Mu-e scattering:
Measuring the leading hadronic contribution to $(g-2)_\mu$

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INFN Padova

KEK
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Outline

- Status of the muon g-2
- Hadronic corrections to the muon g-2: a new approach
- Muon-electron scattering: proposal for a new experiment
Status of the muon $g-2$
The muon g-2: experimental status

BNL E821: \( a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11} \) [0.5ppm].

Future: new muon g-2 experiments at:

- **Fermilab E989**: aims at \( \pm 16 \times 10^{-11} \), ie 0.14ppm. Started taking data at the end of 2017. First result expected in late 2018 with a precision comparable to BNL E821.
- **J-PARC proposal**: phase-1 start with 0.46ppm (TDR 2017).

Are theorists ready for this (amazing) precision? **Not yet!**
The muon g-2: the QED contribution

\[ a_{\mu}^{\text{QED}} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) \]  
Schwinger 1948

+ 0.765857426 (16) \(\left(\frac{\alpha}{\pi}\right)^2\)

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

+ 24.05050988 (28) \(\left(\frac{\alpha}{\pi}\right)^3\)

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04; Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

+ 130.8780 (60) \(\left(\frac{\alpha}{\pi}\right)^4\)


+ 750.80 (89) \(\left(\frac{\alpha}{\pi}\right)^5\) COMPLETED!

Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015 & 2017

Adding up, I get:

\[ a_{\mu}^{\text{QED}} = 116584718.976 (20)(73) \times 10^{-11} \]

from coeffs, mainly from 4-loop unc \(\uparrow\) \(\text{\longleftarrow from } \delta \alpha(\text{Rb})\)

with \(\alpha = 1/137.035998995(85) [0.62 \text{ ppb}]\)
The muon g-2: the electroweak contribution

- **One-loop term:**

\[
a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} \left( 1 - 4\sin^2\theta_W \right)^2 + O \left( \frac{m_{\mu}^2}{M_{Z, W, H}^2} \right) \right] \approx 195 \times 10^{-11}
\]

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

- **One-loop plus higher-order terms:**

\[
a_{\mu}^{\text{EW}} = 153.6 (1) \times 10^{-11}
\]

with \(M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}\)

Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.
The muon g-2: the Hadronic LO contribution (HLO)

\[ a_{\mu}^{HLO} = 6894.6 \pm 32.5 \times 10^{-11} \]
\[ = 6931 \pm 34 \times 10^{-11} \]
\[ = 6932.7 \pm 24.6 \times 10^{-11} \]

\[ K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \]
\[ a_{\mu}^{HLO} = \frac{1}{4\pi^3} \int_{4m^2}^{\infty} ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m^2}^{\infty} \frac{ds}{s} K(s) R(s) \]


Lots of progress in lattice calculations. FNAL - Muon g-2 workshop - June 2017
Capri - FCCP 2017 workshop - Sep 2017
**HNLO: Vacuum Polarization**

\[ a_\mu^{\text{HNLO}}(\text{vp}) = -99.27 (67) \times 10^{-11} \]

O(\alpha^3) contributions of diagrams containing hadronic vacuum polarization insertions:

Krause '96, Alemany et al. '98, Hagiwara et al. 2011, Jegerlehner 2017
**HNLO: Light-by-light contribution**

Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

**This term had a troubled life! Latest values:**

- \(a_{\mu}^{\text{HNLO}(\text{lbl})} = +80\ (40) \times 10^{-11}\) \cite{Knecht2002}
- \(a_{\mu}^{\text{HNLO}(\text{lbl})} = +136\ (25) \times 10^{-11}\) \cite{Melnikov2003}
- \(a_{\mu}^{\text{HNLO}(\text{lbl})} = +105\ (26) \times 10^{-11}\) \cite{Prades2009}
- \(a_{\mu}^{\text{HNLO}(\text{lbl})} = +100\ (29) \times 10^{-11}\) \cite{Jegerlehner2017}

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

**Improvements expected in the \(\pi^0\) transition form factor** \cite{Nyffeler2016}

**The HLbL contribution can be expressed in terms of observables in a dispersive approach.** Colangelo et al, 2014, 15 & 17; Pauk & Vanderhaeghen 2014.

**Progress on the lattice:** +53.5(13.5)x10^{-11}. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.
The muon g-2: the Hadronic NNLO contributions (HNNLO)

- **HNNLO: Vacuum Polarization**
  
  \[ a_\mu^{\text{HNNLO}}(\text{vp}) = 12.4 (1) \times 10^{-11} \]
  
  Kurz, Liu, Marquard, Steinhauser 2014

- **HNNLO: Light-by-light**
  
  \[ a_\mu^{\text{HNNLO}}(\text{lbl}) = 3 (2) \times 10^{-11} \]
  
  Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014

\[ O(\alpha^4) \] contributions of diagrams containing hadronic vacuum polarization insertions:
The muon g-2: SM vs. Experiment

Comparisons of the SM predictions with the measured g-2 value:

\[ a_{\mu}^{\text{EXP}} = 116592091 \pm 63 \times 10^{-11} \]

<table>
<thead>
<tr>
<th>( a_{\mu}^{\text{SM}} \times 10^{11} )</th>
<th>( \Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>116 591 783 (44)</td>
<td>308 (77) ( \times 10^{-11} )</td>
<td>4.0 [1]</td>
</tr>
<tr>
<td>116 591 820 (45)</td>
<td>271 (77) ( \times 10^{-11} )</td>
<td>3.5 [2]</td>
</tr>
<tr>
<td>116 591 821 (38)</td>
<td>270 (74) ( \times 10^{-11} )</td>
<td>3.7 [3]</td>
</tr>
</tbody>
</table>

with the hadronic light-by-light \( a_{\mu}^{\text{HNLO(lbl)}} = 100 (29) \times 10^{-11} \) of F. Jegerlehner arXiv:1705.00263, and the hadronic leading-order of:

A new approach to $a_\mu^{HLO}$

C. Carloni Calame, MP, L. Trentadue, G. Venanzoni
PLB 2015 - arXiv:1504.02228
New space-like proposal for HLO

- At present, the leading hadronic contribution $a_{\mu}^{\text{HLO}}$ is computed via the time-like formula:

\[
a_{\mu}^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m^2_\pi}^{\infty} ds K(s) \sigma^0_{\text{had}}(s)
\]

\[
K(s) = \int_0^1 dx \frac{x^2 (1 - x)}{x^2 + (1 - x) (s/m^2_\mu)}
\]

- Alternatively, exchanging the x and s integrations in $a_{\mu}^{\text{HLO}}$

\[
a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1 - x) \Delta\alpha_{\text{had}}[t(x)]
\]

\[
t(x) = \frac{x^2 m^2_\mu}{x - 1} < 0
\]

which involves $\Delta\alpha_{\text{had}}(t)$, the hadronic contribution to the running of $\alpha$ in the space-like region. It can be extracted from scattering data!
New space-like proposal for HLO (2)

Time-like

F. Jegerlehner, arXiv:1511.04473

Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

Space-like

smooth integrand
\( \Delta \alpha_{\text{had}}(t) \) can be measured via Bhabha scattering:

\[
\begin{align*}
x_{\text{peak}} &\approx 0.914 \\
t_{\text{peak}} &\approx -0.108 \text{ GeV}^2
\end{align*}
\]

The peak occurs at \( x_{\text{peak}} = 0.914 \), \( t_{\text{peak}} = -0.108 \text{ GeV}^2 \approx - (330 \text{ MeV})^2 \)

Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015
Muon-electron scattering

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987
\( \Delta \alpha_{\text{had}}(t) \) can also be measured via the elastic scattering \( \mu e \rightarrow \mu e \).

We propose to scatter a 150 GeV muon beam, available at CERN’s North Area, on a fixed electron target. Modular apparatus, several layers of low Z material (Be) paired to Si strip planes.
With CERN’s 150 GeV muon beam M2, which has an average of $\sim 1.3 \times 10^7 \mu/s$, incident on Be layers for a total thickness of 60cm, and 2 years of data taking with a running time of $2 \times 10^7$ s/yr, one can reach an int. luminosity of $\mathcal{L}_{\text{int}} \sim 1.5 \times 10^7 \text{nb}^{-1}$.
For a 150 GeV muon beam, the scan region extends up to $x=0.932$, i.e. beyond the peak! (the peak is at $x=0.914$)

The integrand in the remaining region $x \in [0.932,1]$ accounts for $\sim 13\%$ of the $a_\mu^{\text{HLO}}$ integral. It cannot be reached by our experiment but it can be determined using pQCD & time-like data, and/or lattice QCD results.

Same detector for signal and normalization ($x \leq 0.3$, $\Delta \alpha_{\text{had}}(t) \leq 10^{-5}$) leads to cancellation of detector effects at first order.
Modular apparatus: one Beryllium target (~1cm thick) coupled to three Silicon planes (thickness: 300μm).

State-of-the-art Silicon strip detectors with hit resolution ~10 μm will provide an expected angular resolution of ~10 μm / 0.5 m = 0.02 mrad

ECAL and μ detector located downstream will solve PID ambiguity below 5 mrad (above that, the angular measurements provide PID).

With $\mathcal{L}_{\text{int}} \sim 1.5 \times 10^7$ nb$^{-1}$ we estimate that we can reach a statistical sensitivity of ~ 0.3% on $a_\mu^{\text{HLO}}$, ie ~ 20 × 10$^{-11}$!
Systematic effects must be known at the level of ≤ 10ppm!

**Systematics**

1. Acceptance
2. Tracking
3. Trigger
4. PID
5. Effects of $E_e$ energy cut
6. Signal/Background:
   - It requires a dedicated event generator.
7. Uncertainty in the location of interaction vertices: Segmented/active target to resolve the vertex position
8. Uncertainty in the muon beam momentum:
   - Scattering kinematics to determine the beam momentum
9. Effects of Multiple Scattering (must be known at ~1%):
   - It requires dedicated work on simulation and measurements (test beam).
10. Theoretical uncertainty on the mu-e cross section (see later)

All the systematic effects must be known to ensure an error on the cross section < 10ppm
Check GEANT MSC prediction and populate the 2D \((\theta_e, \theta_\mu)\) scattering plane

- 27 Sep-3 October 2017 at CERN "H8 Beam Line"
- Adapted UA9 Apparatus
- Beam energy: e- of 12/20 GeV; \(\mu\) of 160 GeV
- \(10^7\) events with C targets of different thickness (2, 4, 8, -20mm)

Data analysis under way.
To extract $\Delta\alpha_{\text{had}}(t)$ from the measured cross section, the SM prediction must be known at NNLO!

Muon-electron scattering: theory

NLO QED corrections known & checked. Pavia group: MC ready!

NNLO QED corrections unknown.

NLO hadronic contributions unknown. NB: the hadronic light-by-light contributions are of higher order!! 😁

Dedicated high-precision MC tools needed.

Possible interplay with lattice calculations.

Explore the new physics sensitivity of this experiment!
State-of-the-art methods required to calculate the 2-loop diagrams.

Two-loop box diagrams:

- Missing Master Integrals for the planar 2-loop box diagrams computed.

Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

The aim of the workshop is to explore the opportunities offered by a recent proposal for a new experiment at CERN to measure the scattering of high-energy muons on atomic electrons of a low-Z target through the process $\mu e \rightarrow \mu e$. The focus will be on the theoretical predictions necessary for this scattering process, its possible sensitivity to new physics signals, and the development of new high-precision Monte Carlo tools. This kickoff workshop is intended to stimulate new ideas for this project.

It is organized and hosted by INFN Padova and the Physics Department.

Organizing Committee
Carlo Carloni Calame - INFN Pavia
Pierpaolo Mastrolia - U. Padova
Guido Montagna - U. Pavia
Oreste Nicrosini - INFN Pavia
Paride Paradisi - U. Padova
Massimo Passera - INFN Padova (Chair)
Fulvio Piccinini - INFN Pavia
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https://agenda.infn.it/internalPage.py?pageId=0&confId=13774
SCIENTIFIC PROGRAMS

**Probing Physics Beyond SM with Precision**
Ansgar Denner \textit{u} Würzburg, Stefan Dittmaier \textit{u} Freiburg, Tilman Plehn \textit{u} Heidelberg

February 26-March 9, 2018

**Bridging the Standard Model to New Physics with the Parity Violation Program at MESA**
Jens Erler \textit{u} UNAM, Mikhail Gorshteyn, Hubert Spiesberger \textit{JGU}

April 23-May 4, 2018

**Modern Techniques for CFT and AdS**
Bartłomiej Czech \textit{IAS Princeton}, Michal P. Heller \textit{MPI for Gravitational Physics}, Alessandro Vichi \textit{EPFL}

May 28-June 8, 2018

**The Dawn of Gravitational Wave Science**

June 4-15, 2018

**String Theory, Geometry and String Model Building**
Philip Candelas, Xenia de la Ossa, Andre Lukas \textit{U Oxford}, Daniel Waldram \textit{Imperial College London}, Gabriele Honecker \textit{JGU}, Duco van Straten

September 10-21, 2018

TOPICAL WORKSHOPS

**The Evaluation of the Leading Hadronic Contribution to the muon anomalous magnetic moment**
Massimo Passera \textit{INFN Padua}, Luca Trentadue \textit{u Parma}, Carlo Carloni Calame \textit{INFN Pavia} Graziano Venanzoni \textit{INFN Frascati}

February 19-23, 2018

**Challenges in Semileptonic B Decays**
Paolo Gambino \textit{u Turin}, Andreas Kronfeld \textit{Fermilab}, Marcello Rotondo \textit{INFN-LNF Frascati}, Christof Schwanda \textit{O EWA Vienna}

April 16-20, 2018

**Tension in LCDM Paradigm**
Cora Dvorkin \textit{u Harvard}, Silvia Galli \textit{iAP Paris}, Fabio Iocco \textit{ICTP-SAIFR}, Federico Marinacci \textit{MIT}

May 14-18, 2018

**Probing Physics Beyond SM with Precision**
Ansgar Denner \textit{u Würzburg}, Stefan Dittmaier \textit{u Freiburg}, Tilman Plehn \textit{u Heidelberg}

February 26-March 9, 2018

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September 10-21, 2018
Experimental plans for 2018

- Build & test a full-scale prototype (2 modules) on a muon beam at Cern:

![Diagram of muon beam setup]

- We have been allocated 1 week (22-29 August 2018) of high-energy muon beam (160 GeV) in H8 (A138).
- We will also run on the M2 line behind COMPASS (start-up in April).
- This proposal is part of the Physics Beyond Colliders activities @ Cern
- Proposal presented in 2017 to INFN’s Committee 1. Funds have been allocated for a full-scale test of a detector prototype in 2018. Letter of Intent planned for 2018-19.
Muon g-2: $\Delta a_\mu \sim 3.5 - 4 \sigma$. New upcoming measurement: QED & EW ready. Lots of progress in the hadronic sector, but not yet ready!

New proposal for an experiment at CERN to measure the leading hadronic contribution to the muon g-2 via $\mu$-$e$ elastic scattering.

$\mu$-$e$ exp: first $\mu$-$e$ testbeam completed at CERN in October. Data analysis under way. One more week of testbeam allocated next August. Test run planned also on CERN's M2 line in April. INFN funds allocated for a test of a detector prototype in 2018.

$\mu$-$e$ th: NLO MC generator ready. Lots of theoretical work needed! NNLO QED & NLO hadronic corrections unknown. First results obtained for the NNLO planar box diagrams contributing to $\mu$-$e$ scattering in QED. Dedicated high-precision MC tools needed. The new physics sensitivity of this experiment must be explored!

JOIN US!
The End
\[ L = 1.5 \times 10^7 \text{ nb}^{-1} \]
\[ \sigma_{LO} = 245.04 \mu \text{b} \]
\[ E^f_{c} > 1 \text{ GeV} \]