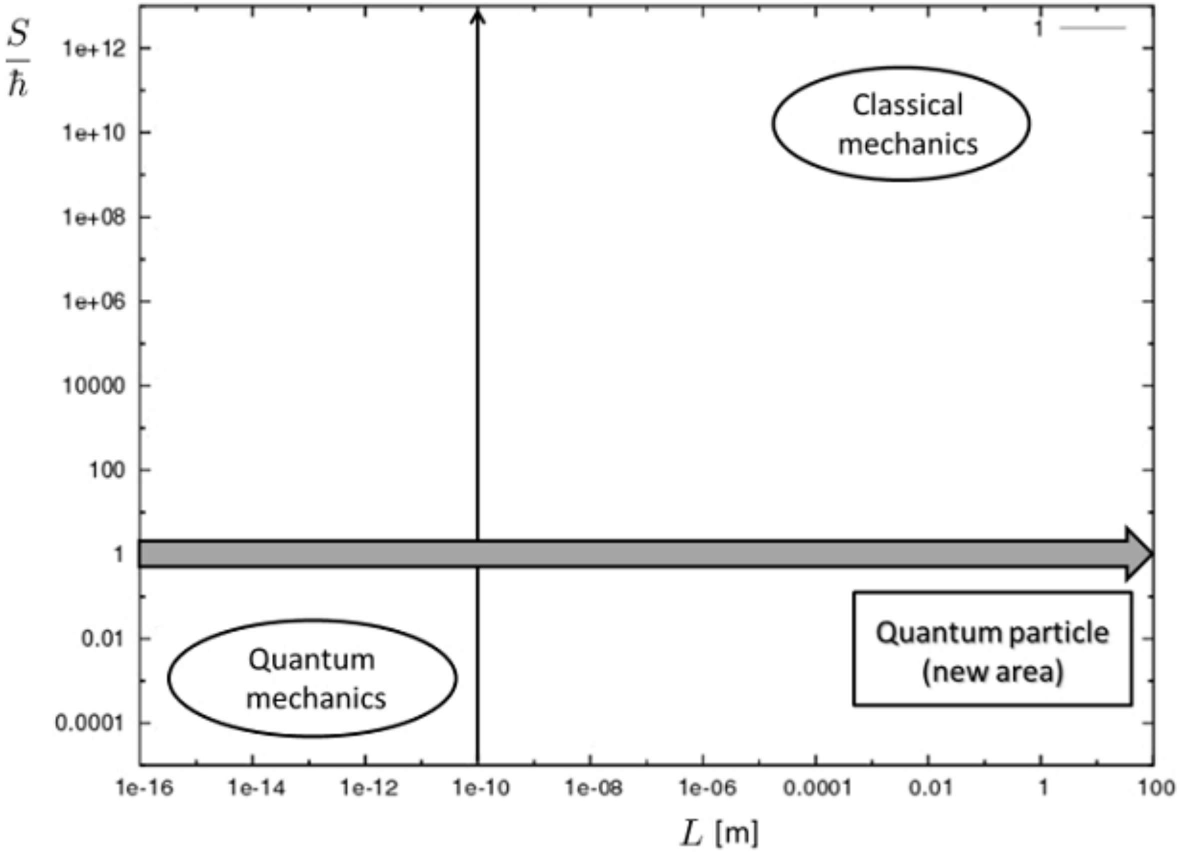
3."Diffraction-corrected neutrino flux and total cross section",in preparation(neutrino flux and total cross section).

4."Quantum particles",in preparation(neutrino orbit and light quanta)

and others

1 Introduction

1-2 parameters



$$m_e = 0.5MeV/c^2 \tag{1}$$
$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e c^2} = r_e\alpha^{-2} = 0.5 \times 10^{-10}M(Bohr\ radius)$$

1-3 Wave function

$$(1).\ \ e^{Et/i\hbar}\psi_E(\vec{x});\ \ stationary\ state \tag{2}$$

$$(2).\ \ \int d\vec{k}e^{(E(\vec{k})t-\vec{k}\vec{x})/i\hbar}f(\vec{k});\ \ non-statinary\ sates \tag{3}$$

Interference pattern occurs,
wave length:λ slit size: d

$$(1)\ \ d \approx \lambda;\ \ stationary\ states \tag{4}$$

$$(2)\ \ d \gg \lambda;\ \ non-stationary\ staes\ \ due\ to\ cancellation\ in\ \ Et-\vec{k}\vec{x}$$

$$\rightarrow distinct\ from\ stationary\ state\ \ : \textbf{neutrino diffraction}$$

1-4 particle's correlation function

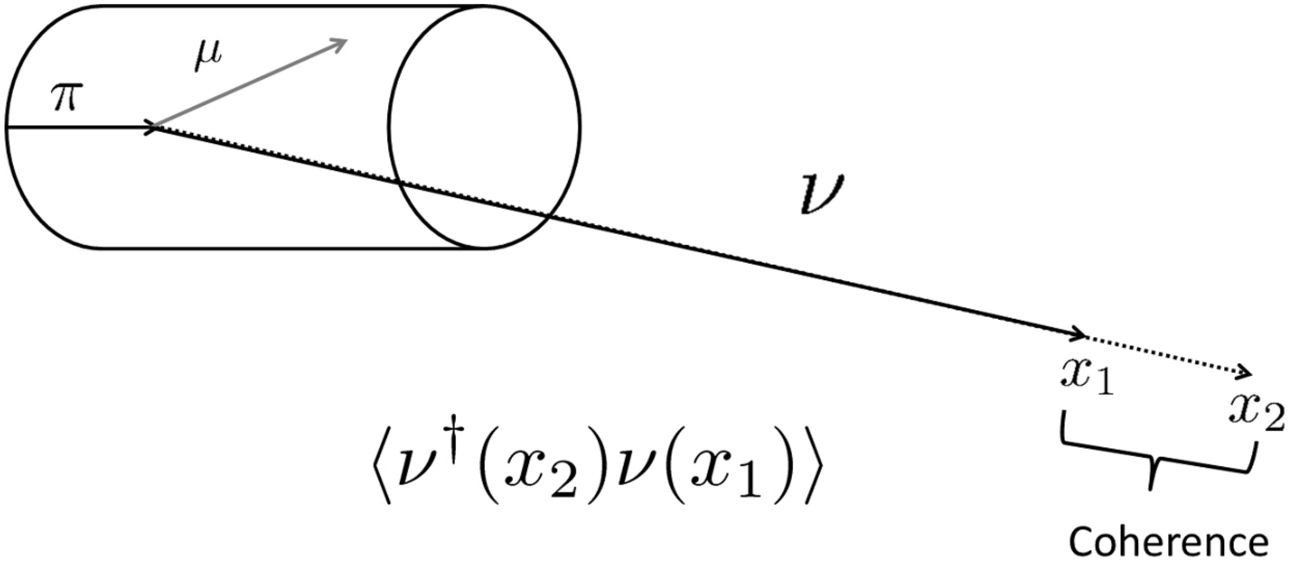
$$\langle\vec{x}_1,t_1|\vec{x}_2,t_2\rangle \approx C\exp(-|\vec{x}_1-\vec{x}_2|/l_0), C\exp(i|\vec{x}_1-\vec{x}_2|/l_0), \tag{5}$$

Wave (quantum) nature disappears in $|\vec{x}_1-\vec{x}_2| > l_0$

$$1.\ l_0 =< 10^{-10}M\ microscopic\ ;\ \ normal\ \ case; electron, \cdots \tag{6}$$

$$2.\ l_0 = L_0 > 1(200)M\ macroscopic\ ;\ \ anomalous\ \ case\ \ \ \textbf{neutrino diffraction}$$

Fig.2



2 Neutrino diffraction induced by light cone singularity

1 Neutrino interference.
Interference phenomena of neutrino are studied.

unique features of neutrino

1. Due to its weak interaction, **single neutrino interference** emerges in a distribution of ensemble of events.

2. Neutrino is observed through its incoherent interaction with nucleus or atom which have microscopic sizes, so its space-time position is identified using wave packet of these sizes. This position-dependent amplitude is that of the **non-stationary state**. The interference of the non-stationary state can have large length scale, much larger than that of the stationary state, de Broglie wave length.

3. **Lorentz invariance** makes the energy and time connect with the momentum and position. Hence the time and space components of the phase are cancelled and wave becomes real at the infinite momentum,which leads the light-cone singularity.

4. Neutrino has so **small mass**. The difference between neutrino's velocity and the light velocity determines the angular velocity of the neutrino along

the light cone.

The neutrino from the decay of high-energy particle reveals a characteristic diffraction pattern in which the length scale is determined by $m_\nu^2/2E_\nu$ as a result of these properties.

2 Position dependent probability.

The pion decay amplitude,

$$T = \int d^4x \, \langle \mu, \nu | H_w(x) | \pi \rangle \tag{7}$$

The wave packet field theory:Ishikawa-Shimomura,Ishikawa-Tobita-ptp, Ishikawa-Tobita;

Neutrino wave packet$\neq$ *field theory*

Kayser,Giunti,Nussinov,Kiers,Stodolsky,Lipkin,Asahara,others.

The state vectors

$$|\pi\rangle = |\vec{p}_\pi, \mathrm{T}_\pi\rangle, \ |\mu, \nu\rangle = |\mu, \vec{p}_\mu; \nu, \vec{p}_\nu, \vec{\mathrm{X}}_\nu, \mathrm{T}_\nu\rangle. \tag{8}$$

The amplitude T

$$\begin{aligned} T = \int d^4x d\vec{k}_\nu \, N \langle 0 | \varphi_\pi(x) | \pi \rangle \bar{u}(\vec{p}_\mu)(1-\gamma_5)\nu(\vec{k}_\nu) \\ \times e^{ip_\mu \cdot x + ik_\nu \cdot (x - \mathrm{X}_\nu) - \frac{\sigma_\nu}{2}(\vec{k}_\nu - \vec{p}_\nu)^2}, \end{aligned} \tag{9}$$

where $N = igm_\mu \, (\sigma_\nu/\pi)^{\frac{4}{3}} \, (m_\mu m_\nu / E_\mu E_\nu)^{\frac{1}{2}}$, $\mathrm{T}_\pi \leq t$.

This depends upon $(\mathrm{T}_\nu, \vec{\mathrm{X}}_\nu)$ and T_π, wave packet size, σ_ν, of the neutrino. σ_ν is computed from using the size of nucleus which the neutrino interacts.

The Gaussian wave packet in this talk and generalization is OK.

The neutrino momentum $\vec{k}_\nu$ integration, the spin summations, this amplitude is expressed with the correlation function and the neutrino wave function in the form

$$\begin{aligned} &\int \frac{d\vec{p}_\mu}{(2\pi)^3} \sum_{s_1, s_2} |T|^2 \\ &= \frac{C}{E_\nu} \int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{x}_i^{\,0})^2} \Delta_{\pi,\mu}(\delta x) e^{i\phi(\delta x)}, \end{aligned} \tag{10}$$

where

$$C = g^2 m_\mu^2 \left(4\pi/\sigma_\nu\right)^{\frac{3}{2}} V^{-1}, \vec{x}_i^{\,0} = \vec{\mathrm{X}}_\nu + \vec{v}_\nu (t_i - \mathrm{T}_\nu), \qquad (11)$$
$$\delta x = x_1 - x_2, \phi(\delta x) = p_\nu \cdot \delta x$$

and the correlation function,

$$\Delta_{\pi,\mu}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d\vec{p}_\mu}{E(\vec{p}_\mu)} (p_\mu \cdot p_\nu) e^{-i(p_\pi - p_\mu)\cdot\delta x}. \qquad (12)$$

The muon momentum is integrated in whole positive energy region, because the energy momentum conservation is approximately satisfied. The effect of these states is computed explicitly and becomes observable.

3 Light-cone singularity.

When the waves are added constructively in $\Delta_{\pi,\mu}(\delta x)$, they show singular behavior. In order to extract leading singular behavior in δx, we write the integral in the four dimensional form with a new variable $q = p_\mu - p_\pi$ that is conjugate to δx.

$$\Delta_{\pi,\mu}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d\vec{p}_\mu}{E(\vec{p}_\mu)} (p_\mu \cdot p_\nu) e^{-i(p_\pi - p_\mu)\cdot\delta x}. \qquad (13)$$

Then $\Delta_{\pi,\mu}(\delta x)$ is decomposed into the integrals in $0 \leq q^0$ and $-p_\pi^0 \leq q^0 \leq 0$. The integral from $0 \leq q^0$ is written in the form, $(p_\pi \cdot p_\nu - ip_\nu \cdot (\frac{\partial}{\partial \delta x}))\tilde{I}_1$, where

$$\tilde{I}_1 = \int d^4q \, \frac{\theta(q^0)}{4\pi^4} \mathrm{Im}\left[\frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon}\right] e^{iq\cdot\delta x}, \qquad (14)$$

and $\tilde{m}^2 = m_\pi^2 - m_\mu^2$. The integrand of $\tilde{I}_1$ is expanded in $p_\pi \cdot q$ and the integration leads the light-cone singularity [?], $\delta(\delta x^2)$, and less singular and regular terms which are described by Bessel functions. The integral from the region $-p_\pi^0 \leq q^0 \leq 0$, I_2, is written with the momentum $\tilde{q} = q + p_\pi$ and has no singularity. Thus the correlation function, $\Delta_{\pi,\mu}(\delta x)$, is written in the form

$$\Delta_{\pi,\mu}(\delta x) = 2i \left\{ p_\pi \cdot p_\nu - ip_\nu \cdot \left(\frac{\partial}{\partial \delta x}\right)\right\}$$
$$\times \left[D_{\tilde{m}} \left(-i\frac{\partial}{\partial \delta x}\right) \left(\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda) + f_{short}\right) + I_2 \right], \qquad (15)$$

where

$$\lambda = (\delta x)^2, D_{\tilde{m}}(-i\frac{\partial}{\partial \delta x}) = \sum_l (1/l!)\big(2p_\pi\cdot(-i\frac{\partial}{\partial \delta x})\frac{\partial}{\partial \tilde{m}^2}\big)^l, \tag{16}$$

$$f_{short} = -\frac{i\tilde{m}^2}{8\pi\xi}\theta(-\lambda)\{N_1(\xi) - i\epsilon(\delta t)J_1(\xi)\} - \frac{i\tilde{m}^2}{4\pi^2\xi}\theta(\lambda)K_1(\xi), \xi = \tilde{m}\sqrt{\lambda}$$

N_1, J_1, and K_1 are Bessel functions. f_{short} has singularity of the form $1/\lambda$ around $\lambda = 0$ and decrease as $e^{-\tilde{m}\sqrt{|\lambda|}}$ or oscillates as $e^{i\tilde{m}\sqrt{|\lambda|}}$ at large $|\lambda|$. The convergence of the series will be studied later.

Integration of spatial coordinates.

Next, Eq.(15) is substituted to

$$\int \frac{d\vec{p}_\mu}{(2\pi)^3}\sum_{s_1,s_2}|T|^2 = \frac{C}{E_\nu}\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i - \vec{x}_i^{\,0})^2}\Delta_{\pi,\mu}(\delta x)e^{i\phi(\delta x)}, \tag{17}$$

From the singular term,

$$\Delta_{\pi,\mu} = \frac{\epsilon(\delta t)}{4\pi}\delta(\lambda) \tag{18}$$

$$J_{\delta(\lambda)} = C_{\delta(\lambda)}\epsilon(\delta t)|\delta t|^{-1}e^{i\bar{\phi}_c(\delta t) - \frac{m_\nu^4}{16\sigma_\nu E_\nu^4}\delta t^2}, C_{\delta(\lambda)} = (\sigma_\nu\pi)^{\frac{3}{2}}\sigma_\nu/2 \tag{19}$$

$$\phi(\delta x) = \bar{\phi}_c(\delta t) = \delta t\ m_\nu^2/2E_\nu \tag{20}$$

the interference pattern is determined by $m_\nu^2/2E_\nu$, but not by de Broglie phase.

Theorem:General cases of non-Gaussian form with spreading effect The singular part $J_{\delta(\lambda)}$ has the slow phase and the magnitude that is inversely proportional to the time difference, of the form Eq. (19). The phase is given by the sum of $\bar{\phi}_c(\delta t)$ and a small correction, which is proportional to $1/E_\nu$ in general systems and becomes $1/E_\nu^2$ if the neutrino wave packet is invariant under the time inversion and is real.

(Proof.)

The wave function $\psi(\vec{x} - \vec{v}t)$ in the general wave packet $\psi_k(k_l, \vec{k}_T)$ with the spreading effect is expressed in the following form

$$\psi(\vec{x} - \vec{v}t) = \int dk_l d\vec{k}_T\, e^{ik_l(x_l - v_\nu t) + i\vec{k}_T\cdot\vec{x}_T + iC_{ij}k_T^i k_T^j t} \times \psi_k(k_l,\ \vec{k}_T),\ C_{ij} = \delta_{ij}/2E, \tag{21}$$

$$J_{\delta(\lambda)} = \pi e^{-\frac{1}{2}} w_0 (1+\gamma) \frac{\epsilon(\delta t)}{2\delta t} e^{i\bar{\phi}_c(\delta t)(1+\delta)} \tag{22}$$

, where

$$w_0 = \int dk_l |\psi_k(k_l, 0)|^2 \tag{23}$$

$$\delta = \frac{d_1}{E} + \frac{d_2}{2E^2},\ \gamma = \frac{d_1}{2E} + \frac{d_2}{2!}\left(\frac{1}{2E}\right)^2 - (1-v_\nu)^2 \delta t^2,$$
$$d_1 = \frac{1}{w_0} \int dk_l k_l |\psi_k(k_l, 0)|^2,\ d_2 = \frac{1}{w_0} \int dk_l k_l^2 |\psi_k(k_l, 0)|^2.$$

The regular terms
are short-range.
First term, $\tilde{L}_1$, is from f_{short} in Eq.(15) and is expressed with Bessel functions. At a large $|\delta t|$ and $\lambda < 0$, introduce

$$r_l = v_\nu \delta t + \tilde{r}_l \tag{24}$$
$$\lambda \approx -2v_\nu \tilde{r}_l \delta t - \tilde{r}_l^2 - \vec{r}_T^{\,2}. \tag{25}$$

usethe asymptotic expression of the Bessel function

$$\tilde{L}_1 = C_1 |\delta t|^{-\frac{3}{4}} e^{i(E_\nu - p_\nu v_\nu)\delta t - \sigma_\nu p_\nu^2 + i\tilde{m}\sqrt{|2v_\nu \sigma_\nu p_\nu \delta t|}}, \tag{26}$$

where

$$C_1 = i\frac{\sigma_\nu}{4}\left(\frac{\sigma_\nu \tilde{m}}{2}\right)^{\frac{1}{2}} (4v_\nu \sigma_\nu p_\nu)^{-\frac{3}{4}} \tag{27}$$

, and oscillates as fast as

$$e^{i\tilde{m}b_1\sqrt{|\delta t|}} \tag{28}$$

$e^{i\tilde{m}b_1\sqrt{|\delta t|}}$ where b_1 is determined by p_ν and σ_ν and is of macroscopic magnitude.

Second term, $\tilde{L}_2$, is from I_2 and is approximately the integral of

$$e^{-i\left(E_\pi - E_\nu - \sqrt{q^2+m_\mu^2}\right)\delta t} \tag{29}$$

over the momentum $\vec{q}$ in the range $1/\sqrt{\sigma_\nu}$. This becomes a short-range correlation of the length, $2\sqrt{\sigma_\nu}$, in the time direction. So the L_2's contribution to the total probability comes from the small δt region, and agrees to that of the standard calculation.

4 Time-dependent probability.

$$\int \frac{d\vec{p}_\mu}{(2\pi)^3} \sum_{s_1,s_2} |T|^2 = N_1 \int dt_1 dt_2 \left[e^{i\frac{m_\nu^2}{2E_\nu}\delta t} \times \frac{\epsilon(\delta t)}{|\delta t|} + 2D_{\tilde{m}}(p_\nu)\frac{\tilde{L}_1}{\sigma_\nu} - \frac{2i}{\pi}\left(\frac{\sigma_\nu}{\pi}\right)^{\frac{1}{2}} \tilde{L}_2 \right], \tag{30}$$

where $N_1 = ig^2 m_\mu^2 \pi^3 \sigma_\nu (8p_\pi\cdot p_\nu/E_\nu)V^{-1}$.

The first term of Eq. (30) is long-range and the rests are short-range. They are separated in a clear manner.

$$\sum_n (-2p_\pi\cdot p_\nu)^n \frac{1}{n!}\left(\frac{\partial}{\partial \tilde{m}^2}\right)^n \tilde{L}_1, \tag{31}$$

converges in

$$2p_\pi\cdot p_\nu \le \tilde{m}^2 \tag{32}$$

.

The power series then oscillates with $\sqrt{|\delta t|}$ rapidly as

$$S_2 = e^{i\tilde{m}\sqrt{|2v_\nu\sigma_\nu p_\nu \delta t|}\left(1-\frac{p_\pi p_\nu}{\tilde{m}^2}\right)} \tag{33}$$

. So the separation of the light-cone singular term from $\tilde{I}_1$ is valid in the region $2p_\pi\cdot p_\nu \le \tilde{m}^2$, and the light-cone singularity exists only in this kinematical region.

In the outside of this region, the power series diverges, and $\Delta_{\pi,\mu}(\delta x)$ has no light-cone singularity.

time integration

Integrating t_1 and t_2 in the finite $\mathrm{T}=\mathrm{T}_\nu-\mathrm{T}_\pi$, we have the slowly decreasing term

$$i\int_0^{\mathrm{T}} dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|} e^{i\omega_\nu \delta t} = \mathrm{T} g(\mathrm{T},\omega_\nu),\ \ \omega_\nu = \frac{m_\nu^2}{2E_\nu}. \tag{34}$$

The slope of $g(\mathrm{T},\omega_\nu)$ at $\mathrm{T}=0$ is $\frac{\partial}{\partial \mathrm{T}} g(\mathrm{T},\omega_\nu)|_{\mathrm{T}=0}=-\omega_\nu$. At $\mathrm{T}=\infty$, $g(\mathrm{T},\omega_\nu)$ becomes $g(\infty,\omega_\nu)=-\pi$ that is cancelled with the short-range term $\tilde{L}_1$ of Eq. (30). So it is convenient to subtract the asymptotic value from $g(\mathrm{T},\omega_\nu)$ and to define $\tilde{g}(\mathrm{T},\omega_\nu)=g(\mathrm{T},\omega_\nu)-g(\infty,\omega_\nu)$. $\tilde{g}(\mathrm{T},\omega_\nu)$ is the diffraction term.

the short-range term

$$\frac{2}{\pi}\sqrt{\frac{\sigma_\nu}{\pi}} \int dt_1 dt_2 \tilde{L}_2(\delta t) = \mathrm{T} G_0 \tag{35}$$

$\delta t \approx 0$, and G_0 is constant in T. This term is independent from σ_ν and agrees to the standard method of using plane waves.

the integration of neutrino's coordinates $\vec{\mathrm{X}}_\nu$ and this is cancelled with V^{-1}, the normalization of the initial pion state.

The total probability

$$P = N_2 \int \frac{d^3 p_\nu}{(2\pi)^3} \frac{p_\pi \cdot p_\nu}{E_\nu} \left[\tilde{g}(\mathrm{T},\omega_\nu)+G_0\right], \tag{36}$$

At $\mathrm{T}\to\infty$, $\tilde{g}\to 0$, and P agrees to G_0, the normal term of standard calculation using plane waves. At finite T, the probability has the diffraction component, which is stable under the varivation of the pion's momentum.

In G_0, $p_\pi \cdot p_\nu = \tilde{m}^2/2$, and the cosine of the angle satisfies

$$1-\cos\theta = \frac{\tilde{m}^2}{2|\vec{p}_\pi||\vec{p}_\nu|} - \frac{m_\pi^2}{2|\vec{p}_\pi|^2} \tag{37}$$

. The transition rate agrees with the value of the ordinary method.

$\tilde{g}(\mathrm{T},\omega_\nu)$, at finite L in the kinematical region,

$$|\vec{p}_\nu|(E_\pi - |\vec{p}_\pi|) \leq p_\pi \cdot p_\nu \leq \tilde{m}^2/2 \tag{38}$$

the wave packet size of the neutrino,

$$\sigma_\nu = A^{\frac{2}{3}}/m_\pi^2 \tag{39}$$

$\sigma_\nu = 6.4/m_\pi^2$ fo ^{16}O nucleus.

An excess of the flux at $\mathrm{L} < 600$ [m],$0.2-0$ of the normal term, has the slope at $\mathrm{L}=0$, ω_ν, and is slowly varying with both the distance and energy.

L_0 is L_0 [m] $= 2E_\nu \hbar c/m_\nu^2 = 400 \times E_\nu[\mathrm{GeV}]/m_\nu^2[\mathrm{eV}^2/c^4]$.

Within Neutrino's energy uncertainty $\Delta E_\nu \approx 0.1 \times E_\nu$, which is about 100 [MeV] for 1 [GeV] and the diffraction components are the same from Fig. 1.

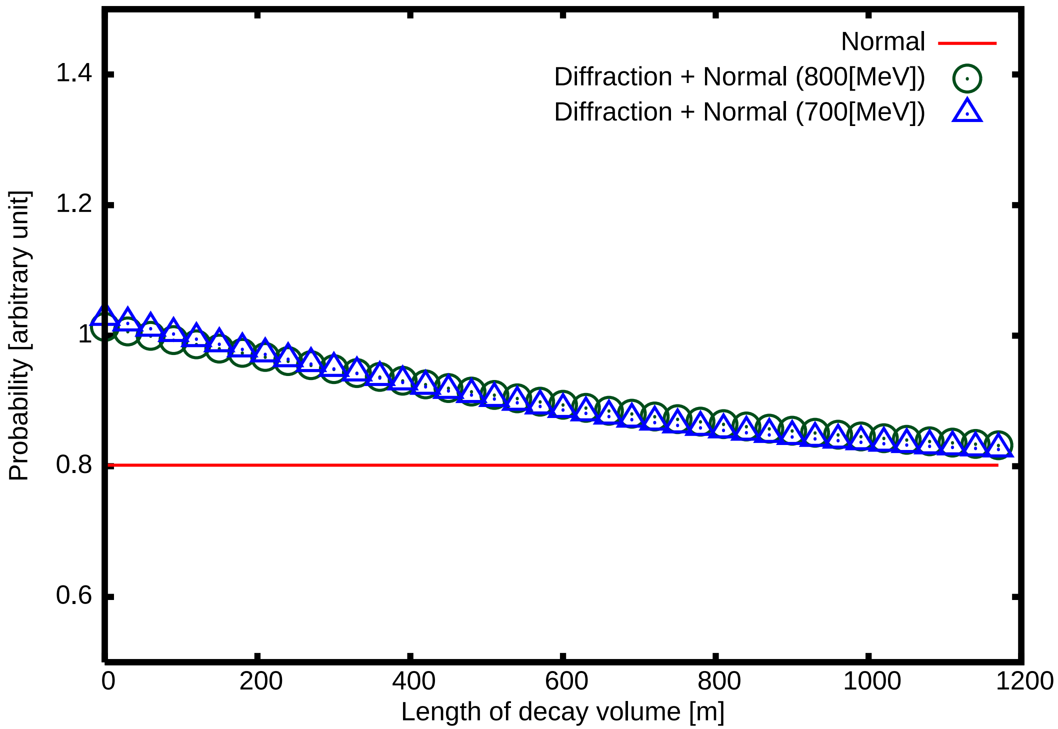


Figure 1: The total probability per time at a finite distance L is given. The constant shows the normal term and the diffraction term is written on top of the normal term. The horizontal axis shows the distance in [m] and the probability is of arbitrary unit. The excess is seen in the distance below 1200m. The neutrino mass, pion energy, neutrino energy are 1.0 [eV/c^2], 4 [GeV], and 700(△) and 800(◯) [MeV].

5 Summary and implications.

The neutrino diffraction ;

(1) manifests the features 1-4 of the introduction,

(2) caused by light-cone singularity of $\Delta_{\pi,\mu}(\delta x)$ and the slow phase $\bar{\phi}_c(\delta t)$, and

(3) decreases with the distance extremely slowly and is added to the normal component.

(4)The length scale is not determined by de Broglie wave length but by $2cE_\nu/m_\nu^2$.

(5)Moreover this component has a sizable magnitude and is stable under the change of the parameters of initial state.

Implications to existing neutrino anomalies.

1. One anomaly is excesses of the neutrino flux at near detectors of ground experiments. Fluxes measured in the near detectors of K2K and

MiniBooNE may show excesses of about $10-20$ percent of the Monte Carlo estimations, whearas the excess is not clear in MINOS .

2. LSND anomaly

$$\delta m^2 =\approx 1eV/c^2 \gg \delta m^2_{solar,atmosph.,reacor,--} \tag{40}$$

3. Total neutrino cross section: E-dependence
4. determination of the absolute neutrino mass.

Intuitive picture of the lepton (small) and neutrino(large).

Inside of length L_0 quantume mechanical natures remain and particle pictures do not hold

3 Resolving LSND anomaly by neutrino diffraction

A diffraction component in the position-dependent neutrino probability leads large electron neutrino fluxes at short base-line experiments, and resolves anomalies of LSND and two neutrino experiment (TWN). The elctron neutrino flux in short distance is attributed to the neutrino diffraction and the one in long distance is attributed to the normal flavor oscillation. Since the former depends upon the average mass-squared $\bar{m}^2$ and the latter depends upon the mass-squared difference δm^2, LSND and TWN are understood in a consistent manner with others and valuable information on the absolute neutrino mass is obtained.

The (V-A)(V-A) interaction:the normal term G_0 and the diffraction term $\tilde{g}(\mathrm{T}, \omega_\nu)$,

$$P = N_3 \int \frac{d^3 p_\nu}{(2\pi)^3} \frac{p_\pi \cdot p_\nu (m_\pi^2 - 2 p_\pi \cdot p_\nu)}{E_\nu} \left[\tilde{g}(\mathrm{T}, \omega_\nu) + G_0\right], \tag{41}$$

The neutrino probability in pion decay in a macroscopic distance, (where charged leptons are unobserved), is

$$P = P_{normal} + P_{diff}. \tag{42}$$

where P_{normal} is the normal term and

$$P_{diff} = CT \sum_i \tilde{g}(T, \omega_i) |U_{i,e}|^2 =, CT \tilde{g}(T, \bar{\omega}) |U_{i,e}|^2 \tag{43}$$

is the new term. Due to this new term neutrino flux is written in the form

$$f = f_{normal} + f_{diff}. \tag{44}$$

Experiment	P_{ν_e}/P_{ν_μ} (Exp)	P_{diff}/P_{normal} (Th)
TWN	0.18	0.17
LSND	$0.0026^{\pm}$	0.0028
CDHSW(N)	?	0.2-0.5
CDHSW(F)	?	$0.02-0.05$
MiniBooNE, KARMEN	0	$\approx 10^{-5}$

Table 1: Experimental values and theoretical values

Experiments

(1)TWN and LSND :from f_{diff}. $\bar{m}^2$

(2)Long baseline,solar,atmosphere,reactor :from f_{normal}. δm^2 Comparisons diffraction component, which is stable under the variation of the pion's momentum.

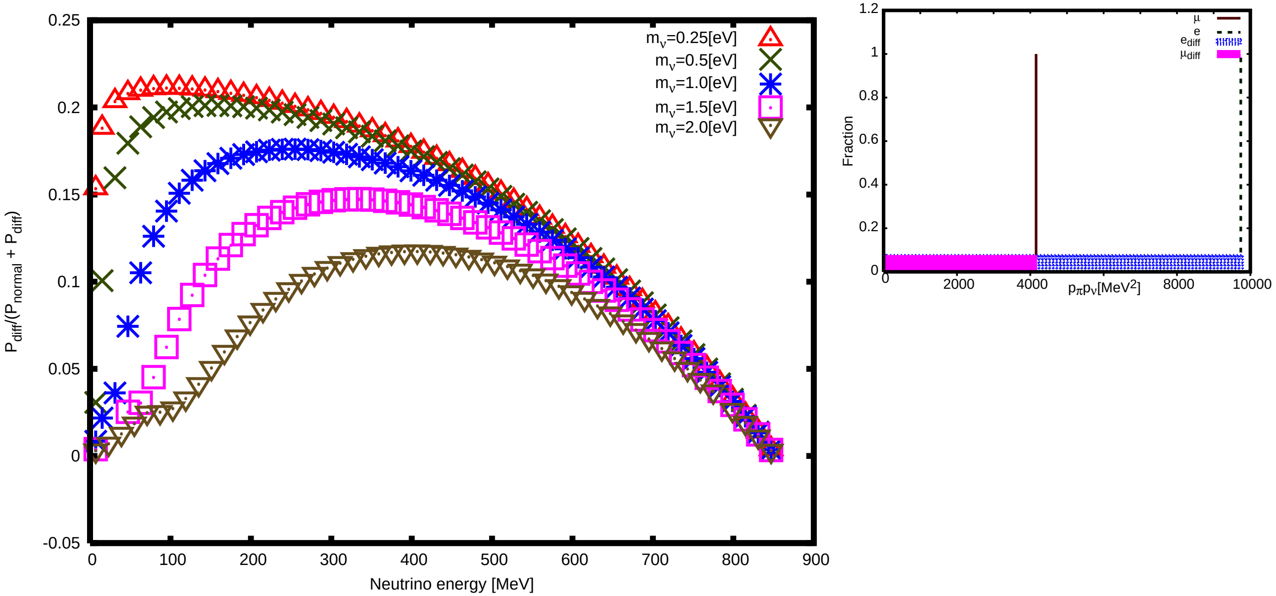


Figure 2: The neutrino energy and $p_\pi p_\nu$ dependences of the fraction of P_{diff} are given in 1-a and 1-b. The horizontal axis shows E_ν in [MeV]. $m_\nu = 0.25 - 2.0$ [eV/c^2], $E_\pi = 2000$ [MeV], and L = 50 [m]. $p_\pi p_\nu$ dependence of the diffraction term is broad and that of the normal term is narrow.

4 Diffraction-corrected Neutrino flux and total cross section

Neutrino diffraction modifies the neutrino fluxes. Using corrected flux, total cross section is obtained. Slowly decreasing νN total cross sections, which are hard to understand in standard model, become energy-independent with the diffraction term of absolute neutrino mass less than $2eV/c^2$.

The total cross sections for the neutrino and anti-neutrino are written as

$$\sigma^{\nu} = \frac{M_N E_\nu G_F^2}{\pi}(Q + 1/3\bar{Q}) \tag{45}$$
$$\sigma^{\bar{\nu}} = \frac{M_N E_\nu G_F^2}{\pi}(\bar{Q} + 1/3Q)$$

using integrals of quark-parton distribution functions $q(x)$ and $\bar{q}(x)$, $Q = \int_0^1 dxxq(x), \bar{Q} = \int_0^1 dxx\bar{q}(x)$. Both are proportional to the neutrino energy and the current values are $\sigma_\nu/E = 0.67 \times 10^{-38}[cm^2/GeV], \sigma_{\bar{\nu}}/E = 0.34 \times 10^{-38}[cm^2/GeV]$. So experiments seem consistent with However, the recent experiments show that the ratio σ_ν/E_ν is decreasing with E_ν very slowly.

Conversly the false cross section is written as

$$\sigma(E)^{false}/E = (1 + r_{diff})(\sigma(E)^{true}/E). \tag{46}$$

$(\sigma(E)^{true}/E)$ is believed constant so the E-dependence of $\sigma(E)^{false}/E$ is due to E-dependence of r_{diff}.

Experiments

Comparisons

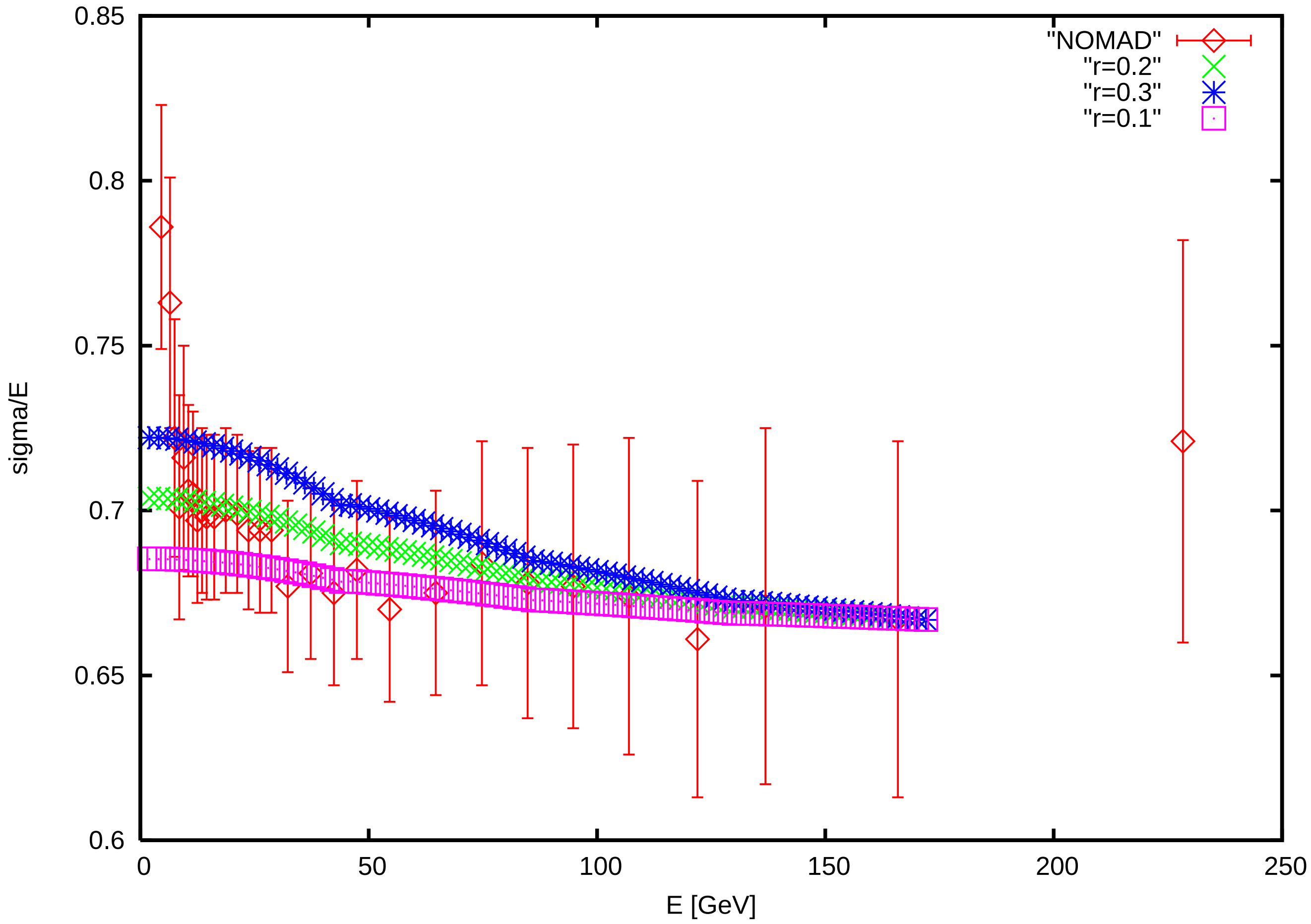


Figure 3: NOMAD

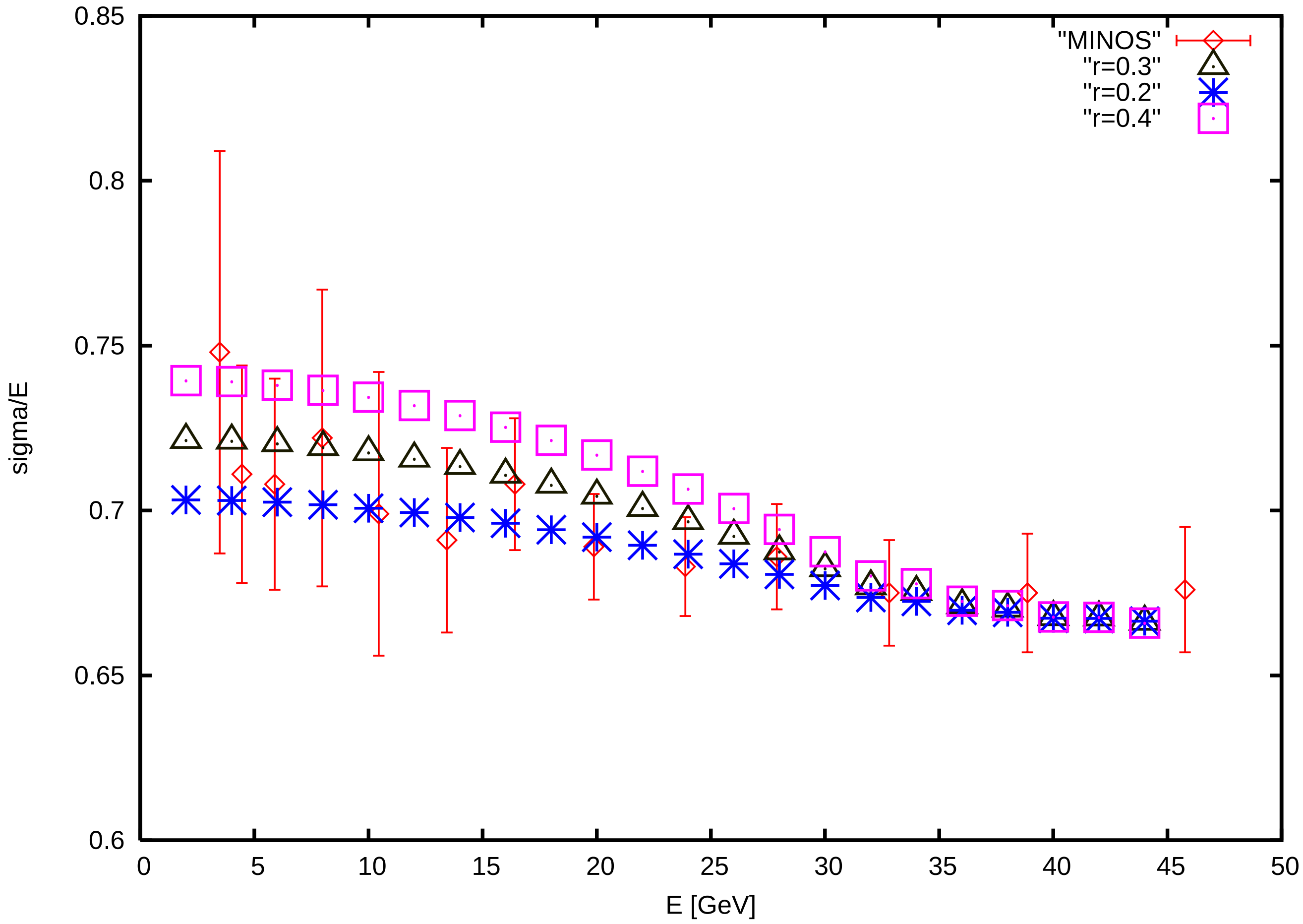


Figure 4: MINOS

T2K:electron neutrino spectrum is sensitive to neutrino mass at around $0.2 eV/c^2$

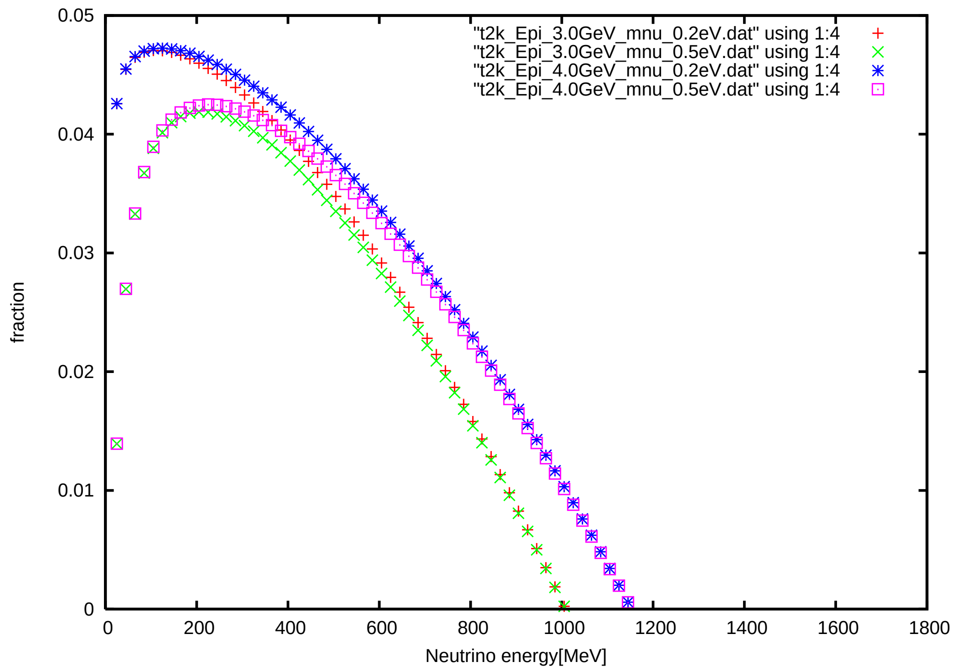


Figure 5: T2K off-axis: E dependence of the electron neutrino;m=0.2,0.5

5 Macroscopic trajectory and energy-momentum conservation of quantum particles

K I and Shimomura,PTP,114(2005),1201,K I and Tobita,PTP,122(2009),1111

generalized vertices of arbitrary wave packets

The product of the wave packets at $(t,\vec{x})$,

$$\Pi_j N_j^* e^{-\frac{1}{2\sigma_j}(\vec{x}-\vec{X}_j-\vec{v}_j(t-T_j))^2+iE(\vec{p}_j)(t-T_j)-i\vec{p}_j(\vec{x}-\vec{X}_j)} \times\Pi_l N_l e^{-\frac{1}{2\sigma_l}(\vec{x}-\vec{X}_l-\vec{v}_l(t-T_l))^2-iE(\vec{p}_l)(t-T_l)+i\vec{p}_l(\vec{x}-\vec{X}_l)} = \Pi_j N_j^* \Pi_l N_l e^{-\frac{1}{2\sigma_s}(\vec{x}-\vec{x}_0(t))^2-\frac{1}{2\sigma_t}(t-t_0)^2} e^{R+i\phi}. \quad (47)$$

Wave packet sizes in the spatial and temporal directions are

$$\frac{1}{\sigma_s}=\Sigma_j\frac{1}{\sigma_j},\frac{1}{\sigma_t}=\Sigma_j\frac{1}{\sigma_j}\vec{v}_j^2-\frac{1}{\sigma_s}\vec{v}_0^2 \quad (48)$$

and the central values of the space-time coordinates are

$$\vec{x}_0(t)=\vec{v}_0 t+\vec{x}_0(0),\vec{v}_0=\sigma_s\Sigma_j\frac{1}{\sigma_j}\vec{v}_j, \quad (49)$$

$$\vec{x}_0(0)=\sigma_s(\Sigma_j\frac{1}{\sigma_j}\tilde{\vec{X}}_j-i(\Sigma_j(\pm)\vec{p}_j)) \quad (50)$$

$$t_0=\sigma_t(\frac{1}{\sigma_s}\vec{v}_0\cdot\vec{x}_0-\Sigma_j\frac{1}{\sigma_j}\vec{v}_j\cdot\tilde{\vec{X}}_j+i\Sigma_j(\pm)E(\vec{p}_j)) \quad (51)$$

$$\tilde{\vec{X}}_j=\vec{X}_j-\vec{v}_jT_j. \quad (52)$$

The real part $\rightarrow$ magnitude $\rightarrow$ the trajectory and energy-momentum .

$$R=R_{trajectory}+R_{momentum}, \quad (53)$$

$$R_{trajectory}=-\Sigma_j\frac{1}{2\sigma_j}\tilde{\vec{X}}_j^{\,2}+2\sigma_s(\Sigma_j\frac{1}{2\sigma_j}\tilde{\vec{X}}_j)^2+2\sigma_t(\Sigma_j(\vec{v}_0-\vec{v}_j)\tilde{\vec{X}}_j)^2, \quad (54)$$

$$R_{momentum}=-\frac{\sigma_t}{2}(\Sigma_j(\pm)(E(\vec{p}_j)-\vec{v}_0\vec{p}_j))^2-\frac{\sigma_s}{2}(\Sigma_j(\pm)\vec{p}_j)^2. \quad (55)$$

The phase factor:

$$\phi = \phi_0 + \phi_1, \tag{56}$$

$$\phi_0 = \Sigma_j(\pm)(\vec{p}_j\vec{X}_j - E(\vec{P}_j)T_j), \tag{57}$$

$$\phi_1 = -2\sigma_t(\Sigma_j \frac{1}{2\sigma_j}(\vec{v}_0 - \vec{v}_j)\tilde{\vec{X}}_j)(\Sigma(\pm)\vec{v}_0(\vec{P}_j - E(\vec{p}_j)))$$
$$-2\sigma_s(\Sigma_j(\pm)\vec{p}_j)(\Sigma_j \frac{1}{2\sigma_j}\tilde{\vec{X}}_j). \tag{58}$$

6 Summary

1.Neutrino's (quantum mechanical) wave nature extends to huge area of few 100 meters.

2.Detector in the inside of this area sees interference (diffraction) component which has peculiar properties that are different from naive particle pictures.

3.Neutrino interferences are becoming important.