$\gamma^*\gamma \rightarrow f_2(1230)$ and $\gamma\gamma \rightarrow G_2\pi^0$ within QCD framework





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Introduction: pQCD description of hard exclusive processes

γ^{*}+γ→ f₂(1270) as example of qq-meson Braun, Kivel, Strohmaier, Vladimirov JHEP 1606 (2016)

> An opportunity to study glueball in γ+γ→ G(2++)+π⁰

> > Kivel, Vanderhaeghen, to be published

Discussion: suggestions, critics, skepticism, etc.

Introduction: QCD factorisation

QCD factorization <=> effective field theory approach Two scales: hard and soft $Q^2 \gg \Lambda^2$ hadronic scale

$$\Lambda/Q$$
 small $\mathcal{A}(Q^2) \simeq H(Q^2) * S(\Lambda)$

Hard part is defined by a hard subprocess $p_h^2 \sim Q^2$

$$H(Q^2) = Q^{-2n} \left\{ h_{\rm LO} \left(\ln \frac{Q^2}{\Lambda^2} \right)^{\gamma_{LO}} + h_{\rm NLO} \alpha_s(Q^2) \left(\ln \frac{Q^2}{\Lambda^2} \right)^{\gamma_{NLO}} + \mathcal{O}(\alpha_s^2) \right\}$$

scaling behavior Q^{-2n} is the model independent QCD prediction (!) which can be checked by experiment

Log corrections can be systematically computed in pQCD $\alpha_s(Q^2) \sim \ln^{-1}Q^2/\Lambda^2 \ll 1$

Introduction: QCD factorisation

QCD factorization <=> effective field theory approach

Two scales: hard and soft $Q^2 \gg \Lambda^2_{QCD}$

$$Q \to \infty$$
 $\mathcal{A}(Q^2) \simeq H(Q^2) * S(\Lambda)$

Soft part is associated with a soft subprocess $p_s^2 \sim \Lambda^2$ defined as a matrix element in QCD process independent (universality)

can be estimated only in the framework of some nonperturbative approach

or

can be constrained from the experimental data

Gamma-f₂ transition FF's

 $\gamma^*(q)\gamma(q') \to f_2(p)$ $\Gamma[f_2] = 185 \text{MeV}$ $Br[f_2 \to \pi\pi] \sim 85\%$

assignment of known mesons to quark-model states.

$n^{2s+1}L_J$	J^{PC}	I = 1	$I = \frac{1}{2}$	I = 0	I = 0	$ heta_q$	$ heta_l$
		$u \bar{d} \cdots$	$u\overline{s}\cdots$	f	f'		
$1^{1}S_{0}$	0^{-+}	π	K	η	η '	-11.5°	-24.6°
$1^{3}S_{1}$	1	ρ	K^*	ω	ϕ	38.7°	36.0°
$1^{1}P_{1}$	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1380)$		
$1^{3}P_{0}$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1370)$	$f_0(1710)$		
$1^{3}P_{1}$	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1(1420)$		
$1^{3}P_{2}$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2(1270)$	$f_2'(1525)$	29.6°	28.0°

flavor mixing in SU(3) $|8\rangle = \frac{1}{\sqrt{6}} (u\bar{u} + d\bar{d} - 2s\bar{s}) |1\rangle = \frac{1}{\sqrt{3}} (u\bar{u} + d\bar{d} + s\bar{s})$

 $f = \cos \theta |1\rangle + \sin \theta |8\rangle$ $f' = \cos \theta |8\rangle - \sin \theta |1\rangle$ $|f_2\rangle = 0.70|u\bar{u} + d\bar{d}\rangle + 0.11|s\bar{s}\rangle$

γ f₂ transition Form Factors $\gamma^*(q)\gamma(q') \rightarrow f_2(1270)$

There are 3 amplitudes = transitions FFs

 $T^{\mu\nu} = i \int d^4x \, e^{-i(qx)} \langle f_2(P,\lambda) | T\{j^{\mu}_{\rm em}(x)j^{\nu}_{\rm em}(0)\} | 0 \rangle = T_0^{\mu\nu} + T_1^{\mu\nu} + T_2^{\mu\nu}$

$$T_0^{\mu\nu} = (-g_{\perp}^{\mu\nu}) \, e_{\alpha\beta}^{(\lambda)*} (q-q')^{\alpha} (q-q')^{\beta} \frac{m^2}{(2qq')^2} T_0(Q^2) \, .$$

$$T_1^{\mu\nu} = e_{\alpha\beta}^{(\lambda)*} \ \left(-g_{\perp}^{\alpha\nu}\right) (q-q')^{\beta} \left[q^{\mu} - q'^{\mu} \frac{q^2}{(qq')}\right] \frac{m^2}{(2qq')^2} T_1(Q^2) \,,$$

$$T_2^{\mu\nu} = e_{\alpha\beta}^{(\lambda)*} \left[g_{\perp}^{\alpha\mu} g_{\perp}^{\beta\nu} - \frac{1}{2} g_{\perp}^{\mu\nu} \frac{m^2}{(2qq')^2} (q - q')^{\alpha} (q - q')^{\beta} \right] T_2(Q^2)$$

 $q_{\mu}g_{\perp}^{\mu\nu} = q_{\nu}'g_{\perp}^{\mu\nu} = 0$ transverse q & q' $-q^2 \equiv Q^2 > 0$

Y f₂ transition Form Factors $\gamma^*(q)\gamma(q') \rightarrow f_2(1270)$

There are 3 amplitudes = transitions FFs

 $\gamma^*(\pm)\gamma(\pm) \to f_2(0)$ $\gamma^*(0)\gamma(\pm) \to f_2(\mp)$ $\gamma^*(\mp)\gamma(\pm) \to f_2(\mp 2)$ $T_0(Q^2)$ $T_1(Q^2)$ $T_2(Q^2)$

$$\begin{aligned} \mathbf{Q}^{2} = \mathbf{0} \qquad \Gamma[f_{2} \to \gamma \gamma] &= \frac{\pi \alpha^{2}}{5m} \left(\frac{2}{3} |T_{0}(0)|^{2} + |T_{2}(0)|^{2} \right) = 3.03(40) \,\mathrm{keV} \\ \\ \frac{\Gamma_{\gamma\gamma}^{\Lambda=0}}{\Gamma_{\gamma\gamma}^{\Lambda=2}} &\simeq (3.7 \pm 0.3) \times 10^{-2} \qquad |T_{0}(0)| \ll |T_{2}(0)| \qquad \text{Belle 2008, Uehara et a} \end{aligned}$$

$$|T_2(0)| \simeq \sqrt{\frac{5m}{\pi\alpha^2}} \Gamma[f_2 \to \gamma\gamma] = 339(22) \,\mathrm{MeV}$$

Y f2 transition Form Factors



f₂ transition FF's in pQCD



twist-2

twist-4 kinematical

Light-cone distribution amplitudes

$$xp \longrightarrow p \quad \sim \int_{|k_{\perp}| < \mu} d^2 k_{\perp} \Psi_{BS}(x, k_{\perp}) = \phi(x, \mu)$$

describes the momentum-fraction distribution of partons at zero transverse separation in a 2-particle Fock state

$$|\bar{q}q({}^{1}S_{0})\rangle \quad \langle f_{2}(p)|\bar{\psi}(z)\not\notz\psi(0)|0\rangle |_{z_{-}=z_{\perp}=0} \sim \int_{0}^{1} dx \, e^{ixz_{+}p_{-}}\phi_{2}(x,\mu)$$

 $V_{\pm} = V_{0} \pm V_{3}$

 μ is the factorization scale (renormalization of the operator in QCD)

suppressed by powers of 1/Q:

multiparticle states: qqg

 $q\bar{q}$ with orbital momentum

gamma-f₂: T₀

 $T_0(Q^2) \qquad \gamma^*(\pm)\gamma(\pm) \to f_2(0)$



 $T_0(Q^2) \simeq \langle f_q \rangle \int dx \frac{\phi_2(x)}{x} \qquad \langle f_q \rangle = \frac{4}{9} f_u(\mu) + \frac{1}{9} f_d(\mu) + \frac{1}{9} f_s(\mu)$

simplest model $\phi_2(x) = 30x(1-x)(2x-1)$

normalization constant

$$\frac{1}{2} \langle f_2(P,\lambda) | \bar{q} \left[\gamma_\mu i \stackrel{\leftrightarrow}{D}_\nu + \gamma_\nu i \stackrel{\leftrightarrow}{D}_\mu \right] q | 0 \rangle = f_q m^2 e_{\mu\nu}^{(\lambda)*}$$

 $f_u(1 {\rm GeV}) = f_d(1 {\rm GeV}) = 101(10) {\rm MeV}$ $f_s(1 {\rm GeV}) \approx 0$ Aliev, Shifman 1982 (QCD SR, TM dom.) Terazawa, 1990, Suzuki 1993 (TM dom.) Cheng, Koike, Yang 2010 (QCD SR, TM dom.)

for comparison

 $f_{\pi} = 130 MeV$ $f_{\rho} = 221 MeV$ $f_{\omega} = 198 MeV$ $f_{\phi} = 161 MeV$

 $T_0(Q^2)$

NLO & LLog resummation involves gluons



Gluon DA: $|g(\pm)g(\mp)(^{1}S_{0})\rangle$

$$\langle f_2(P,\lambda) | z^{\alpha} z^{\beta} G^a_{\alpha\mu}(z) G^a_{\beta\mu}(0) | 0 \rangle |_{z_-=z_\perp=0} \sim \int_0^1 dx \, e^{ixp_-z_+} \phi_g^S(x)$$

simplest model
$$\phi_g^S(x) = f_g^S m^2 30 x^2 (1-x)^2$$

normalization constant

$$\langle f_2(P,\lambda)|z^{\alpha}z^{\beta}G^a_{\alpha\mu}(0)G^a_{\beta\mu}(0)|0\rangle \left|_{z_-=z_\perp=0}\right. = -f_g^S m^2 z^{\alpha}z^{\beta}e^{(\lambda)*}_{\alpha\beta} \left|_{z_-=z_\perp=0}\right.$$

expected to be small for $q\bar{q}$ mesons at low normalization

gamma-f₂ T₀ FF: coupling to gluons

rich gluon process

00000000



$$f_g^S(\mu^2 = 4m_b^2) = (14.9 \pm 0.8) \,\mathrm{MeV}$$

 $\Upsilon(1S) \to \gamma f_2 \qquad M_{\Upsilon} = 9.5 \text{GeV} \qquad m_b \simeq 4.5 \text{GeV}$

QCD evolution mixes f_q and f_g^S

therefore this result compatible with

 $f_g^S(1\,{\rm GeV}) \approx 0$

i.e. the meson only consists from quarks at low normalization point





 $T_1(Q^2)$

 $|\bar{q}q(^1P_1)\rangle$

 $T_1(Q^2) \qquad \gamma^*(0)\gamma(\pm) \to f_2(\mp)$



DAs of twist-3:

$$\begin{aligned} & \left| \bar{q}q({}^{1}S_{0})g \right\rangle & \stackrel{\alpha_{1}p}{\longrightarrow} & f_{q}m^{2}360\alpha_{1}\alpha_{2}^{2}\alpha_{3} \left[\zeta_{3} + \frac{1}{2}\omega_{3}(7\alpha_{2} - 3) + \dots \right] \\ & \alpha_{3}p \exp \left[\alpha_{2}p + \alpha_{3}p + \alpha_{3}p$$

$$\langle f_2(P,\lambda) | z^{\alpha} \bar{q}(z) \gamma_{\alpha} \{ \overrightarrow{D}_{\mu}^{\perp} - \overleftarrow{D}_{\mu}^{\perp} \} q(0) | 0 \rangle |_{z_- = z_\perp = 0} \rightarrow e_{-\mu_\perp}^{(\lambda)*} \int_0^u dv \, \frac{\phi_2(v)}{\bar{v}} + \int_u^1 dv \, \frac{\phi_2(v)}{\bar{v}}$$

$$\langle f_2(P,\lambda) | z^{\alpha} \bar{q}(z) \gamma_{\alpha} \gamma_5 \{ \overrightarrow{D}_{\mu}^{\perp} - \overleftarrow{D}_{\mu}^{\perp} \} q(0) | 0 \rangle |_{z_- = z_\perp = 0} \rightarrow e_{-\mu_\perp}^{(\lambda)*} \int_0^u dv \, \frac{\phi_2(v)}{\bar{v}} - \int_u^1 dv \, \frac{\phi_2(v)}{\bar{v}}$$

Wandzura-Wilczek part



gamma-f₂ T₂ FF



one more gluon DA of twist-2: do not mix with quarks!

$$|gg({}^{5}S_{2})\rangle \qquad \langle f_{2}(P)|z^{\alpha}z^{\beta}G_{\alpha\{\mu}(z)G_{\beta\nu\}}(0)|0\rangle|_{z_{-}=z_{\perp}=0} = f_{g}^{T}e_{\{\mu\nu\}}^{\perp}\int_{0}^{1}dx\,e^{ixp_{-}z_{+}}\phi_{g}^{T}(x)$$

simplest model $\phi_g^T(x) = 30x^2(1-x)^2$ unknown parameter f_g^T

$$\begin{split} |\bar{q}q(^{1}D_{2})\rangle \\ \langle f_{2}(P)|\bar{\psi}(z)\overleftrightarrow{D}_{\{\perp\mu}\overleftrightarrow{D}_{\perp\nu\}}\not{z}\psi(0)|0\rangle &\sim \frac{\Lambda^{2}}{Q^{2}} & \longrightarrow &\sim \frac{m^{2}}{Q^{2}}\langle f_{q}\rangle + \dots \\ \\ \mathsf{QCD} \ \mathsf{EOM} \end{split}$$

gamma-f₂ T₂ FF

$$\begin{array}{c}
\text{LO }^{1}\text{D}_{2} \text{ quarks} & \text{virtual } c-\text{quarks, small} \\
T_{2}(Q^{2}) = \frac{20}{3} \frac{m^{2}}{Q^{2}} \langle f_{q} \rangle + \frac{5}{3} \frac{\alpha_{s}(Q^{2})}{\pi} f_{g}^{T} \left[1 + \frac{8}{3} \lambda(m_{c}^{2}/Q^{2}) \right] \\
\text{data BELLE 2015 } 3.5 \text{GeV}^{2} \leq Q^{2} \leq 24 \text{GeV}^{2} \\
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gamma-f₂ T₂ FF

virtual c-quarks, small

$$T_2(Q^2) = \frac{20}{3} \frac{m^2}{Q^2} \langle f_q \rangle + \frac{5}{3} \frac{\alpha_s(Q^2)}{\pi} f_g^T \left[1 + \frac{8}{3} \lambda (m_c^2/Q^2) \right]$$

data BELLE 2015 $3.5 \text{GeV}^2 \le Q^2 \le 24 \text{GeV}^2$



 $\frac{\alpha_s(Q^2)}{\pi} = 0.1 - 0.06$

Scaling behavior

$$F_{\gamma f_2}^{\text{eff}}(\boldsymbol{Q^2}) = \sqrt{\frac{2}{3} \left| \frac{T_0(\boldsymbol{Q^2})}{T_2(0)} \right|^2 + \frac{\boldsymbol{Q^2}m^2}{(m^2 + \boldsymbol{Q^2})^2} \left| \frac{T_1(\boldsymbol{Q^2})}{T_2(0)} \right|^2 + \left| \frac{T_2(\boldsymbol{Q^2})}{T_2(0)} \right|^2}$$

good scaling behavior for $Q^2>4-5$ GeV²



One more way to use tensor coupling: glueball production $\gamma\gamma \to \pi^0 G(2^{++})$

$$\frac{d\sigma_{\gamma\gamma}[\pi^0 G(2^{++})]}{d\cos\theta} = \frac{1}{64\pi} \frac{s+m^2}{s^2} \left(|\overline{A_{++}}|^2 + |\overline{A_{+-}}|^2 \right)$$





all terms are of order α_s

 $A_{\pm\pm}: \gamma(\pm)\gamma(\pm) \to M(\pm 2)$ only tensor gluons

 $A_{\pm\mp}:\gamma(\pm)\gamma(\mp)\to M(0)$

quarks & scalar gluons

Predictions & evidence for glueballs

Lattice (Morningstar, Peardon, 1999)



Predictions & evidence for glueballs

Experiment BES III, Ablikim et al, PRD 93(2016)

TABLE I. Mass, width, $\mathcal{B}(J/\psi \to \gamma X \to \gamma \phi \phi)$ (B.F.) and significance (Sig.) of each component in the baseline solution. The first errors are statistical and the second ones are systematic.

Resonance	$M(MeV/c^2)$	$\Gamma({ m MeV}/c^2)$	B.F.($\times 10^{-4}$)	Sig.	PDG
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5σ	1
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4σ	~
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11σ	√

first observed in $\pi^- + p \rightarrow \phi \phi n$

Etkin et al, PRL(1978), PLB(1985), PLB(1988)

Predictions & $\chi^2/n_{ev}=2.01$ o -1 -0.5 0 0.5 1 Experiment BES III, Ablikim et al. PRD 93(2016) COS $\theta(\phi_1)^2$





Predictions & evidence for glueballs

Experiment BELLE, Uehara et al, PTEP (2013)



 $f_2(2300)$ $M = 2297 \pm 28 \text{ MeV}$ $\Gamma = 149 \pm 40 \text{ MeV}$

Glueball or qq-state ?

Glueball cross section at large energy

$$\frac{d\sigma_{\gamma\gamma}[\pi^0 G_2]}{d\cos\theta} = \frac{1}{64\pi} \frac{s+m^2}{s^2} \left(|A_g^{++}|^2 + \frac{8}{3}|A_q^{+-} + A_g^{+-}|^2 \right)$$



$$s \to \infty \qquad \frac{d\sigma_{\gamma\gamma}[\pi^0 G(2^{++})]}{d\cos\theta} \sim \frac{1}{s} \frac{f_\pi^2 f_G^2}{s^2} f(\theta) \qquad f_\pi = 130 MeV$$





m	odels	for t	he I	DAs		
$egin{array}{c} 5 \ \phi \\ \phi _g \end{array}$	$(x) \ge$	$\phi_g^S(x)$	= 30	$x^2 \bar{x}^2$	2	
$\mathcal{D}_{\pi(y)}$	$\simeq 6 y \bar{y}$	$+6a_{2}()$	u)yyC	3/2	2y -	1)
-50	$a_2(\mu$	= 1G	eV) =	0.2	20	
	-0.6-	0.4 - 0.2	2 0.0	0.2	0.4	0.6



$$A_q^{+-} \sim \frac{f_\pi f_q}{s} \, \alpha \alpha_s I_q^{+-}(\cos \theta)$$

$$I_q^{+-}(\eta) = \int_0^1 dy \; \frac{\phi_\pi(y)}{y\bar{y}} \int_0^1 dx \; \frac{\phi_2(x)}{x\bar{x}} \frac{\eta(1-\eta^2)(y-x)(1-x-y)^2}{[(1-x-y)^2(1-\eta^2)+4x\bar{x}y\bar{y}]}$$

models for the DAs

$$\phi_g^S(x) = 30x^2\bar{x}^2$$

$$\phi_2(x) = 30x\bar{x}(2x-1)$$

 $\phi_{\pi}(y) \simeq 6y\bar{y} + 6a_2(\mu)y\bar{y}C_2^{3/2}(2y-1)$ $a_2(\mu = 1 \text{GeV}) = 0.20$



 $|I_g^{++}| \gg |I_g^{+-}| \gg |I_q^{+-}|$ at large angles G₂ is produced in tensor polarization

Glueball cross section

 $M_G = 2.3 \text{ GeV}$



 $|t| \& |u| > 2.5 \text{ GeV}^2$

tensor channel is dominant





s = 13GeV² $|t| \& |u| > 2.5 \text{ GeV}^2$



Conclusions

QCD predicts the scaling behavior of the $\gamma^*\gamma \rightarrow f_2$ form factors

The QCD predictions are in agreement with BELLE data. More precise data is required in order to better constrain meson DAs

Production of tensor glueball at large energies and momentum transfers in QCD $\gamma\gamma \rightarrow \pi^0 G(2^{++})$

Dominance of the tensor polarization

Direct coupling to gluon wave function (no quark mixing, model independent)

Probably, can be observed in BELLE II

(Thank you!