

$b \rightarrow s\ell\ell$ decays, Standard Model and New Physics: facts and fantasy

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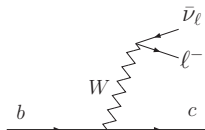
KEK, Sep 5th 2017



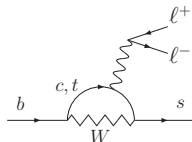
B-physics anomalies

Interesting times for B -physics

$$b \rightarrow c \ell \bar{\nu}_\ell$$



$$b \rightarrow s \ell^+ \ell^-$$

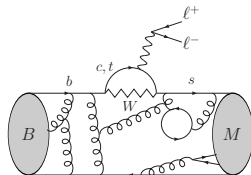
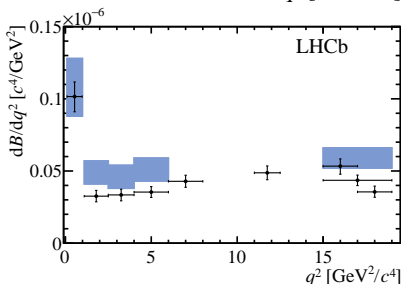
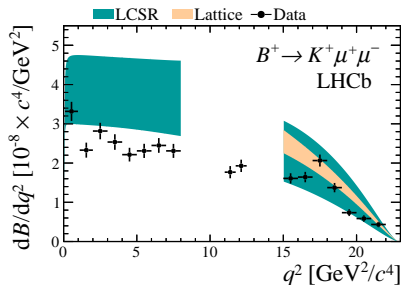


SM	tree (charged) ($V - A$)	loop (neutral)
Spin 0	$\bar{B} \rightarrow D \ell \bar{\nu}_\ell$	$B \rightarrow K \ell \ell$
Spin 1	$\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell$	$B \rightarrow K^* \ell \ell, B_s \rightarrow \phi \ell \ell$
Observables	Total Br + P_τ $\ell = \tau, \mu, e$	$d\Gamma/dq^2$ + Angular obs $\ell = \mu, e$
Tensions	$R_{D^{(*)}} = \frac{Br(B \rightarrow D^{(*)} \tau \nu)}{Br(B \rightarrow D^{(*)} \ell \bar{\nu}_\ell)}$	$R_{K^{(*)}} = \frac{Br(B \rightarrow K^{(*)} \mu \mu)}{Br(B \rightarrow K^{(*)} e e)}$ $Br(K, K^*, \phi + \mu \mu)$ angular obs (e.g., P'_5)

Two transitions exhibiting interesting patterns of **deviations from SM**
hinting at **Lepton Flavour Universality Violation**

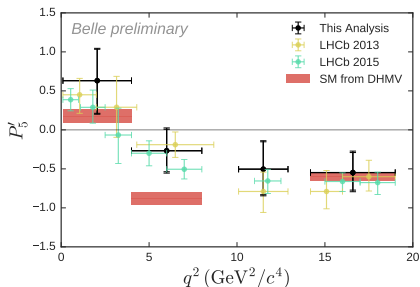
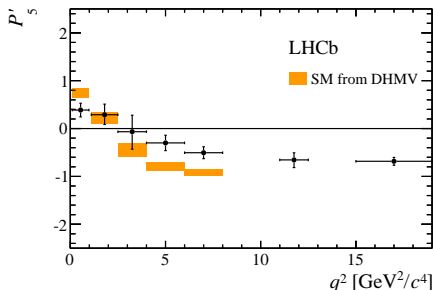
Here, focus on $b \rightarrow s \ell \ell$

Anomalies in branching ratios



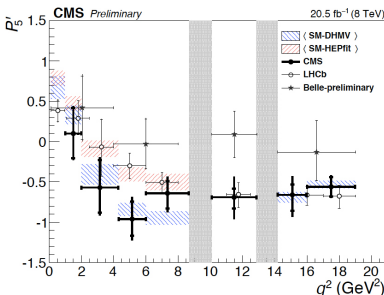
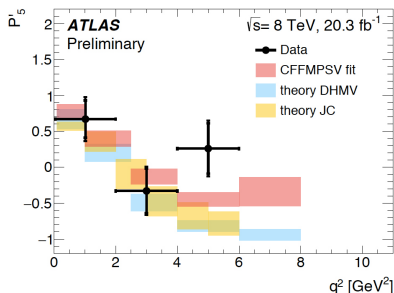
- $Br(B \rightarrow K \mu \mu)$ (up),
 $Br(B \rightarrow K^* \mu \mu)$ (down),
 $Br(B_s \rightarrow \phi \mu \mu)$ **too low** wrt SM
- q^2 invariant mass of $\ell \ell$ pair
- removing bins dominated by J/ψ and ψ' resonances
- large hadronic uncertainties from form factors at
 - Large-meson recoil/low q^2 : light-cone sum rules
 - Low-meson recoil/large q^2 : lattice QCD

Anomalies in angular observables (1)



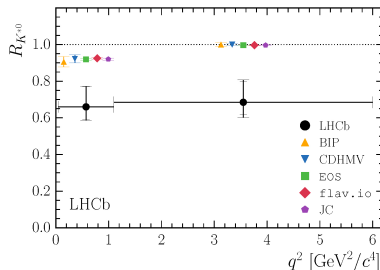
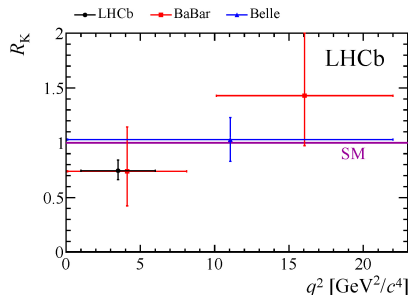
- Basis of optimised observables P_i (angular coeffs)
with **reduced hadronic uncertainties**
[Matias, Krüger, Becirevic, Schneider, Mescia, Virto, SDG, Ramon, Hurth; Hiller, Bobeth, Van Dyk...]
- Measured at LHCb with 1 fb⁻¹ (2013) and 3 fb⁻¹ (2015)
- Discrepancies for some (but not all) observables,
in particular two bins for P'_5 deviating from SM by **2.8 σ** and **3.0 σ**
- ... confirmed by Belle in 2016 (with larger uncertainties)

Anomalies in angular observables (2)



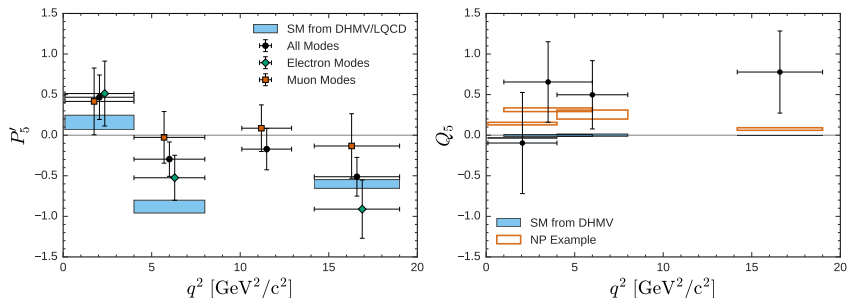
- ATLAS and CMS in 2017, but with larger uncertainties
- ATLAS: full basis, deviation in P'_5 (OK with LHCb) and P'_4 (not OK)
- CMS: only P_1 and P'_5 using input on F_L from earlier analyses (not clear why) leading to lower P'_5 than others
- There is more to $B \rightarrow K^* \mu\mu$ than just P'_5
 - P_2 also interesting deviations in LHCb 1 fb⁻¹ data in [2,4] bin (but not seen at 3 fb⁻¹ due to too large F_L leading to large uncert.)
 - useful that other optimised observables in agreement with SM

Anomalies in lepton flavour universality : Br



- LFU-test ratios $R_K = \frac{Br(B \rightarrow K \mu \mu)}{Br(B \rightarrow K e e)}$ and $R_{K^*} = \frac{Br(B \rightarrow K^* \mu \mu)}{Br(B \rightarrow K^* e e)}$ for LHCb
- hadronic uncertainties/effects cancel largely in the SM ($V - A$ interaction only) and for $q^2 \geq 1$ GeV² (m_ℓ effects negligible)
- in SM, a single form factor cancel in $R_K = 1$, but several polarisations and form factors in R_{K^*} (small q^2 -dep.)
- small effects of QED radiative corrections (1-3 %)
- LHCb: **2.6 σ for $R_{K[1,6]}$, 2.3 and 2.6 σ for $R_{K^*[0.045,1.1]}$ and $R_{K^*[1.1,6]}$**

Anomalies in LFU: angular observables



Belle also compared $b \rightarrow s e e$ and $b \rightarrow s \mu \mu$ in 2016

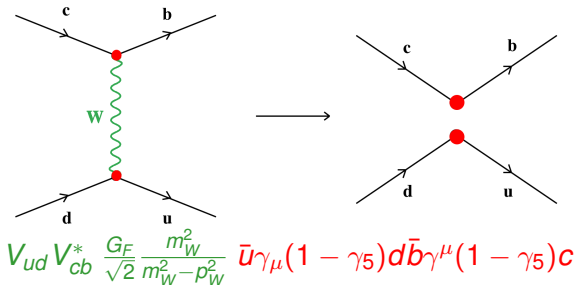
- different systematics from LHCb
- 2.6σ deviation for $\langle P'_5 \rangle_{[4,8]}^\mu$ versus 1.3σ deviation for $\langle P'_5 \rangle_{[4,8]}^e$
- same indication by looking at $Q_5 = P_5^{\mu'} - P_5^{e'}$, deviating from SM
- more data needed to confirm this hint of LFU violation (LFUV)

A global framework for the anomalies

Effective approaches

Fermi-like approach (for decoupling th): separation of different scales

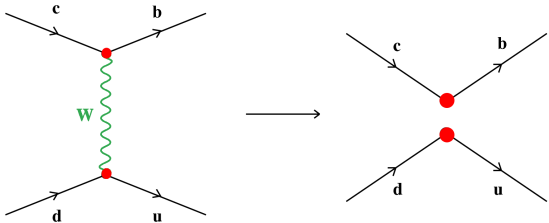
Short dist/Wilson coefficients and Long dist/local operator



Effective approaches

Fermi-like approach (for decoupling th): separation of different scales

Short dist/Wilson coefficients and Long dist/local operator


$$V_{ud} V_{cb}^* \frac{G_F}{\sqrt{2}} \bar{u} \gamma_\mu (1 - \gamma_5) d \bar{b} \gamma^\mu (1 - \gamma_5) c + O(1/M_W^2)$$

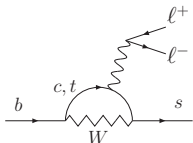
Fermi theory carries some info on the underlying (electroweak) theory

- G_F : scale of underlying physics
- O_i : interaction with left-handed fermions, through charged spin 1
- Losing some info (gauge structure, Z^0 ...)
but a good start if no particle ($=W, Z$) yet seen

Model-independent approach: \mathcal{H}_{eff}

$$b \rightarrow s \gamma^{(*)} : \mathcal{H}_{\Delta F=1}^{SM} \propto \sum V_{ts}^* V_{tb} \mathcal{C}_i \mathcal{O}_i + \dots$$

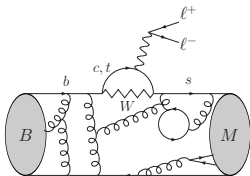
to separate short and long distances ($\mu_b = m_b$)



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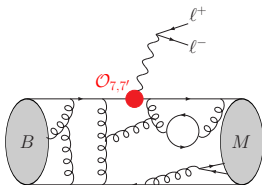
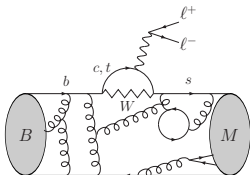


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- $\mathcal{O}_7 = \frac{e}{g^2} m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) F_{\mu\nu} b$ [real or soft photon]

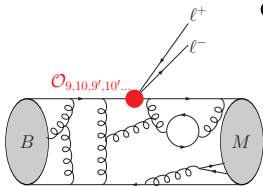
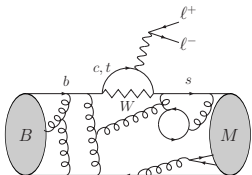


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- $\mathcal{O}_{10} = \frac{e^2}{g^2} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell$ [$b \rightarrow s\mu\mu$ via Z]

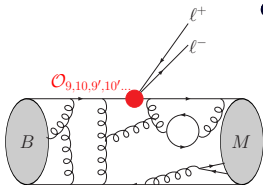
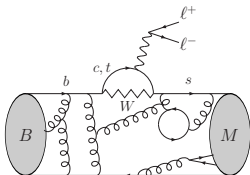


Model-independent approach: \mathcal{H}_{eff}

$$b \rightarrow s\gamma^{(*)} : \mathcal{H}_{\Delta F=1}^{\text{SM}} \propto \sum V_{ts}^* V_{tb} \mathcal{C}_i \mathcal{O}_i + \dots$$

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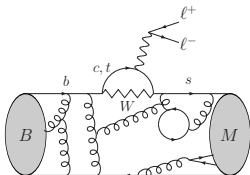
$$\mathcal{C}_7^{\text{SM}} = -0.29, \mathcal{C}_9^{\text{SM}} = 4.1, \mathcal{C}_{10}^{\text{SM}} = -4.3$$

$A = \mathcal{C}_i$ (short dist) \times Hadronic qties (long dist)

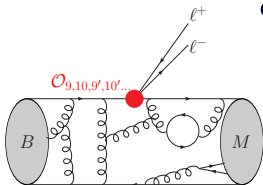
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NP changes short-distance \mathcal{C}_i or add new operators \mathcal{O}_i

- Chirally flipped ($W \rightarrow W_R$) $\mathcal{O}_7 \rightarrow \mathcal{O}_{7'} \propto \bar{s} \sigma^{\mu\nu} (1 - \gamma_5) F_{\mu\nu} b$
- (Pseudo)scalar ($W \rightarrow H^+$) $\mathcal{O}_9, \mathcal{O}_{10} \rightarrow \mathcal{O}_S \propto \bar{s} (1 + \gamma_5) b \bar{\ell} \ell, \mathcal{O}_P$
- Tensor operators ($\gamma \rightarrow T$) $\mathcal{O}_9 \rightarrow \mathcal{O}_T \propto \bar{s} \sigma_{\mu\nu} (1 - \gamma_5) b \bar{\ell} \sigma_{\mu\nu} \ell$

Global analysis of $b \rightarrow s\ell\ell$ anomalies

175 observables in total (no CP-violating obs) [\[Capdevila, Crivellin, SDG, Matias, Virto\]](#)

- $B \rightarrow K^* \mu\mu$ (Br, $P_{1,2}$, $P'_{4,5,6,8}$, F_L in large- and low-recoil bins)
- $B \rightarrow K^* ee$ ($P_{1,2,3}$, $P'_{4,5}$, F_L in large- and low-recoil bins)
- R_K , R_{K^*} , $Q_{4,5}$ (large-recoil bins)
- $B_s \rightarrow \phi \mu\mu$ (Br, P_1 , $P'_{4,6}$, F_L in large- and low-recoil bins)
- $B^+ \rightarrow K^+ \mu\mu$, $B^0 \rightarrow K^0 \mu\mu$ (Br in several bins)
- $B \rightarrow X_s \gamma$, $B \rightarrow X_s \mu\mu$, $B_s \rightarrow \mu\mu$, $B_s \rightarrow \phi \gamma$ (Br), $B \rightarrow K^* \gamma$ (Br, A_I , $S_{K^* \gamma}$)

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Various computational approaches

- inclusive: OPE
- excl large-meson recoil: QCD fact, Soft-collinear effective theory
- excl low-meson recoil: Heavy quark eff th, Quark-hadron duality

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Frequentist analysis

- $C_i(\mu_{ref}) = C_i^{SM} + C_i^{NP}$, with C_i^{NP} assumed to be real (no CPV)
- Experimental correlation matrices provided
- Theoretical inputs (form factors...) with correlation matrix computed treating all theo errors as Gaussian random variables

1D and 2D fits for NP in $b \rightarrow s\mu\mu$ only

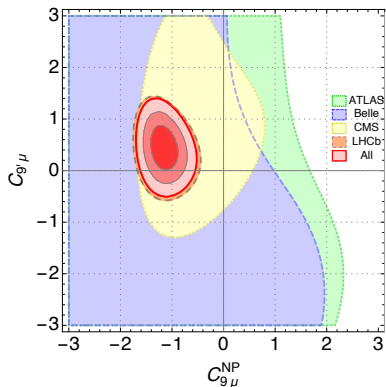
- All: 175 obs
- LFUV: 17 obs ($b \rightarrow s\mu\mu$ LFUV, $b \rightarrow s\gamma$, $B_s \rightarrow \mu\mu$, $B \rightarrow X_s\mu\mu$)
- Hypotheses “NP in some \mathcal{C}_i only” to be compared with SM

1D Hyp.	All				LFUV			
	Bfp	1 σ	Pull _{SM}	p-value %	Bfp	1 σ	Pull _{SM}	p-value %
$C_{9\mu}^{\text{NP}}$	-1.11	[-1.28, -0.94]	5.8	68	-1.76	[-2.36, -1.23]	3.9	69
$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}}$	-0.62	[-0.75, -0.49]	5.3	58	-0.66	[-0.84, -0.48]	4.1	78
$C_{9\mu}^{\text{NP}} = -C_{9'\mu}$	-1.01	[-1.18, -0.84]	5.4	61	-1.64	[-2.13, -1.05]	3.2	32

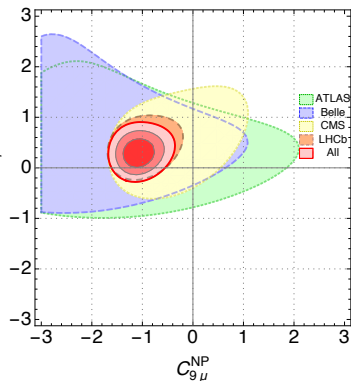
2D Hyp.	All			LFUV		
	Best fit	Pull _{SM}	p-value %	Best fit	Pull _{SM}	p-value %
$(C_{9\mu}^{\text{NP}}, C_{10\mu}^{\text{NP}})$	(-1.01, 0.29)	5.7	72	(-1.30, 0.36)	3.7	75
$(C_{9\mu}^{\text{NP}}, C_7')$	(-1.13, 0.01)	5.5	69	(-1.85, -0.04)	3.6	66
$(C_{9\mu}^{\text{NP}}, C_{9'\mu})$	(-1.15, 0.41)	5.6	71	(-1.99, 0.93)	3.7	72
$(C_{9\mu}^{\text{NP}}, C_{10'\mu})$	(-1.22, -0.22)	5.7	72	(-2.22, -0.41)	3.9	85

- p -value : χ_{\min}^2 considering N_{dof} (SM: All 11.3%, LFUV 4.4%)
 \Rightarrow **goodness of fit**: does the hypothesis give an overall good fit ?
- Pull_{SM} : $\chi_{\min}^2(C_i = 0) - \chi_{\min}^2$
 \Rightarrow **metrology**: how much does the hyp. solve SM deviations ?

Some favoured scenarios



NP in $C_{9\mu}^{NP}, C_{9\mu}^{NP}$



NP in $C_{9\mu}^{NP}, C_{10\mu}^{NP}$

- NP in C_9 only: p -value=68%, $\text{pull}_{SM} = 5.8\sigma$, $[-1.28, -0.94]$ at 1σ
- $C_9^{NP} = -C_{10}^{NP}$ good scenario (NP models obeying $SU(2)_L$)
- $C_9^{NP} \simeq -1$ favoured in all “good” scenarios
- 3σ regions, apart from combination with 1,2,3 σ
- LHCb dominates the field !

Improving on the main anomalies

Largest pulls	$\langle P'_5 \rangle_{[4,6]}$	$\langle P'_5 \rangle_{[6,8]}$	$R_K^{[1,6]}$	$R_{K^*}^{[0.045,1.1]}$
Experiment	-0.30 ± 0.16	-0.51 ± 0.12	$0.745^{+0.097}_{-0.082}$	$0.66^{+0.113}_{-0.074}$
SM pred.	-0.82 ± 0.08	-0.94 ± 0.08	1.00 ± 0.01	0.92 ± 0.02
Pull (σ)	-2.9	-2.9	+2.6	+2.3
Pred. $\mathcal{C}_{9\mu}^{\text{NP}} = -1.1$	-0.50 ± 0.11	-0.73 ± 0.12	0.79 ± 0.01	0.90 ± 0.05
Pull (σ)	-1.0	-1.3	+0.4	+1.9

Largest pulls	$R_{K^*}^{[1.1,6]}$	$\mathcal{B}_{B_s \rightarrow \phi \mu^+ \mu^-}^{[2,5]}$	$\mathcal{B}_{B_s \rightarrow \phi \mu^+ \mu^-}^{[5,8]}$
Experiment	$0.685^{+0.122}_{-0.083}$	0.77 ± 0.14	0.96 ± 0.15
SM pred.	1.00 ± 0.01	1.55 ± 0.33	1.88 ± 0.39
Pull (σ)	+2.6	+2.2	+2.2
Pred. $\mathcal{C}_{9\mu}^{\text{NP}} = -1.1$	0.87 ± 0.08	1.30 ± 0.26	1.51 ± 0.30
Pull (σ)	+1.2	+1.8	+1.6

⇒ Not all anomalies “solved”, but many are alleviated

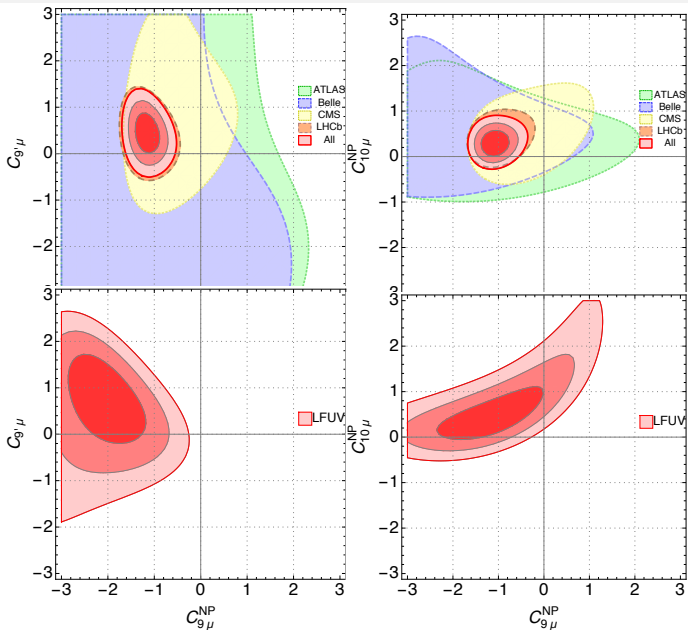
$b \rightarrow s_{\mu\mu}$: 6D hypothesis

Letting all 6 Wilson coefficients for muons vary (but only real)

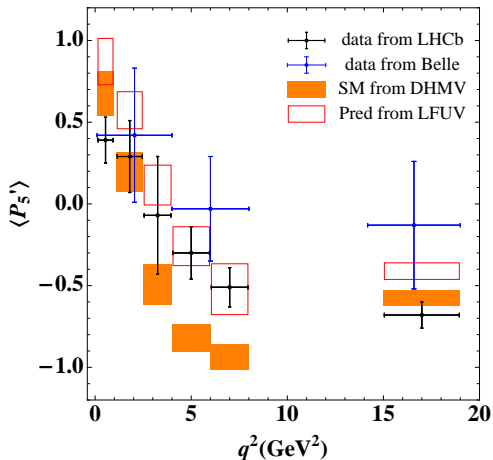
	Best fit	1σ	2σ
C_7^{NP}	+0.03	$[-0.01, +0.05]$	$[-0.03, +0.07]$
$C_{9\mu}^{\text{NP}}$	-1.12	$[-1.34, -0.88]$	$[-1.54, -0.63]$
$C_{10\mu}^{\text{NP}}$	+0.31	$[+0.10, +0.57]$	$[-0.08, +0.84]$
$C_{7'}$	+0.03	$[+0.00, +0.06]$	$[-0.02, +0.08]$
$C_{9'\mu}$	+0.38	$[-0.17, +1.04]$	$[-0.59, +1.58]$
$C_{10'\mu}$	+0.02	$[-0.28, +0.36]$	$[-0.54, +0.68]$

- Pattern: $C_7^{\text{NP}} \gtrsim 0$, $C_{9\mu}^{\text{NP}} < 0$, $C_{10\mu}^{\text{NP}} > 0$, $C_{7'} \gtrsim 0$, $C_{9'\mu} > 0$, $C_{10'\mu} \gtrsim 0$
- C_9 is consistent with SM only above 3σ
- All others are consistent with zero at 1σ except for C_{10} at 2σ
- Pull_{SM} for the 6D fit is 5.0σ (used to be 3.6σ)

Consistency between fits to All and LFUV obs

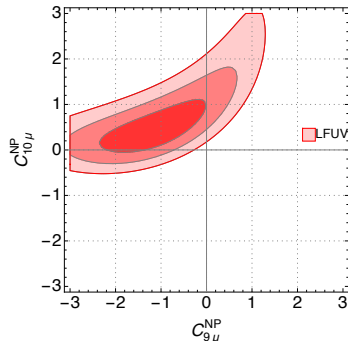
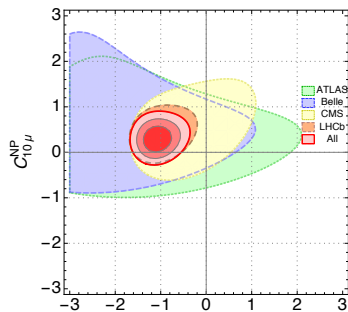


Consistency: P'_5 from LFUV obs



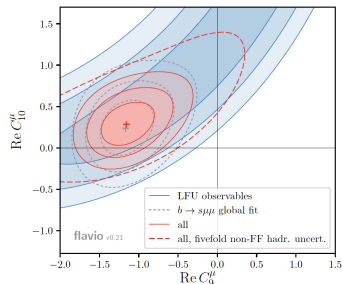
- Fit to LFUV obs only to determine $\mathcal{C}_{9\mu}^{NP}$
- ... then predict value of P'_5
- Confirms the very good agreement between fits to LFUV only and the other observables
- Disagreements with Standard Model in $b \rightarrow s\ell\ell$
obey a pattern

Consistency with analysis of (Altmannshofer, Stangl, Straub)



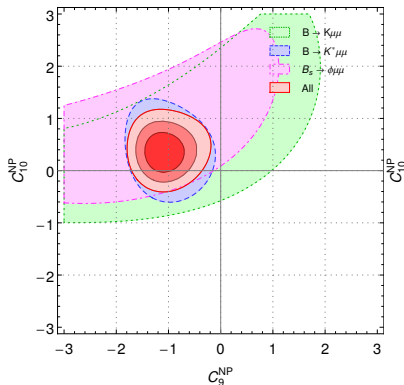
[Capdevila, Crivellin, SDG, Matias, Vito]

- Different observables (P_i or J_i)
- Different form factor inputs
- Different treatments of hadronic corrections
- Same NP scenarios favoured (higher significances for ASS)

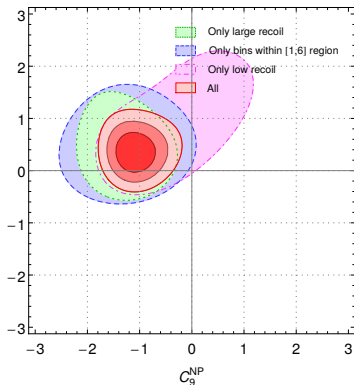


[Altmannshofer, Stangl, Straub]

Consistency: by channels, low versus large recoil



Split by decay channel



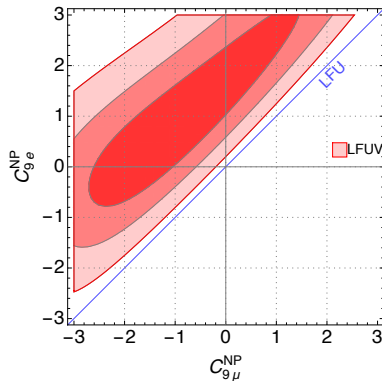
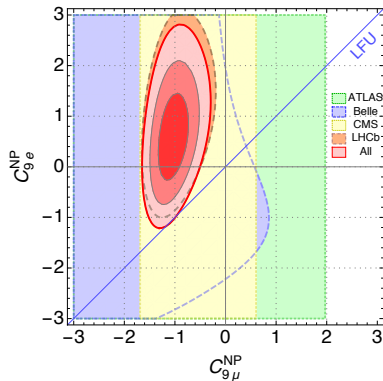
Split by q^2 region

- Analysis prior to R_{K^*} , with only LHCb data
- Different processes, kinematic ranges, theoretical tools
- $B \rightarrow K^*\mu\mu$ tighter than $B_s \rightarrow \phi\mu\mu$, tighter than $B \rightarrow K\mu\mu$
- Large and low recoil bins both favour points away from SM

[SDG, Hofer, Matias, Virto]

[Horgan et al., Bouchard et al., Altmannshofer and Straub]

NP in both $b \rightarrow s\mu\mu$ and $b \rightarrow see$



NP in $C_{9e}^{NP}, C_{9\mu}^{NP}$

- Up to now, only NP in $b \rightarrow s\mu\mu$, what about $b \rightarrow see$?
- Necessity to have a NP contribution for $C_{9\mu}$ but no need for C_{9e}
- But not forbidden either: for instance, $C_{9\mu} = -3C_{9e}$ very good ($U(1)$ models for neutrino mixing [Bhatia, Chakraborty, Dighe])

Confirming the interpretation

$$C_9^{\text{NP}} = C_9^{\text{New Physics}} \text{ or } C_9^{\text{Non Perturbative}} ?$$

Anomalies can be a sign from many things

- unlucky statistical fluctuations

Collect more data (more runs)

- underestimated syst in the experimental analysis

Cross-checks from different experiments (LHCb vs Belle/Belle II)

- underestimated syst in the theoretical computation

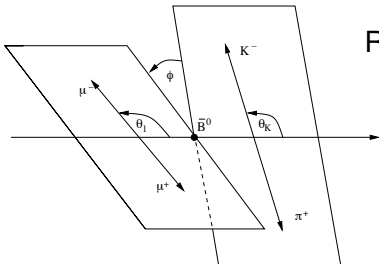
Check and recheck the hypotheses of computation

- something really new...

Add more observables, and interpret

Exclusive $b \rightarrow s\mu\mu$ decays play an important role in global fits
necessary to cross-checks SM computations !

$$B \rightarrow K^*(\rightarrow K\pi)\mu\mu$$

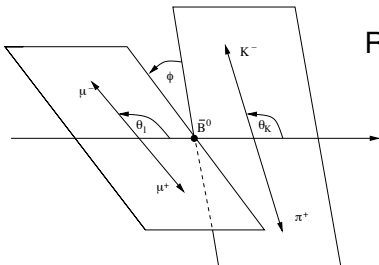


Rich kinematics

- differential decay rate in terms of 12 **angular coeffs** $J_i(q^2)$
with $q^2 = (p_{\ell^+} + p_{\ell^-})^2$
- interferences between 8 **transversity amplitudes** for $B \rightarrow K^*(\rightarrow K\pi)V^*(\rightarrow \ell\ell)$

[Ali, Hiller, Matias, Krüger, Mescia, SDG, Virto, Hofer, Bobeth, van Dyck, Buras, Altmanshoffer, Straub, Bharucha, Zwicky, Gratrex, Hopfer, Becirevic, Sumensari, Zukanovic-Funchal ...]

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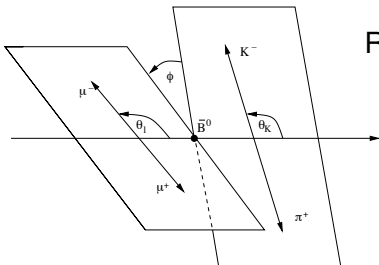
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- Transversity amplitudes (K^* polarisation, $\ell\ell$ chirality)
in terms of Wilson coefficients and 7 form factors $A_{0,1,2}$, V , $T_{1,2,3}$
- EFT relations between form factors in limit $m_B \rightarrow \infty$,
either when K^* very soft or very energetic (low/large-recoil)

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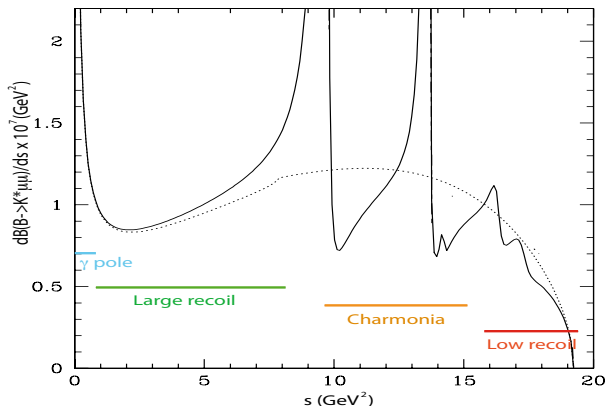
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- EFT relations between form factors in limit $m_B \rightarrow \infty$,
either when K^* very soft or very energetic (low/large-recoil)
- Build ratios of J_i where form factors cancel in these limits
- Optimised observables P_i with **reduced hadronic uncertainties**

[Matias, Krüger, Becirevic, Schneider, Mescia, Virto, SDG, Ramon, Hurth; Hiller, Bobeth, Van Dyk]

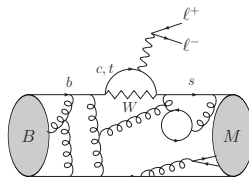
Low and large K^* recoils for $B \rightarrow K^* \mu \mu$



- Very large K^* -recoil ($4m_\ell^2 < q^2 < 1 \text{ GeV}^2$) γ almost real
- Large K^* -recoil ($q^2 < 9 \text{ GeV}^2$) energetic K^* ($E_{K^*} \gg \Lambda_{QCD}$)
Light-Cone Sum Rules, QCD factorisation, SCET
- Charmonium region ($q^2 = m_{\psi, \psi' \dots}^2$ between 9 and 14 GeV^2)
- Low K^* -recoil ($q^2 > 14 \text{ GeV}^2$) soft K^* ($E_{K^*} \simeq \Lambda_{QCD}$)
Lattice QCD, OPE, HQET

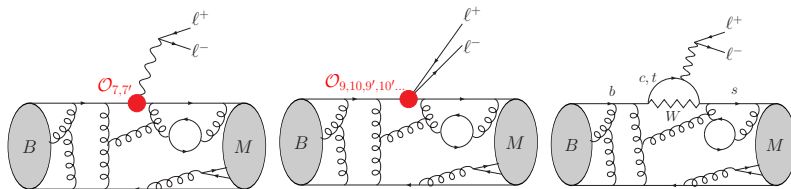
Two sources of hadronic uncertainties

$$A(B \rightarrow K^* \ell \ell) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* [(A_\mu + T_\mu) \bar{u}_\ell \gamma^\mu v_\ell + B_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell]$$



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Form factors (local)

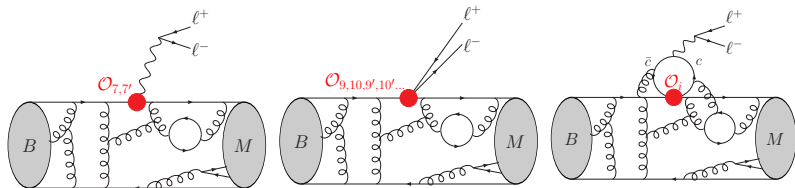
- Local contributions (more terms if NP in non-SM \mathcal{C}_i): **form factors**

$$A_\mu = -\frac{2m_b q^\nu}{q^2} C_7 \langle V_\lambda | \bar{s} \sigma_{\mu\nu} P_R b | B \rangle + C_9 \langle V_\lambda | \bar{s} \gamma_\mu P_L b | B \rangle$$

$$B_\mu = C_{10} \langle V_\lambda | \bar{s} \gamma_\mu P_L b | B \rangle \quad \lambda : K^* \text{ helicity}$$

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Form factors (local)

Charm loop (non-local)

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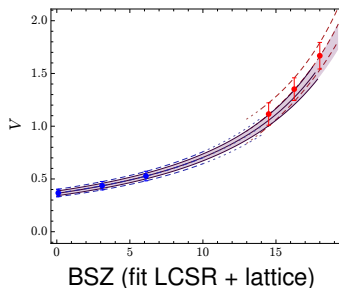
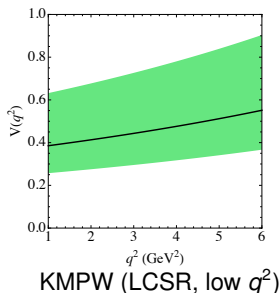
$$B_\mu = C_{10} \langle V_\lambda | \bar{s} \gamma_\mu P_L b | B \rangle \quad \lambda : K^* \text{ helicity}$$

- Non-local contributions (charm loops): **hadronic contris.**

T_μ contributes like $O_{7,9}$, but depends on q^2 and external states

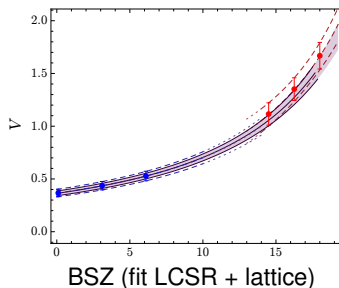
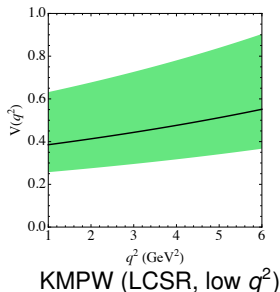
Form factors

- low K^* recoil: **lattice**, with correlations [Horgan, Liu, Meinel, Wingate]
- large K^* recoil: **B-meson Light-Cone Sum Rule**, large error bars and no correlations [Khodjamirian, Mannel, Pivovarov, Wang]
- all: fit K^* -meson LCSR + lattice, small errors bars, correlations [Bharucha, Straub, Zwicky]



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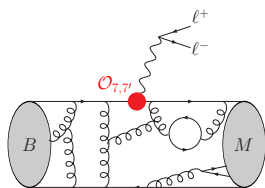


Reduce uncertainties and restore correlations

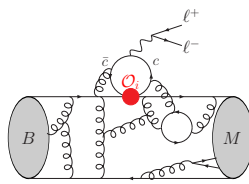
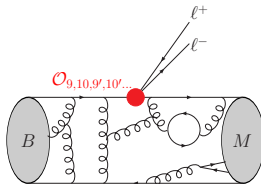
using **EFT correlations** arising in $m_b \rightarrow \infty$, e.g., at large K^* recoil

$$\xi_{\perp} = \frac{m_B}{m_B + m_{K^*}} V = \frac{m_B + m_{K^*}}{2E_{K^*}} A_1 = T_1 = \frac{m_B}{2E_{K^*}} T_2 + O(\alpha_s, \Lambda/m_b) \text{ corr}$$

Form factors and power corrections

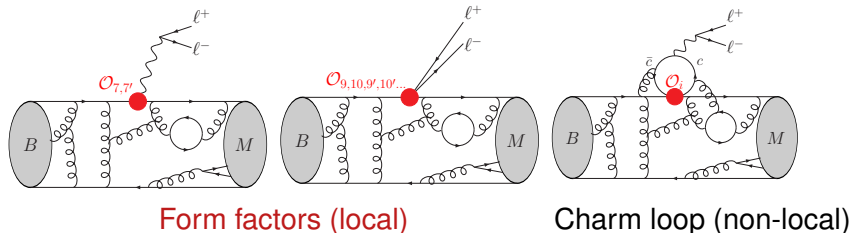


Form factors (local)



Charm loop (non-local)

Form factors and power corrections

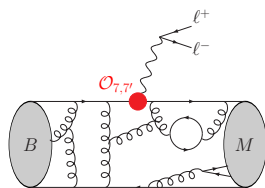


Uncertainties in form factors ?

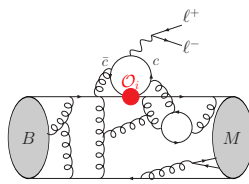
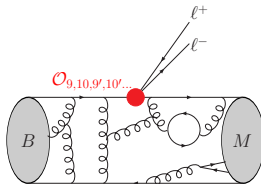
[Camalich, Jäger; Matias, Virto, Hofer, Capdevilla, SDG]

- EFT with limit $m_b \rightarrow \infty$ useful to correlate form factors
but $O(\Lambda/m_b)$ **power corrections** to this limit
- Power corrs with large impact on optimised observables ?

Form factors and power corrections



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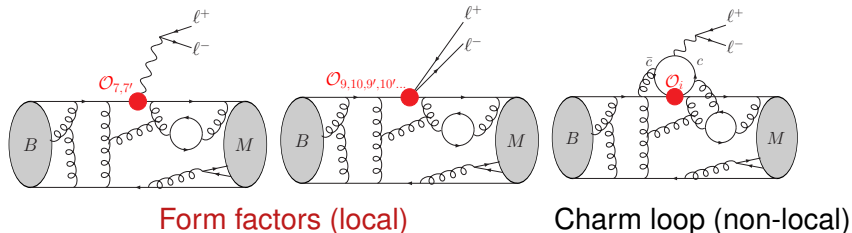
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- Power corrs with large impact on optimised observables ?
- No, but accurate predictions require
 - appropriate def of soft form factors $\xi_{\perp,||}$ in $m_b \rightarrow \infty$ limit (scheme)
 - correlations from EFT (heavy-quark sym.) among form factors
 - power corrections varied in agreement with info on form factors

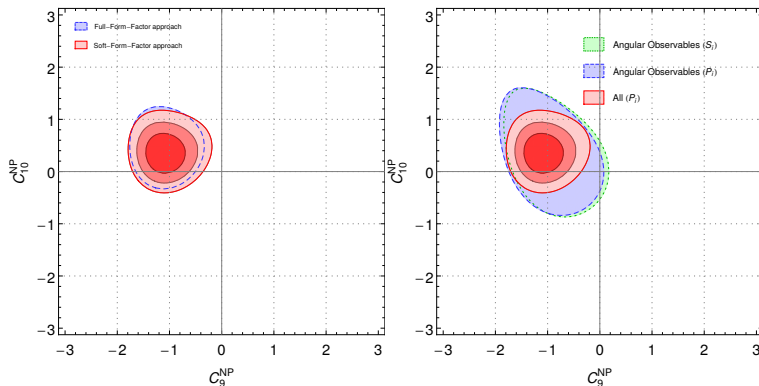
Form factors and power corrections



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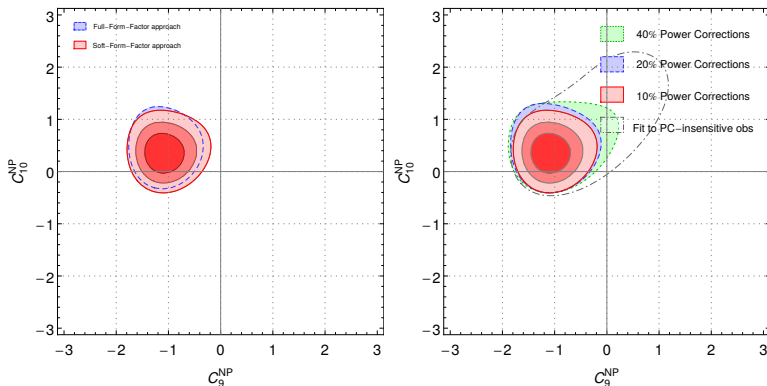
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 - power corrections varied in agreement with info on form factors
- [Camalich, Jäger] artefacts from ill-advised scheme/variation for pcs



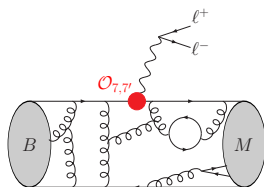
- Soft form factor approach ([Khodjamirian et al.] ff + EFT correls) vs full ff ([Altmannshofer, Straub] with [Bharucha et al.] ff with correls and small errors)
- Similar results using either optimised or angular coeffs (if correlations of form factors included through EFT)

Cross-checks: F. factors & power corrs (SDG, Hofer, Matias, Virto)

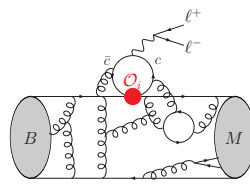
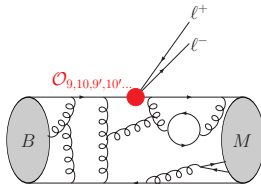


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- Increasing power corrections weakens role of large recoil, but low recoil enough to pull fit away from the SM

Charm-loop contribution

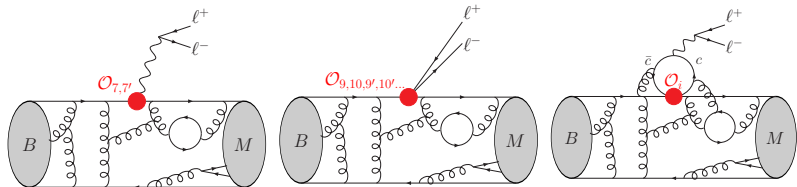


Form factors (local)



Charm loop (non-local)

Charm-loop contribution



Form factors (local)

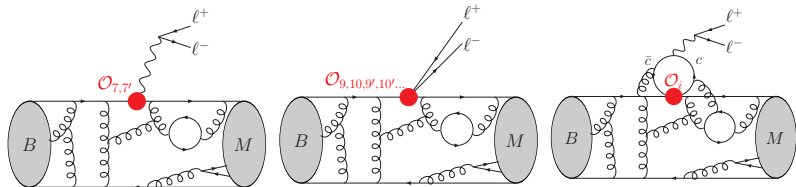
Charm loop (non-local)

Uncertainties from charm loops ?

[Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli]

- Effect well-known (loop process, charmonium resonances)
- Yields q^2 - and hadron-dependent contrib with $\mathcal{O}_{7,9}$ -like structures
 - Contribution $\Delta C_9^{BK(*)}$ from LCSR computation [Khodjamirian et al.]
 - Global fits use this result as order of magn, or $O(\Lambda/m_b)$ estimates

Charm-loop contribution



Form factors (local)

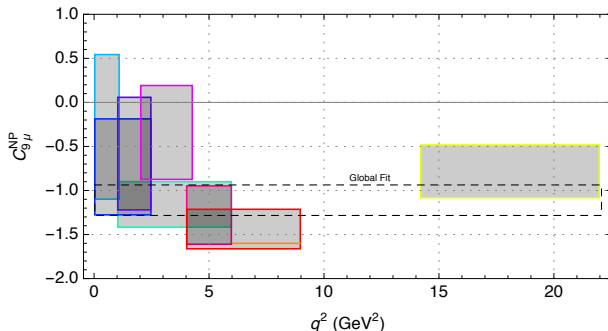
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 - Global fits use this result as **order of magn**, or $O(\Lambda/m_b)$ estimates
- Bayesian extraction from $B \rightarrow K^* \mu\mu$ performed by [Ciuchini et al.]
 - q^2 dependence in agreement with $\Delta\mathcal{C}_9^{BK(*)} + \text{constant } \mathcal{C}_9^{\text{NP}}$
 - no need for extra q^2 -dep. contribution (no missed hadronic contrib)
 - actually not contradicting results of global fits, though less precise

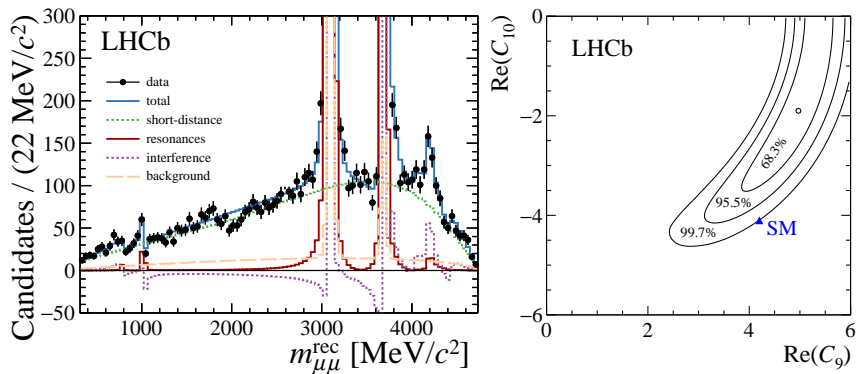
[Matias, Virto, Hofer, Capdevilla, SDG; Hurth, Mahmoudi, Neshatpour]



[Capdevila, Crivellin, Matias, Virto, SDG]

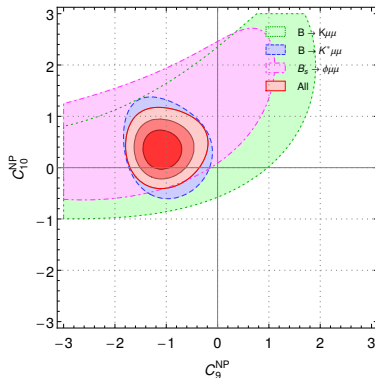
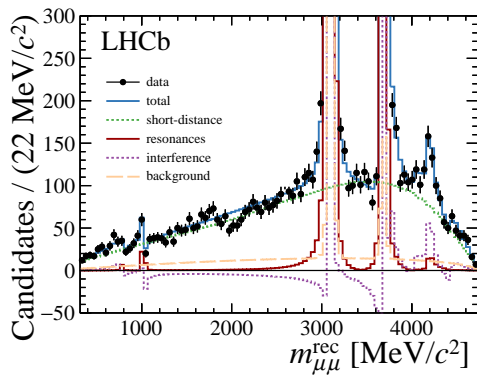
- Fit to $\mathcal{C}_9^{\text{NP}}$ from individual bins of $b \rightarrow s\mu\mu$ data (NP only in $\mathcal{C}_{9\mu}$)
 - NP in \mathcal{C}_9 from short distances, q^2 -independent
 - Hadronic physics in \mathcal{C}_9 related to $c\bar{c}$ dynamics, (likely) q^2 -dependent
- No indication of additional q^2 -dependence missed by the fit
- Can be checked for other NP scenarios
- In agreement with similar findings in [Altmanshoffer, Straub]

Charm loop from resonances in $B \rightarrow K\ell\ell$ data



- $C_9^{\text{eff}} = C_9^{\text{SD}} + \text{sum of resonant Breit-Wigner } (\omega, \rho^0, \phi, \text{ charmonia})$
- LHCb data driven fit to couplings and phases, as well as C_9, C_{10}
- 4 equivalent sols, with tiny contrib from resonances below J/ψ

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- LHCb data driven fit to couplings and phases, as well as C_9, C_{10}
- 4 equivalent sols, with tiny contrib from resonances below J/ψ
- agrees with (tiny) ΔC_9^{BK} [Khodjamirian et al.] (C_9, C_{10}) OK with global fits

Data-driven charm loop contribution (1)

[Bobeth, Chrzaszcz, Van Dyk, Virto]

Rather than fitting unphysical polynomial with arbitrary coefficients

- Known **analytic structure** of charm loop contribution
 - Analytical up to poles and a cut starting $q^2 = 4M_D^2$
 - Inherit all singularities from form factors (M_{B_s} pole for instance)
- Appropriate **parametrisation valid up to $D\bar{D}$ cut**
 - z -expansion (better conv below cut, mapped into disc $|z| \leq 1$)
 - Poles for J/ψ and ψ' + good asymptotic behaviour

$$\eta_\alpha^* \mathcal{H}^{\alpha\mu} = i \int d^4x e^{iq \cdot x} \langle \bar{K}^*(k, \eta) | T \{ j_{\text{em}}^\mu(x), \mathcal{C}_2 \mathcal{O}_2(y) \} | \bar{B}(p) \rangle$$

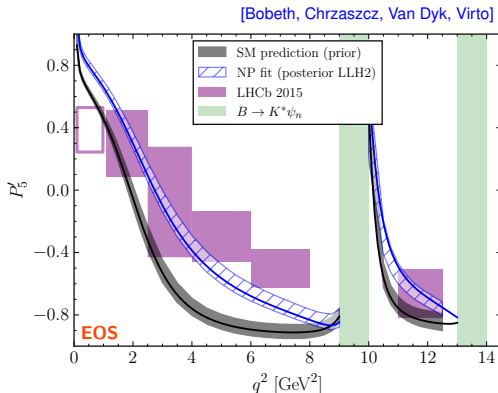
$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = 4M_D^2, \quad t_0 = t_+ - \sqrt{t_+(t_+ - M_{\psi(2S)}^2)}$$

$$\mathcal{H}_\lambda(z) = \frac{1 - z z_{J/\psi}^*}{z - z_{J/\psi}} \frac{1 - z z_{\psi(2S)}^*}{z - z_{\psi(2S)}} \left[\sum_{k=0}^{K \leq 2} \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)$$

Data-driven charm loop contribution (2)

- Exploit info to **determine the coefficients**

- Experimental info: discarded LHCb bins to fix J/ψ and ψ' residues
- Theoretical info: LCSR for $q^2 \leq 0$ (most accurate)



- Compute the observables

- $c\bar{c}$ contribution in agreement with earlier estimates
- P'_5 for SM in disagreement with LHCb data
- Agreement if $C_9^{NP} \simeq -1.1$
- Access to intermediate region between J/ψ and ψ'
- Extension possible to other $b \rightarrow s\ell\ell$ modes

Moving forward

The need for more observables

A few interesting outcomes of the analysis

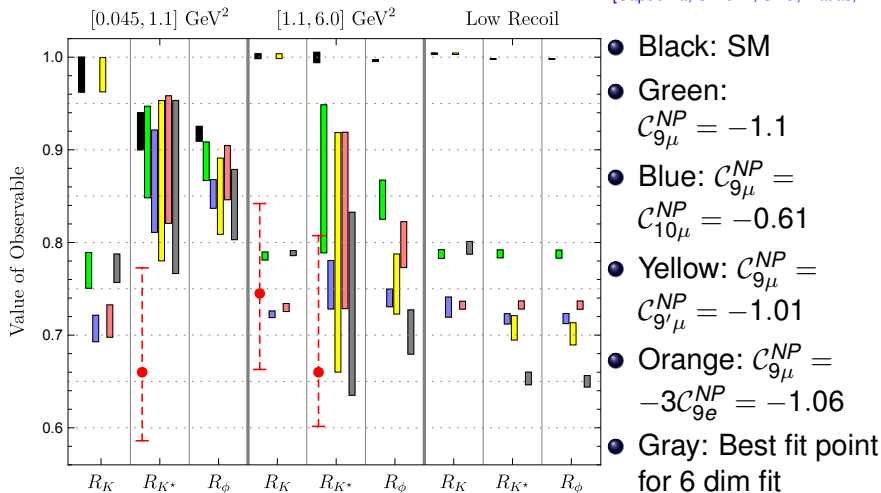
- Large deviation for $C_{9\mu}$ from SM
- **Potential deviations** for $C_{9'\mu}$ and $C_{10\mu}$
- Small (or vanishing) deviations for $b \rightarrow$ see Wilson coefficients

Useful to have more observables to

- **reduce uncertainties** in determination of Wilson coefficients
- identify subleading deviations wrt SM in $C_{9'\mu}$ and $C_{10\mu}$
(cannot be mimicked by long-distance contribution to $c\bar{c}$ loops)
- **confirm LFUV** and exploit it to build new observables

LFUV in branching ratios

[Capdevila, Crivellin, SDG, Matias, Virto]



R_{K^*} with conservative [Khodjamirian et al] but R_ϕ computed with [Bharucha et al]

LFUV in angular observables: Q_i, B_i, M

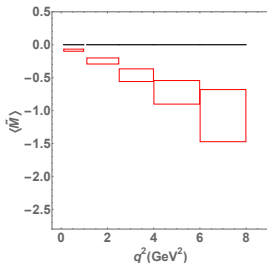
[Capdevilla, Matias, Virto, SDG]

Expecting measurements of BR and angular coefficients for $B \rightarrow K^* e e$

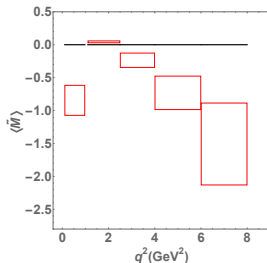
- null SM tests (up to m_ℓ effects): $Q_i = P_i^\mu - P_i^e$, $B_i = \frac{J_i^\mu}{J_i^e} - 1$
- angular coeffs J_5 and J_{6s} with only a linear dependence on \mathcal{C}_9

$$M = (J_5^\mu - J_5^e)(J_{6s}^\mu - J_{6s}^e)/(J_{6s}^\mu J_5^e - J_{6s}^e J_5^\mu)$$

- cancellation of hadronic contris in \mathcal{C}_9 if NP in $\mathcal{C}_{9\mu}$ only
- different sensitivity to NP scenarios compared to $R_{K(*)}$



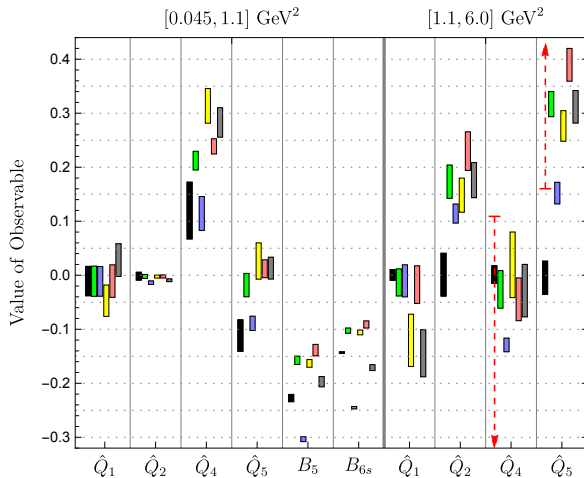
$$\mathcal{C}_{9\mu}^{\text{NP}} = -1.1, \mathcal{C}_{ie}^{\text{NP}} = 0$$



$$\mathcal{C}_{9\mu}^{\text{NP}} = \mathcal{C}_{10\mu}^{\text{NP}} = -0.65, \mathcal{C}_{ie}^{\text{NP}} = 0$$

LFUV in angular observables: Q_i, B_i

[Capdevila, Crivellin, SDG, Matias, Virto]



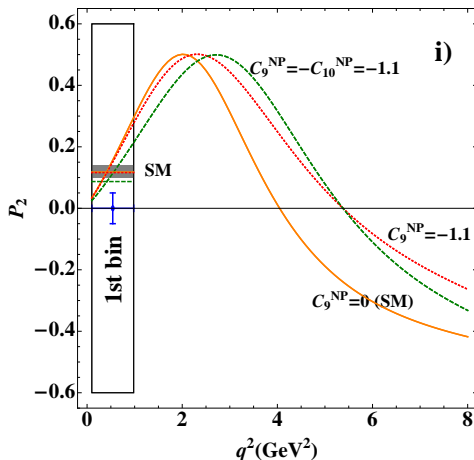
- Black: SM
- Green: $C_{9\mu}^{NP} = -1.1$
- Blue: $C_{9\mu}^{NP} = -0.61$
- Yellow: $C_{9\mu}^{NP} = -1.01$
- Orange: $C_{9\mu}^{NP} = -1.06$
- Gray: Best fit point for 6 dim fit

- Precise measurement of Q_5 in $[1,6]$ can discard $C_{9\mu}^{NP} = -C_{10\mu}^{NP}$
- Other useful to separate various scenarios

Additional observables: P_1 and P_2 at very low q^2

At very low q^2 , C_9 kinematically suppressed in P_1 and P_2

⇒ way of probing other Wilson coefficients



Probes of other Wilson coefficients

- $P_1 \leftrightarrow C_{7(\prime)}$ (not competitive with $B \rightarrow X_S \gamma$)
- $P_2 \leftrightarrow C_7 C_{10}, C_{7'} C_{10'}$ (interesting for $C_{10(\prime)}$)

[Becirevic, Schneider, Capdevila, Hofer, Matias, SDG]

NP interpretations

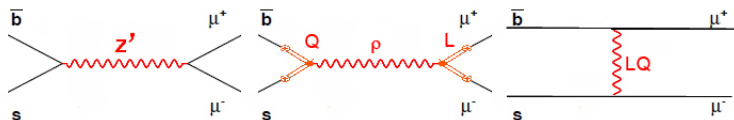
No consistent global alternative from SM/long-dist. for $b \rightarrow s\ell\ell$

- hadronic effects ($B \rightarrow K^* \mu\mu$, $B_s \rightarrow \phi\mu\mu$ at low and large recoils)
- statistical fluctuation and/or pb with e/mu (R_K , R_{K^*})
- bad luck (short-distance scenarios can accomodate all discrepancies very well by chance)

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NP models with new scale around TeV

- Z' boson and leptoquarks are favourite
- Partial compositeness and NP in $b \rightarrow c \bar{c} s$ also investigated
- but susy (MSSM) not favoured (hard to generate C_9 -like contribution without having flavour problems in other places)

[Buras, De Fazio, Girschbach, Blanke, Altmannshofer, Straub, Crivellin, D'Ambrosio,

Becirevic, Sumensari, Isidori, Greljo, Jäger, Lenz. . .]

B physics anomalies

- $b \rightarrow s\ell^+\ell^-$ with many obs., more or less sensitive to hadronic unc.
- Interesting deviations from SM expectations
- Indications of violation of lepton flavour universality
- Global fit supports large $C_{9\mu}^{NP}$ with very good consistency (Br vs angular vs R , channels, recoil regions, LFUV and All obs. . .)
- Does not seem to favour hadronic explanations (power corrections for form factors, charm loop contributions)

Where to go ?

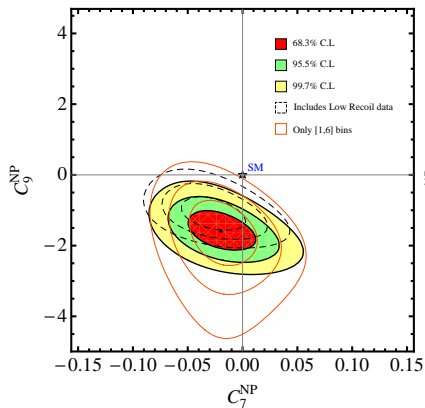
- Other LFU violating observables: R_ϕ , Q_i . .
- Charm loops (estimates, data-driven info on resonances, new obs)
- More determinations of form factors to control uncertainties
- More accurate constraints on other Wilson coefficients ($C_{9'}$, C_{10})
- Model building to connect with other anomalies (like $b \rightarrow c\ell\nu_\ell$)

A lot of (interesting) work on the way !

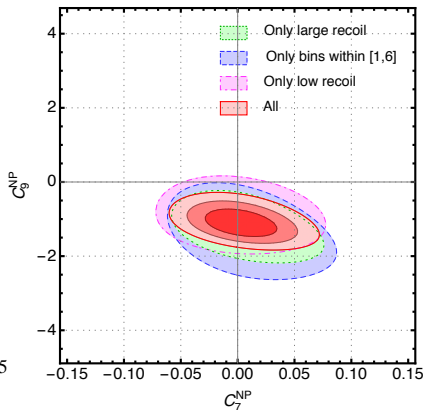
Thank you for your attention !

From 2013 to 2016

Many improvements from experiment and theory, but...



[SDG, J. Matias, Virto] (2013)



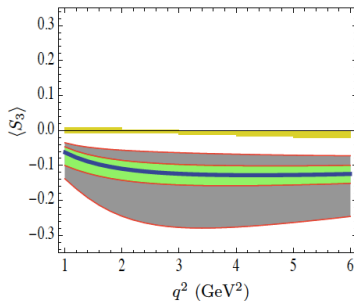
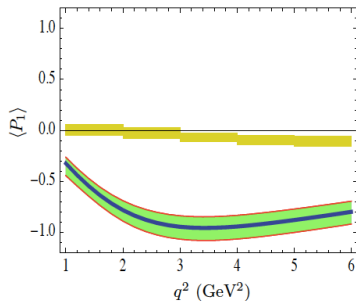
[SDG, L. Hofer J. Matias, Virto] (2016)

A few recent global fits (before R_{K^*})

Statistical approach	[SDG, Hofer Matias, Virto] Frequentist $\Delta\chi^2$	[Straub, Stangl & Altmannshofer] Frequentist $\Delta\chi^2$	[Hurth, Mahmoudi, Neshatpour] Frequentist $\Delta\chi^2$ & χ^2
Data	LHCb	Averages	LHCb
$B \rightarrow K^* \mu\mu$ data	P_i , Max likelihood	S_i , Max likelihood	S_i , Max l.& moments
Form factors	B-meson LCSR [Khodjamirian et al.] + lattice QCD	[Bharucha, Straub, Zwicky] fit light-meson LCSR + lattice QCD	[Bharucha, Straub, Zwicky]
Theo approach	soft and full ff	full ff	soft and full ff
$c\bar{c}$ large recoil	magnitude from [Khodjamirian et al.]	polynomial param	polynomial param
C_9^μ 1D 1σ pull _{SM}	[-1.22,-0.79] 4.2 σ	[-1.54,-0.53] 3.7 σ	[-0.27,-0.13] 4.2 σ
“good scenarios”	see before	$C_9^{\text{NP}}, C_{9'}^{\text{NP}} = -C_{10}^{\text{NP}}$ $(C_9^{\text{NP}}, C_{9'}^{\text{NP}}), (C_9, C_{10}^{\text{NP}})$	$(C_9^{\text{NP}}, C_{9'}^{\text{NP}}), (C_9^{\text{NP}}, C_{10}^{\text{NP}})$

⇒ Good overall agreement for the results of the three fits

Sensitivity of observables to form factors



- P_i designed to have limited sensitivity to form factors
- S_i CP-averaged version of J_i

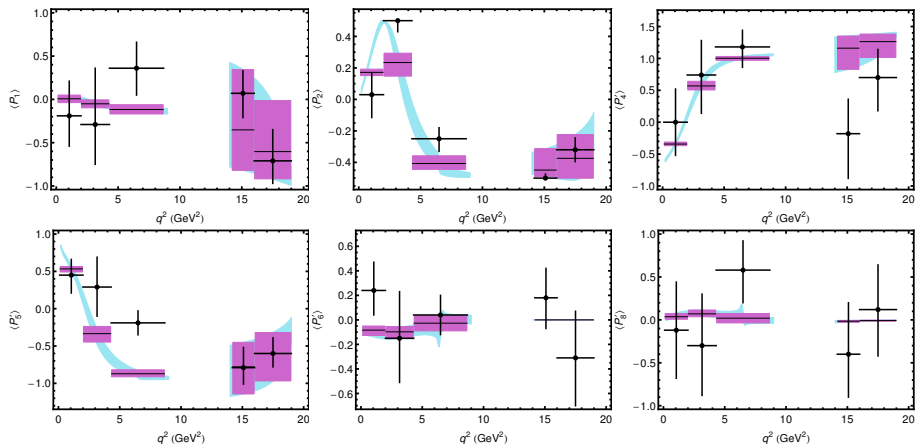
$$P_1 = \frac{2S_3}{1 - F_L} \quad F_L = \frac{J_{1c} + \bar{J}_{1c}}{\Gamma + \bar{\Gamma}} \quad S_3 = \frac{J_3 + \bar{J}_3}{\Gamma + \bar{\Gamma}}$$

Illustration for arbitrary NP point for two sets of LCSR form factors:

green [Ball, Zwicky] versus gray [Khodjamirian et al.]

more or less easy to discriminate against yellow (SM prediction)

SM predictions and LHCb results at 1 fb^{-1}



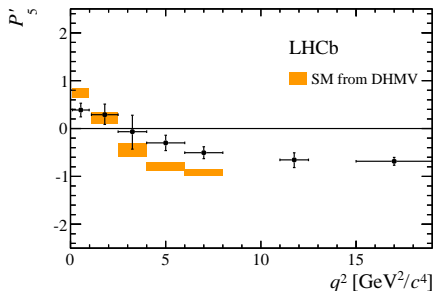
Meaning of the discrepancy in P_2 and P_5' ?

[SDG, Matias, Virto]

- P_2 same zero as A_{FB} , related to C_9/C_7
- $P_5' \rightarrow -1$ as q^2 grows due to $A_{\perp,||}^R \ll A_{\perp,||}^L$ for $C_9^{SM} \simeq -C_{10}^{SM}$
- A negative shift in C_7 and C_9 can move them in the right direction

Focus on P'_5

[SDG, J. Matias, M. Ramon, J. Virto]



$B \rightarrow K^* \mu \mu$ with $A_{\text{transversity}}^{\ell\ell}$ chirality

$$P'_5 = \sqrt{2} \frac{\text{Re}(A_0^L A_{\perp}^{L*} - A_0^R A_{\perp}^{R*})}{\sqrt{|A_0|^2(|A_{\perp}|^2 + |A_{\parallel}|^2)}}$$

LHCb measurements (crosses)
significantly away from SM
(boxes) in the large-recoil region

In large recoil limit with no right-handed current, with $\xi_{\perp, \parallel}$ ffs

$$A_{\perp, \parallel}^L \propto \pm \left[C_9 - C_{10} + 2 \frac{m_b}{s} C_7 \right] \xi_{\perp, \parallel}(s) \quad A_{\perp, \parallel}^R \propto \pm \left[C_9 + C_{10} + 2 \frac{m_b}{s} C_7 \right] \xi_{\perp, \parallel}(s)$$

$$A_0^L \propto - \left[C_9 - C_{10} + 2 \frac{m_b}{m_B} C_7 \right] \xi_{\parallel}(s) \quad A_0^R \propto - \left[C_9 + C_{10} + 2 \frac{m_b}{m_B} C_7 \right] \xi_{\parallel}(s)$$

- In SM, $C_9 \simeq -C_{10}$ leading to $|A_{\perp, \parallel}^R| \ll |A_{\perp, \parallel}^L|$
- If $C_9^{\text{NP}} < 0$, $|A_{0, \parallel, \perp}^R|$ increases, $|A_{0, \parallel, \perp}^L|$ decreases, $|P'_5|$ gets lower
- For P'_4 , sum with $A_{0, \parallel}$, so not sensitive to C_9 in the same way

Power corrections

- Factorisable power corrections (form factors)

- Parametrize power corrections to form factors (at large recoil):

$$F(q^2) = F^{\text{soft}}(\xi_{\perp,\parallel}(q^2)) + \Delta F^{\alpha_s}(q^2) + a_F + b_F \frac{q^2}{m_B^2} + \dots$$

- Fit a_F, b_F, \dots to the full form factor F (taken e.g. from LCSR)
 - Respect correlations among a_{F_i}, b_{F_i}, \dots and kinematic relations
 - Choose appropriate definition of $\xi_{\parallel,\perp}$ from form factors (scheme) or take into account correlations among form factors
- Vary power corrections as 10% of the total form factor around the central values obtained for $a_F, b_F \dots$

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- Nonfactorisable power corrections (extra part from amplitudes)

- Extract from $\langle K^* \gamma^* | H_{\text{eff}} | B \rangle$ the part not associated to form factors
- Multiply each of them with a complex q^2 -dependent factor

$$\mathcal{T}_i^{\text{had}} \rightarrow (1 + r_i(q^2)) \mathcal{T}_i^{\text{had}}, \quad r_i(s) = r_i^a e^{i\phi_i^a} + r_i^b e^{i\phi_i^b} (s/m_B^2) + r_i^c e^{i\phi_i^c} (s/m_B^2)^2.$$

- Vary $r_i^{a,b,c} = 0 \pm 0.1$ and phase $\phi_i^{a,b,c}$ free for $i = 0, \perp, \parallel$

Correlating form factors

Implement correlations among form factors

- **Soft form factor approach**

[Matias, Virto, Hofer, Mescia, SDG...]

- Decompose, e.g., $V = \frac{m_B + m_{K^*}}{m_B} \xi_\perp + \Delta V^{\alpha_s} + \Delta V^\Lambda$
with hard gluons ΔV^{α_s} , power corrections $\Delta V^\Lambda = O(\Lambda/m_B)$
- Define soft form factors by setting some $\Delta = 0$
- (Factorisable) power corrs. from fit to full form factors,
embedding correlations from large-recoil
- $B \rightarrow V \ell \ell$ from soft form factors + hard gluons + power corrections

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[Buras, Ball, Bharucha, Altmannshofer, Straub...]

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- $B \rightarrow V\ell\ell$ from correlated full form factors
+ hard gluons & power corrs. not from form factors (nonfactorisable)

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Choice of observables

- **optimised observables** P_i with limited sensitivity to form factors
- averaged angular coefficients S_i with larger sensitivity

Very large power corrections ? (1)

- **Scheme:** choice of definition for the two soft form factors
(all equivalent for $m_B \rightarrow \infty$)

$$\{\xi_{\perp}, \xi_{\parallel}\} = \{V, \alpha A_1 + \beta A_2\}, \{T_1, A_0\}, \dots$$

- **Power corrections** for the other form factors from dimensional estimates or fit to available determinations (LCSR)

$$F(q^2) = F^{\text{soft}}(\xi_{\perp, \parallel}(q^2)) + \Delta F^{\alpha_s}(q^2) + a_F + b_F \frac{q^2}{m_B^2} + \dots$$

- For some schemes, large(r) uncertainties found for some optimised observables

[Camalich, Jäger]

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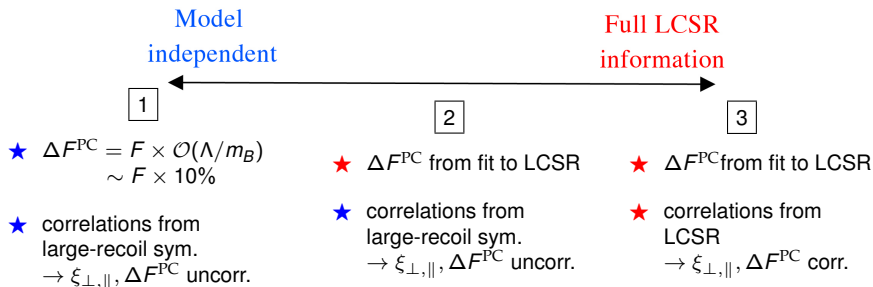
[Camalich, Jäger]

Observables are scheme independent, but

procedure to compute them can be either **scheme dependent or not**

- a) Include all correlations among errors for power corr
more accurate, but hinges on detail of ff determination
- b) Assign 10% uncorrelated uncertainties for power corrs a_F, b_F
depends on scheme (setting $a_F = b_F = 0$ for two form factors)

Very large power corrections ? (2)



Very large power corrections ? (2)

Model
independent

Full LCSR
information

1

★ $\Delta F^{\text{PC}} = F \times \mathcal{O}(\Lambda/m_B)$
 $\sim F \times 10\%$

★ correlations from
large-recoil sym.
 $\rightarrow \xi_{\perp, \parallel}, \Delta F^{\text{PC}} \text{ uncorr.}$

2

★ ΔF^{PC} from fit to LCSR

★ correlations from
large-recoil sym.
 $\rightarrow \xi_{\perp, \parallel}, \Delta F^{\text{PC}} \text{ uncorr.}$

3

★ ΔF^{PC} from fit to LCSR

★ correlations from
LCSR
 $\rightarrow \xi_{\perp, \parallel}, \Delta F^{\text{PC}} \text{ corr.}$

$P_5'[4.0, 6.0]$	scheme 1	scheme 2
1	-0.72 ± 0.05	-0.72 ± 0.12
2	-0.72 ± 0.03	-0.72 ± 0.03
3	-0.72 ± 0.03	-0.72 ± 0.03
full BSZ	-0.72 ± 0.03	

errors only from pc with BSZ form factors

[Capdevilla, SDG, Hofer, Matias]

- [Bharucha, Straub, Zwicky] as example (correl provided)
- scheme indep. restored if ΔF^{PC} from fit to LCSR, with expected magnitude
- sensitivity to scheme can be understood analytically
- no uncontrolled large power corrections for P_5'

Scheme dependence of observables

Using the connection between full and soft form factors at large recoil, keeping power corrections

$$P'_5(6 \text{ GeV}^2) = P'_{5|\infty}(6 \text{ GeV}^2) \left(1 + 0.18 \frac{2a_{V-} - 2a_{T-}}{\xi_{\perp}} - 0.73 \frac{2a_{V+}}{\xi_{\perp}} + 0.02 \frac{2a_{V_0} - 2a_{T_0}}{\tilde{\xi}_{\parallel}} + \text{nonlocal terms} \right) + O\left(\frac{m_{K^*}}{m_B}, \frac{\Lambda^2}{m_B^2}, \frac{q^2}{m_B^2}\right).$$

$$P_1(6 \text{ GeV}^2) = -1.21 \frac{2a_{V+}}{\xi_{\perp}} + 0.05 \frac{2b_{T+}}{\xi_{\perp}} + \text{nonlocal terms} + O\left(\frac{m_{K^*}}{m_B}, \frac{\Lambda^2}{m_B^2}, \frac{q^2}{m_B^2}\right),$$

- scheme dependence of P'_5 not fully taken into account in [\[Camalich,Jäger\]](#)
- allows to understand the scheme dependence of P_i
- P'_5 and P_1 with reduced unc. if ξ_{\perp} defined from V ($a_{V+} = 0$)

Charm-loop effects: large recoil

- Short-distance (hard gluons)
 - $\mathcal{C}_9 \rightarrow \mathcal{C}_9 + Y(q^2) = \mathcal{C}_9 + \delta\mathcal{C}_{9,\text{SD}}^{BK^{(*)}}(q^2)$, dependence on m_c
 - higher-order short-distance QCD via QCDF/HQET

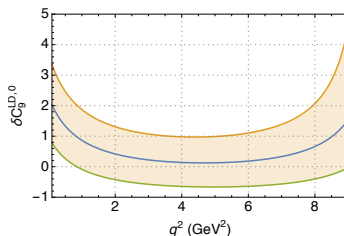
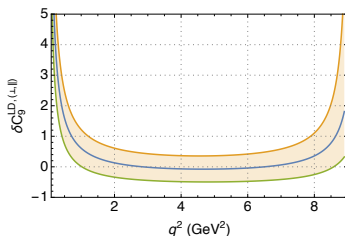
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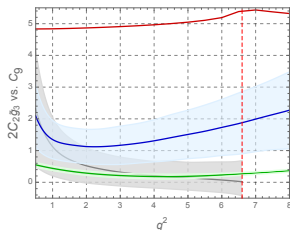
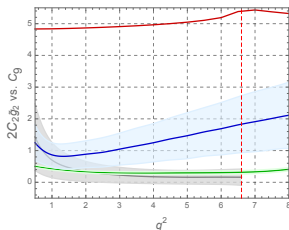
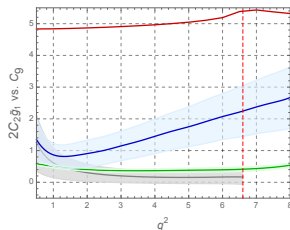
- Long-distance (soft gluons)

- $\Delta\mathcal{C}_9^{BK(*)},i > 0$ ($i = 0, ||, \perp$) using LCSR [Khodjamirian, Mannel, Pivovarov, Wang]
- Computed for $q^2 < 0$ and small, then extrapolated through dispersion relation reincluding J/ψ (but many unknown parameters)
- For us, order of magnitude: $\Delta\mathcal{C}_9^{BK*}|_{KMPW} = \delta\mathcal{C}_{9,\text{SD}}^{BK(*)} + \delta\mathcal{C}_{9,\text{LD}}^{BK(*)}$
taking $\Delta\mathcal{C}_9^{BK*},i = \delta\mathcal{C}_{9,\text{SD}}^{BK(*)},i + \textcolor{red}{s}_i \delta\mathcal{C}_{9,\text{LD}}^{BK(*)},i$ with $\textcolor{red}{s}_i = 0 \pm 1$



Charm-loop fit to $B \rightarrow K^* \ell \ell$ (1)

- $c\bar{c}$ contributions to 3 K^* helicity amplitudes $g_{1,2,3}$ as q^2 -polynomial
- params from Bayesian fit to data [Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli]

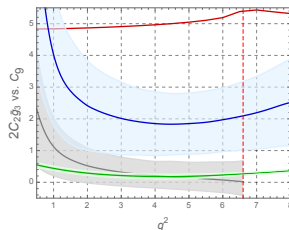
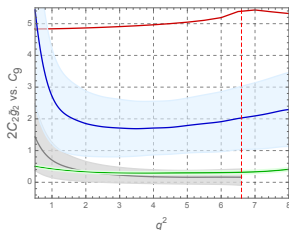
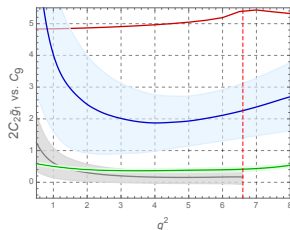


In units of C_9 : Short-Dist, QCDF, fit, KMPW $\Delta C_9^{BK^*}$

- constrained fit: imposing SM + $\Delta C_9^{BK^*}$ [Khodjamirian et al.] at $q^2 < 1 \text{ GeV}^2$ yields q^2 -dependent $c\bar{c}$ contribution, with “large” coefs for q^4

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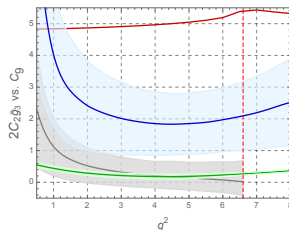
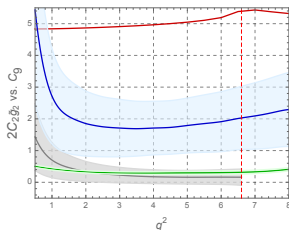
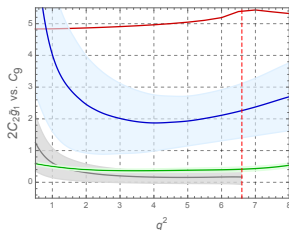


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- unconstrained fit: polynomial agrees with $\Delta C_9^{BK^*}$ + large cst C_9^{NP} identical for all 3 helicity amplitudes

Charm-loop fit to $B \rightarrow K^* \ell \ell$ (1)

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- params from Bayesian fit to data [Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli]

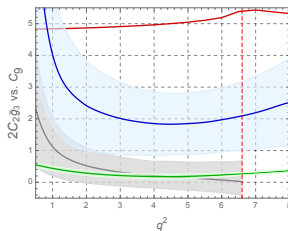
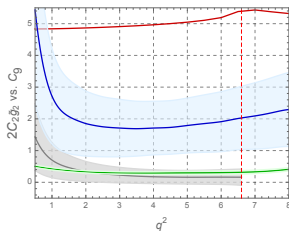
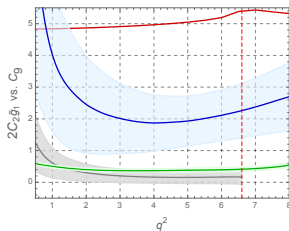


In units of C_9 : Short-Dist, QCD fit, KMPW $\Delta C_9^{BK^*}$

- constrained fit: imposing SM + $\Delta C_9^{BK^*}$ [Khodjamirian et al.] at $q^2 < 1 \text{ GeV}^2$ yields q^2 -dependent $c\bar{c}$ contribution, with “large” coefs for q^4
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- no dynamical hadronic explanation for enhancement at high q^2

Charm-loop fit to $B \rightarrow K^* \ell \ell$ (2)

Problem related to q^4 contribution ? [\[Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli\]](#)

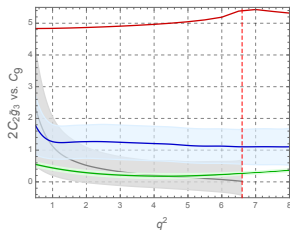
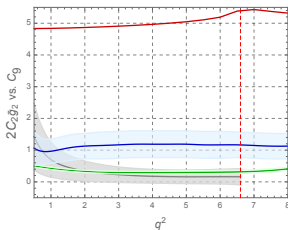
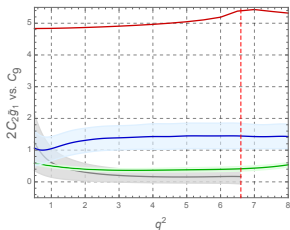
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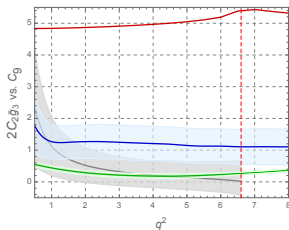
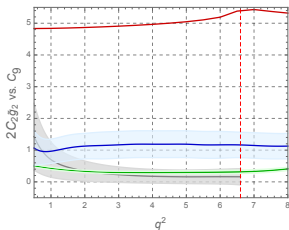
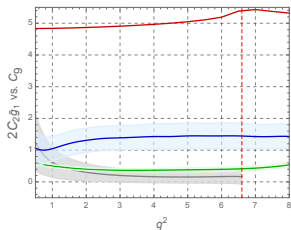
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- Frequentist fits indicate no improvement by adding q^4 term, and adding \mathcal{C}_9 better pull than 12 independent coefficients

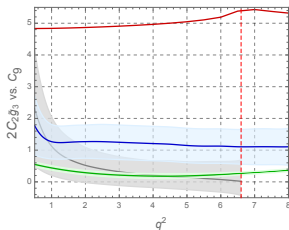
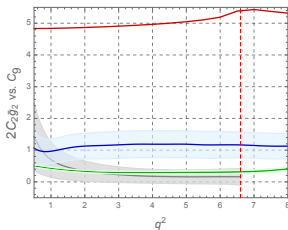
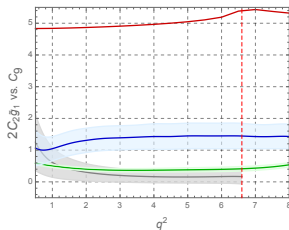
[Capdevila, Hofer, Matias, SDG; Hurth, Mahmoudi, Neshatpour]

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[Capdevila, Hofer, Matias, SDG; Hurth, Mahmoudi, Neshatpour]

- if $c\bar{c}$, why same constant C_9^{NP} for all mesons and helicities, which explanation for $R_{K^{(*)}}$, what causes deviations in low-recoil BRs ?

Charm-loop fit to $B \rightarrow K^* \ell \ell$ (3)

(Capdevila, Hofer, Matias, SDG)

$$A_{L,R}^0 = A_{L,R}^0(s_i = 0) + \frac{N}{q^2} \left(h_0^{(0)} + \frac{q^2}{1 \text{ GeV}^2} h_0^{(1)} + \frac{q^4}{1 \text{ GeV}^4} h_0^{(2)} \right),$$

$$A_{L,R}^{\parallel} = A_{L,R}^{\parallel}(s_i = 0) + \frac{N}{\sqrt{2}q^2} \left[(h_+^{(0)} + h_-^{(0)}) + \frac{q^2}{1 \text{ GeV}^2} (h_+^{(1)} + h_-^{(1)}) + \frac{q^4}{1 \text{ GeV}^4} (h_+^{(2)} + h_-^{(2)}) \right],$$

$$A_{L,R}^{\perp} = A_{L,R}^{\perp}(s_i = 0) + \frac{N}{\sqrt{2}q^2} \left[(h_+^{(0)} - h_-^{(0)}) + \frac{q^2}{1 \text{ GeV}^2} (h_+^{(1)} - h_-^{(1)}) + \frac{q^4}{1 \text{ GeV}^4} (h_+^{(2)} - h_-^{(2)}) \right],$$

- $s_i = 0$ means no contrib from long-distance $c\bar{c}$
- n order of the polynomial added, coeffs fit in frequentist framework
- testing nested hyp: pull from $\chi_{\min}^{2(n-1)} - \chi_{\min}^{2(n)} \quad (\chi_{\min}^{2(-1)} = \text{SM})$

n	0	1	2	3
$B \rightarrow K^* \mu \mu, C_9^{\mu, \text{NP}} = 0$	2.88 (0.8 σ)	17.90 (3.5 σ)	0.08 (0.0 σ)	0.34 (0.1 σ)
$B \rightarrow K^* \mu \mu, C_9^{\mu, \text{NP}} = -1.1$	4.79 (1.3 σ)	9.73 (2.3 σ)	0.20 (0.0 σ)	0.39 (0.1 σ)
$b \rightarrow s \ell \ell, C_9^{\mu, \text{NP}} = 0$	1.55 (0.4 σ)	21.40 (3.9 σ)	0.61 (0.1 σ)	

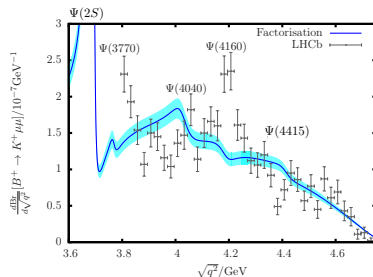
No need for high-order polyn or strong q^2 -dep impossible with short distance contrib, contrary to claims by [\[Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli\]](#)

Charm-loop effects : resonances (1)

- Low recoil: quark-hadron duality
 - Average “enough” resonances to equate quark and hadron levels
 - Model estimate yield a few % for $BR(B \rightarrow K\mu\mu)$ [Beylich, Buchalla, Feldmann]

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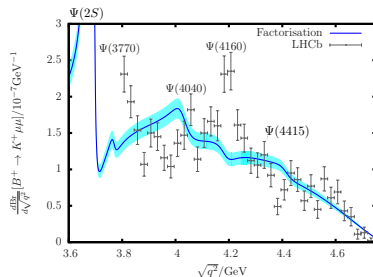


- Probably (?) effect of similar size for $B \rightarrow K^*\mu\mu$ (BR and angular obs.)
- OPE corrections + NLO QCD corrections + complex correction of 10% for each transversity amplitude
- Difficulties to explain $B \rightarrow K\ell\ell$ low-recoil spectrum using $\sigma(e^+e^- \rightarrow \text{hadrons})$ and naive factorisation [Lyon, Zwicky]

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- Large recoil

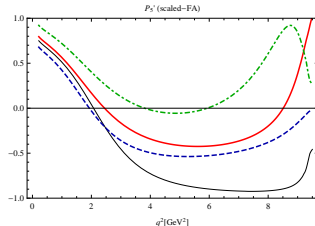
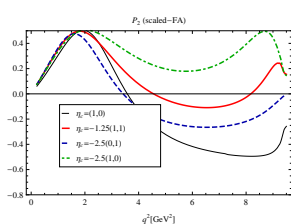
- $q^2 \leq 7\text{-}8 \text{ GeV}^2$ to limit the impact of J/ψ tail
- Still need to include the effect of $c\bar{c}$ loop (tail of resonances + nonresonant)
- LHCb on $B \rightarrow K\mu\mu$: resonance tails have very limited impact

Charm-loop effects : resonances (2)

On the basis of a model for $c\bar{c}$ resonances for **low-recoil** $B \rightarrow K\mu\mu$
 [Zwicky and Lyon] proposed very large $c\bar{c}$ contrib for **large-recoil** $B \rightarrow K^*\mu\mu$

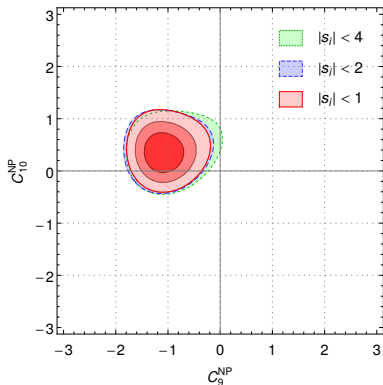
$$\mathcal{C}_9^{\text{eff}} = \mathcal{C}_9^{\text{SM}} + \mathcal{C}_9^{\text{NP}} + \eta h(q^2) \text{ and } \mathcal{C}_{9'} = \mathcal{C}_{9'}^{\text{NP}} + \eta' h(q^2)$$

where $\eta + \eta' = -2.5$ where conventional expectations are $\eta = 1, \eta' = 0$



- P_2 and P'_5 could have more zeroes for $4 \leq q^2 \leq 9 \text{ GeV}^2$
- $P'_{5[6,8]}$ would be above or equal to $P'_{5[4,6]}$, whereas global effects (like $\mathcal{C}_9^{\text{NP}}$) predicts $P'_{5[6,8]} < P'_{5[4,6]}$ in agreement with experiment
- Not in agreement with LHCb findings for $B \rightarrow K\ell\ell$
- R_K and R_{K^*} unexplained since it would affect identically $\ell = e, \mu$

Cross-checks: Charm-loop dependence



- For each $B \rightarrow K^* \mu \mu$ transversity $\Delta \mathcal{C}_9^{BK(*),i} = \delta \mathcal{C}_{9,\text{pert}}^{BK(*),i} + s_i \delta \mathcal{C}_{9,\text{non pert}}^{BK(*),i}$
- Ditto for $B_s \rightarrow \phi$, with all 6 s_i independent
- For $B \rightarrow K \mu \mu$, $c\bar{c}$ estimated as very small
- Increasing the range allowed for s_i makes low-recoil and $B \rightarrow K \mu \mu$ dominate more and more

- Does not alter the pull, and does not explain LFUV