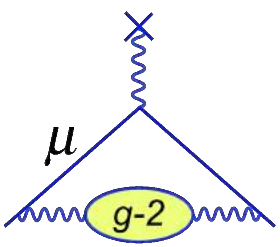
The muon g-2: theory and experiment



Alex Keshavarzi

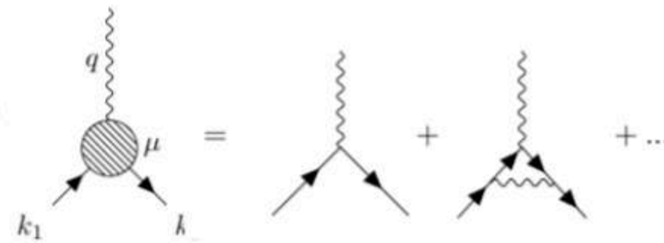
KEK Seminar

30th September 2019

The anomalous magnetic moment

Magentic moment: $\vec{\mu} = -\frac{e}{2m} g \vec{S}$: $\quad g = 2$ (Dirac) $\quad \Rightarrow$ bare (no quantum effects)

$\Rightarrow$ Renormalise the QED vertex: $\Gamma^{\mu}_{\rm phys}(k_1, k_2) =$



$\Rightarrow$ By virtue of Gordon decomposition: $\Gamma^{\mu}(k_1, k_2) = -ie\left[\gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2)\right]$

$\Rightarrow$ In NR limit, with $k_2 - k_1 = q \to 0$: $\left(\gamma^{\mu}(\partial_{\mu} - ieA_{\mu}) - m + \frac{eF_2(0)}{4m} F_{\mu\nu}\sigma^{\mu\nu}\right)\psi = 0$

$\rightarrow$ New $F_2(0)$ form factor term in Dirac equation from radiative corrections

$\Rightarrow$ Effective Hamiltonian: $H = \left[\frac{1}{2m}(\vec{p} - e\vec{A})^2 + eA^0 + \frac{e}{2m}(1 + F_2(0))\vec{\sigma}.\vec{B}\right]$

$\rightarrow$ Magnetic potential $U = -\vec{\mu} \cdot \vec{B} = \frac{e}{2m}(1 + F_2(0))\vec{\sigma}.\vec{B} = \frac{e}{2m} g \vec{S}.\vec{B} = \frac{e}{4m} g \vec{\sigma}.\vec{B}$

$\Rightarrow$ Implies $g = 2 + 2F_2(0)$ with quantum effects $\longrightarrow a = F_2(0) = (g-2)/2$

A quick history recap…

1948 – Kusch and Foley measure g_e = **2.00238 +/- 0.00006**

PHYSICAL REVIEW VOLUME 74, NUMBER 3 AUGUST 1, 1948

The Magnetic Moment of the Electron†

P. KUSCH AND H. M. FOLEY
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

A comparison of the g_J values of Ga in the ${}^2P_{3/2}$ and ${}^2P_{1/2}$ states, In in the ${}^2P_{1/2}$ state, and Na in the ${}^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L=1$ and $g_S=2(1.00119\pm0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

1947 – Schwinger calculates $g_e \approx 2(1+\frac{\alpha}{2\pi}) \approx$ **2.00232**

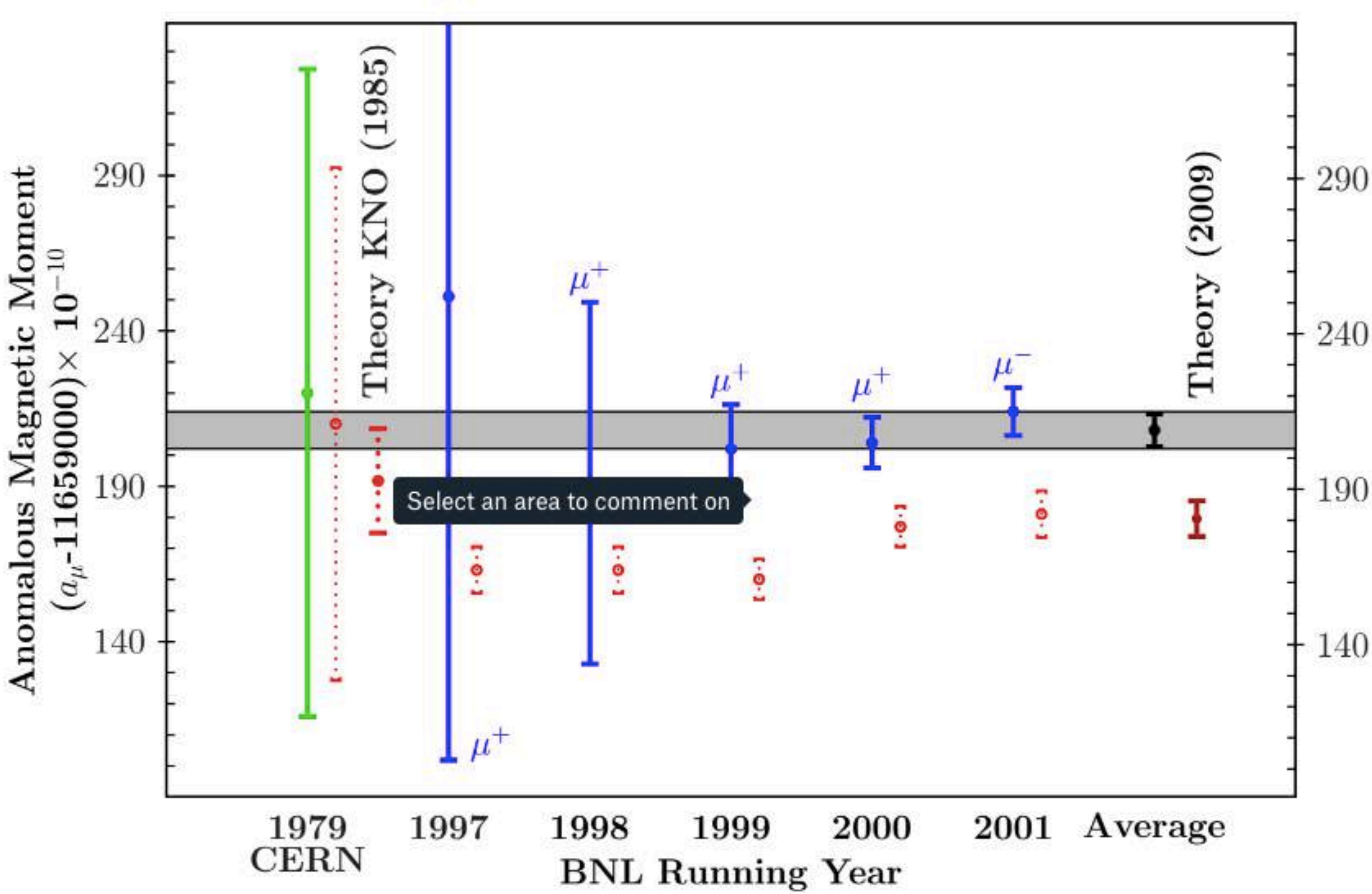
On Quantum-Electrodynamics and the Magnetic Moment of the Electron

JULIAN SCHWINGER
Harvard University, Cambridge, Massachusetts

December 30, 1947

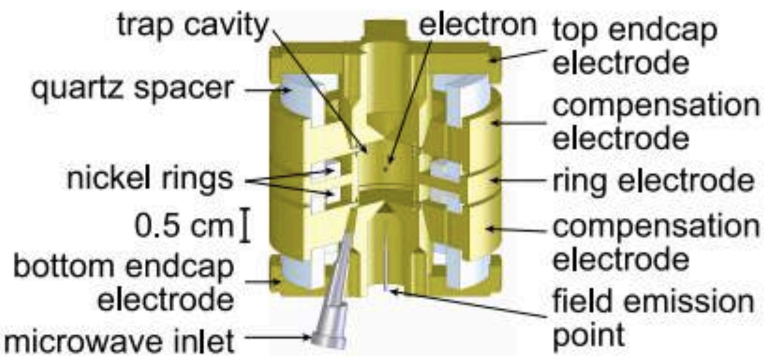
ATTEMPTS to evaluate radiative corrections to electron phenomena have heretofore been beset by divergence difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from

A quick history recap…



Magnetic moments: a_e vs. a_μ

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]
Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801



one electron quantum cyclotron

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]
Bennet et al., PRD 73(2006)072003

- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales

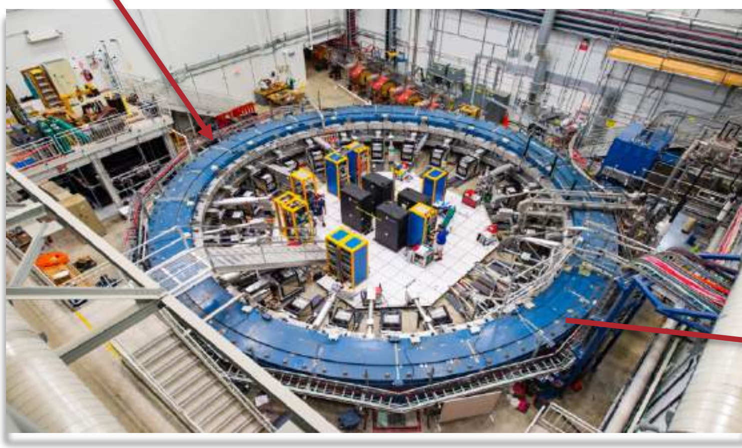
- dimensional analysis: **sensitivity to NP** (at high scale Λ_{NP}): $a_\ell^{\mathrm{NP}} \sim C\, m_\ell^2/\Lambda_{\mathrm{NP}}^2$

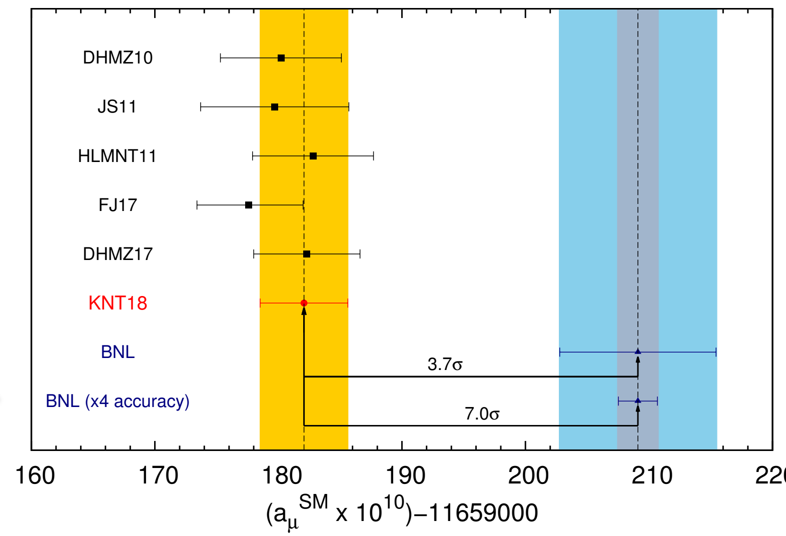
→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, but a_e provides precise determination of α

Why do we care about the muon anomaly?

Currently, there is a $> 3\sigma$ discrepancy between theory and experiment (new physics?!)…

Fermilab experiment is set to improve the uncertainty on a_μ by 4x compared to BNL





Keshavarzi, Nomura & Teubner (KNT18), Phys. Rev. D. **97** 114025 (2018).

- BNL experiment achieved 540ppb precision.
- Fermilab experiment targeted to reach 140ppb precision.
- Requires taking 20x statistics compared to BNL.
- If mean value is unchanged, this would result in a $\sim 7\sigma$ discrepancy between theory and experiment.
- Therefore, theory estimates are further improving as we will see…

The Muon g-2 theory initiative

A year of meetings/workshops of the Muon $g-2$ theory initiative...

→ First full workshop, 3-6 June 2017, Fermilab/Q Centre

→ HVP workshop, 12-14 February 2018, KEK, Japan

→ HLbL workshop, 12-14 March 2018, University of Connecticut

→ Second full workshop, 18-22 June 2018, JGU Mainz

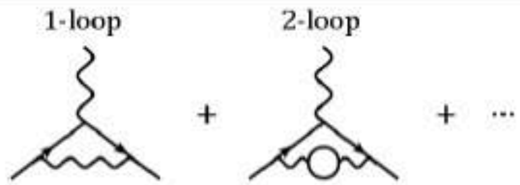
Proposed paper structure (one ~300 page paper and one ~20 page overview)

- Intro, QED, EW
- HVP
  - Data-driven HVP
  - Lattice HVP
  - Future of HVP
- HLbL
  - Model LbL
  - Lattice LbL
  - Analytical LbL
- BSM $g-2$

The theoretical determination of a_μ

$$a_\mu^{\text{Theory}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had, VP}} + a_\mu^{\text{had, LbL}} + a_\mu^{\text{BSM??}}$$

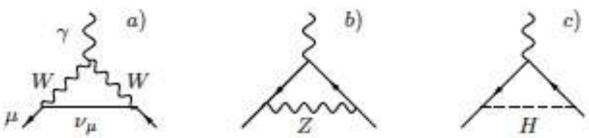
QED



Known to five-loop (12,672 diagrams)

99.99% of a_μ^{SM} ~0.001% of δa_μ^{SM}

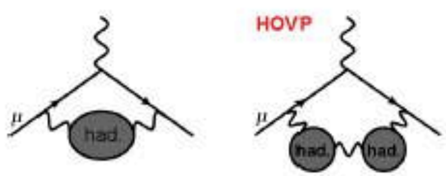
EW



Known to two-loop (with m_H known)

0.0001% of a_μ^{SM} ~0.2% of δa_μ^{SM}

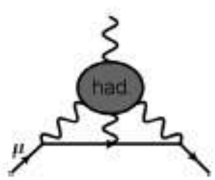
HVP



Non-perturbative (data input + lattice)

0.006% of a_μ^{SM} ~47% of δa_μ^{SM}

HLbL



Non-perturbative (data input + model/lattice)

0.0001% of a_μ^{SM} ~53% of δa_μ^{SM}

BSM

? ? ? ? ? ? ?

QED: a five-loop success

⇒ All contributions due to photons and leptons alone

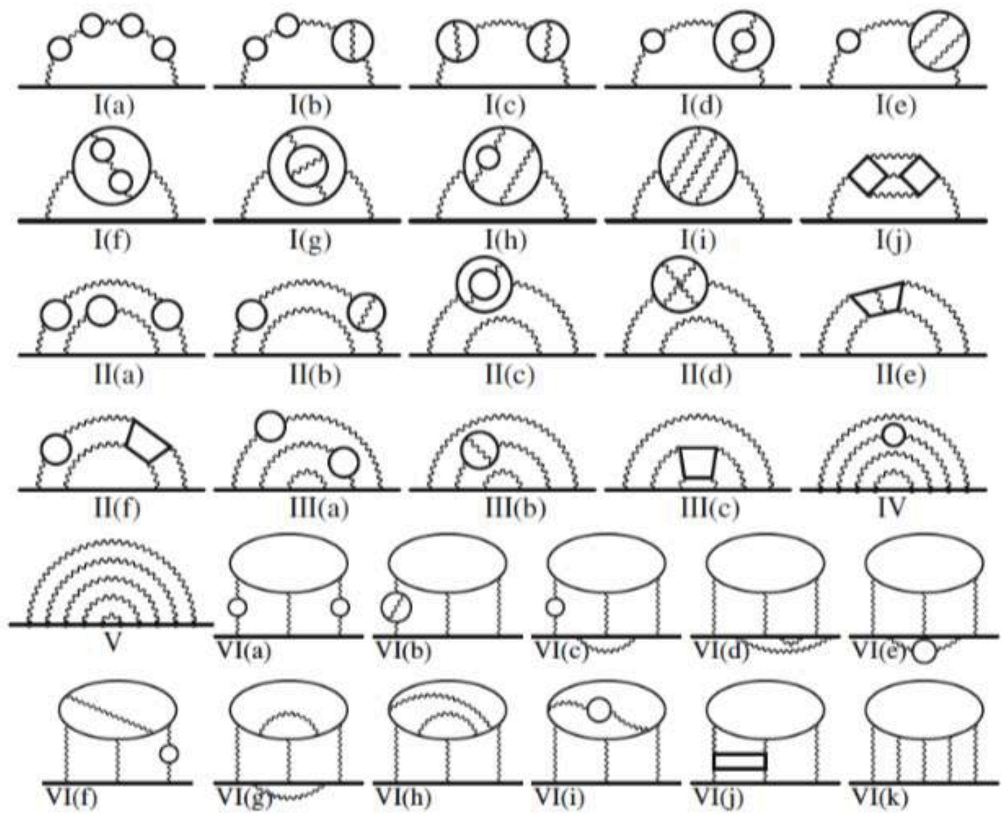
- All one, two, three, four and five-loop contributions have now been accurately calculated
- All 12,762 five-loop diagrams determined numerically
- Huge success for perturbative QFT and computing
- Independent checks crucial...

Phys. B 877 (2013) 647.
Nucl. Phys. B 879 (2014) 1.
Phys. Rev. D 92 (2015), 073019.
Phys. Rev. D 93 (2016) 053017.
Phys. Lett. B 772 (2017) 232.

...and all corroborate Kinoshita's results

⇒ QED results safe

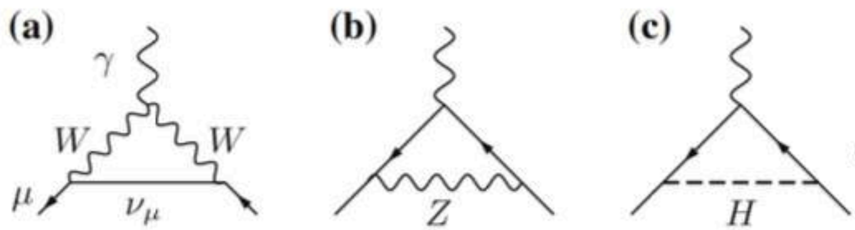
Phys. Rev. Lett. 109 (2012) 111808, Phys. Rev. D 97 (2018) 036001.



$$a_\mu^{\mathrm{QED}} = 11\ 658\ 471.8971\ (0.0007)_{m_l}(0.0017)_{41}(0.0006)_{51}(0.0072)_{\alpha(R_b)} \times 10^{-10}$$

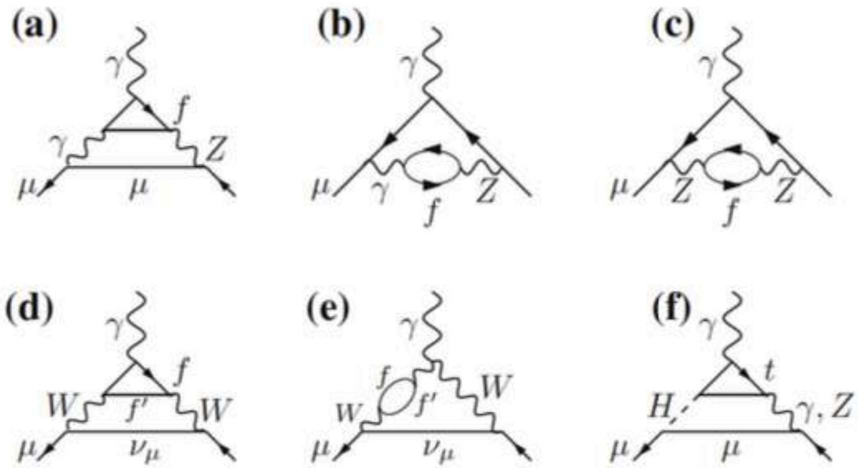
EW contributions

⇒ 1-loop:



$$a_\mu^{\mathrm{EW,1-loop}} = (19.5 \pm 0.0) \times 10^{-10}$$

⇒ 2-loop:



$$a_\mu^{\mathrm{EW,2-loop}} = (-4.1 \pm 0.1) \times 10^{-10}$$
$$\delta a_\mu[m_H, m_t] = 0.022 \times 10^{-10}$$
$$\delta a_\mu[\mathrm{had}] = 0.072 \times 10^{-10}$$

[F. Jegerlehner, Springer Tracts Mod. Phys. 274 (2017) 1.]

Total

$$a_\mu^{\mathrm{EW}} = (15.4 \pm 0.1) \times 10^{-10}$$

[Phys. Rev. D 88 (2013) 053005.]

⇒ Total uncertainty has halved now with m_H known

⇒ Relatively small contribution when compared to a_μ^{QED}

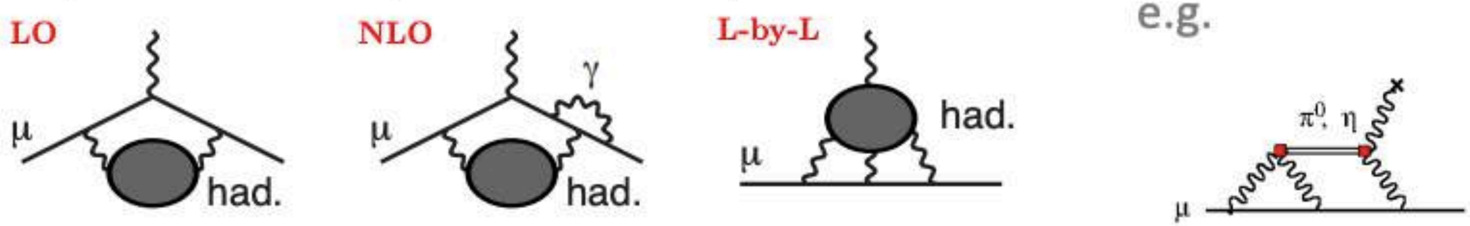
⇒ Uncertainty is not negligible considering projected experimental accuracy, but small compared to hadronic and well under control

⇒ **EW results safe**

Hadronic contributions

- Hadronic: non-perturbative, the limiting factor of the SM prediction ✗ → ✓

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$



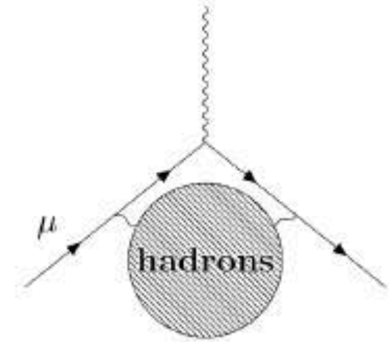
- L-by-L: - so far use of model calculations (+ form-factor data and pQCD constraints),
  - but very good news from lattice QCD, and
  - from new dispersive approaches

- For the moment, still use the `updated Glasgow consensus':
  (original by Prades+deRafael+Vainshtein) $a_\mu^{\text{had,L-by-L}} = (98 \pm 26) \times 10^{-11}$

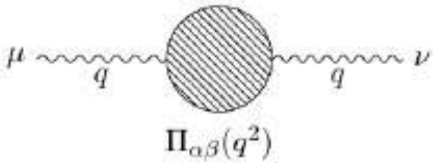
- But first results from new approaches confirm existing model predictions and
- indicate that L-by-L prediction will be improved further
- with new results & progress, tell politicians/sceptics: L-by-L _can_ be predicted!

$a_\mu^{\text{had, VP}}$: theoretical setup

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

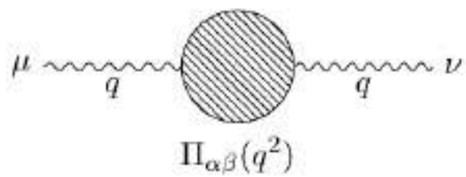


1) Feynman rules for HVP insertion to photon propagator:



$$= \frac{-ig^{\mu\alpha}}{(q^2 - i\varepsilon)}(-ie)i\Pi_{\alpha\beta}(q^2)(-ie)\frac{-ig^{\beta\nu}}{(q^2 - i\varepsilon)}$$

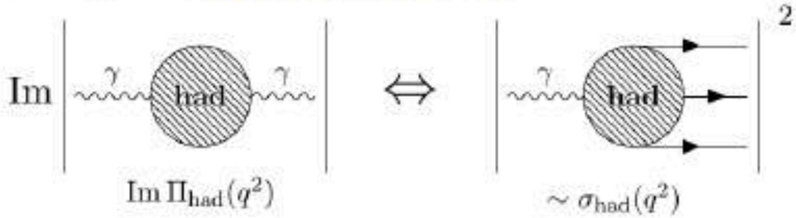
2) Employ analyticity:



$$= \frac{ie^2 g_{\mu\nu}}{(q^2 - i\varepsilon)^2}\frac{q^4}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\Pi(s)}{s(s - q^2 - i\varepsilon)}$$

3) Insert to vertex correction, solve for a_μ: $a_\mu^{\text{had, LO VP}} = \frac{\alpha}{\pi^2}\int_{s_{th}}^{\infty} \frac{\mathrm{d}s}{s}\mathrm{Im}\,\Pi_{\text{had}}(s)K(s)$

4) Utilise optical theorem:



5) Arrive at equation for $a_\mu^{\text{had, LO VP}}$:

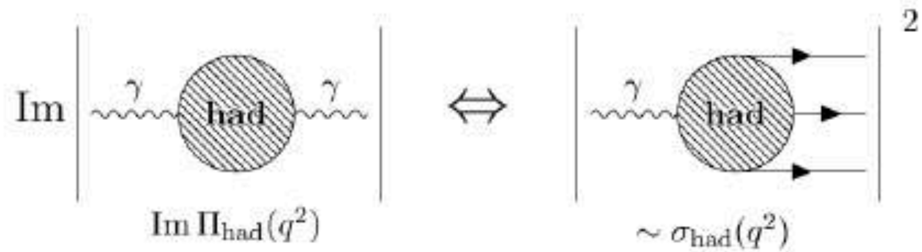
$$a_\mu^{\text{had, LO VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} \mathrm{d}s\, \sigma^0_{\text{had},\gamma}(s)K(s)$$

$\sigma^0_{\text{had},\gamma}$ = bare cross section, FSR included

⇒ Similar dispersion integrals for NLO and NNLO HVP

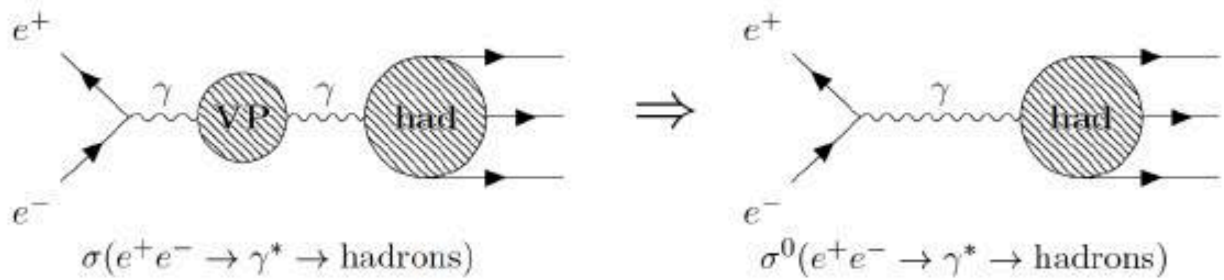
$\sigma^0_{\text{had},\gamma}$: VP corrections

⇒ Reconsider the optical theorem:



⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\text{had, VP}}$

→ Must subtract VP:



⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0], available for distribution

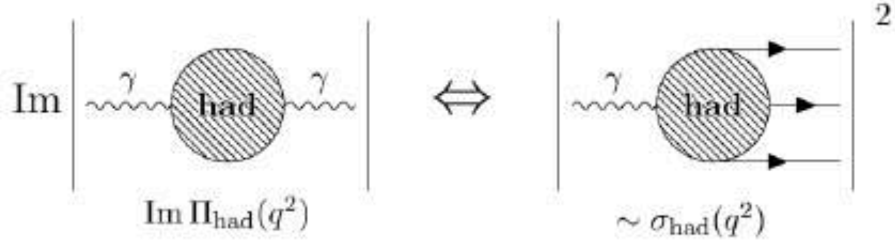
→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\text{had}}(s) = \sigma_{\text{had}}(s)|1 - \Pi(s)|^2$

⇒ If correcting data, apply corresponding radiative correction uncertainty

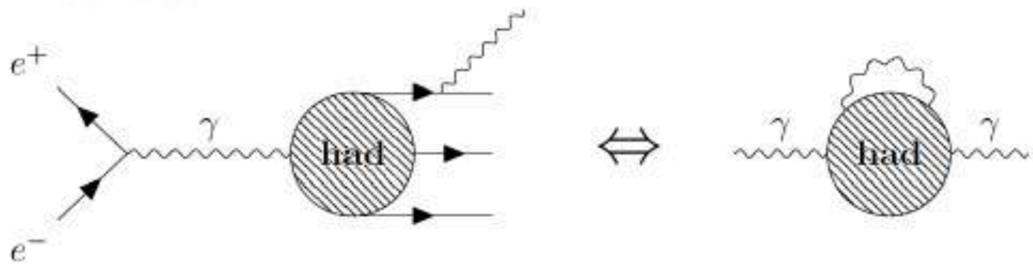
→ Take $\frac{1}{3}$ of total correction per channel as conservative extra uncertainty

$\sigma^0_{\mathrm{had},\gamma}$: FSR corrections

⇒ Reconstructing the optical theorem:



⇒ Photon FSR formally higher order corrections to $a_\mu^{\mathrm{had,\,VP}}$



⇒ Cannot be unambiguously separated, not accounted for in HO contributions

→ Must be included as part of 1PI hadronic blobs

⇒ Experiment may cut/miss photon FSR → Must be added back

⇒ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For higher multiplicity states, difficult to estimate correction

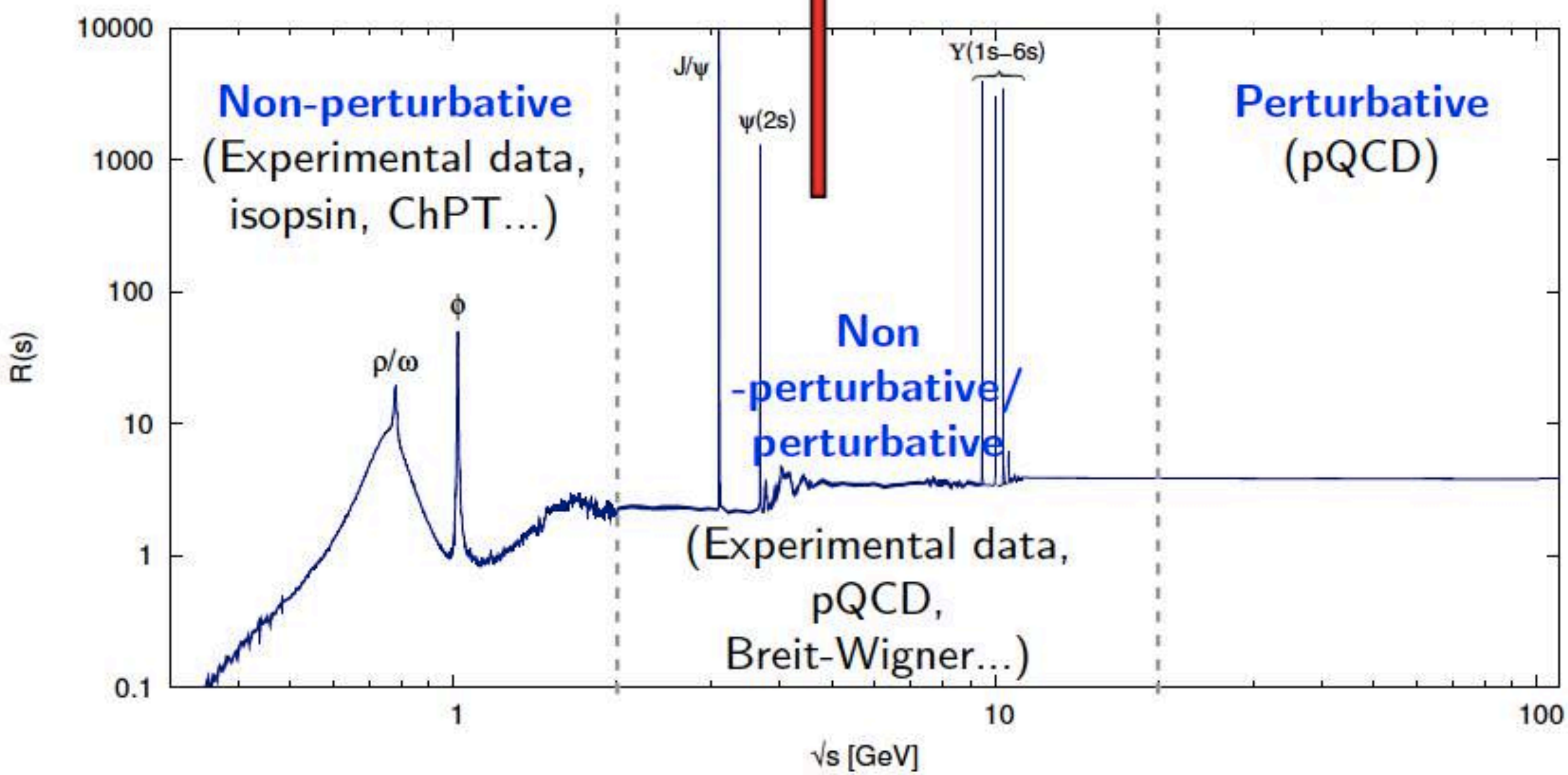
∴ Apply conservative uncertainty

Need new, more developed tools to increase precision here

(e.g. - CARLOMAT 3.1 [Eur.Phys.J. C77 (2017) no.4, 254]?)

Building the hadronic *R*-ratio

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



$a_\mu^{\text{had, VP}}$ from KNT (now being updated)

The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis

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Abstract

This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon, $a_\mu^{\text{had, VP}}$ and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\text{had}}(M_Z^2)$, from the combination of $e^+e^- \to$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^- \to$ hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic R-ratio and its covariance matrix in the energy range $m_\pi \leq \sqrt{s} \leq 11.2$ GeV. Using these combined data and perturbative QCD above that range results in estimates of the hadronic vacuum polarisation contributions to $g-2$ of the muon of $a_\mu^{\text{had, LO VP}} = (693.26 \pm 2.46) \times 10^{-10}$ and $a_\mu^{\text{had, NLO VP}} = (-9.82 \pm 0.04) \times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\text{SM}} = (11\,659\,182.04 \pm 3.56) \times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$, resulting in $\alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$. Detailed comparisons with results from similar related works are given.

Building the hadronic *R*-ratio

$m_\pi \leq \sqrt{s} \leq 2$ GeV

- Input experimental hadronic cross section data*
- Combine all available data in exclusive hadronic final states $(\pi^+\pi^-, K^+K^-, \ldots)$
- Sum ~ 35 exclusive channels
- Detailed data analysis
- Robust treatment of experimental errors
- Estimate missing data input (isospin relations, ChPT...)

$2 \leq \sqrt{s} \leq 11.2$ GeV

- Can use experimental inclusive R data* or pQCD
- Must use data at quark flavour thresholds
- Combine all available R data
- Robust treatment of experimental errors
- Include narrow resonances

$11.2 \leq \sqrt{s} < \infty$ GeV

- Calculate R using pQCD (`rhad`)

$^*\sigma_{\rm had}$ experiments

- KLOE
- BaBar
- SND
- CMD-(2/3)
- KEDR
- BESIII

Question: for reliable precision, how are data correlated and how should those correlations be implemented?

Data combination: setup

⇒ Re-bin data into *clusters*

→ Scan cluster sizes for preferred solution (error, χ^2, check by sight...)

⇒ Correlated data beginning to dominate full data compilation...

→ Non-trivial, energy dependent influence on both mean value and error estimate

KNT18 prescription

- Construct full covariance matrices for each channel & entire compilation
  ⇒ Framework available for inclusion of any and all inter-experimental correlations
- If experiment does not provide matrices...
  → Statistics occupy diagonal elements only
  → Systematics are 100% correlated
- If experiment does provide matrices...
  → Use all information provided
- Use correlations to full capacity

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

$\Rightarrow$ When combining data...

- $\rightarrow$ ...how to best combine large amounts of data from different experiments
- $\rightarrow$ ...the correct implementation of correlated uncertainties (statistical and systematic)
- $\rightarrow$ ...finding a solution that is free from bias

d'Agostini bias [Nucl.Instrum.Meth. A346 (1994) 306-311]

$$x_1 = 0.9 \pm \delta x_1 \quad C_{\text{sys}} = \begin{pmatrix} p^2 x_1^2 & p^2 x_1 x_2 \\ p^2 x_2 x_1 & p^2 x_2^2 \end{pmatrix}$$
$$x_2 = 1.1 \pm \delta x_2$$

$\Rightarrow \bar{x} \simeq 0.98$ (systematic bias)

(Normalisation uncertainties defined by data)

Effect worsened with full, iterative data combination

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

⇒ When combining data...

- → ...how to best combine large amounts of data from different experiments
- → ...the correct implementation of correlated uncertainties (statistical and systematic)
- → ...finding a solution that is free from bias

Fixed matrix method [R. D. Ball et al. [NNPDF Collaboration], JHEP 1005 (2010) 075.]

$$x_1 = 0.9 \pm \delta x_1 \qquad C_{\text{sys}} = \begin{pmatrix} p^2\bar{x}^2 & p^2\bar{x}^2 \\ p^2\bar{x}^2 & p^2\bar{x}^2 \end{pmatrix} \quad \Rightarrow \bar{x} = 1.00 \text{ (systematic bias)}$$
$$x_2 = 1.1 \pm \delta x_2$$

(Normalisation uncertainties defined by estimator)

Redefinition repeated at each stage of iterative data combination

Linear χ^2 minimisation

⇒ Clusters are defined to have linear cross section

→ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

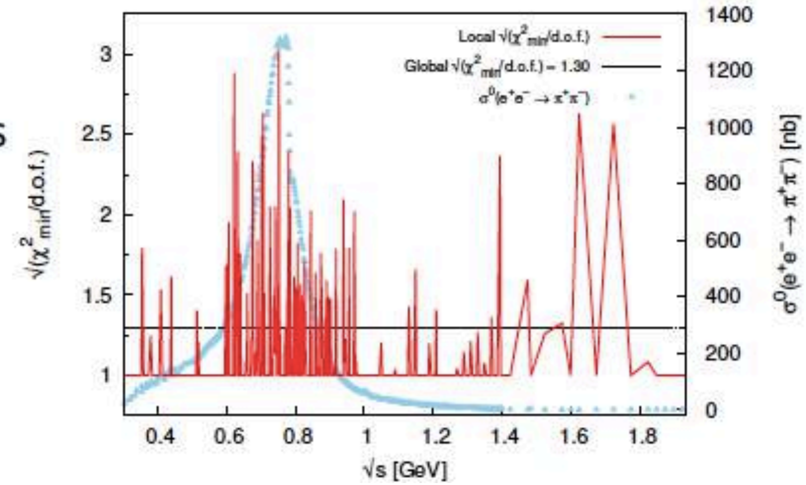
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^i\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right) \left(R_j^{(n)} - \mathcal{R}_n^j\right)$$

⇒ Through correlations and linearisation, result is the minimised solution of all available uncertainty information

→ ... through a method that has been shown to avoid d'Agostini bias

⇒ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

→ ... and any data tensions are reflected in a local and global $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation

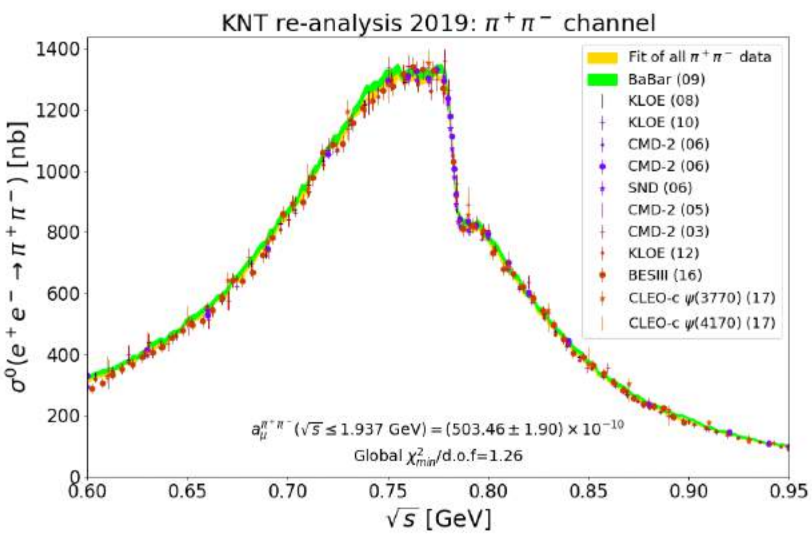


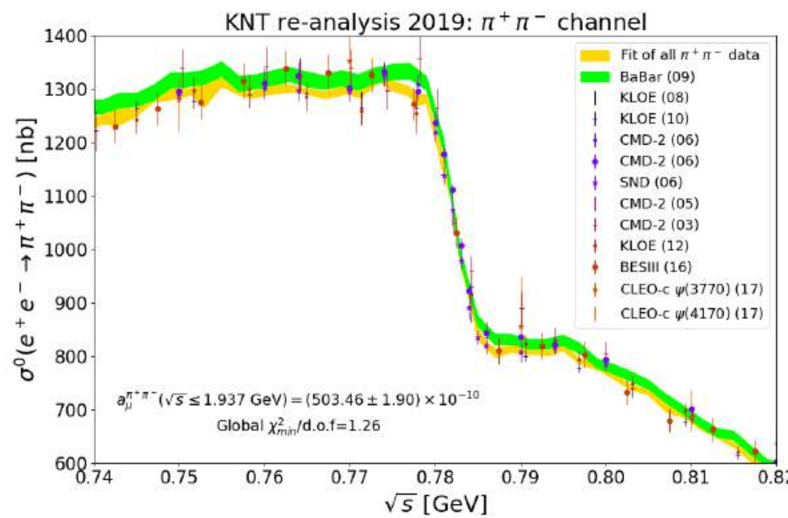
The $\pi^+\pi^-$ channel [preliminary]

$\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\text{had, LOVP}}$

→ Combines ~30 measurement totalling over 1000 data points

KNT 2019 update: combination now includes CLEO-c (2017) data [Phys.Rev. D97 (2018) 032012]





→ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 503.46 \pm 1.14_{stat} \pm 1.52_{sys} \pm 0.05_{vp} \pm 0.14_{fsr}$$

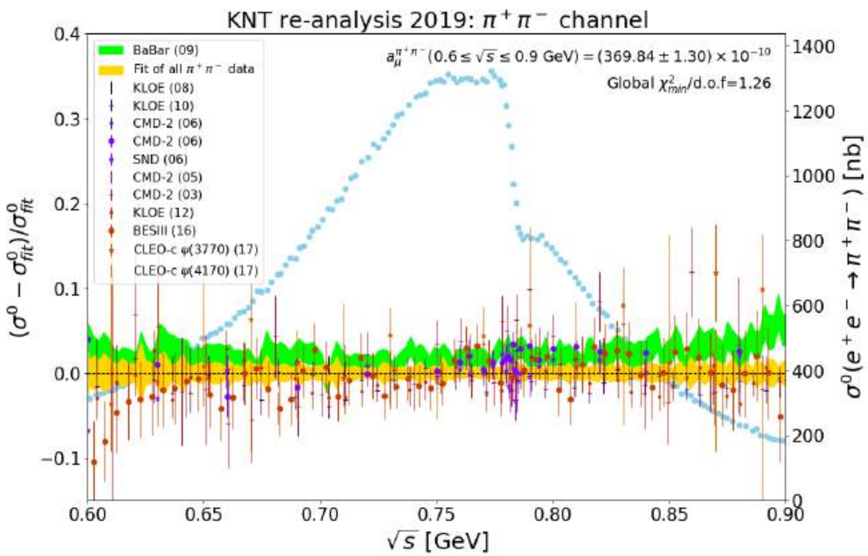
$$= 503.46 \pm 1.91_{tot} \quad \text{KNT18: } 502.97 \pm 1.97_{tot}$$

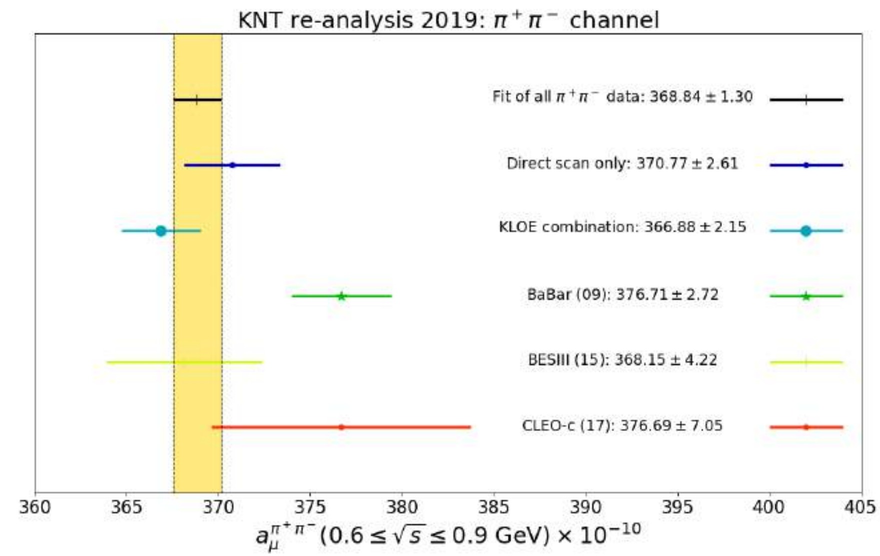
→ 14% local χ^2_{min}/d.o.f. error inflation due to tensions in clustered data

The $\pi^+\pi^-$ channel [preliminary]

Tension between BaBar and other data slightly alleviated by CLEO-c data [Phys.Rev. D97 (2018) 032012]

- However, large difference between KNT vs. BaBar and KLOE vs. BaBar is still evident





Compared to $a_\mu^{\pi^+\pi^-} = 503.5 \pm 1.9 \rightarrow a_\mu^{\pi^+\pi^-}$(BaBar data only) = 513.2 ± 3.8

Simple weighted average of all data $\rightarrow a_\mu^{\pi^+\pi^-}$(weighted average) = 509.2 ± 2.9

(i.e. – no correlations in determination of mean value)

BaBar data dominate when no correlations are accounted for in the mean value.

- Highlights the importance of incorporating available correlated uncertainties in fit.

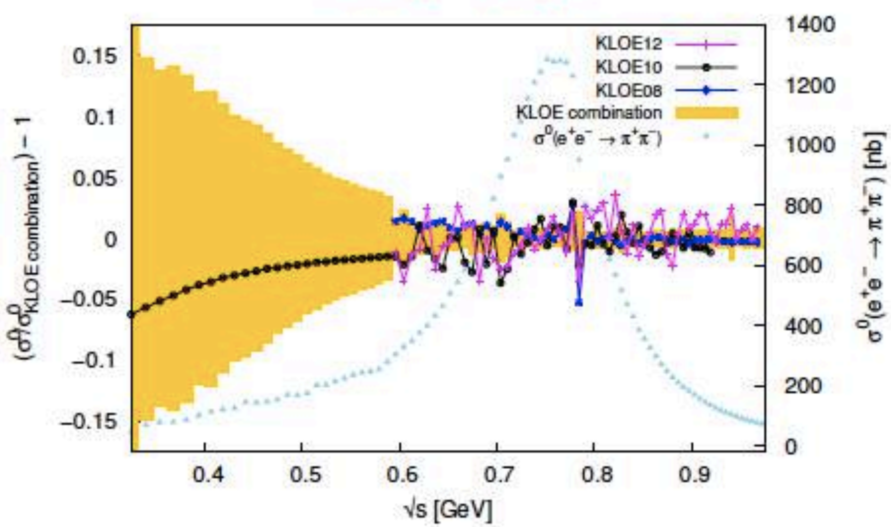
KNT vs. DHMZ: the use of correlations

Differences are dominated by the choices of how to use the correlations

$\Rightarrow$ Consider the combination of the KLOE $\pi^+\pi^-$ data sets:

KNT18

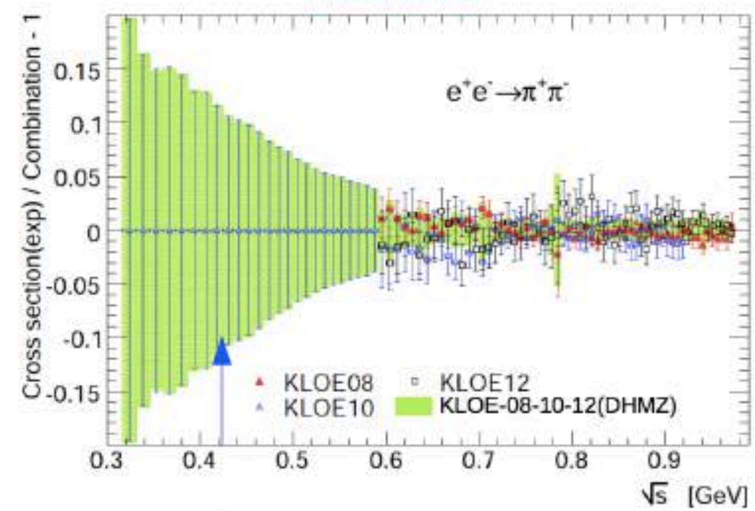
Use available uncertainty and correlation information to fully influence over entire energy range



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 503.74 \pm 1.96$$

DHMZ17

Only allow correlations to only influence the combination locally inside defined energy intervals

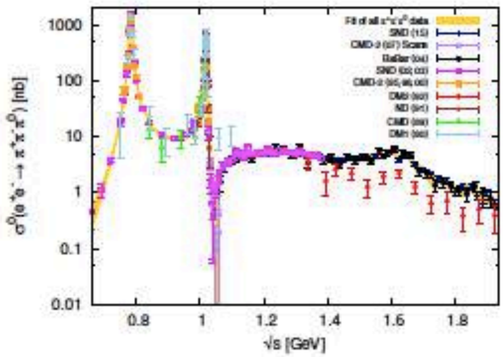


$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 507.14 \pm 2.58$$

Take-home message: correlations are important and the choices of how to use them are not trivial

Other notable channels

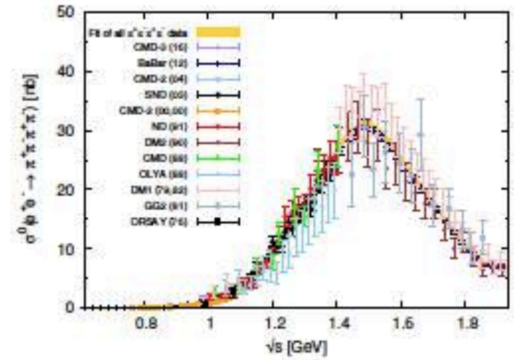
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

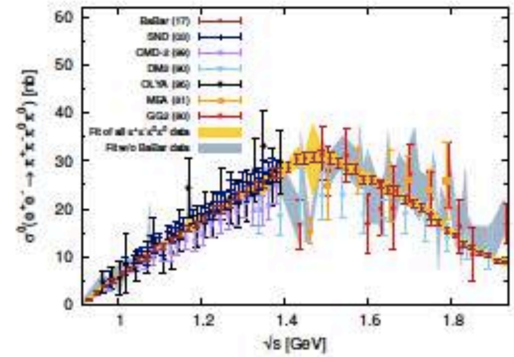
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

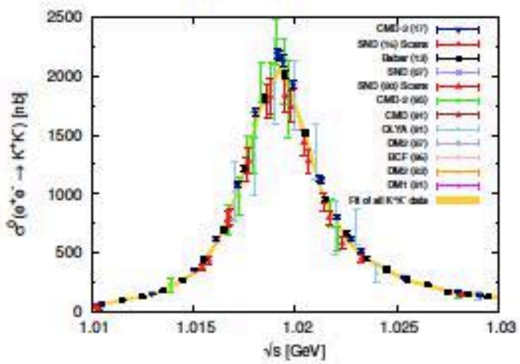
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

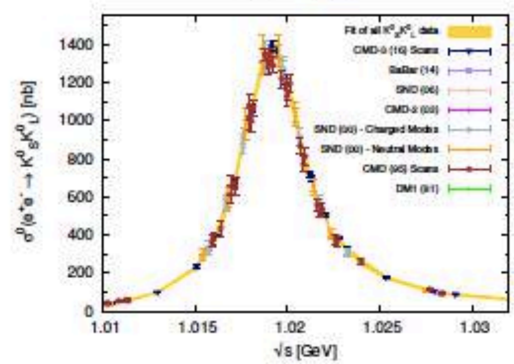
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

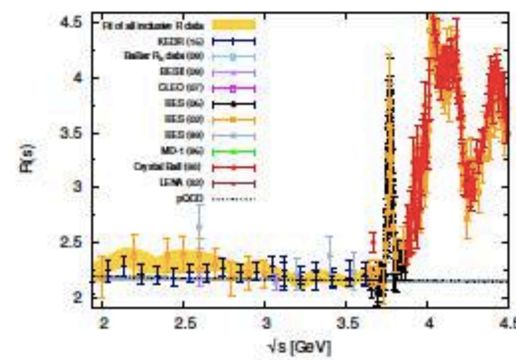
$K_S^0 K_L^0$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)

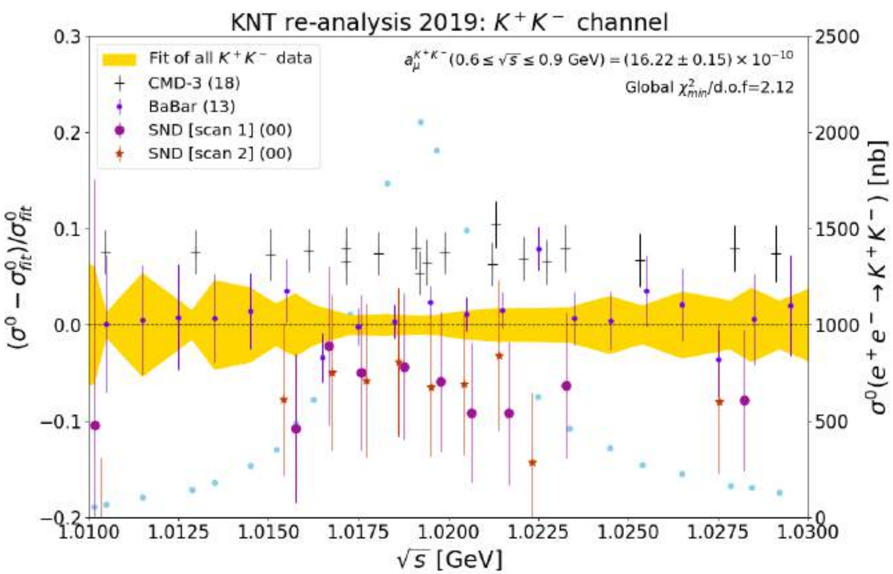


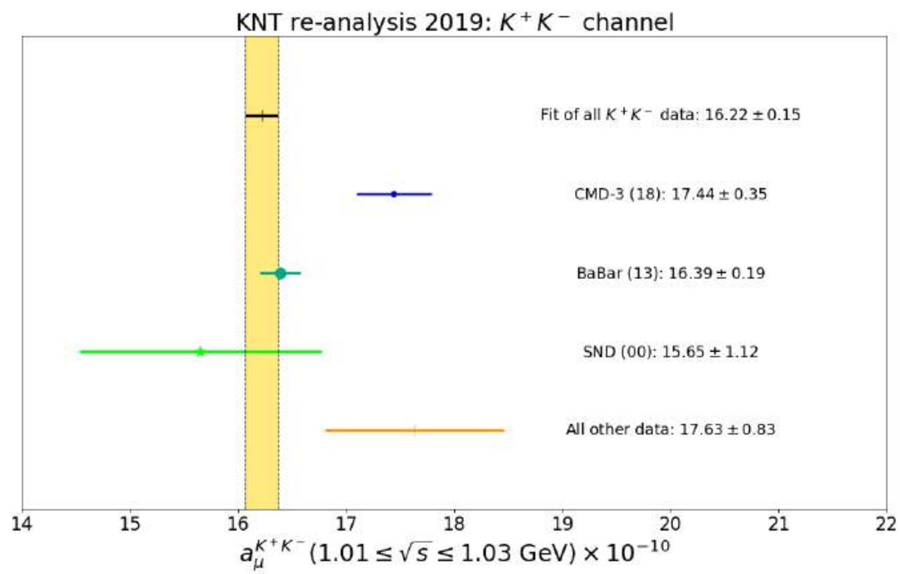
HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

Highlight: data tensions in K^+K^- channel

Notable tension also now exists on ϕ in $\boldsymbol{K^+K^-}$ channel:





Most recent CMD-3 data is higher than BaBar data…

… and BaBar was already relatively high compared to than previous direct scan data

(Note: previously used CMD-2 data under reanalysis and therefore omitted).

$$a_\mu^{K^+K^-}[0.9875 \le \sqrt{s} \le 1.937\ \text{GeV}] = 23.03 \pm 0.08_{stat} \pm 0.20_{sys} \pm 0.03_{vp} \pm 0.00_{fsr}$$
$$= 23.03 \pm 0.22_{tot}$$

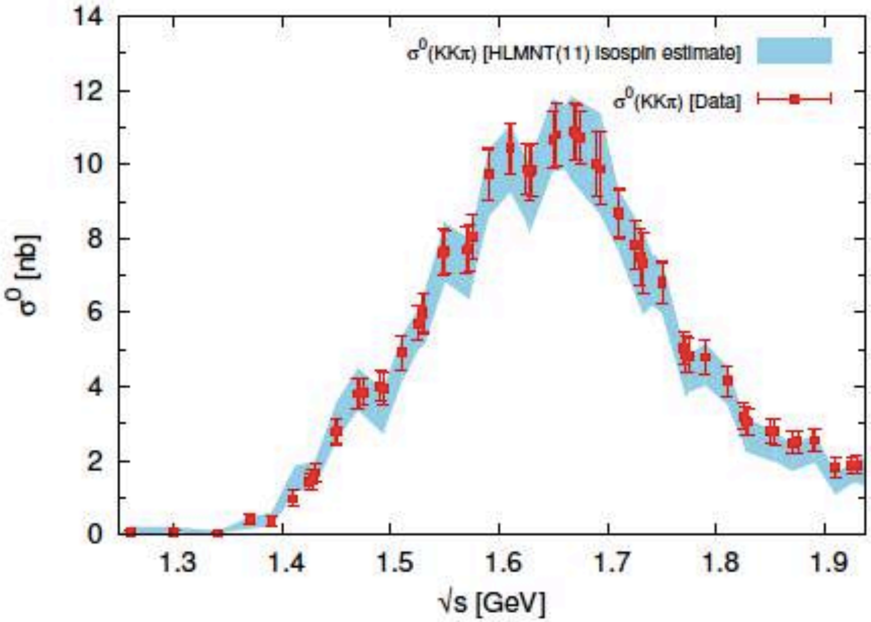
- Tensions results in 20 % local χ^2_{min}/d.o.f. error inflation

$KK\pi, KK\pi\pi$ & isospin

$\Rightarrow$ New data for $KK\pi$ and $KK\pi\pi$
removes reliance on isopsin (only $K_S^0 \cong K_L^0$)

$\underline{KK\pi}$

$K_S^0 K_L^0 \pi^0$ [Phys.Rev. D95 (2017), 052001, arXiv:1711.07143]

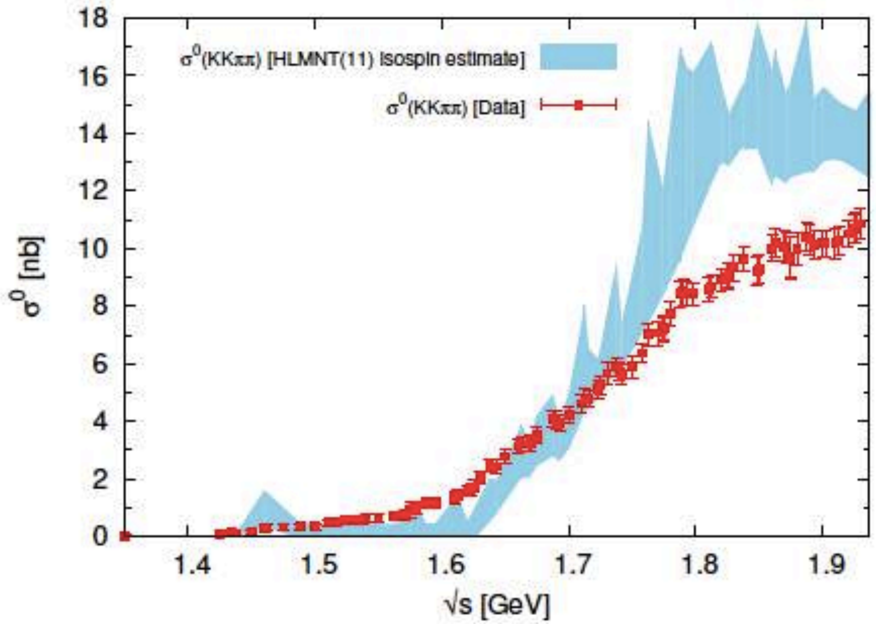


HLMNT11: 2.65 ± 0.14

KNT18: 2.71 ± 0.12

$\underline{KK\pi\pi}$

$K_S^0 K_L^0 \pi^+ \pi^-$ [Phys.Rev. D80 (2014), 092002]
$K_S^0 K_S^0 \pi^+ \pi^-$ [Phys.Rev. D80 (2014), 092002],
$K_S^0 K_L^0 \pi^0 \pi^0$ [Phys.Rev. D95 (2017), 052001]
$K_S^0 K^\pm \pi^\mp \pi^0$ [Phys.Rev. D95 (2017), 092005]

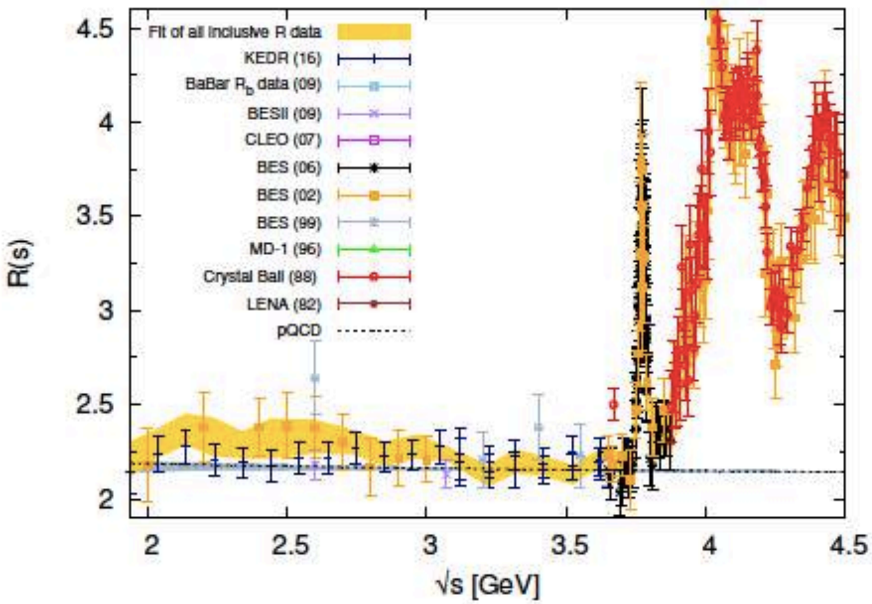


HLMNT11: 2.51 ± 0.35

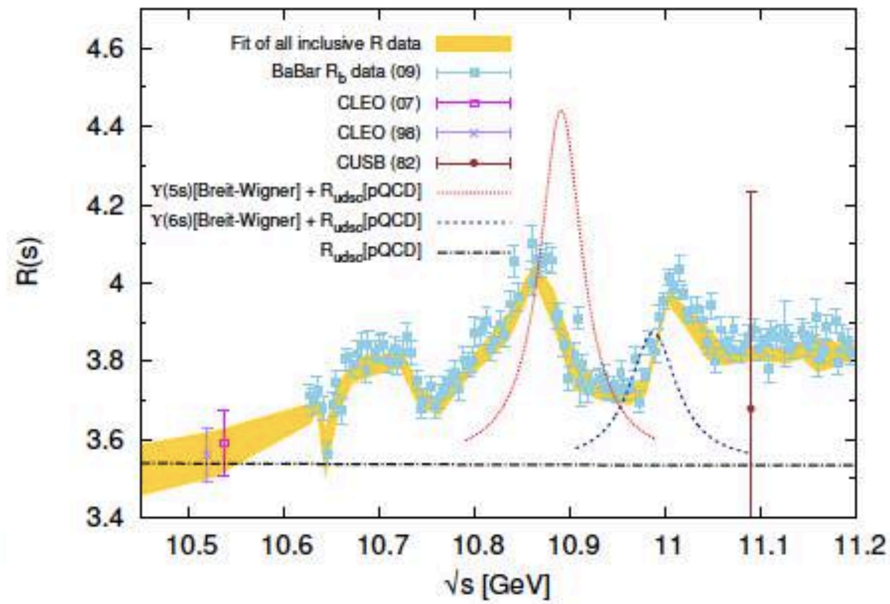
KNT18: 1.93 ± 0.08

Inclusive channel

⇒ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

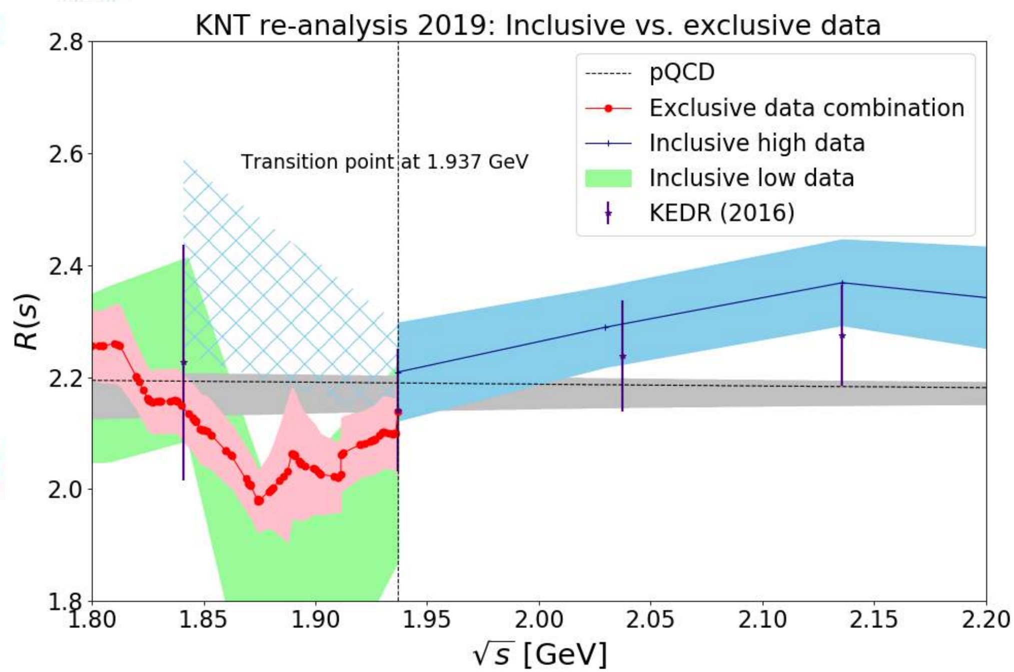
$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

Exclusive/inclusive transition point

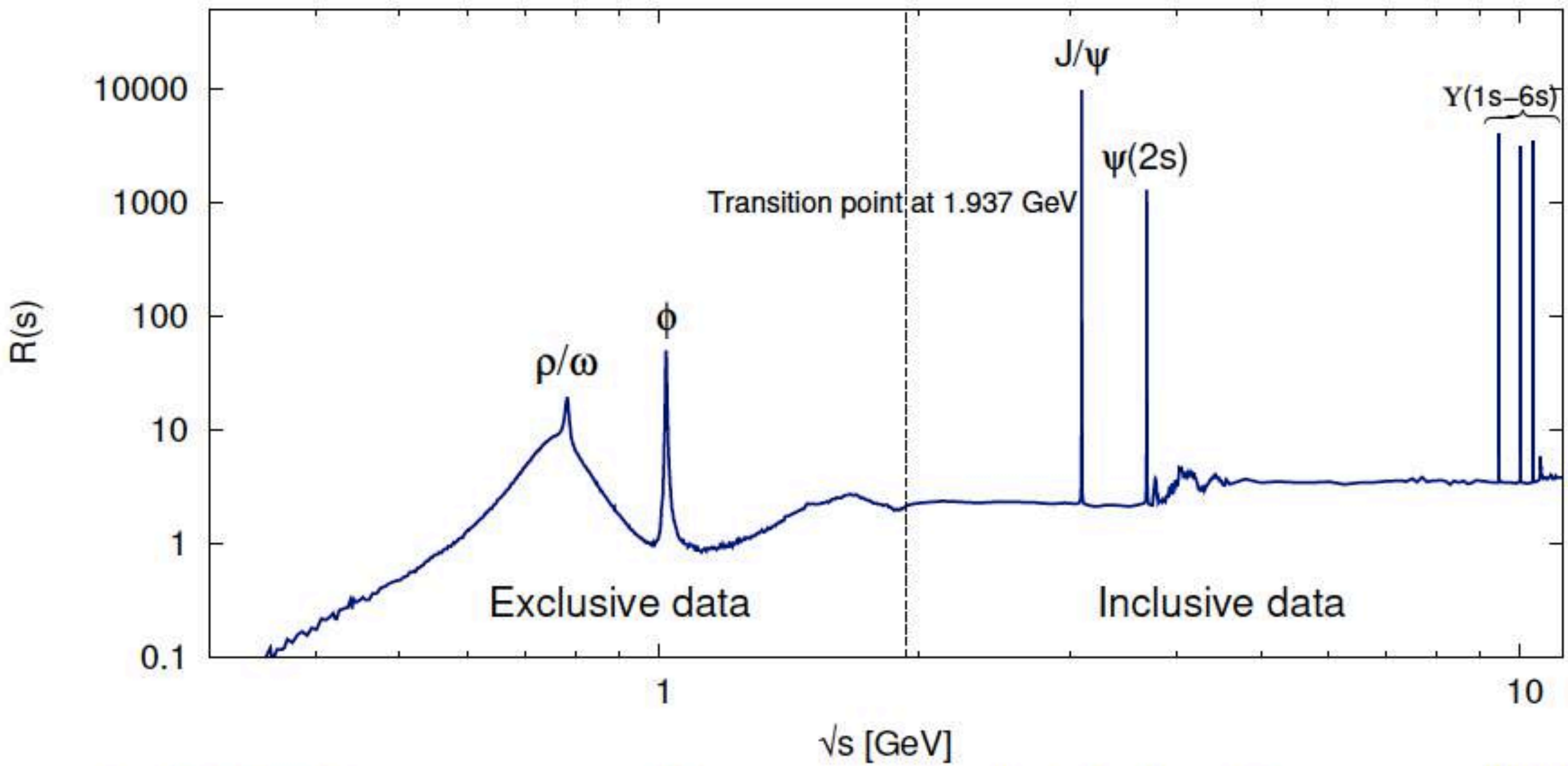
$\Rightarrow$ New KEDR data allow reconsideration of exclusive/inclusive transition point

- $\rightarrow$ KNT18 aim to avoid use of pQCD and keep a data-driven analysis
- $\rightarrow$ Disagreement between sum of exclusive states and inclusive data/pQCD
- $\rightarrow$ New $\pi^+\pi^-\pi^0\pi^0$ data result in reduction of the cross section
- $\rightarrow$ Previous transition point at 2 GeV no longer the preferred choice
- $\rightarrow$ More natural choice for this transition point at 1.937 GeV



Input	$a_\mu^{\mathrm{had,\,LO\,VP}}[1.841 \le \sqrt{s} \le 2.00\ \mathrm{GeV}] \times 10^{10}$
Exclusive sum	6.06 ± 0.17
Inclusive data	6.67 ± 0.26
pQCD	6.38 ± 0.11
Exclusive ($<$ 1.937 GeV) + inclusive ($>$ 1.937 GeV)	6.23 ± 0.13

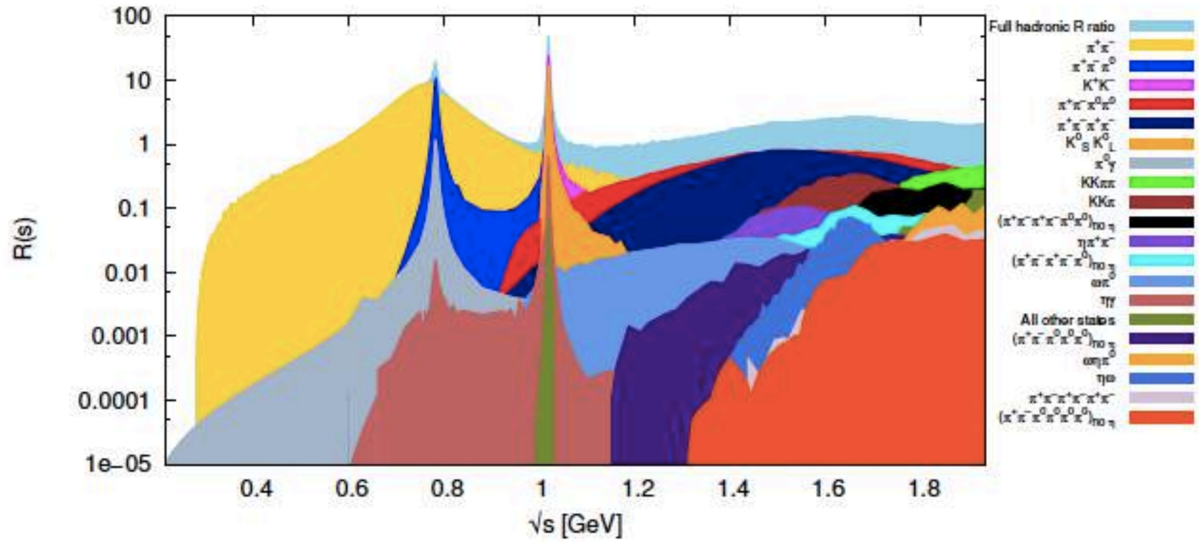
$R(s)$ for $m_\pi \leq \sqrt{s} \leq 11.2$ GeV



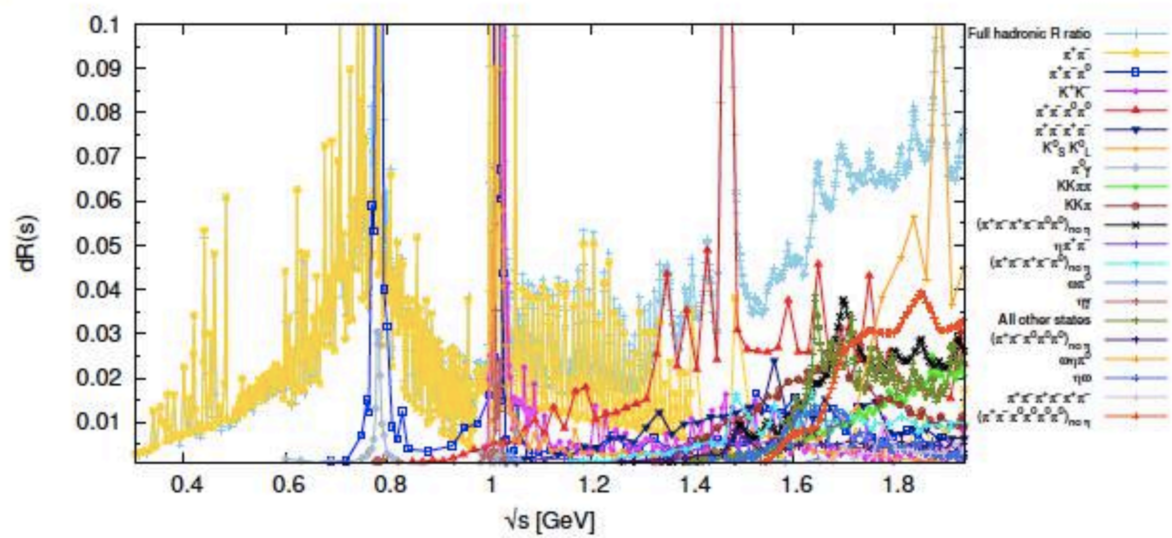
$\Rightarrow$ Full KNT18 compilation data set for hadronic R-ratio now available...

$\Longrightarrow$ **...complete with full covariance matrix**

Contributions below 2 GeV



→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty



→ Large improvement to cross section and uncertainty from new 4π data

$a_\mu^{\text{had, LO VP}}$ from KNT [preliminary]

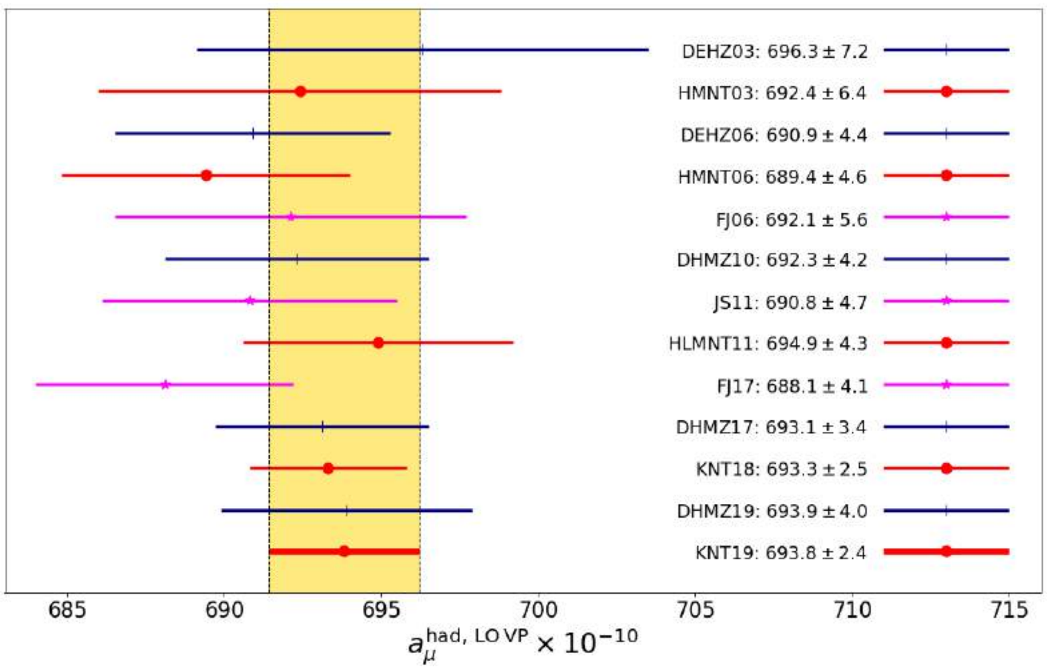
KNT18: $a_\mu^{\text{had, LOVP}} = 693.26 \pm 2.46_{tot}$

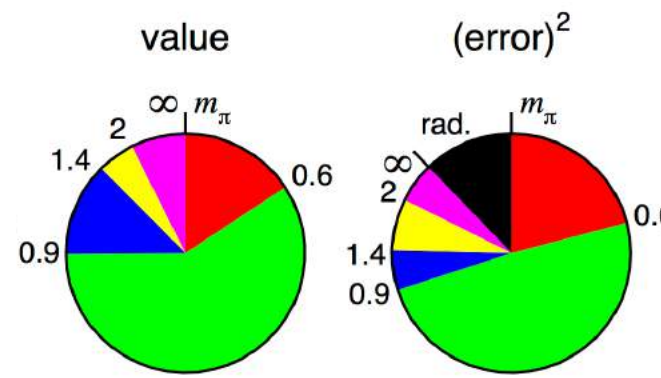
Preliminary: $a_\mu^{\text{had, LOVP}} = 693.84 \pm 1.19_{stat} \pm 1.96_{sys} \pm 0.22_{vp} \pm 0.71_{fsr}$

$= 693.84 \pm 2.29_{exp} \pm 0.74_{rad}$

$= 693.84 \pm 2.41_{tot}$

- Precision better than 0.4% (uncertainties include all available correlations and χ^2 inflation)





- Clear $\pi^+\pi^-$ dominance

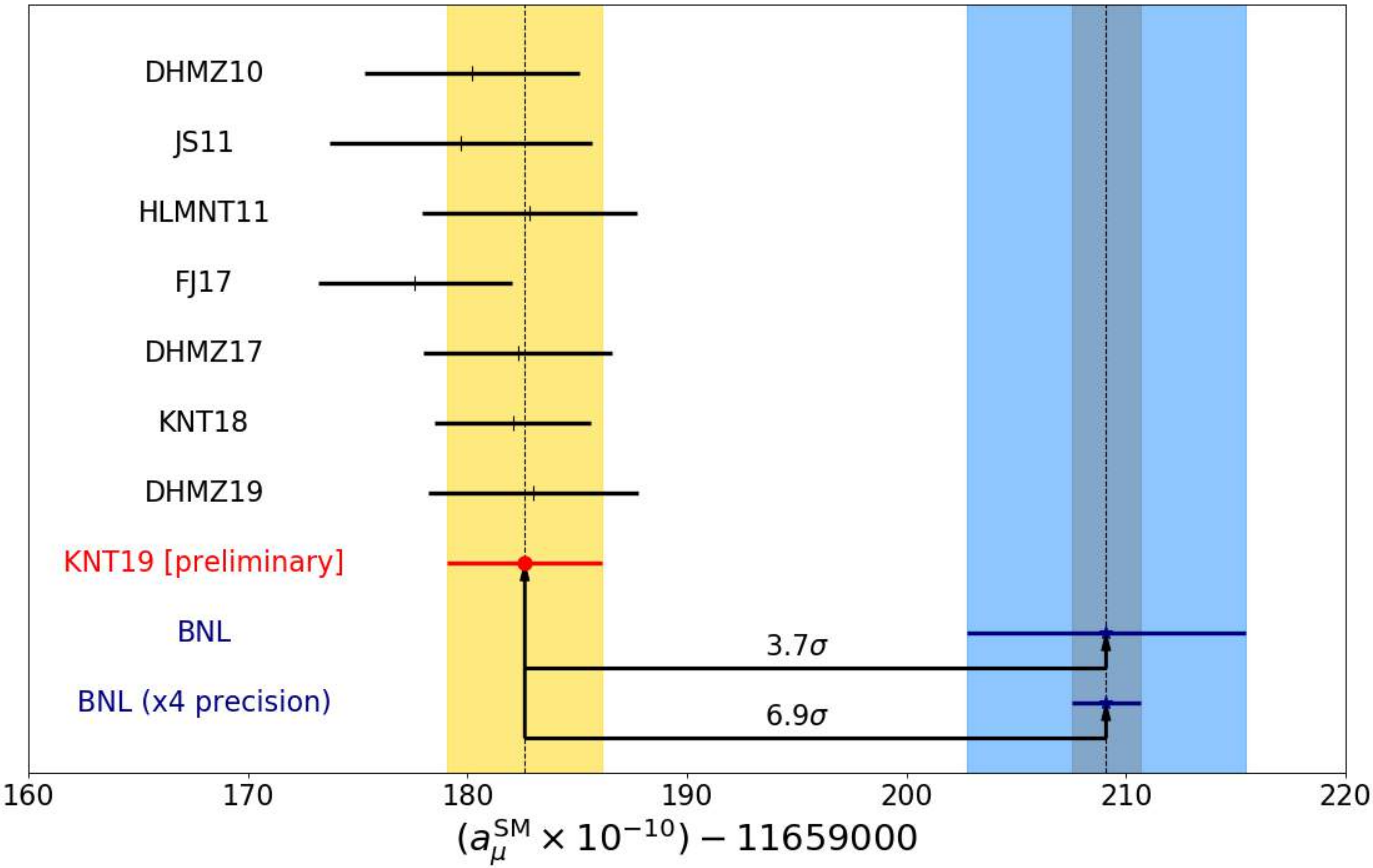
a_μ^{SM} from KNT19 [preliminary]

Updating only the LO & NLO HVP wrt to KNT18…

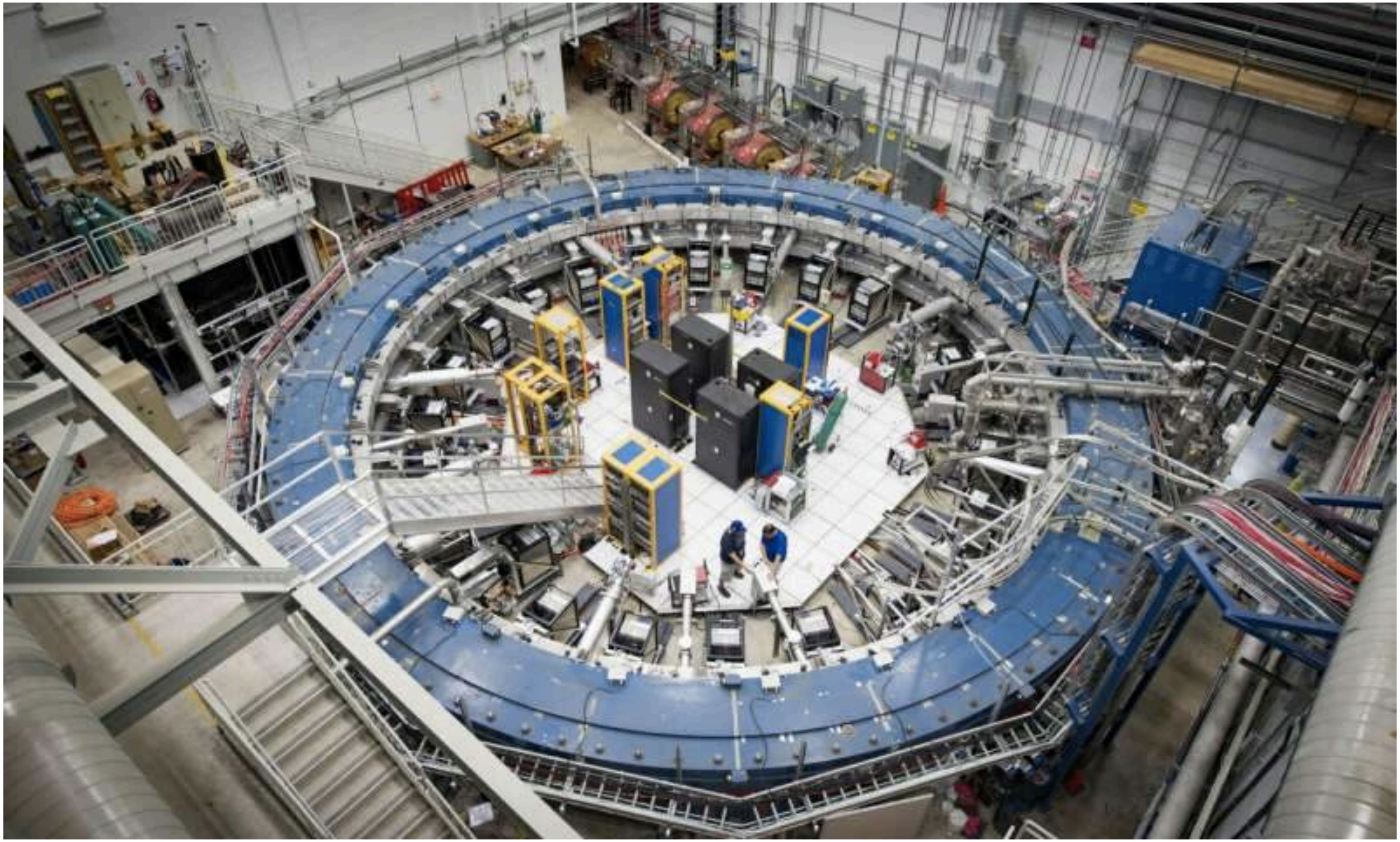
	2018		2019	
QED			11658471.90 (0.01)	[Phys. Rev. D 97 (2018) 036001]
EW			15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL			9.80 (2.60)	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	KNT18 [Phys.Rev. D97 (2018) 114025]		KNT19 [preliminary]	
LO HVP	693.27 (2.46)	→	693.84 (2.41)	This work
NLO HVP	-9.82 (0.04)	→	-9.83 (0.04)	This work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.04 (3.56)	→	11659182.61 (3.52)	This work
Experiment			11659209.10 (6.33)	World average
Exp - Theory	27.1 (7.3)	→	26.5 (7.2)	This work
Δa_μ	3.7σ	→	3.7σ	This work

a_μ^{SM} from KNT19 [preliminary]

Updating only the LO & NLO HVP wrt to KNT18…



The Muon g-2 Experiment at Fermilab



How do we measure a_μ?

Inject polarised muons in a magnetic storage ring (dipole B-field → 1.45T).

- Measure the difference between the muon cyclotron and spin frequencies:

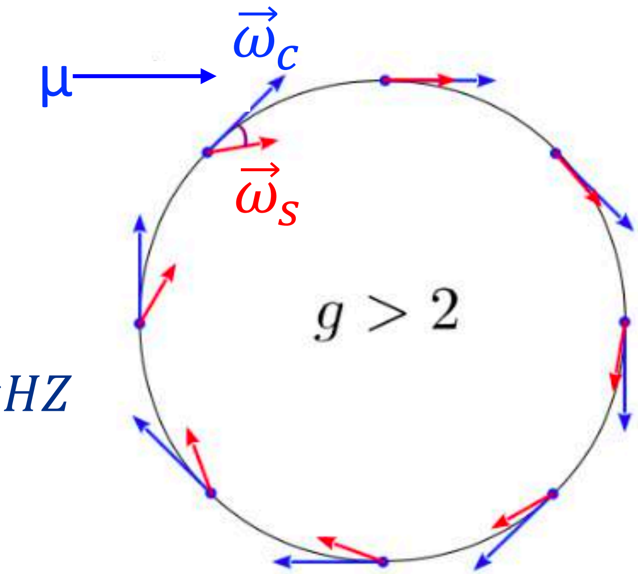
Spin frequency: $\vec{\omega}_s = \frac{ge\vec{B}}{2mc} + (1-\gamma)\frac{e\vec{B}}{\gamma mc}$

Cyclotron frequency: $\vec{\omega}_c = \frac{e\vec{B}}{\gamma m}$

Anomalous prececssion frequency:

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = \left(\frac{g-2}{2}\right)\frac{e\vec{B}}{mc} = \boldsymbol{a_\mu}\frac{e\vec{B}}{mc} \approx 229kHZ$$

(Note that if $a_\mu = 0$, then $g = 2$ and $\vec{\omega}_s = \vec{\omega}_c$.)



Therefore, the Fermilab Muon g-2 experiment will measure two quantities:

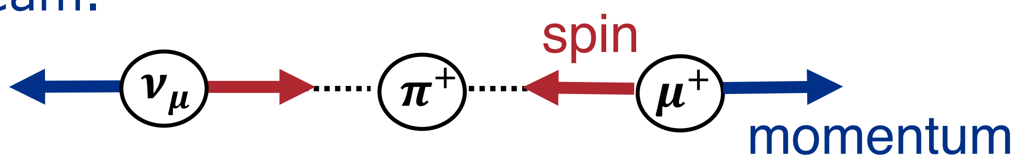
1. The anomalous precession frequency, $\vec{\omega}_a$ to ± 100 ppb (stat) ± 70 ppb (syst).
2. Magnetic field $\vec{B}$ in terms of proton NMR frequency to ± 70 ppb (syst).

How do we measure a_μ?

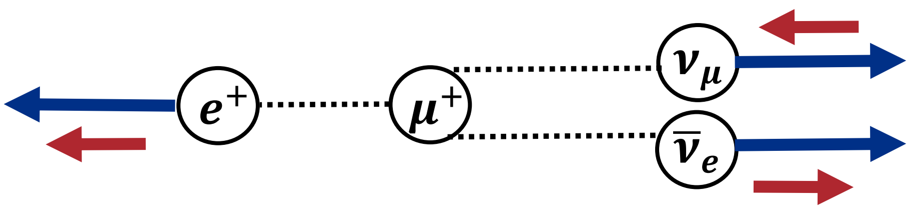
→ We need to know the spin of the muon…

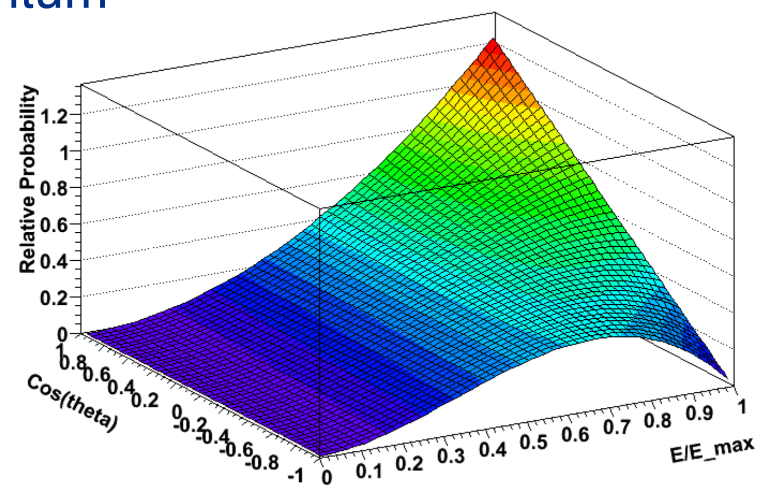
In the weak decay of a pion, the neutrino spin must be opposite of momenta.

➢ The same must be true for the muon, resulting in a polarised muon beam.



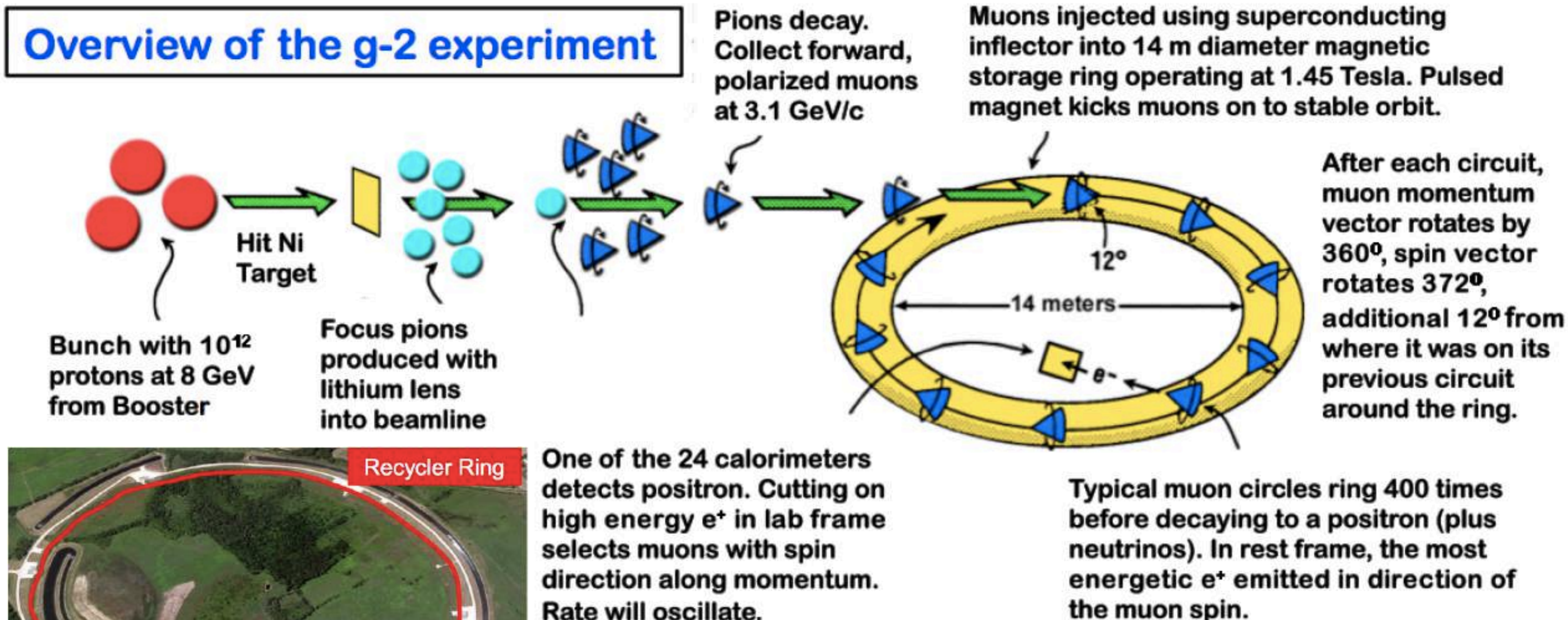
Then, the highest energy positrons are emitted along the direction of the spin of the muon…





So, by detecting positrons above a certain energy threshold using calorimeters, we know the spin of the parent muon.

Producing the muons



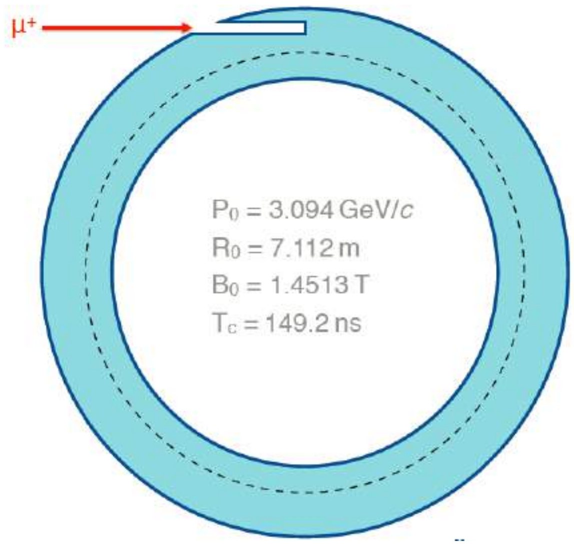
<u>Fermilab statistics advantages</u>

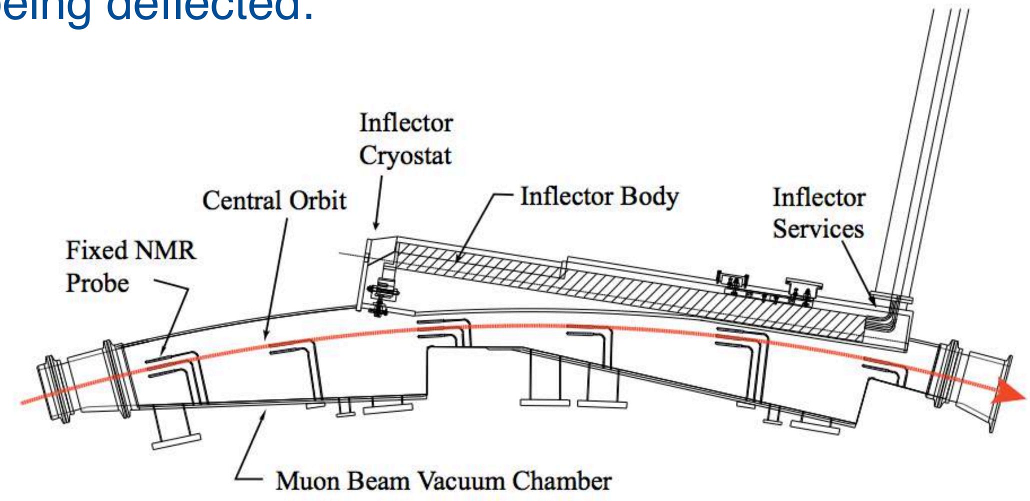
- Long decay channel for $\pi \rightarrow \mu$
- Reduced p and π in ring
- Factor 20 reduction in hadronic flash
- 4x higher fill frequency than BNL

→ 21 times more positrons detected than at BNL

Storing the beam: the inflector

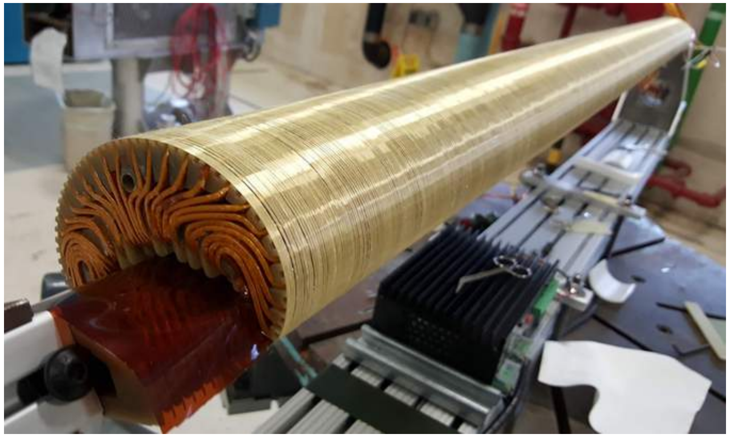
A superconducting inflector magnet at injection cancels the 1.45 T storage field to allow the muon to enter without being deflected:





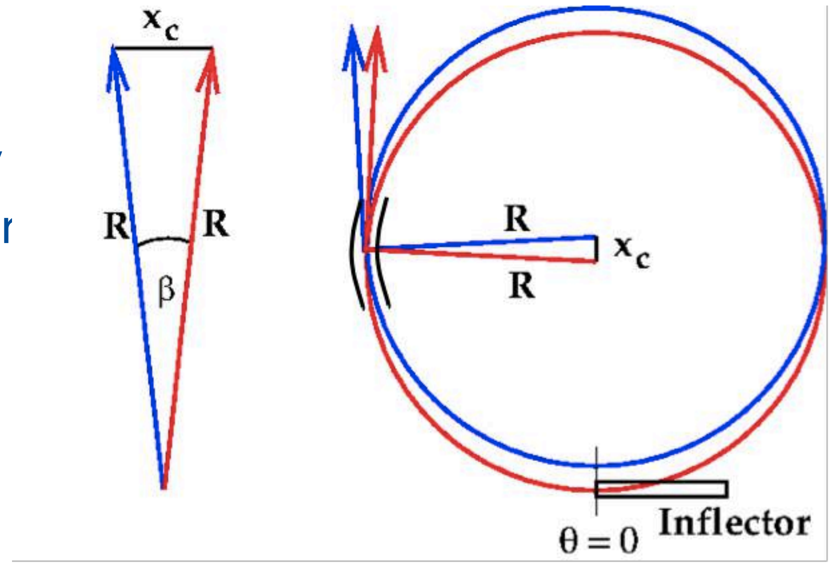
Note: new open-ended inflector upgrade being installed in summer of this year.

→ Projected 40% gain in statistics.



Storing the beam: the kicker

- Beam enters the ring displaced by 11mrads from ideal orbit.
- Kicker magnets inside ring require 65kv pulse to produce 300 Gauss $\vec{B}$ field over 4 metres for 100 ns at 100 Hz.
  → "Kick" muons onto correct orbit.



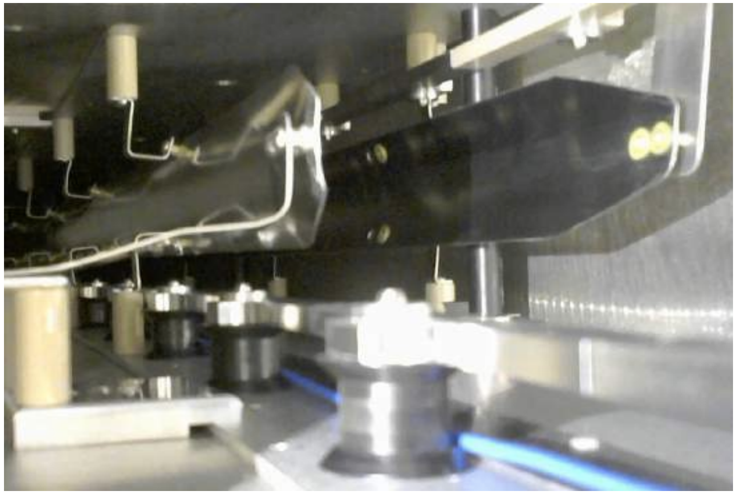
Run-2 upgrades

Run-1 kicker performance problems:

- 30% less kick strength than necessary.
- Kick reflection due to impedance mismatching.

This has lead to a **full kicker system upgrade,** which has just been completed ready for Run-2 data taking.

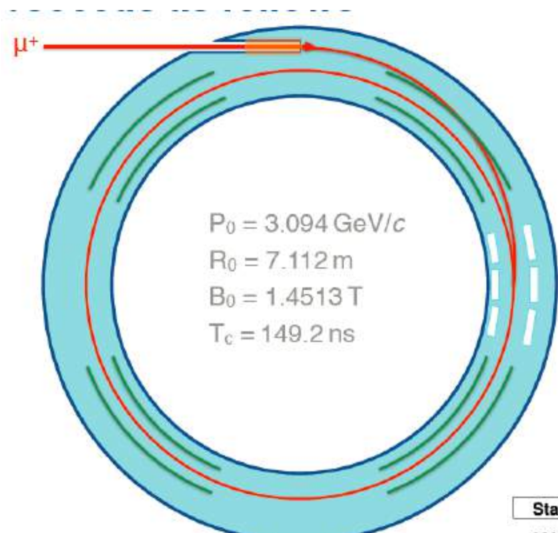
➢ Projected to give us up to 30% better storage efficiency.

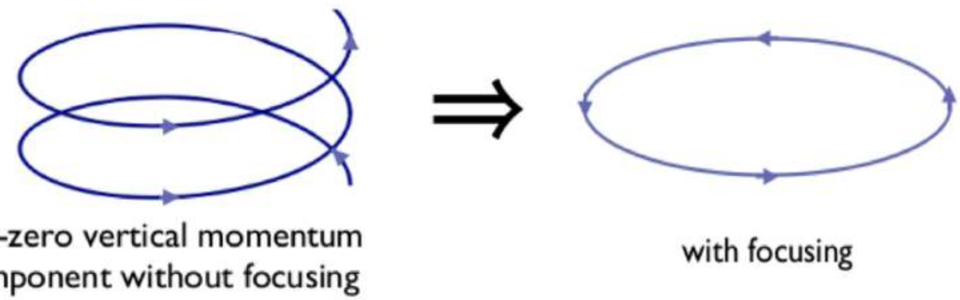


Storing the beam: electrostatic quadrupoles

→ Storage ring B-field **only provides radial focusing.**

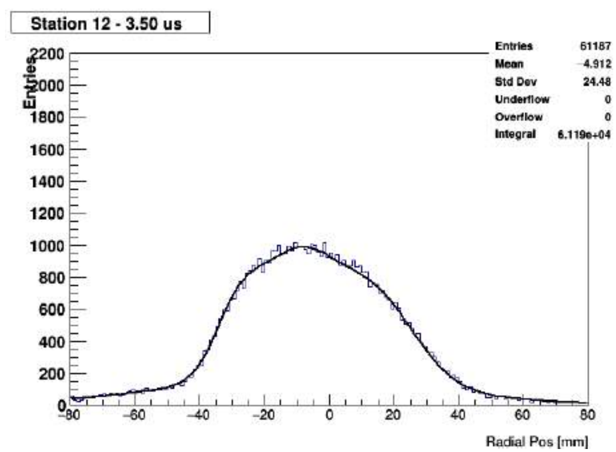
→ Use electric field (electrostatic quadrupoles) to provide vertical focusing (to counteract vertical pitch angle).

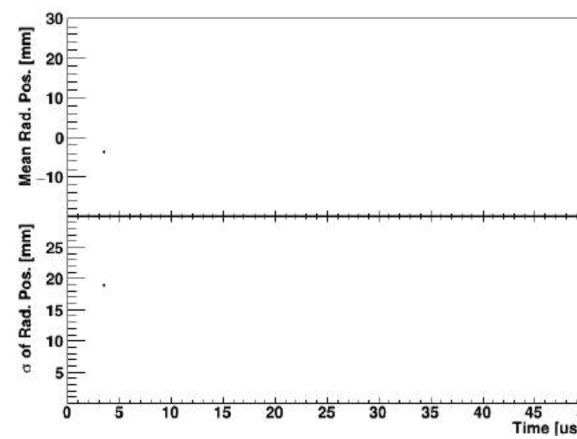




However, combination of E and B field leads to 2D SHM about closed orbit (in the form of betatron oscillations)

The amplitude, frequency and damping time of these beam oscillations are critical to the measurement

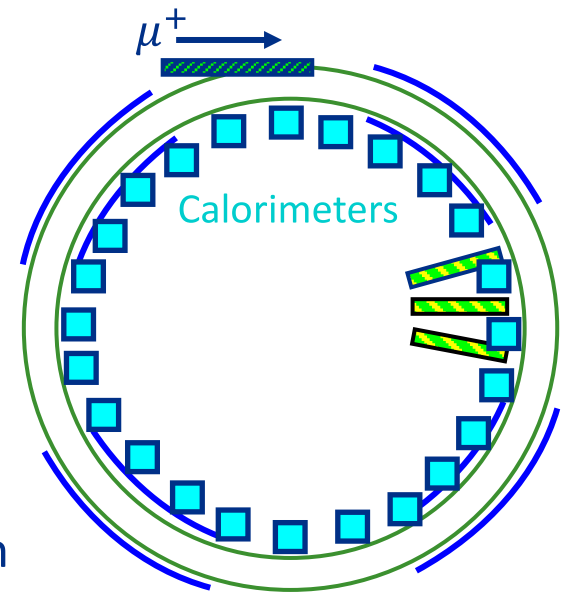




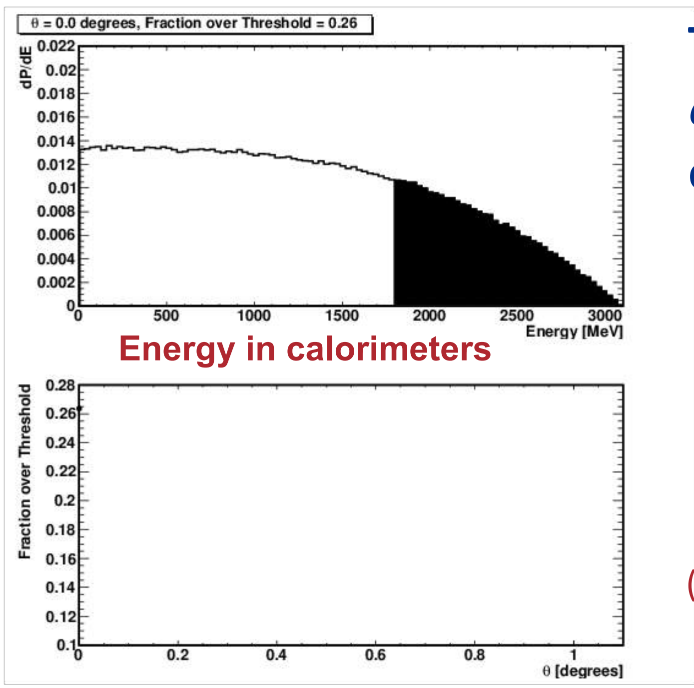
Measuring the decay positrons

24 calorimeters located equidistantly around the storage ring measuring arrival time and energy of decay positrons:

➔ Each calorimeter has 54 Cherenkov PbF_2 crystals with very fast SiPMs.



The muons pass the calorimeters at cyclotron frequency, so the oscillation occurs at the difference frequency ω_a:



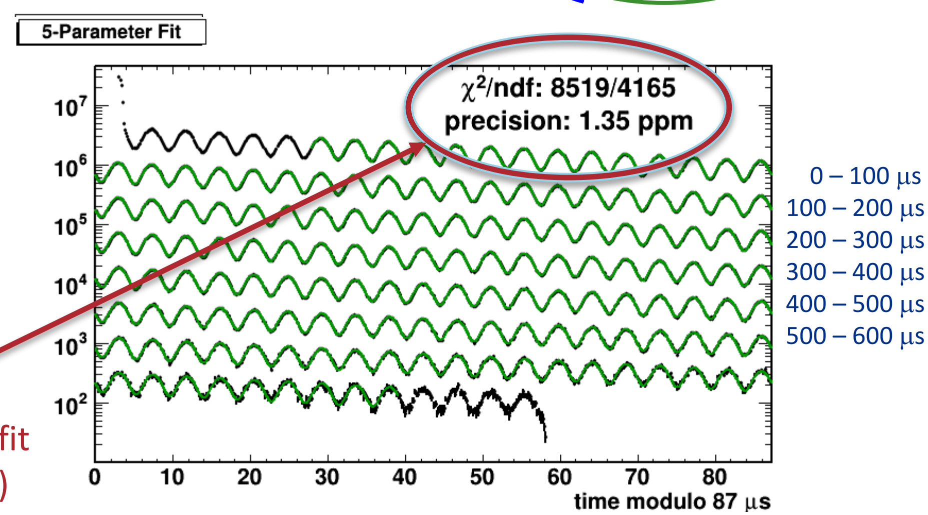
Energy in calorimeters

Direction/phase of muon spin

The wiggle plot: no. of e^+ (>1.8GeV) as a function of time.

Run-1 '60 hour' data set

Not good (not enough fit parameters)

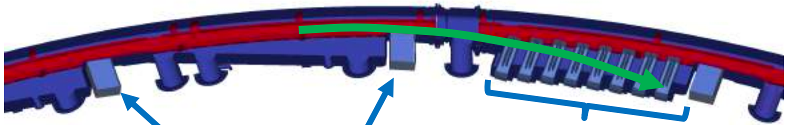


$$N_e(t) \simeq N_0 e^{-\frac{t}{\gamma\tau}}[1 - A\cos(\omega_a t + \phi_a)]$$

Trackers and fiber harps

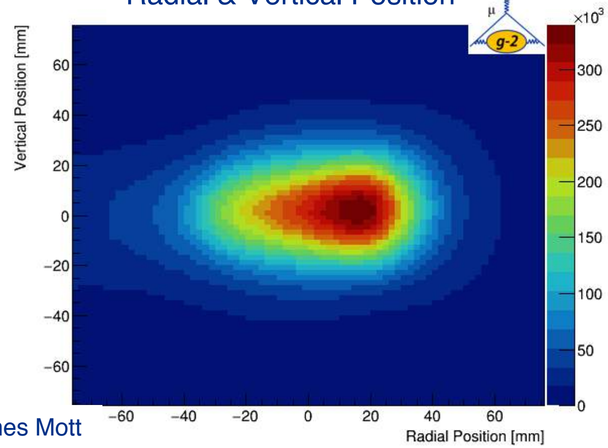
We have two other detectors that we use to monitor the beam dynamics:

Straw trackers (non-destructive)



Provides essential information for:

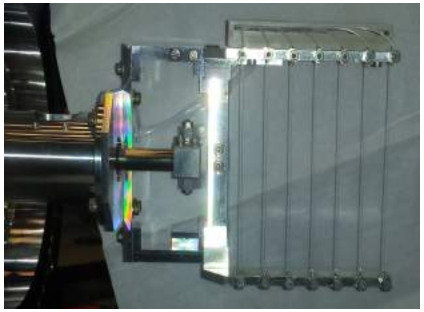
- Weighting magnetic field data by muon distribution.
- Acceptance corrections for calorimeter due to beam oscillations.
- Pitch correction C_p to ω_a.



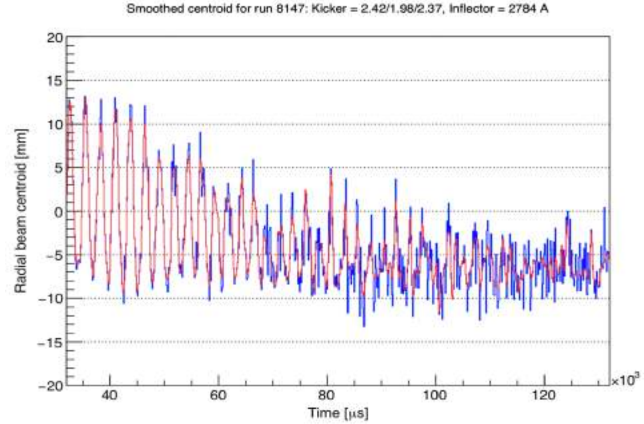
James Mott

Fiber harps (destructive)

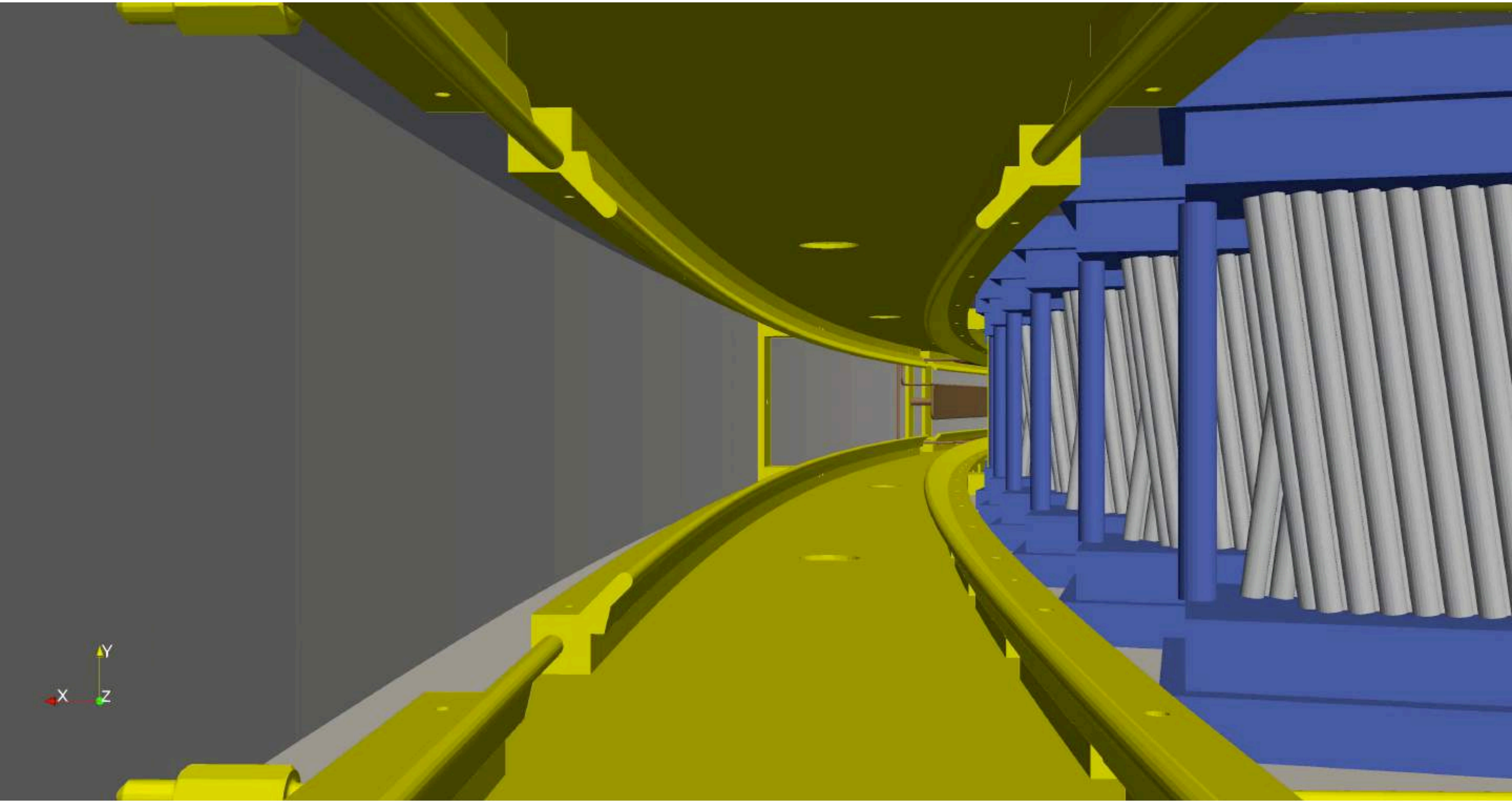
Fiber profile beam monitor measure vertical position of beam at 180° and 270° around ring:



…and provides information on Coherent Betatron Motion amplitude:



The muon's view



Dealing with a less than ideal world…

In addition, our expression for $\vec{\omega}_a$ now includes two more terms:

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \underbrace{\left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}} - \underbrace{a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}}\right]$$

→ Choosing the "magic momentum" $\gamma = 29.3$ ($p = 3.094$ GeV) cancels the electric field term to first order.

→ This leaves two effects that we have to correct for:

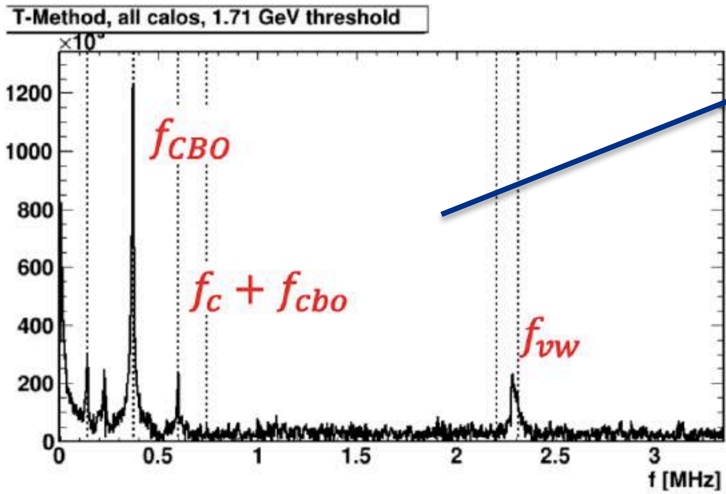
Electric-field correction

- Not all muons are at the magic momentum.
- Have to correct $\vec{\omega}_a$ for those muons.
- This E-field correction, C_E, can be determined via the **'Fast Rotation' analysis**.
- This results in a systematic uncertainty.

Pitch correction

- Some muons still have a small amount of vertical pitching.
- Have to correct $\vec{\omega}_a$ for those muons.
- This Pitch correction, C_P, can be determined from **straw tracker data**.
- This results in a systematic uncertainty.

Fitting all the relevant beam dynamics



FFT of fit residuals shows other systematic effects in frequency space

Physical frequency	Variable	Frequency (MHz)	Period (μs)
Anomalous precession	f_a	0.23 MHz	4.37
Cyclotron	f_c	6.70 MHz	0.149
Horizontal Betatron	f_x	6.34 MHz	0.158
Vertical Betatron	f_y	2.2 MHz	0.455
CBO	f_{CBO}	0.37 MHz	2.7
Vertical Waist	f_{VW}	2.3 MHz	0.435

→ Fit function must account for all these effects: CBO, vertical waist, pileup, muon losses, in-fill gain changes...

And so, five-parameter function:

$$N_e(t) \simeq N_0 e^{-\frac{t}{\gamma\tau}} [1 - A\cos(\omega_a t + \phi_a)]$$

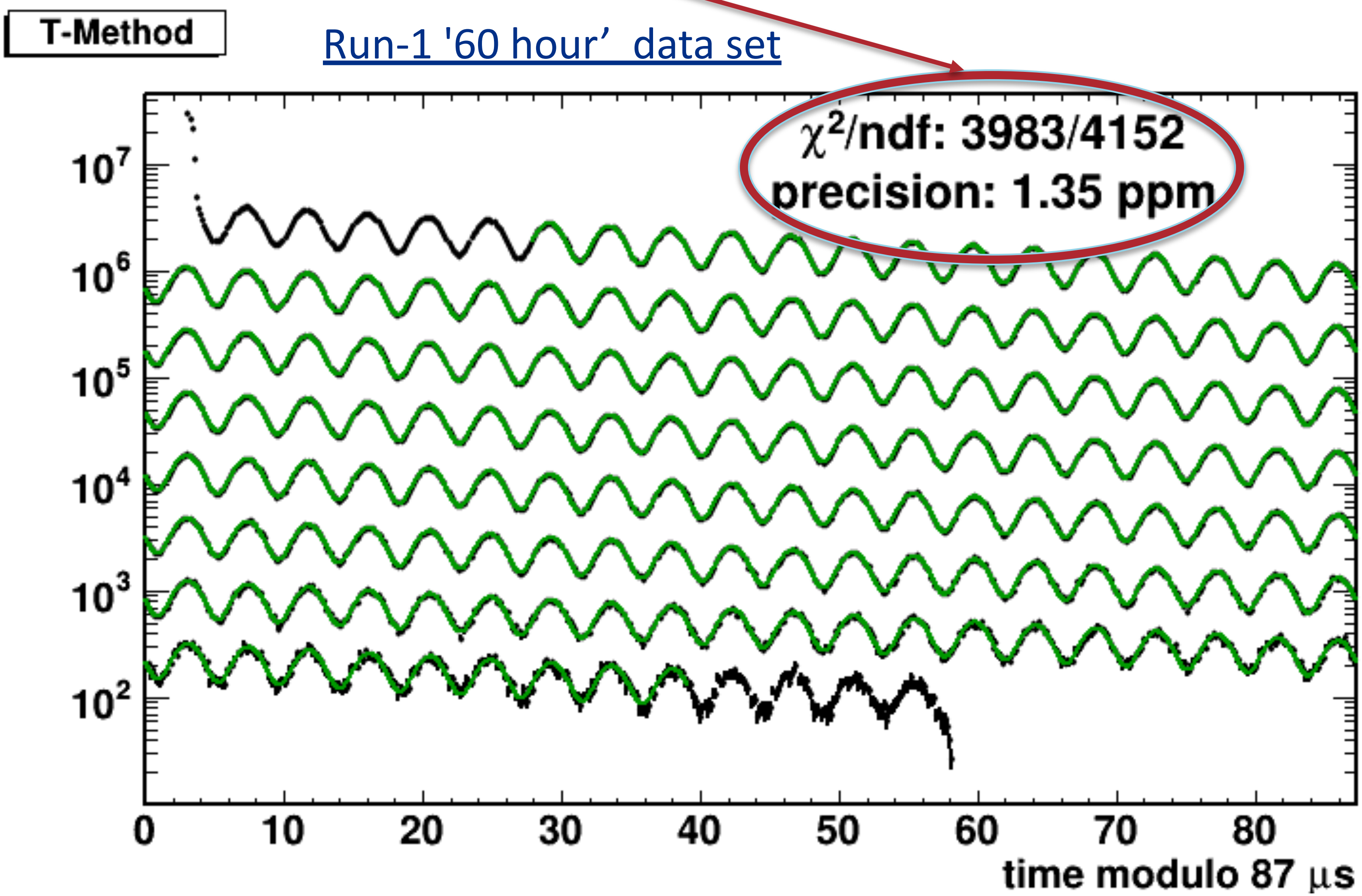
… becomes 17-parameter function:

$$N(t) = N_0 N_{CBO}(t) N_{2CBO}(t) N_{VW}(t) L(t) \exp(-t/\tau) \left[1 + A_0 A_{CBO}(t) \cos\left(\omega_a(R)t + \phi(t)\right)\right]$$

... that fully describes the beam dynamics.

Fitting all the relevant beam dynamics

And the fit is complete…



Blinding

The experiment is both hardware and software blinded:

<u>Software blinding</u>

- Analysis package applies two frequency offsets to ω_a and ω_p:

$$\omega_a = 2\pi \cdot 0.2291 \text{ MHz} \cdot [1 - (R - \Delta R) \times 10^{-6}]$$

→ Each analyser has an individual, unknown personal offset ΔR.

→ We are currently **fitting for $\boldsymbol{R}$** and are very close to a relative unblinding of the first data set.

<u>Hardware blinding</u>

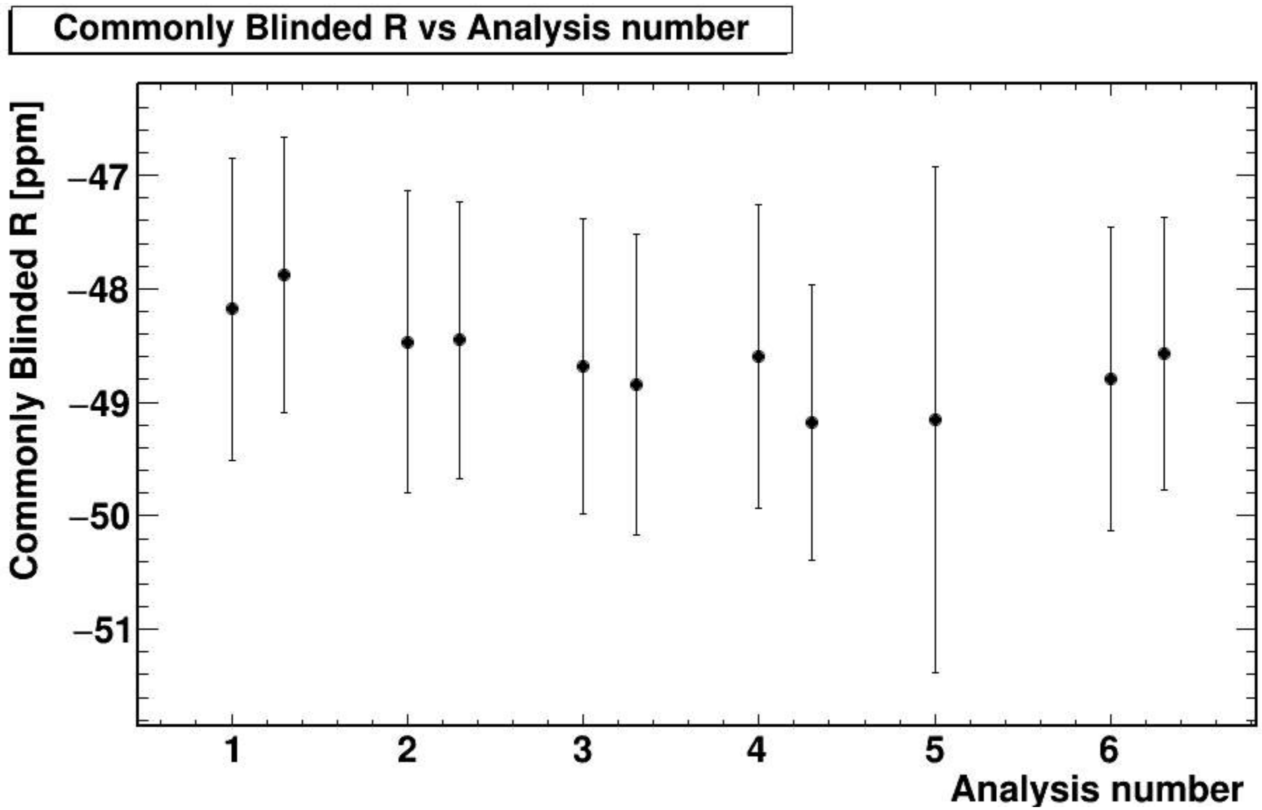
- A 40MHz clock drives the calorimeter digitizers, straw tracker and NMR digitisers.
- This has been shifted by a small amount in the range +/- 25ppm.
- The offset is known only to two people (not part of the experiment).

Take-home message:

We can't say anything about the final result (yet), despite recent rumours…

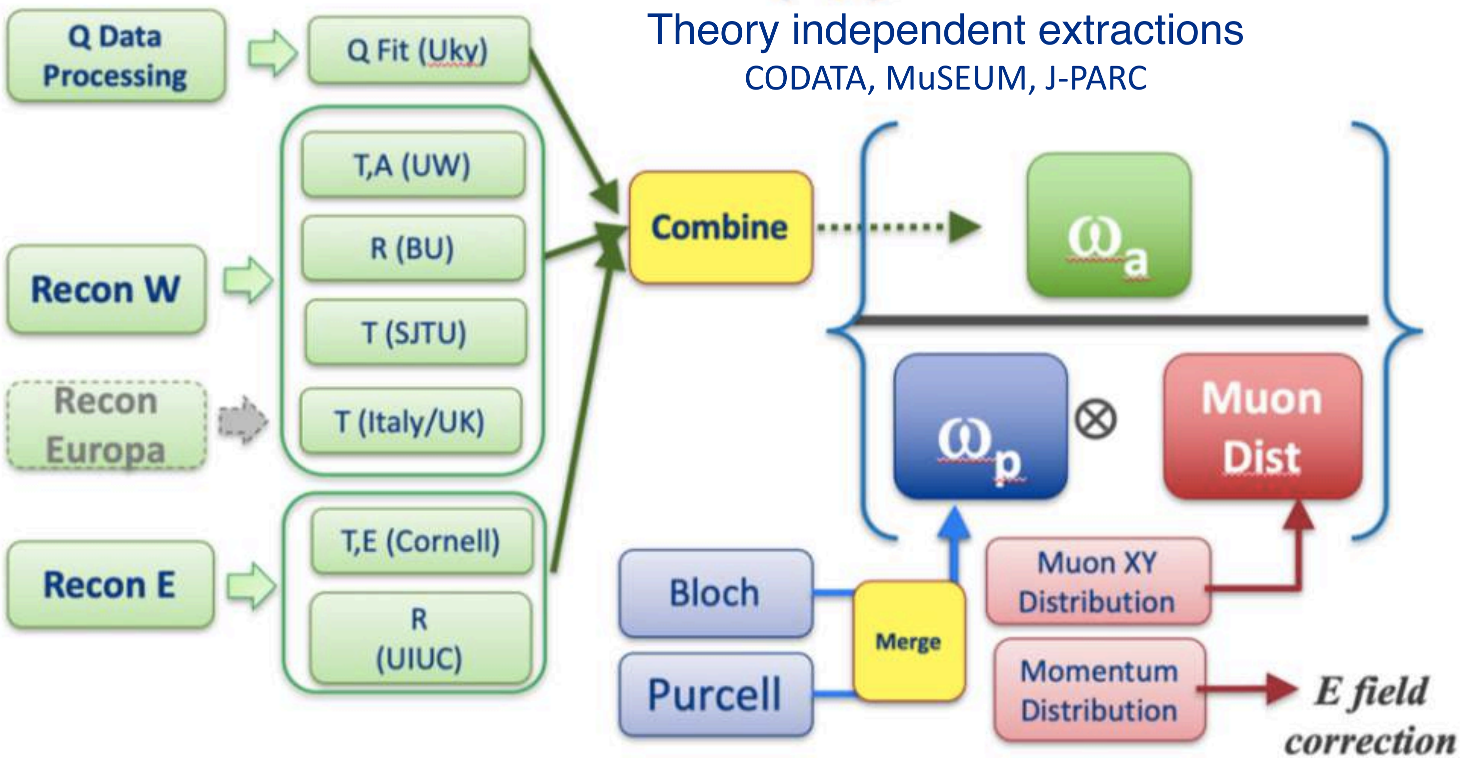
A sneak preview…

Relative unblinding of '60 hour' data set confirmed 6 precession frequency analyses are consistent



The full picture (after unblinding)

2018 Analysis Structure $a_\mu = \left\{\frac{\omega_a}{\tilde{\omega}_p}\right\} \frac{\mu_p}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2} + \boldsymbol{E \,\&\, \mathrm{pitch}}$

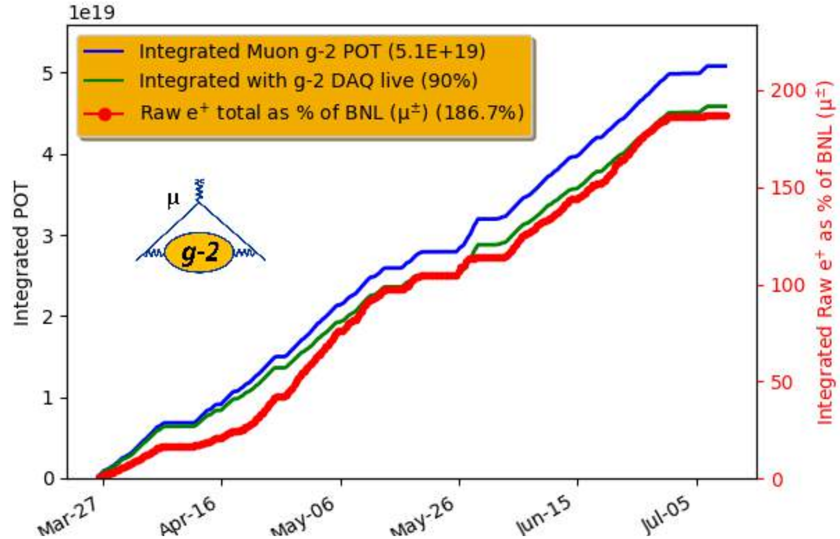


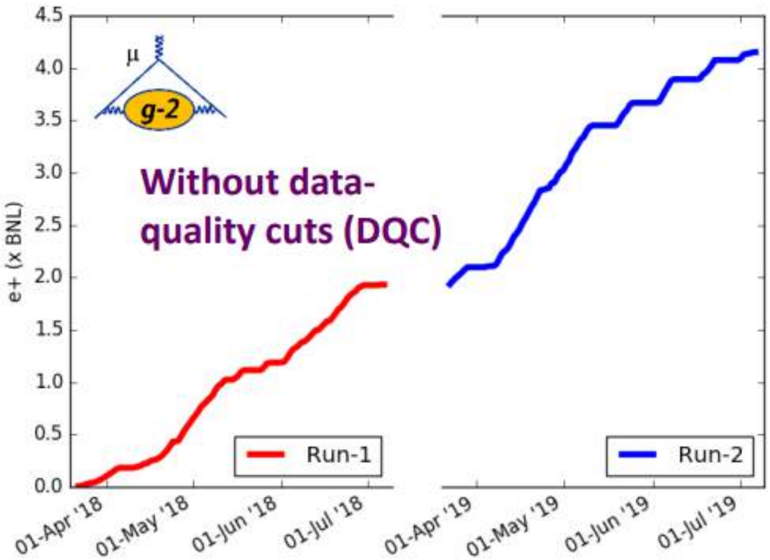
Result from 1st physics run with BNL level statistics planned for early 2020.

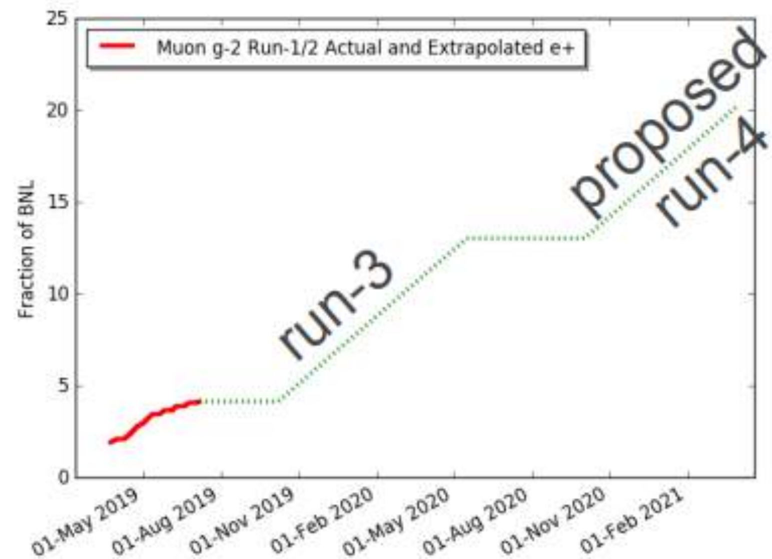
Reaching 100ppb statistics…

- In **Run-1**, we recorded 17.5B e^+ (x2 Brookhaven dataset).
- **Run 2** was stable, but started late. Greatly reduced running at end because of lab budget
- **Run 3** is planned to be a direct continuation of Run 2; no major changes
- **Run 4** is still far away …

→ Over the next 3 years, we will increase the current dataset by factor of 5.







Systematic uncertainty budget

ω_a

- New calorimeters, trackers, techniques to reduce uncertainties by a factor of 2.6 compared to BNL.
- Upgrades have drastically reduced systematics issues in Run-1.

Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]
Gain changes	120	Better laser calibration	
		low-energy threshold	20
Pileup	80	Low-energy samples recorded	
		calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher n value (frequency)	
		Better match of beamline to ring	< 30
E and pitch	50	Improved tracker	
		Precise storage ring simulations	30
Total	180	Quadrature sum	70

Source of uncertainty	1999	2000	2001	E989
Systematics of calibration probes	50	50	50 →	35
Calibration of trolley probes	200	150	90 →	30
Trolley measurements of B_0	100	100	50 →	30
Interpolation with fixed probes	150	100	70 →	30
Uncertainty from muon distribution	120	30	30 →	10
Inflector fringe field uncertainty	200	–	–	–
Time dependent external B fields	–	–	– →	5
Others †	150	100	100 →	30
Total systematic error on ω_p	400	240	170 →	70
Muon-averaged field [Hz]: $\omega_p/2\pi$	61 791 256	61 791 595	61 791 400	–

ω_p

- New electronics, new probes, new techniques reduce uncertainties factor 2.5
- Temperature issues in Run-1 now alleviated via magnet insulation and new hall cooling.

Conclusions

- Accuracy of a_μ^{SM} limited by hadronic contributions.
- Hadronic VP contributions can be determined from dispersion relations and hadronic cross section input.
- New data combination method + new data yields improvements in all channels due to increased fit flexibility.
- Correlations have large effect on mean value and uncertainty and all available information should be correctly incorporated.
- $a_\mu^{\text{had, LOVP}}$ accuracy better than 0.4% and improvement in HVP yields g-2 discrepancy of 3.7σ
- Overall HVP uncertainty now better than HLbL uncertainty
- Fermilab Muon g-2 experiment on track to ascertain whether current discrepancy with SM is well established.
- The experiment will measure two frequencies, ω_a and ω_p, to an unpresented precision.
- Major upgrade work has taken place over the shutdown to ensure that the experiment reaches its statistics and systematics goals (with more planned for summer 2019).
- Run-1 (2018) data is currently being analysed, but **is currently fully blinded.**
- The blinding is applied for both hardware and software, for both ω_a and ω_p.
- First result from Run-1 with BNL level statistics is planned for early 2020.
- Run-2, Run-3 and Run-4 will ensure we reach the 20x BNL statistics goal, and systematics are currently very well under control.

Thank you.

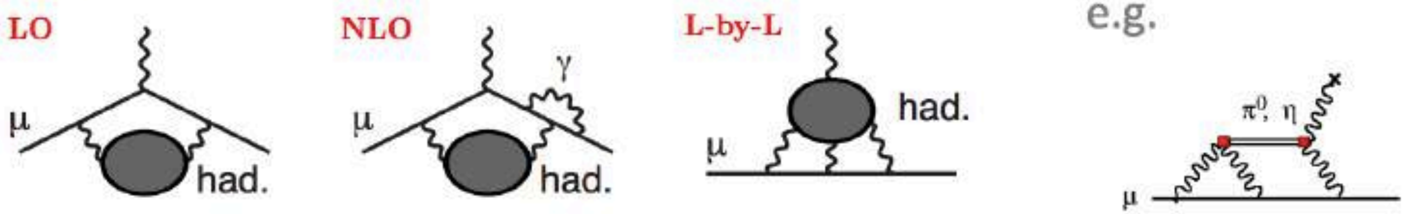
Backup slides

Hadronic contributions

⇒ Uncertainty on $a_\mu^{\rm SM}$ dominated by hadronic contributions

→ Non-perturbative, low energy region of hadronic resonances

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$



⇒ LbL contributions $(\mathcal{O}(\alpha^3))$, so far only *fully* determined using model calculations

→ Difficult to quantify/control uncertainties from models

→ Huge progress from lattice and dispersive approaches

→ So far, no indication of unpleasant surprises

→ But, big improvements expected in near future Phys. Rev. D 94 (2016) 053006.

⇒ LO LbL, updated 'Glasgow consensus' estimate: $a_\mu^{\rm had,\,LO\,LbL} = (9.8 \pm 2.6) \times 10^{-10}$

→ NLO LbL estimated to be $a_\mu^{\rm had,\,NLO\,LbL} = (0.3 \pm 0.2) \times 10^{-10}$ Phys. Lett. B 735 (2014) 90.

$$a_\mu^{\rm had,\,LbL} = (10.1 \pm 2.6) \times 10^{-10}$$

Avoiding systematic bias

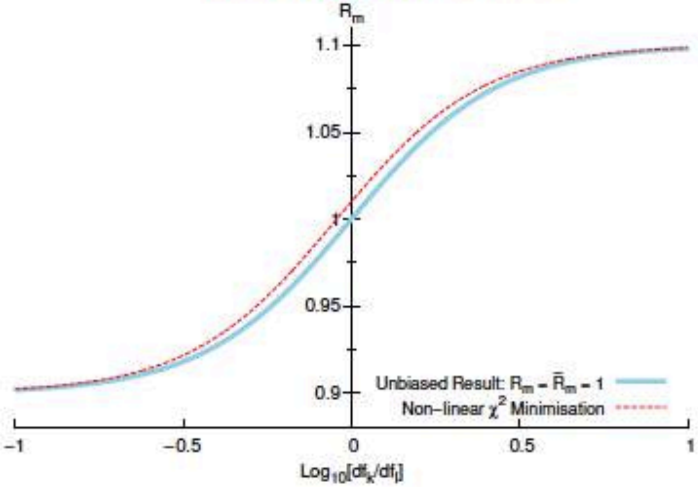
⇒ Data is re-binned using an adaptive clustering algorithm

⇒ Iterative fit of covariance matrix as defined by data → D'Agostini bias

[Nucl.Instrum.Meth. A346 (1994) 306-311]

HLMNT11

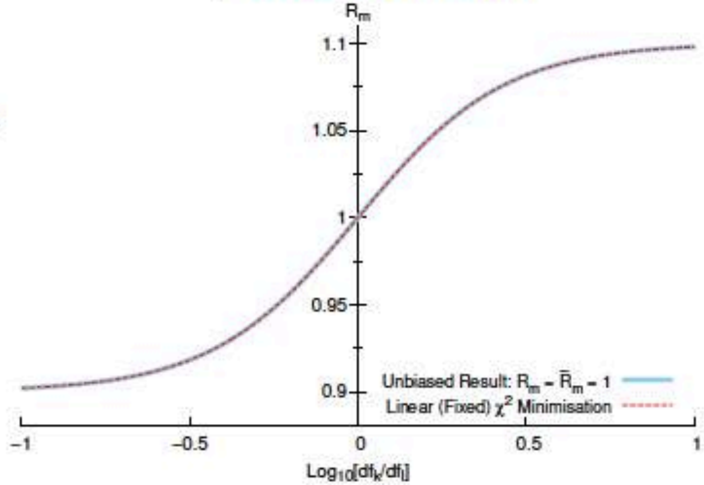
⇒ Non-linear χ^2 minimisation fitting nuisance parameters
→ Penalty trick bias



⟹

KNT18

⇒ Fix the covariance matrix in an iterative χ^2 minimisation
→ Free from bias



Allows for increased fit flexibility and full use of energy dependent, correlated uncertainties

Fixing the covariance matrix

⇒ Apply a procedure to fix the covariance matrix

$$\mathrm{C}_I\left(i^{(m)}, j^{(n)}\right) = \mathrm{C}^{\mathrm{stat}}\left(i^{(m)}, j^{(n)}\right) + \frac{\mathrm{C}^{\mathrm{sys}}\left(i^{(m)}, j^{,n)}\right)}{R_i^{(m)} R_j^{(n)}} R_m R_n \quad ,$$

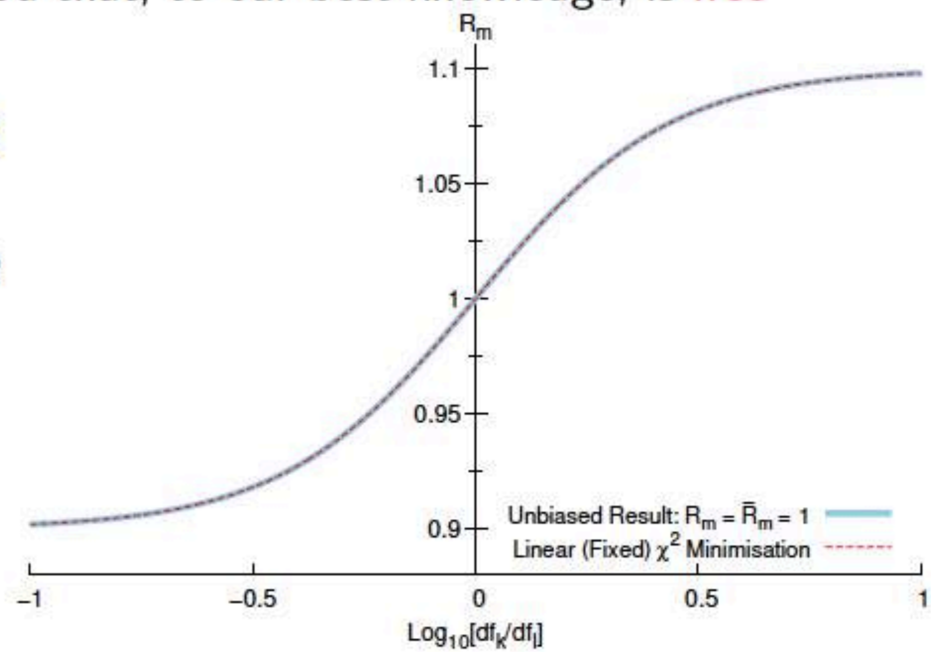
in an iterative χ^2 minimisation method that, to our best knowledge, is free from bias

⇒ Fixing with theory value regulates influence

⇒ Can be shown from toy models to be free from bias

⇒ Swift convergence

⇒ Comparison with past results shows HLMNT11 estimates are largely unaffected



Allows for increased fit flexibility and full use of energy dependent, correlated uncertainties

Properties of a covariance matrix

Any covariance matrix, $\mathcal{C}_{ij}$, of dimension $n \times n$ must satisfy the following requirements:

- As the diagonal elements of any covariance matrix are populated by the corresponding variances, all the diagonal elements of the matrix are positive. Therefore, the trace of the covariance matrix must also be positive

$$\text{Trace}(\mathcal{C}_{ij}) = \sum_{i=1}^{n} \sigma_{ii} = \sum_{i=1}^{n} \text{Var}_i > 0$$

- It is a symmetric matrix, $\mathcal{C}_{ij} = \mathcal{C}_{ji}$, and is, therefore, equal to its transpose, $\mathcal{C}_{ij} = \mathcal{C}_{ij}^{T}$
- The covariance matrix is a positive, semi-definite matrix,

$$\mathbf{a}^T \mathcal{C} \ \mathbf{a} \geq 0 \ ; \ \mathbf{a} \in \mathbf{R^n},$$

  where $\mathbf{a}$ is an eigenvector of the covariance matrix $\mathcal{C}$
- Therefore, the corresponding eigenvalues $\lambda_{\mathbf{a}}$ of the covariance matrix must be real and positive and the distinct eigenvectors are orthogonal

$$\mathbf{b} \ \mathcal{C} \ \mathbf{a} = \lambda_{\mathbf{a}}(\mathbf{b} \cdot \mathbf{a}) = \mathbf{a} \ \mathcal{C} \ \mathbf{b} = \lambda_{\mathbf{b}}(\mathbf{a} \cdot \mathbf{b})$$

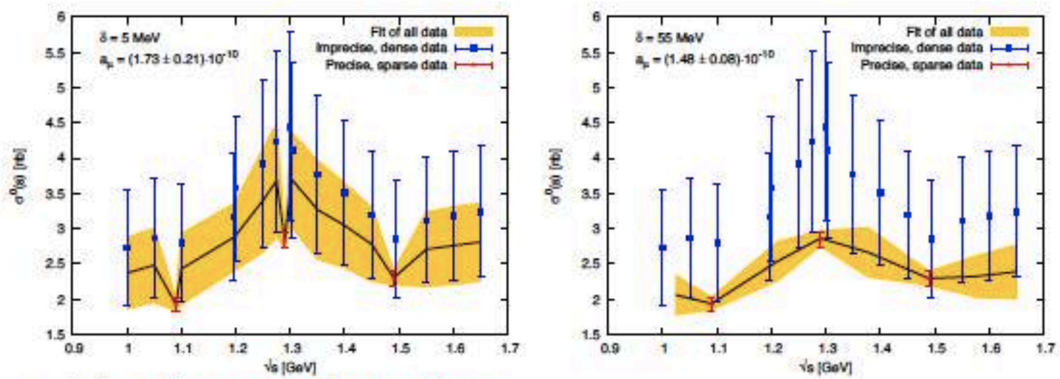
$$\therefore \text{if } \lambda_{\mathbf{a}} \neq \lambda_{\mathbf{b}} \Rightarrow (\mathbf{a} \cdot \mathbf{b}) = 0$$

- The determinant of the covariance matrix is positive: $\text{Det}(\mathcal{C}_{ij}) \geq 0$

Clustering data

⇒ Re-bin data into *clusters*

Better representation of data combination through adaptive clustering algorithm



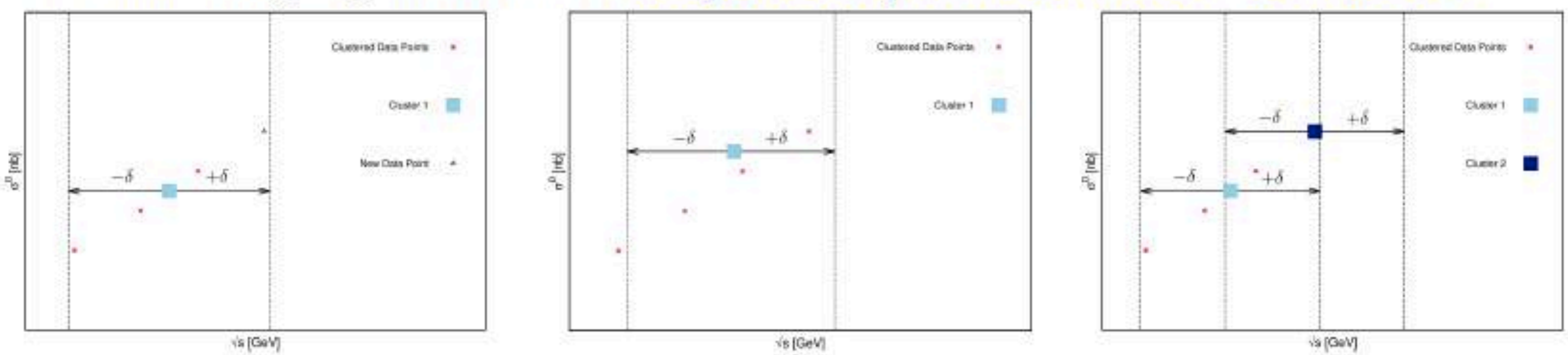
→ More and more data ⇒ risk of over clustering

⇒ loss of information on resonance

→ Scan cluster sizes for optimum solution (error, χ^2, check by sight...)

⇒ Scanning/sampling by varying bin widths

→ Clustering algorithm now adaptive to points at cluster boundaries

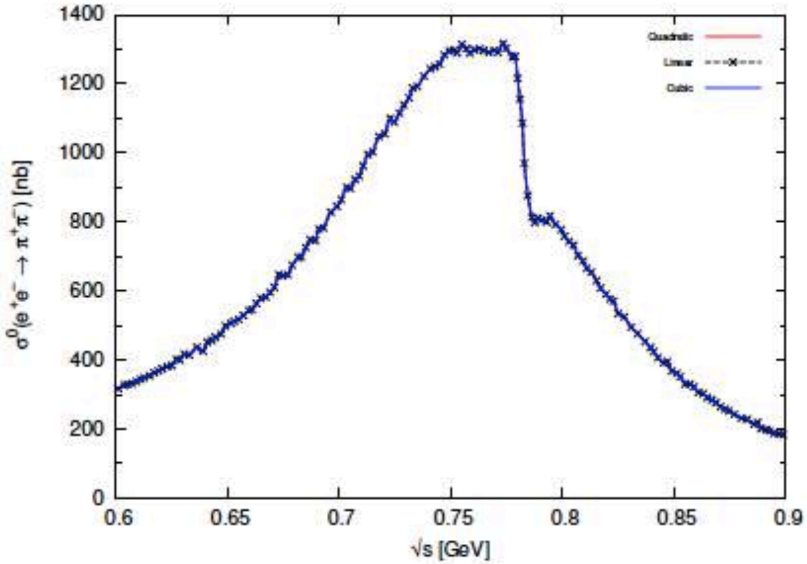


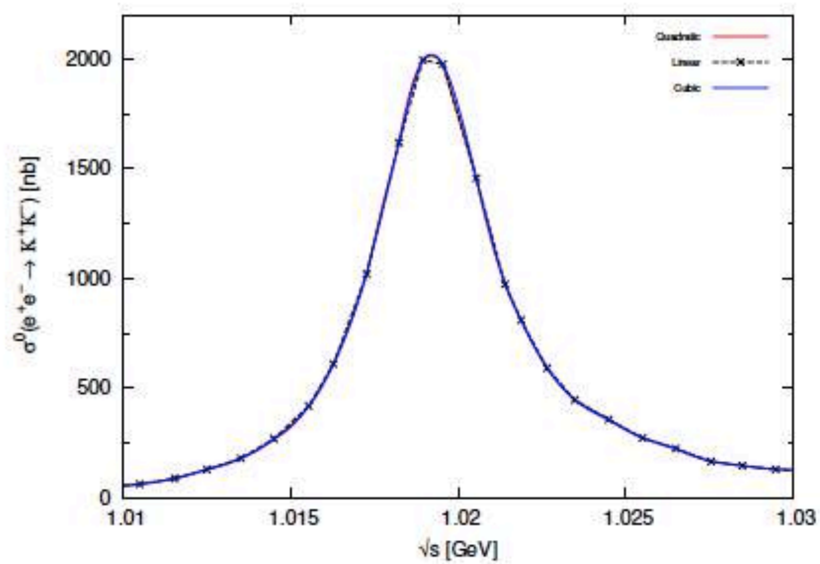
Integration

⇒ Trapezoidal rule integral

→ Consistency with linear cluster definition

→ High data population ∴ Accurate estimate from linear integral





→ Higher order polynomial integrals give (at maximum) differences of $\sim 10\%$ of error

⇒ Estimates of error non-trivial at integral borders

⟶ Extrapolate/interpolate covariance matrices

2π CLEO-c data [Phys.Rev. D97 (2018) 032012]

$\Rightarrow$ New 2π data from CLEO-c should be used with caution

- $\rightarrow$ Two measurements taken at different COM energies ($\psi(3770)/\psi(4170)$) have very different cross sections
- $\rightarrow$ Large statistical and systematic errors compared to other radiative return sets
- $\rightarrow$ VP correction has been applied with FJ03VP (needs updated version) and only subtracts real part
- $\rightarrow$ The values for $a_\mu^{\pi^+\pi^-}$ given in the paper only calculated using weighted average
  - $\rightarrow$ Systematics will be highly correlated and should be incorporated
- $\rightarrow$ The authors have fitted the data to Gounaris-Sakurai parametrisation
  - $\rightarrow$ Unreliable representation of cross section at high s
- $\rightarrow$ The authors find (with FJ03VP):

$$a_\mu^{\pi^+\pi^-}(\psi(3770)) = 489.6 \pm 4.5_{\rm stat},\ a_\mu^{\pi^+\pi^-}(\psi(4170)) = 503.6 \pm 5.9_{\rm stat}$$
$$a_\mu^{\pi^+\pi^-}(\text{Weighted average}) = 500.4 \pm 3.6_{\rm stat} \pm 7.5_{\rm sys}$$

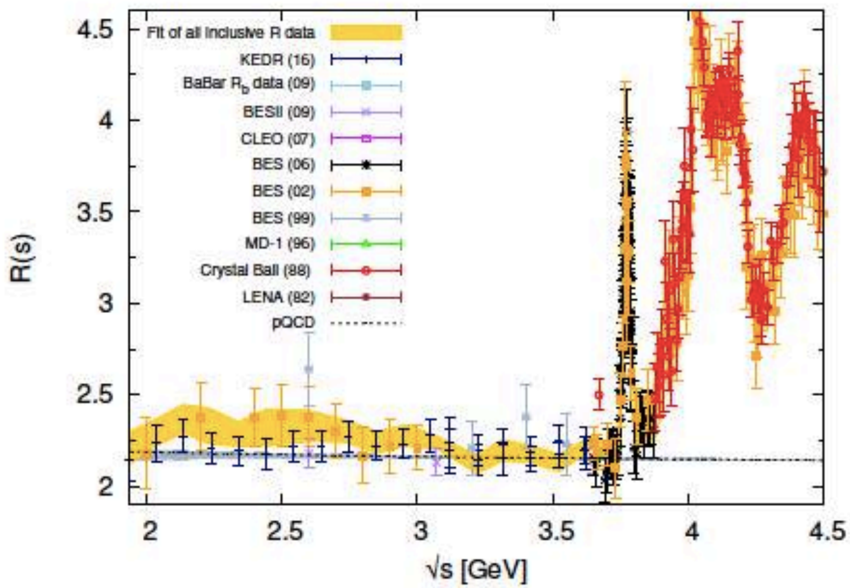
- $\rightarrow$ I find (with KNT18VP):

$$a_\mu^{\pi^+\pi^-}(\psi(3770)) = 499.6 \pm 4.5_{\rm stat} \pm 7.5\text{sys},\ a_\mu^{\pi^+\pi^-}(\psi(4170)) = 504.3 \pm 5.9_{\rm stat} \pm 7.6_{\rm sys}$$
$$a_\mu^{\pi^+\pi^-}(\text{Fit} - \text{w/o correlated systematics}) = 500.9 \pm 4.0_{\rm stat} \pm 5.9_{\rm sys}$$
$$a_\mu^{\pi^+\pi^-}(\text{Fit} - \text{with correlated systematics}) = 500.7 \pm 4.0_{\rm stat} \pm 8.3_{\rm sys}$$

KEDR $R(s)$ with covariance matrix

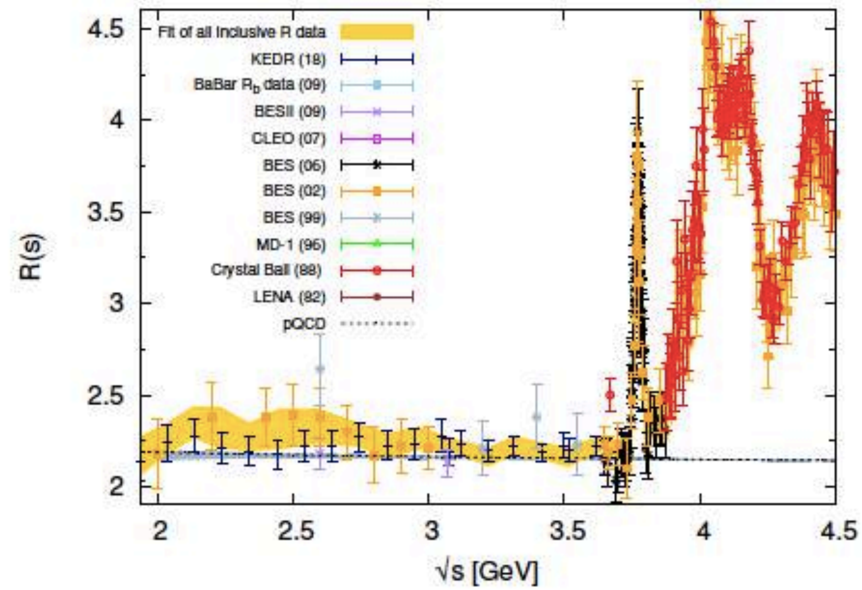
⇒ New precise KEDR update [arXiv:1805.06235] with systematic covariance matrix for all measurements provided by experiment

KNT18



$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.51$$
$$\sqrt{\chi^2_{\text{min}}/\text{d.o.f.}} = 1.44$$

KNT18 + new KEDR data



$$a_\mu^{\text{Inclusive}} = 43.54 \pm 0.51$$
$$\sqrt{\chi^2_{\text{min}}/\text{d.o.f.}} = 1.47$$

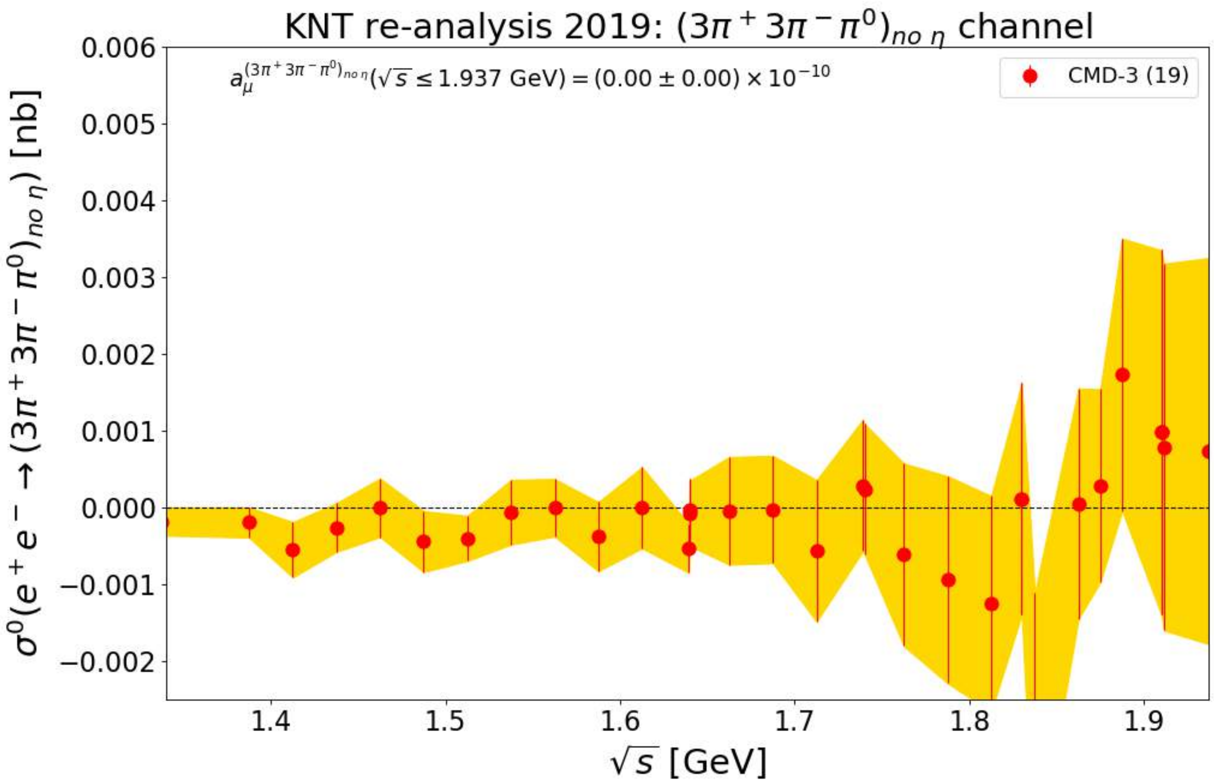
Note: Uncertainties quoted here do not include radiative correction uncertainties

⇒ Observe **very small** changes due to including correlations (slightly closer to pQCD)

New data updates [preliminary]

$3\pi^+3\pi^-\pi^0$channel – CMD-3 (Phys.Lett. B792 (2019) 419-423)

- This is the first measurement of a 7-pion final state below 2 GeV.



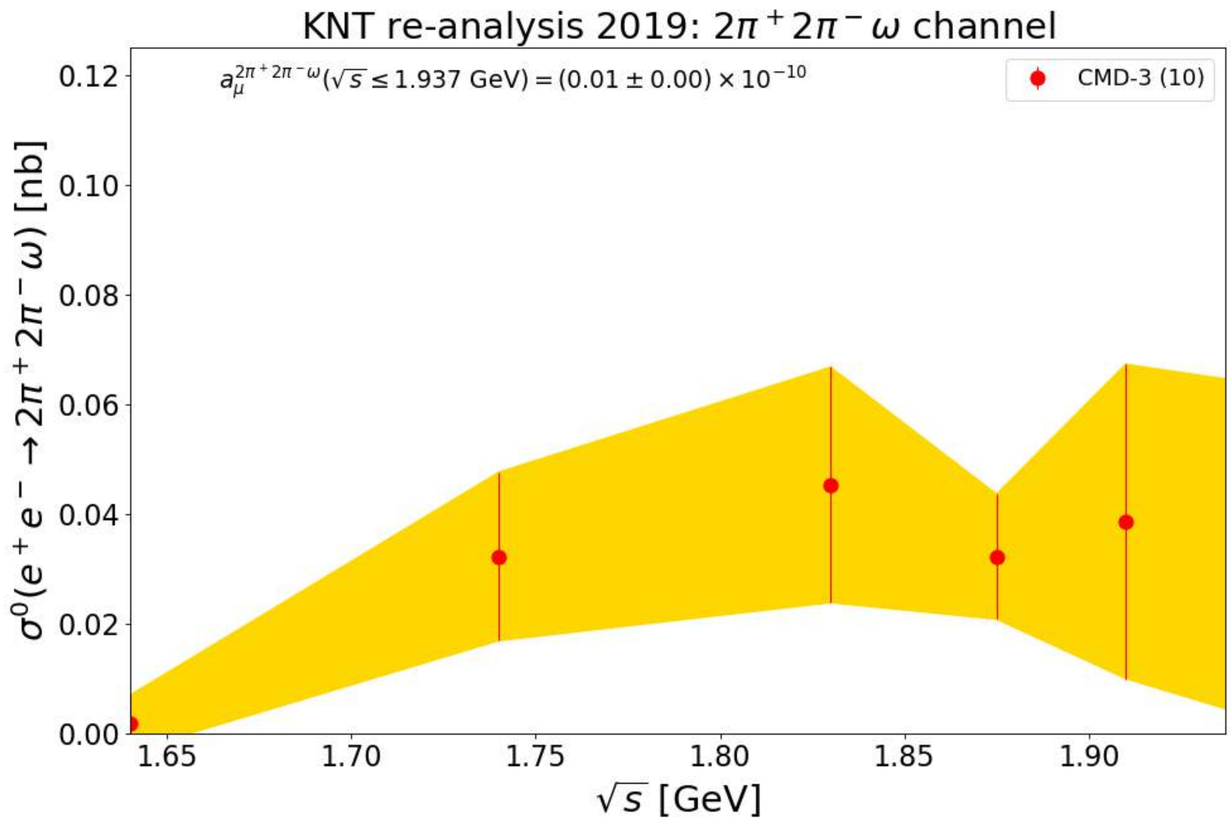
New [preliminary]: $a_\mu^{(3\pi^+3\pi^-\pi^0)_{no\ \eta\omega}} = 0.00\ \pm 0.00$

→ After subtracting η and ω contributions to avoid double counting, entirely consistent with zero!

New data updates [preliminary]

$2\pi^+2\pi^-\omega$ channel – CMD-3 (Phys.Lett. B792 (2019) 419-423)

- First measurement of this mode as a production mechanism for $3\pi^+3\pi^-\pi^0$.



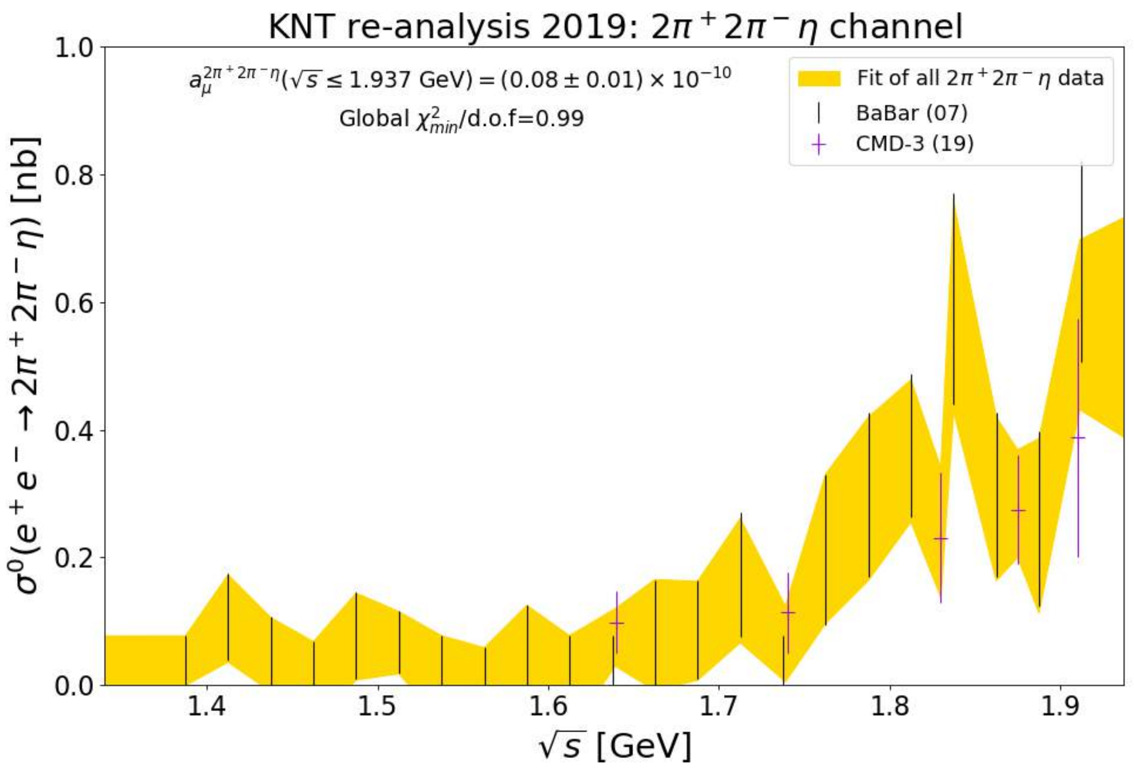
New [preliminary]: $a_\mu^{2\pi^+2\pi^-\omega} = 0.01 \pm 0.00$

→ Very small new contribution from this channel.

New data updates [preliminary]

$2\pi^+2\pi^-\eta$ channel – CMD-3 (Phys.Lett. B792 (2019) 419-423)

- New addition to compliment lone measurement in this channel.



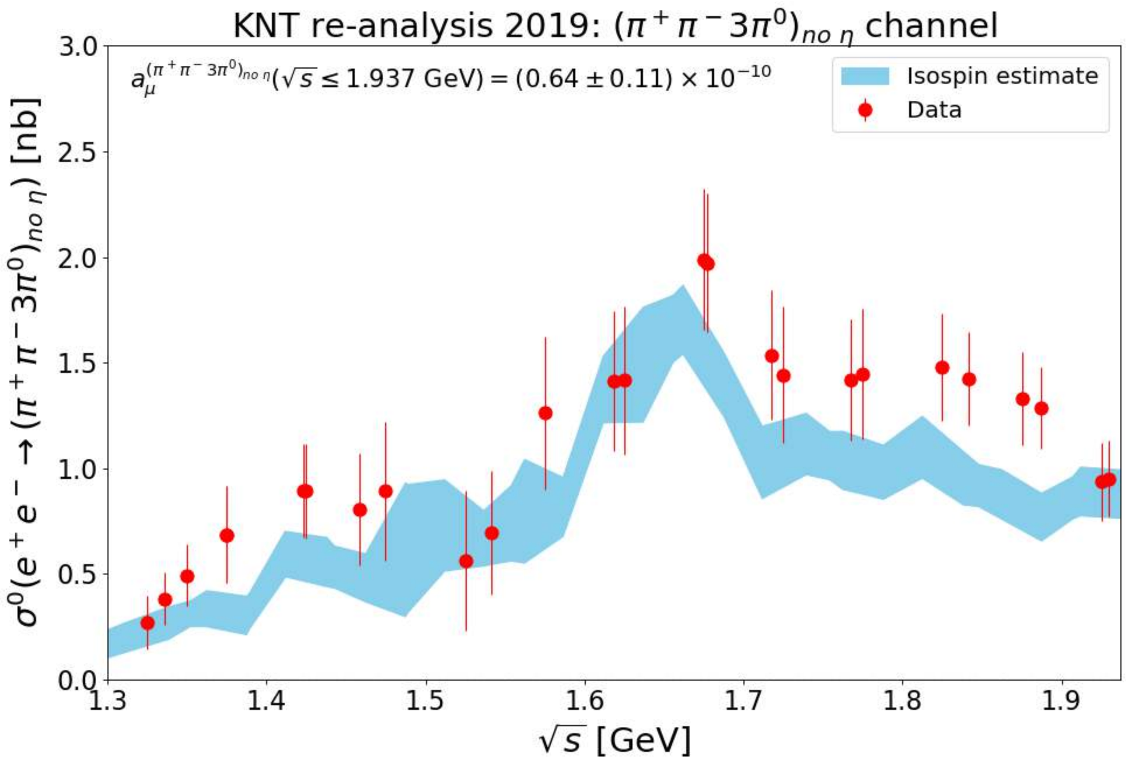
KNT18: $a_\mu^{2\pi^+2\pi^-\eta} = 0.08 \pm 0.01$

Now: $a_\mu^{2\pi^+2\pi^-\eta} = 0.08 \pm 0.01, \chi^2_{\text{min}}/\text{d.o.f.} = 0.99$
[preliminary]

New data updates [preliminary]

$\pi^+\pi^-3\pi^0$ channel - BaBar (Phys.Rev. D98 (2018) 112015)

- This channel was previously estimated via isospin relations:



KNT18: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\pi^0} = 0.50 \pm 0.04$

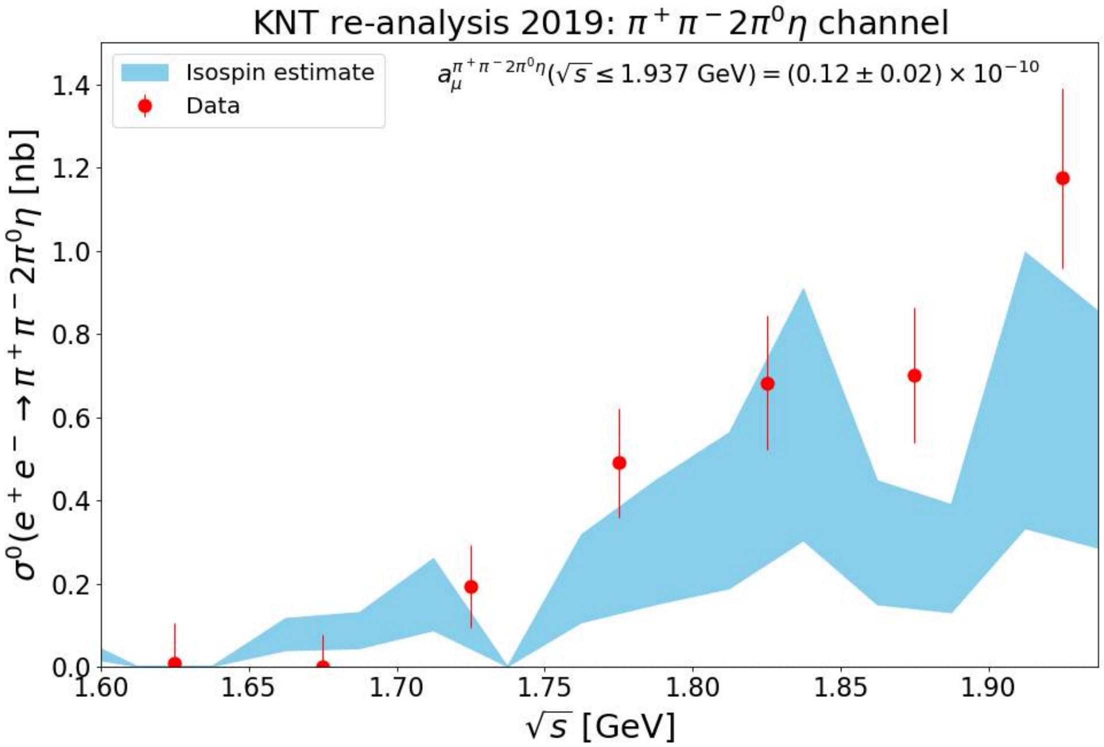
Now: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\pi^0} = 0.64 \pm 0.11$

[preliminary]

New data updates [preliminary]

$\pi^+\pi^-\pi^0\pi^0\eta$ channel - BaBar (Phys.Rev. D98 (2018) 112015)

- This channel was previously estimated via isospin relations:



KNT18: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\eta} = 0.08 \pm 0.04$

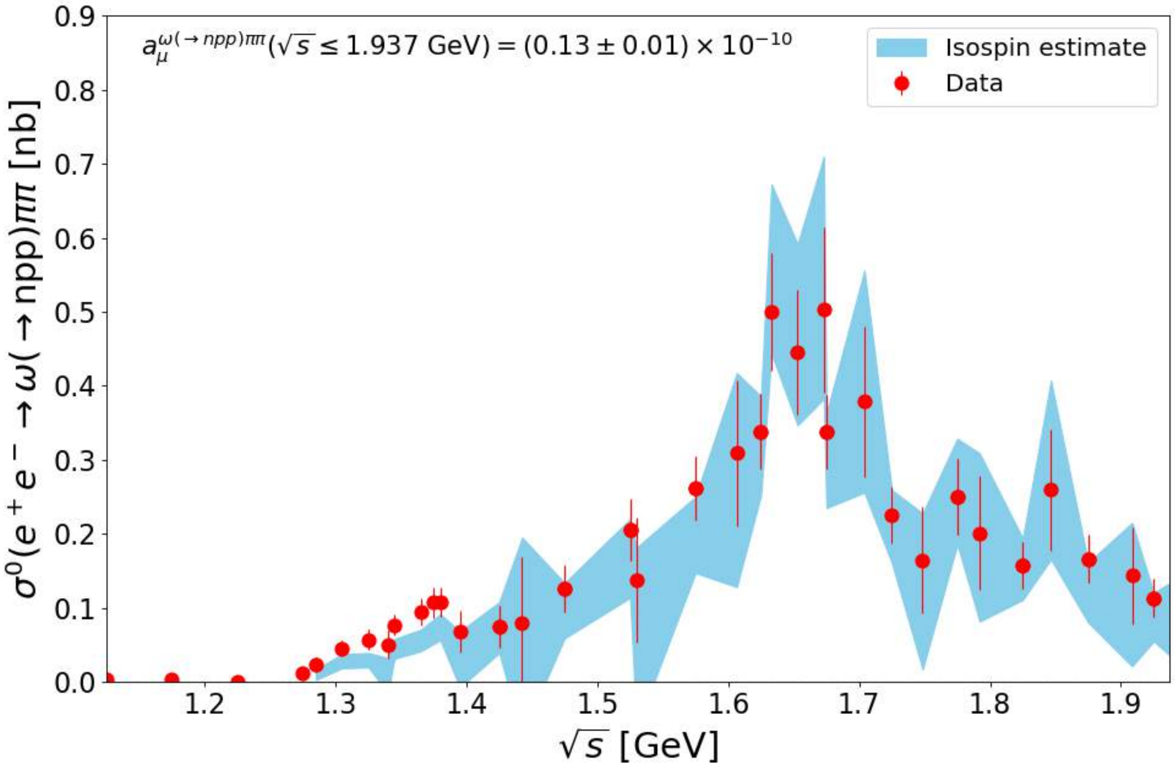
Now: $a_\mu^{\pi^+\pi^-\pi^0\pi^0\eta} = 0.12 \pm 0.02$

[preliminary]

New data updates [preliminary]

$\omega(\to \mathrm{npp})\pi\pi$ channel - $\omega\pi^0\pi^0$ from BaBar (Phys.Rev. D98 (2018) 112015)

- This channel was previously estimated via isospin relations:



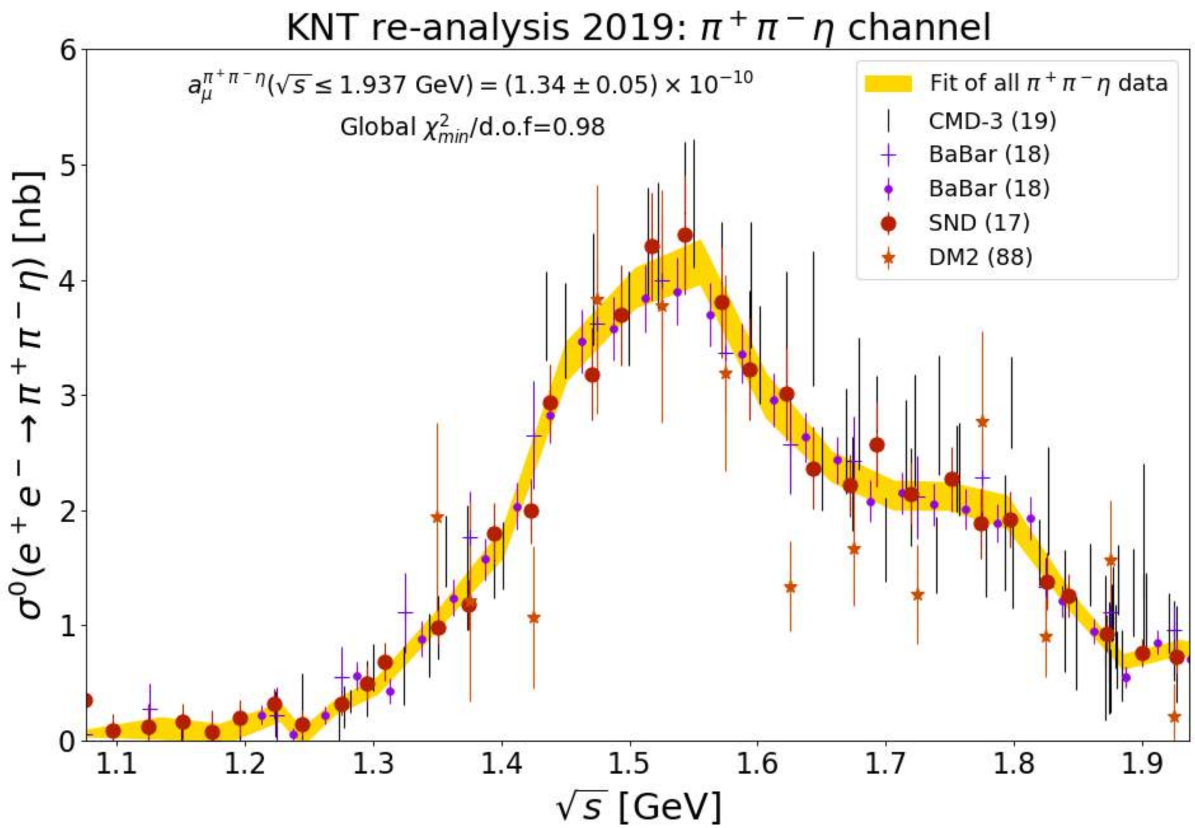
KNT18: $a_\mu^{\omega(\to \mathrm{npp})\pi\pi} = 0.10 \pm 0.02$

Now: $a_\mu^{\omega(\to \mathrm{npp})\pi\pi} = 0.13 \pm 0.01$

[preliminary]

New data updates [preliminary]

$\pi^+\pi^-\eta$ channel - BaBar (Phys.Rev. D98 (2018) 112015) & CMD-3 (arXiv:1907.08002)

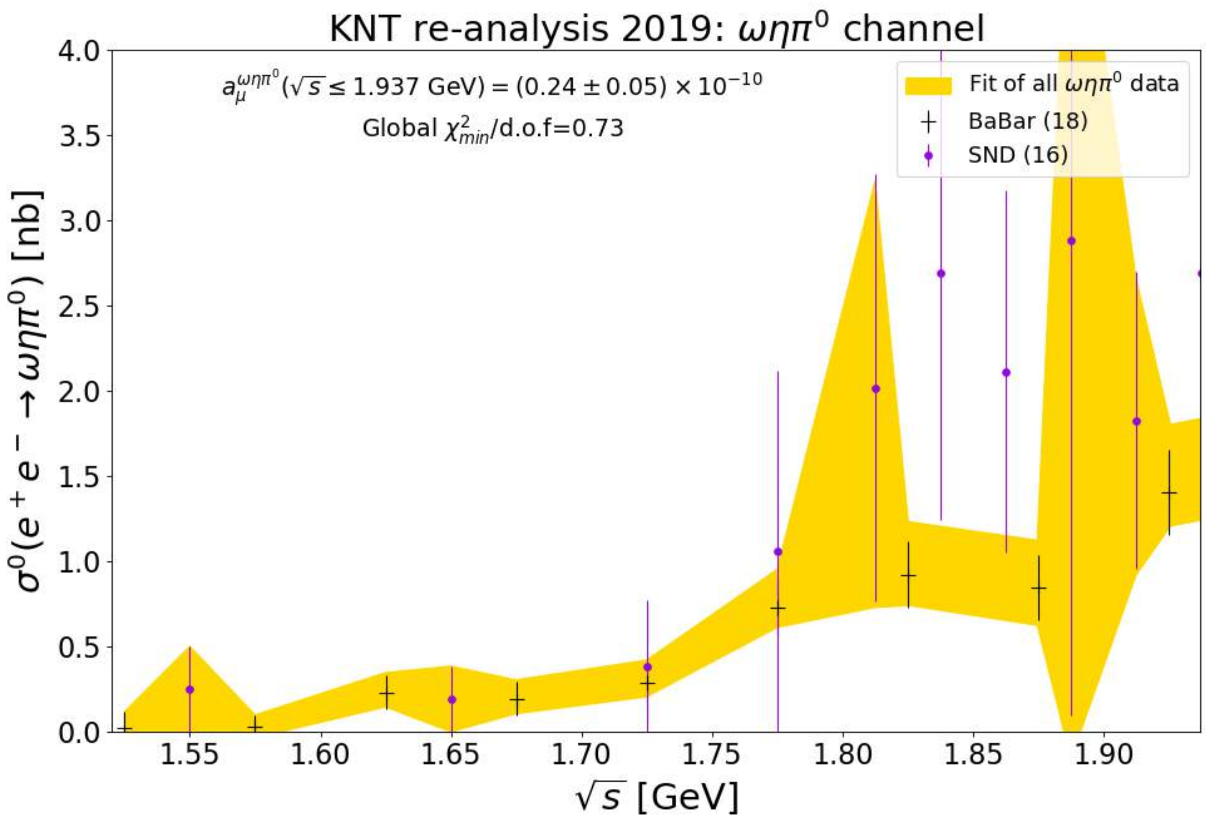


KNT18: $a_\mu^{\pi^+\pi^-\eta} = 1.29 \pm 0.06$, $\chi^2_{\min}/\text{d.o.f.} = 1.06$

Now: $a_\mu^{\pi^+\pi^-\eta} = 1.34 \pm 0.05$, $\chi^2_{\min}/\text{d.o.f.} = 0.98$
[preliminary]

New data updates [preliminary]

$\omega\eta\pi^0$ channel - BaBar (Phys.Rev. D98 (2018) 112015)



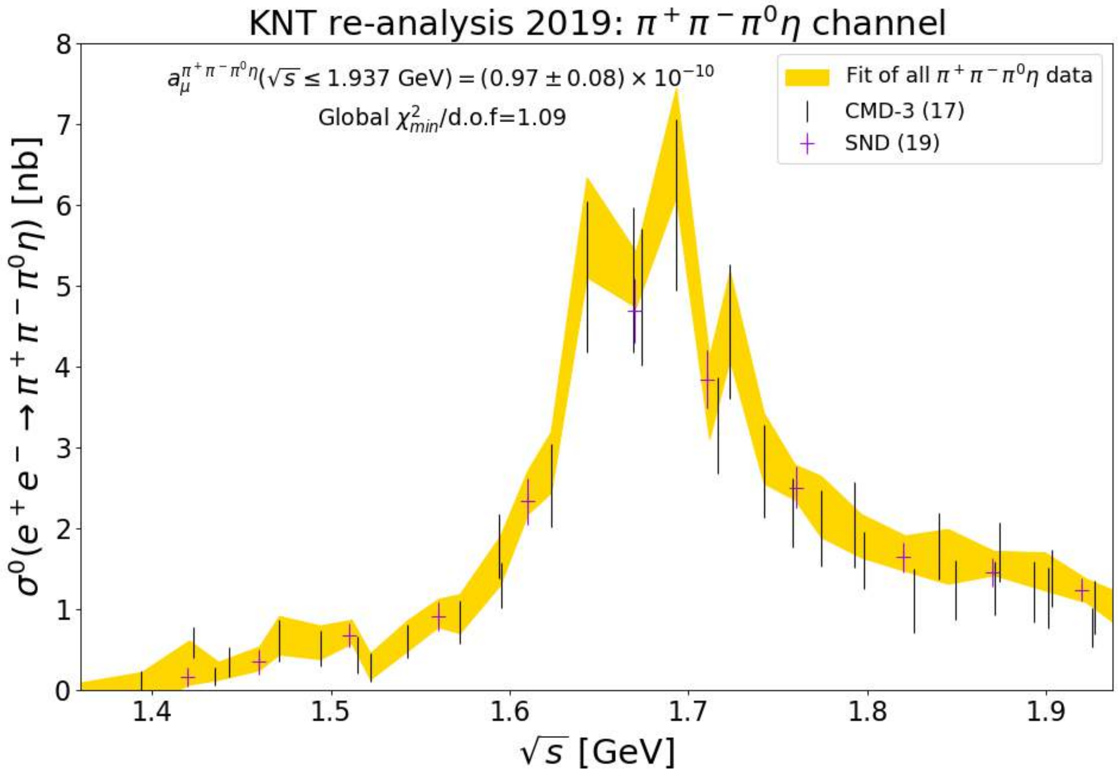
KNT18: $a_\mu^{\omega\eta\pi^0} = 0.35 \pm 0.09$

Now: $a_\mu^{\omega\eta\pi^0} = 0.24 \pm 0.05, \chi^2_{\mathrm{min}}/\mathrm{d.o.f.} = 0.73$
[preliminary]

New data updates [preliminary]

$\pi^+\pi^-\pi^0\eta$ channel – SND (Phys. Rev. D 99 (2019) 112004)

- New addition to compliment single CMD-3 measurement in this channel.



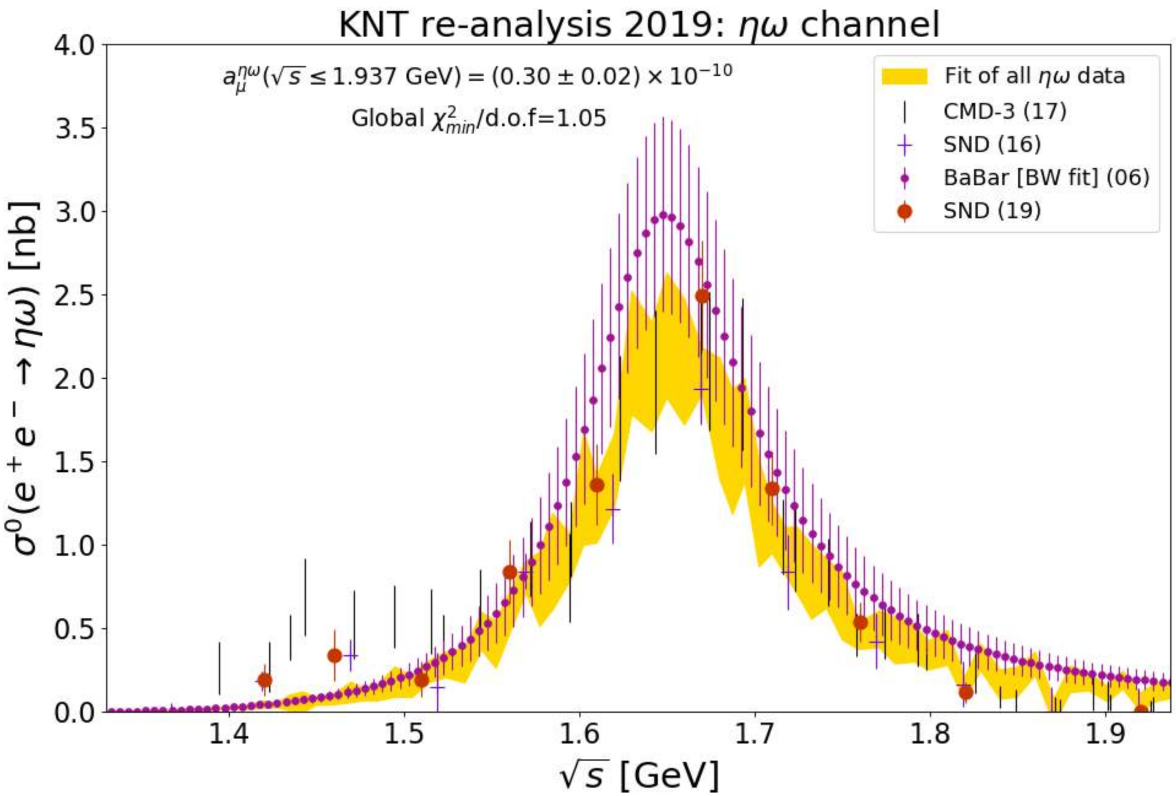
KNT18: $a_\mu^{(\pi^+\pi^-\pi^0\eta)_{no\,\omega}} = 0.60 \pm 0.15$

Now: $a_\mu^{(\pi^+\pi^-\pi^0\eta)_{no\,\omega}} = 0.71 \pm 0.08, \chi^2_{\text{min}}/\text{d.o.f.} = 1.09$

[preliminary]

New data updates [preliminary]

$\eta\omega$ channel – SND (Phys. Rev. D 99 (2019) 112004)



KNT18: $a_\mu^{(\pi^+\pi^-\pi^0\eta)_{no\ \omega}} = 0.31 \pm 0.03,\ \chi^2_{\text{min}}/\text{d.o.f.} = 1.82$

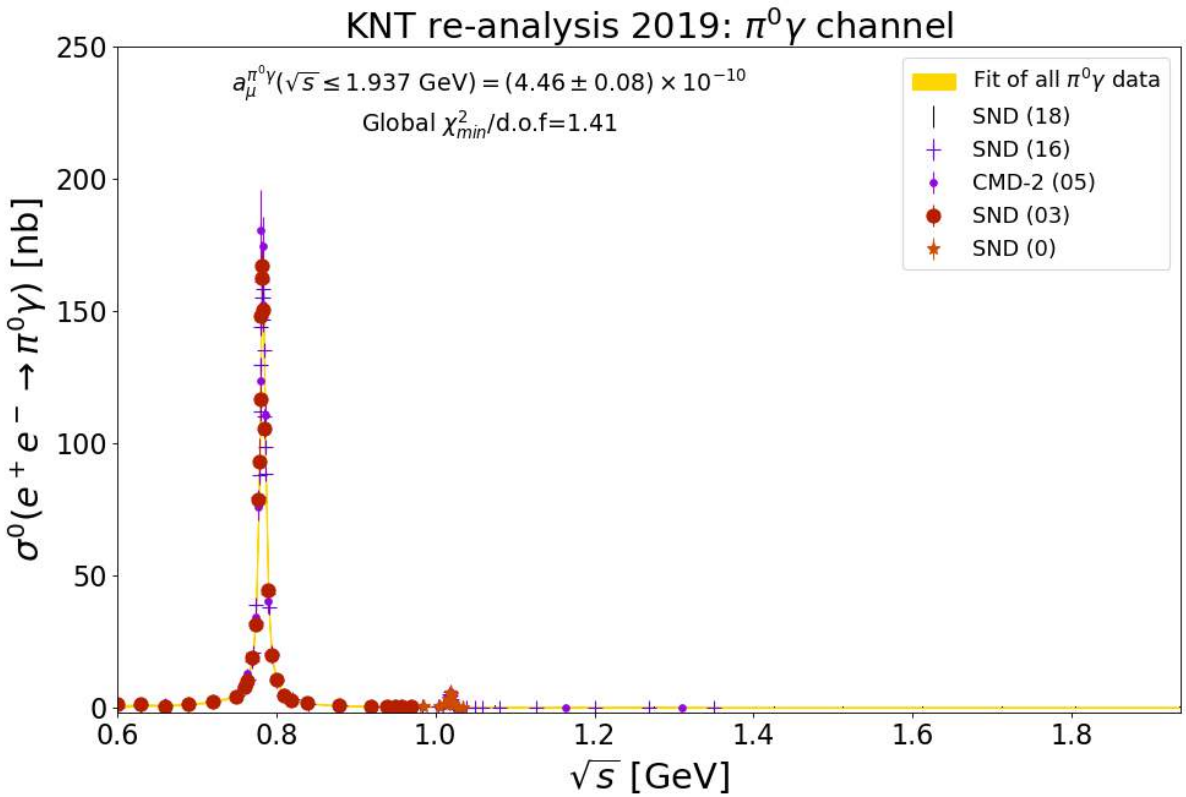
Now: $a_\mu^{(\pi^+\pi^-\pi^0\eta)_{no\ \omega}} = 0.30 \pm 0.02,\ \chi^2_{\text{min}}/\text{d.o.f.} = 1.05$

[preliminary]

New data updates [preliminary]

$\pi^0\gamma$ channel - SND (Phys.Rev. D98 (2018) 112001)

➢ This extends the upper border of the pi0 gamma data from 1.35 GeV to 1.935 GeV.



KNT18: $a_\mu^{\pi^0\gamma} = 4.46 \pm 0.08, \chi^2_{\text{min}}/\text{d.o.f.} = 1.44$

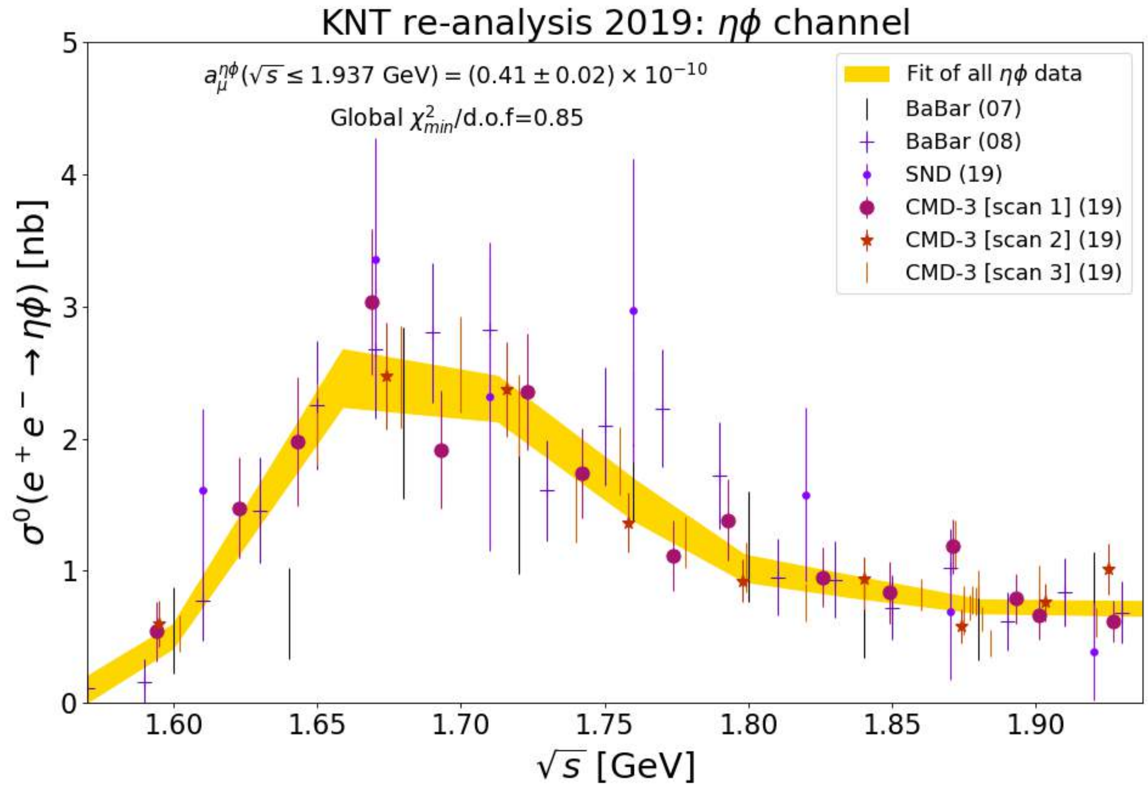
Now [preliminary]: $a_\mu^{\pi^0\gamma} = 4.46 \pm 0.08, \chi^2_{\text{min}}/\text{d.o.f.} = 1.41$

→ Negligible changes, consolidation of previous estimate

New data updates [preliminary]

$\eta\phi$ channel – SND (Phys. Rev. D 99 (2019) 112004) & CMD-3 (arXiv:1906.08006)

- Three new data scans from CMD-3; systematics taken to be 100% correlated.

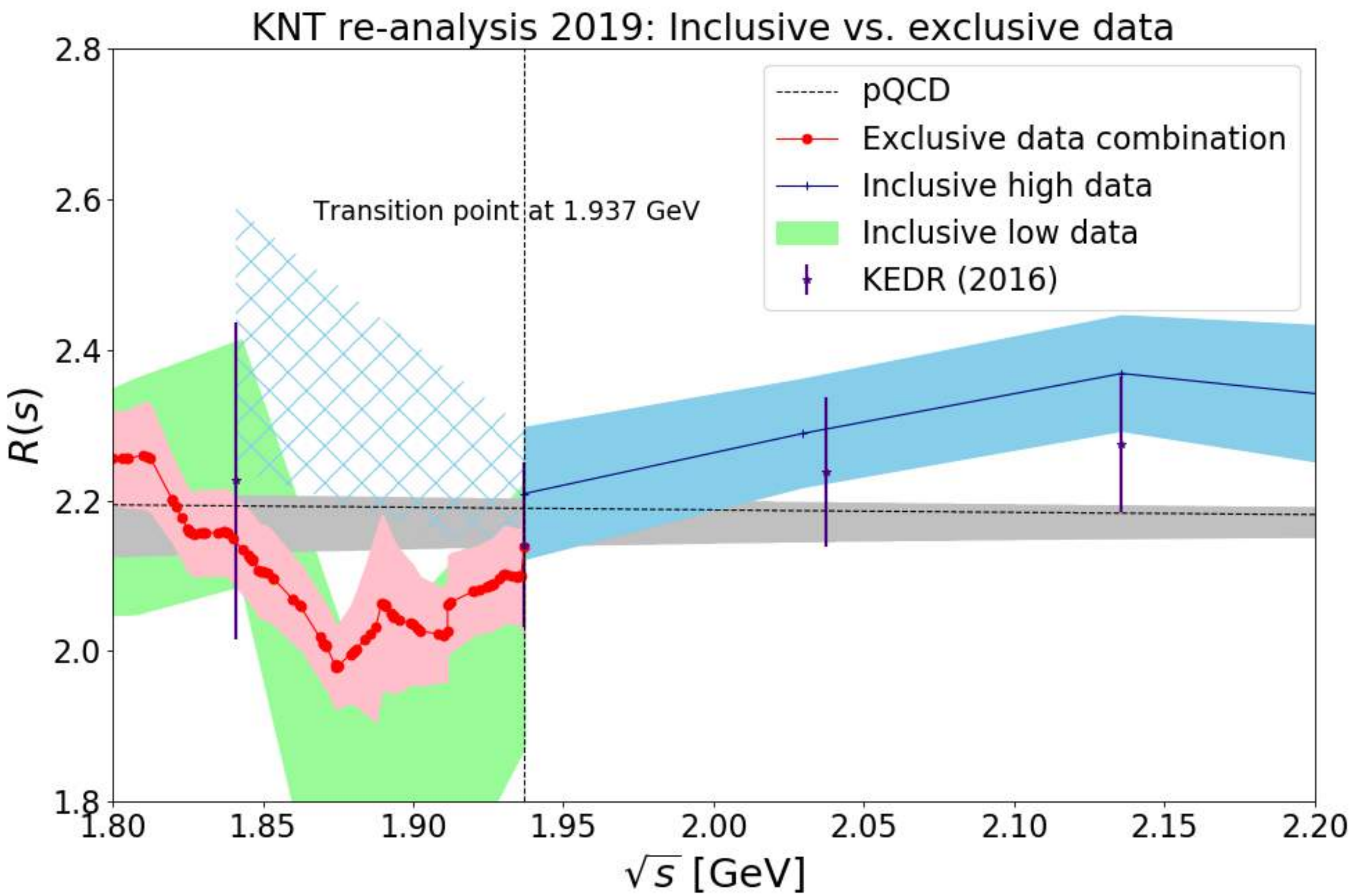


KNT18: $a_\mu^{\eta\phi} = 0.42 \pm 0.03,\ \chi^2_{\text{min}}/\text{d.o.f.} = 0.54$

Now [preliminary]: $a_\mu^{\eta\phi} = 0.41 \pm 0.02,\ \chi^2_{\text{min}}/\text{d.o.f.} = 0.85$

Exclusive vs. Inclusive data [preliminary]

How have these data changes affected the KNT18 exclusive/inclusive transition region?



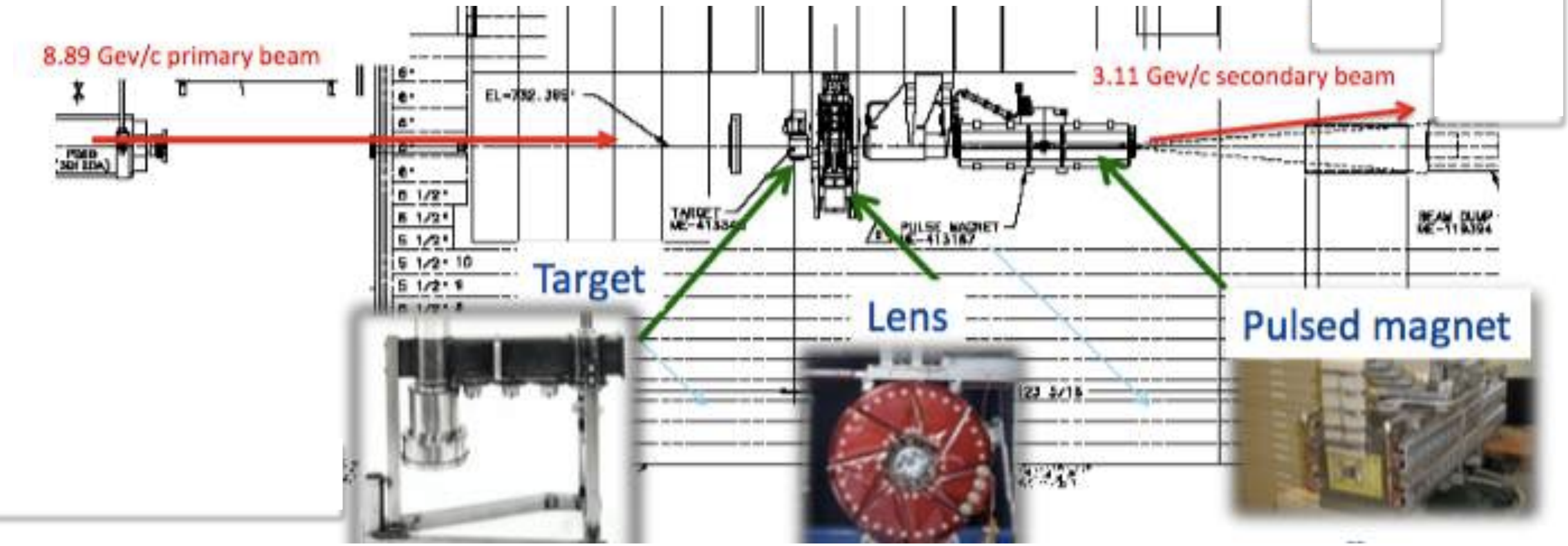
Changes are negligible.

1.937 GeV is still the preferred transition region.

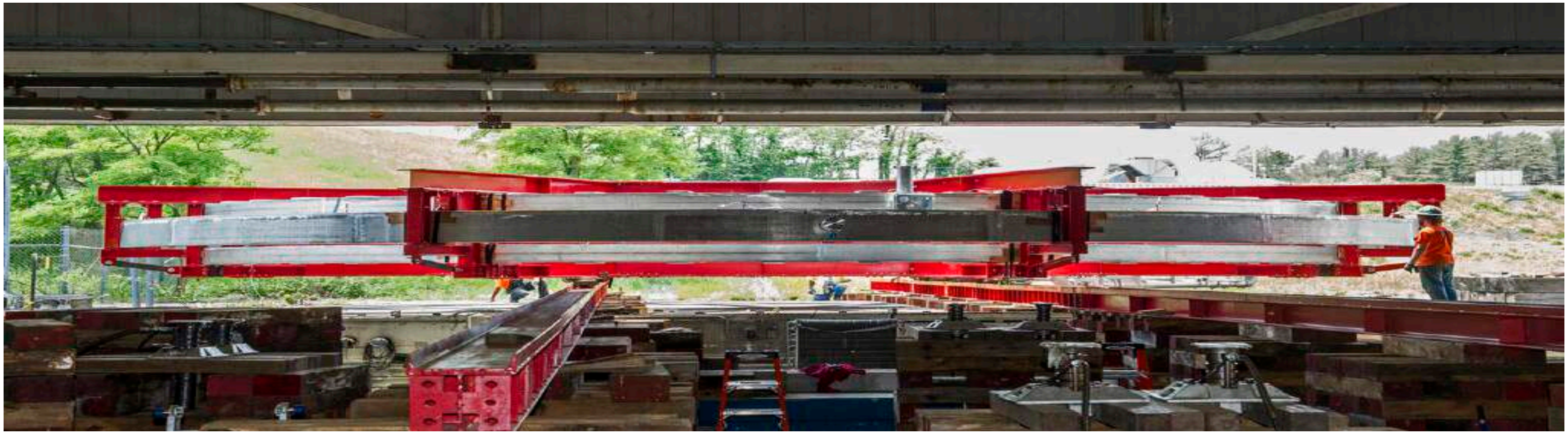

MAIN INJECTOR/RECYCLER RINGS
TEVATRON (RIP)
DR
BOOSTER
LINAC

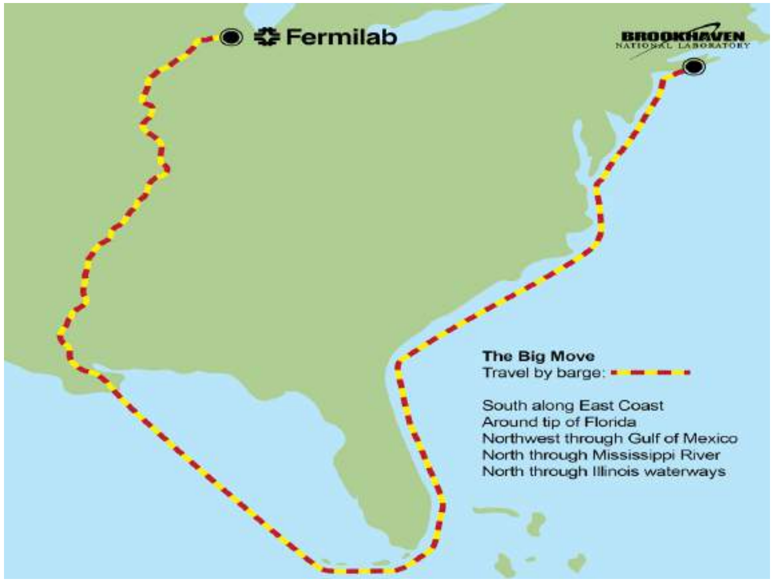

CHICAGO
BOOSTER
Mu2e
g-2
MUON TARGET & DELIVERY RING

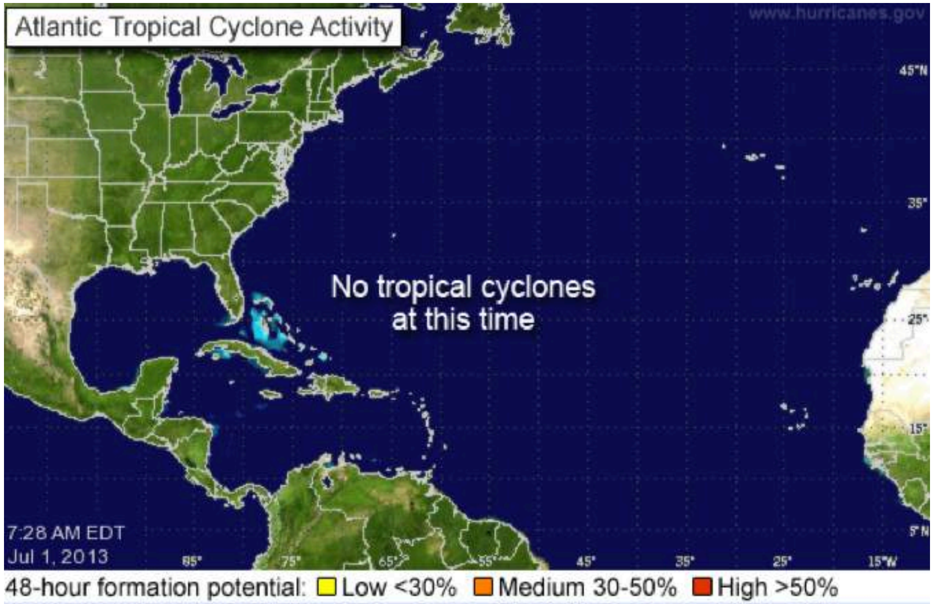
900m of instrumented beamline



From BNL to FNAL

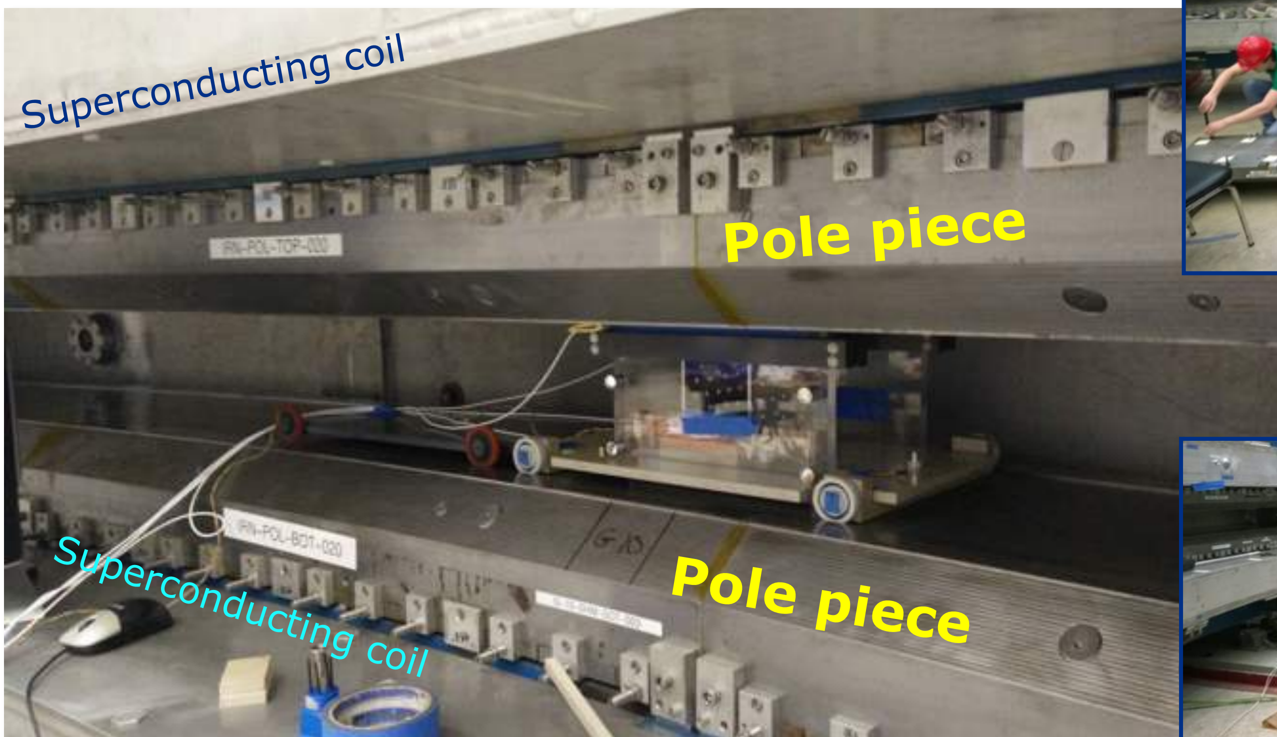






2.5 years to get magnet field uniformity

It took 2.5 years to shim the magnetic field to achieve the ppm uniformity required …



Anatomy of the magnet

Not simply a coil & 72 pole pieces but:

864 wedges

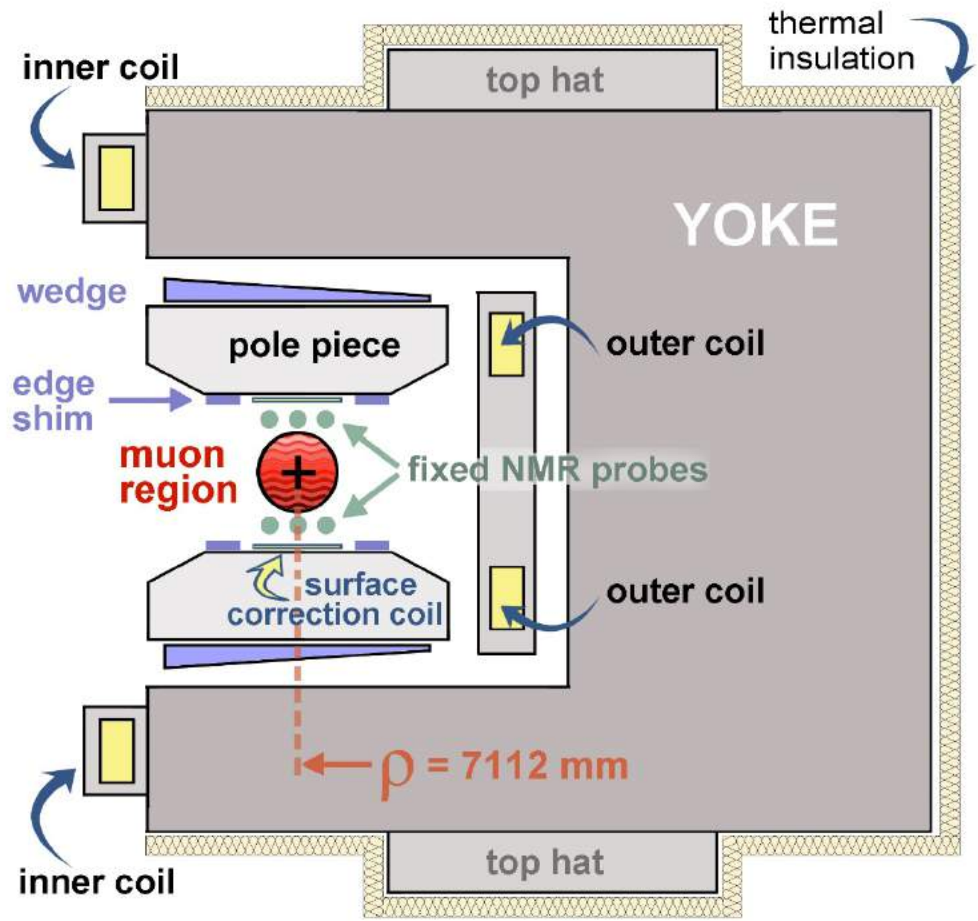
48 iron “top hats”

144 edge shims

8000 surface iron foils

100 active surface coils

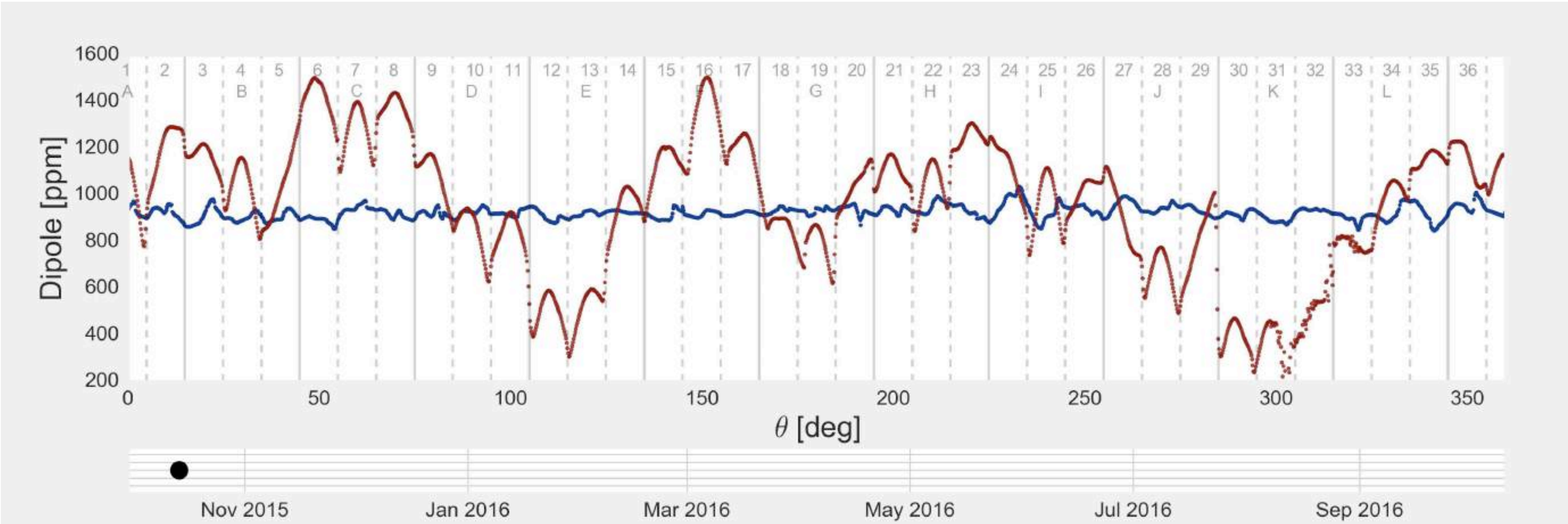
requiring precision alignment & “shimming”



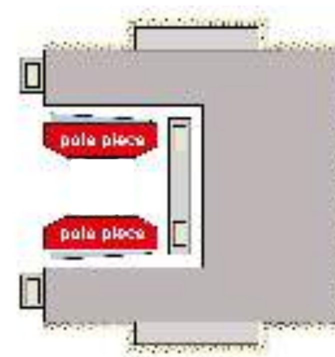
Yoke : 26 tons to 125 microns….

Shimming the magnet

→ Progress towards a uniform magnetic field from Oct 2015 to Sep 2016:



Red = Initial dipole field starting point at Fermilab
Blue = typical BNL final field *after* shimming



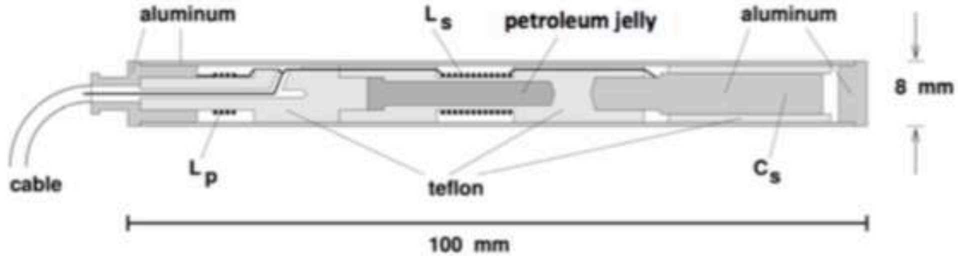
→ Final Fermilab Result is better than BNL by a factor of ~3 (p-p & RMS)

→ Shimming checked between runs to ensure uniformity.

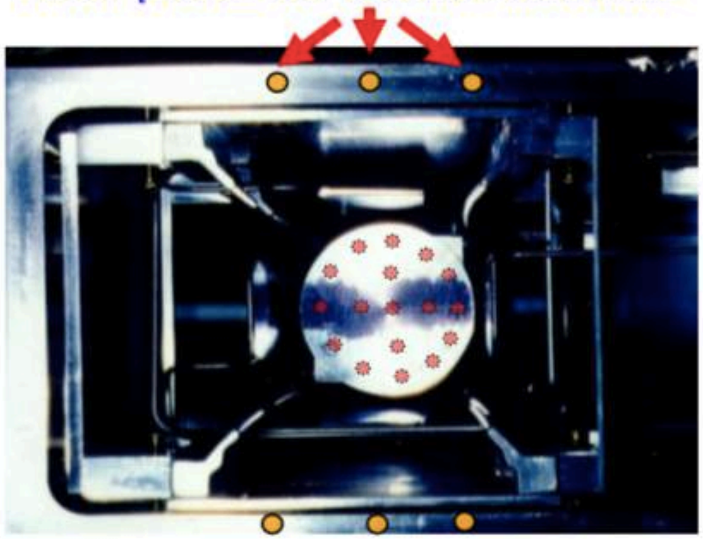
James Mott, SSP 2018, Aachen, 12th June 2018

Measuring the B-Field to 70 ppb

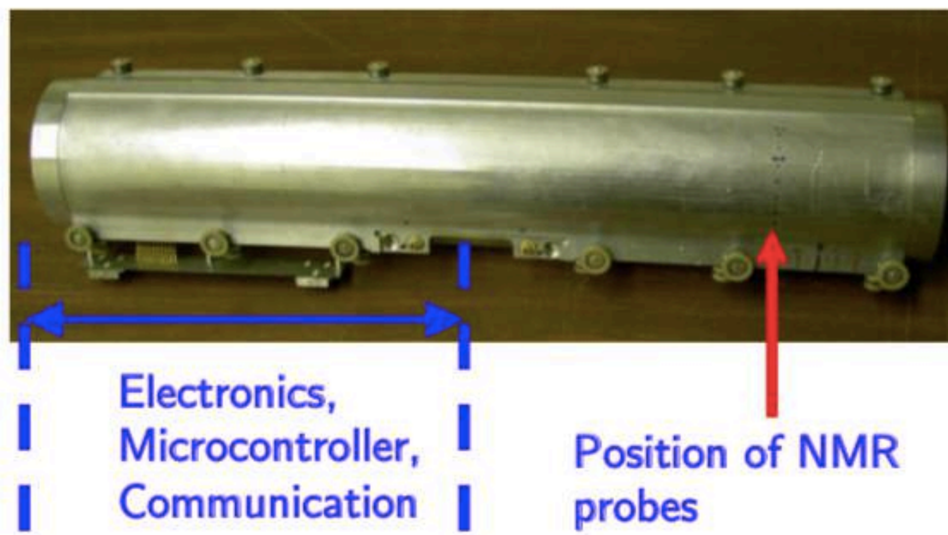
- 387 Fixed NMR probes outside storage volume measure field while muons stored
- Field inside storage volume measured by NMR trolley periodically
- Fixed probes calibrated when trolley passes; can infer field inside storage volume



Fixed probes on vacuum chambers



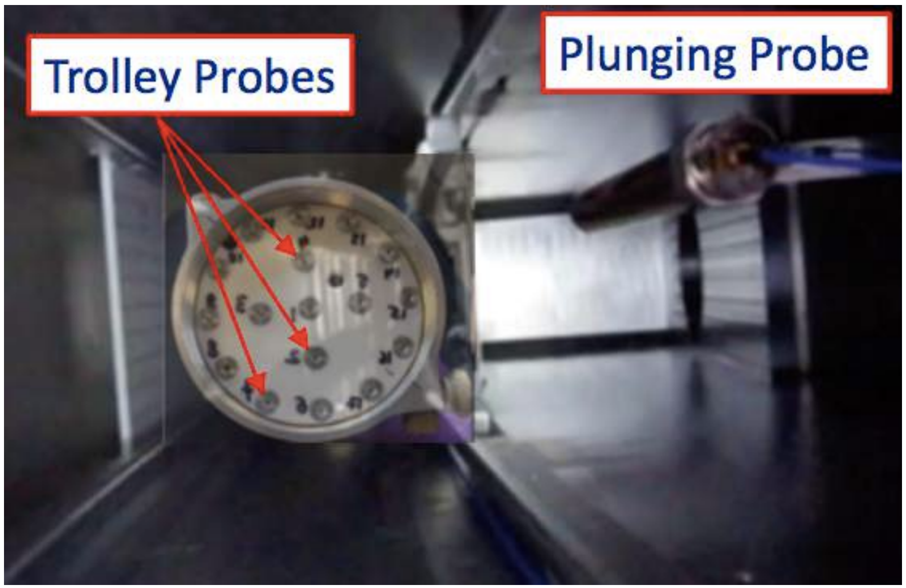
Trolley with matrix of 17 NMR probes

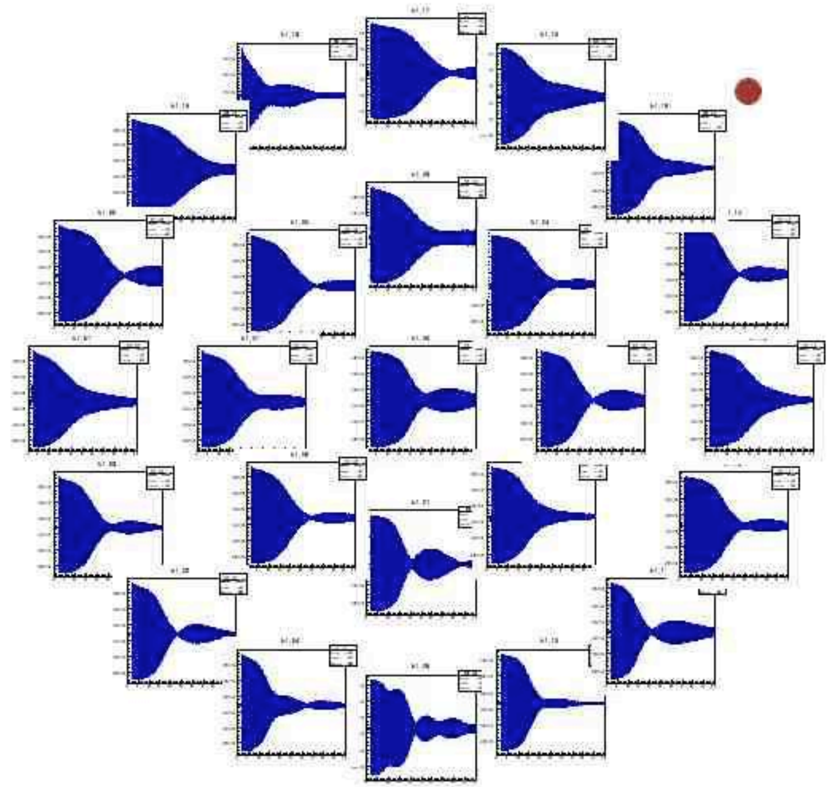


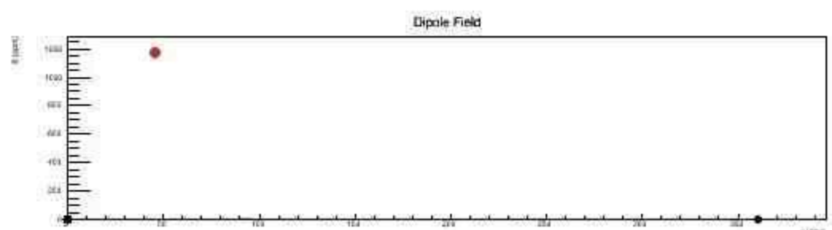
Dave Kawall, Fermilab Measurement of Muon g-2, g-2 Theory Initiative Workshop in Mainz, June 18-22, 2018

Mapping the field seen by the muons…

→ The NMR trolley maps the B-field inside the storage region:







Mark Lancaster, UCL Schuster Colloquim, 5th December 2018

The kicker magnet

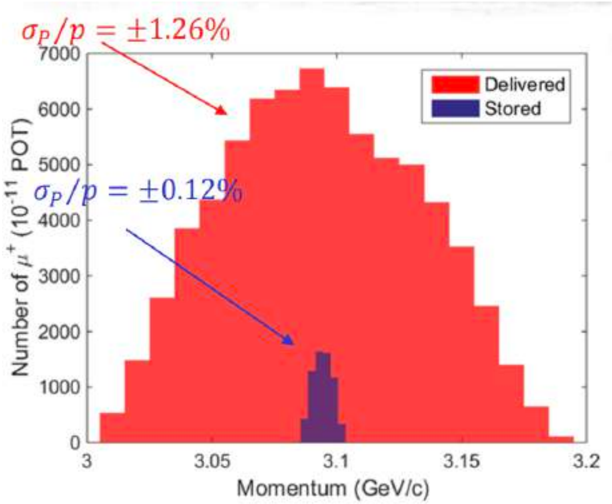
Shutdown performance issues

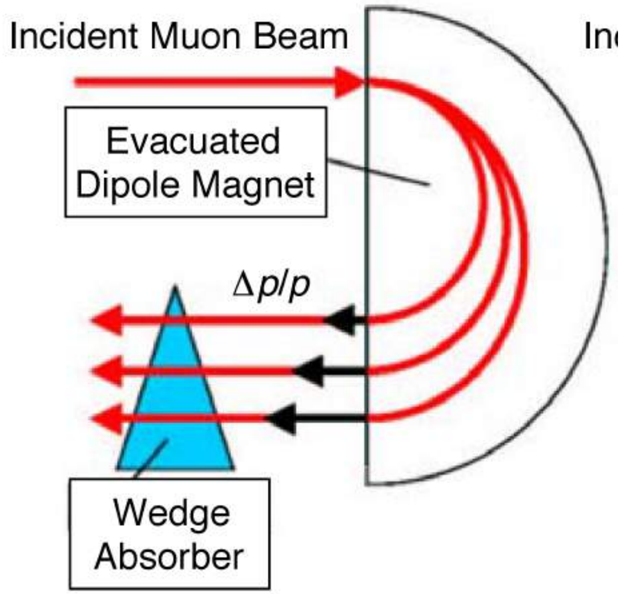
- Shutdown 2018 had a few key improvements to improve the number of muons we store:

System	Improvement	Gain
Accelerator	Beam Wedge	20%
	Power Supplies & Vacuum Window	11%
Kicker	Rework to provide higher strength	10%
Quads	More reliable operation at higher voltage	10%
Total		60%

- Total expected improvement is 1.6x run 1 storage rate
- Next year, will likely install new inflector (+40%)

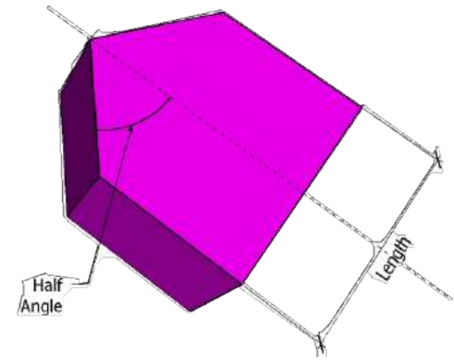
Beamline wedges

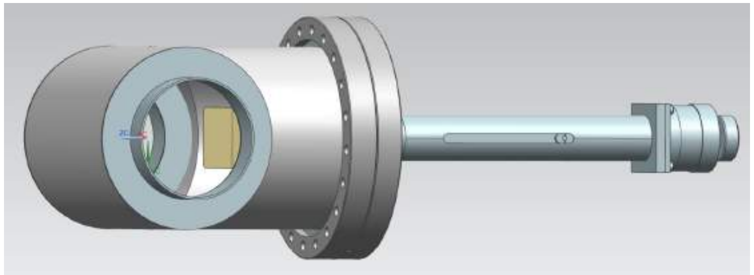




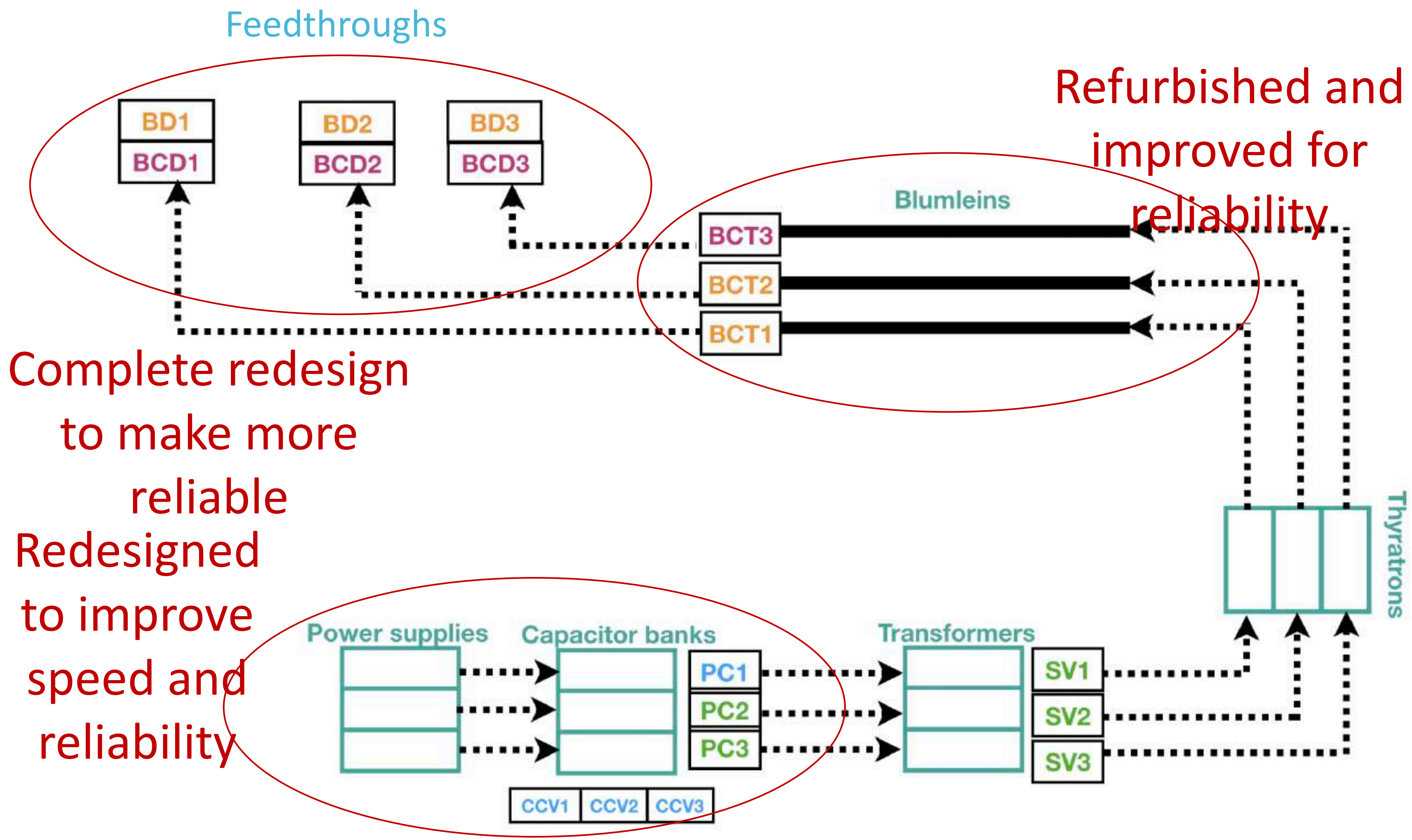
Only store a small fraction of delivered muons
Upstream wedges placed in region with dispersion to compactify momentum (during 2018 shutdown)

Simulations indicate gain of ~20%





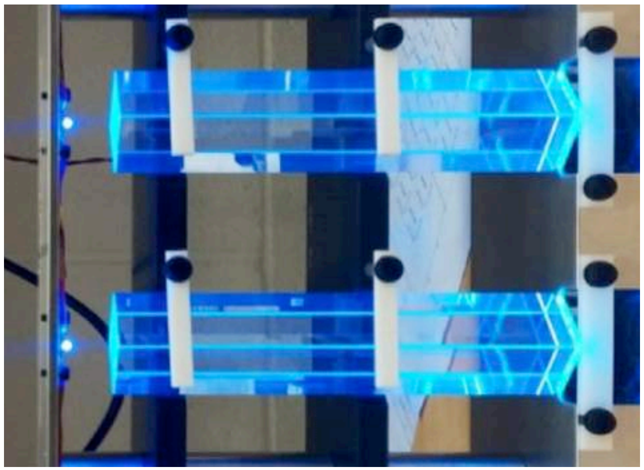
Kicker upgrade

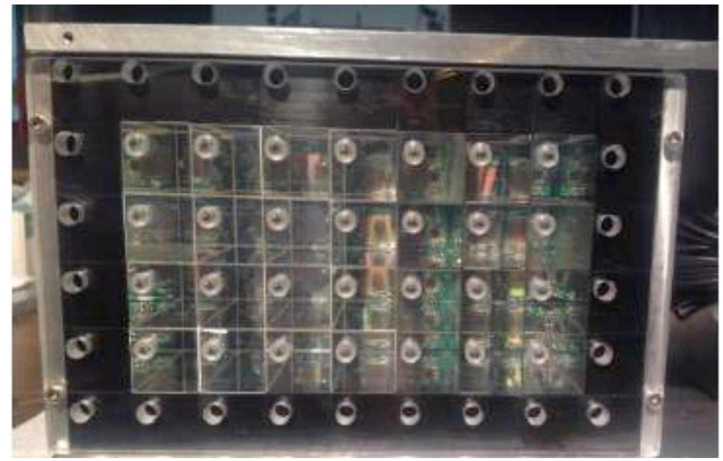


Expect +15% from more stored muons and better reliability

PbF_2 calorimeter

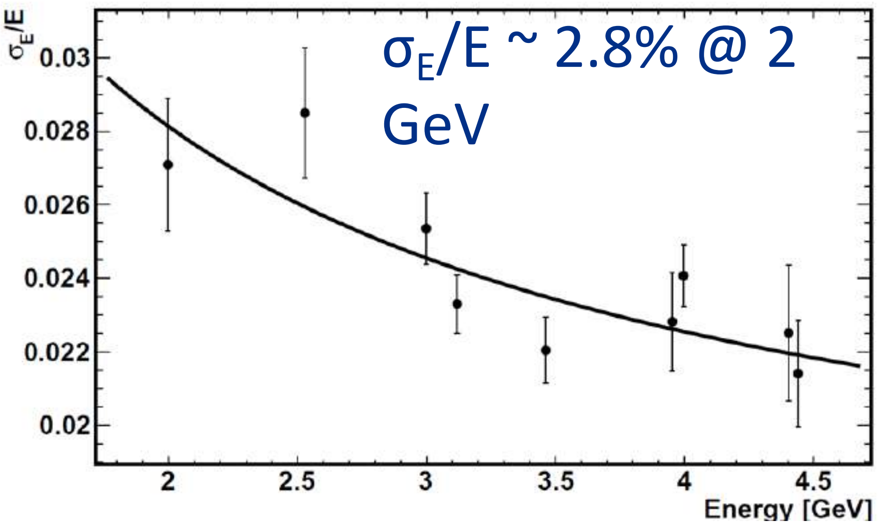
- Each calorimeter is array of 54 PbF_2 crystals - 2.5 x 2.5 cm^2 x 14 cm ($15X_0$)
- Readout by SiPMs to 800 MHz WFDs (1296 channels)



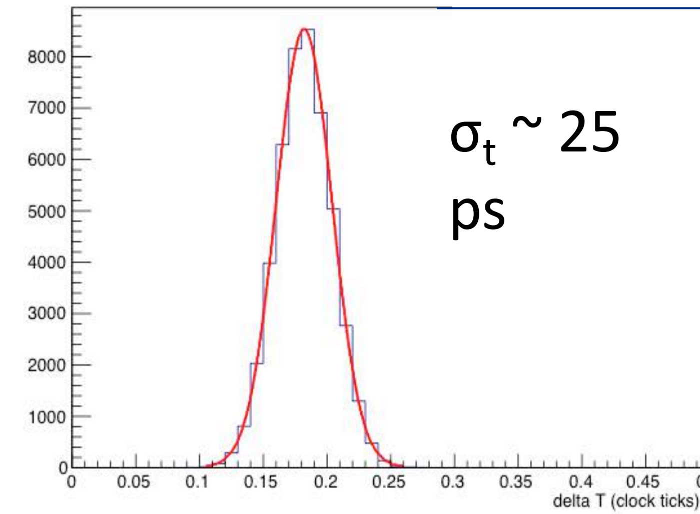


Calorimeter performance

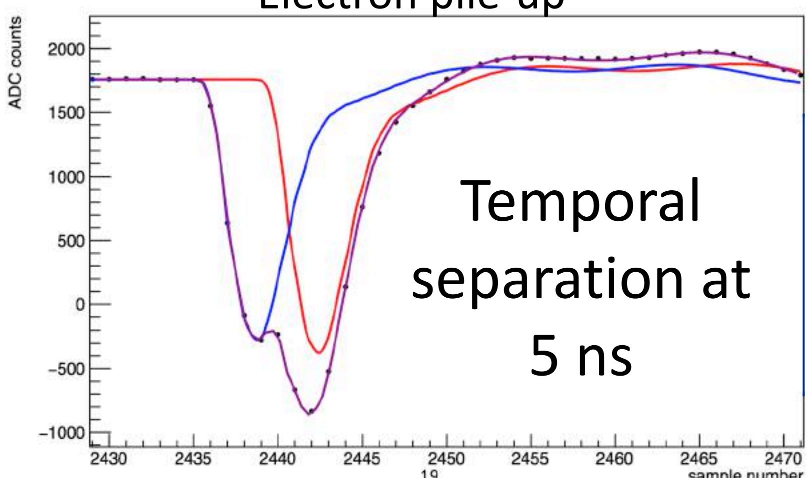
Energy Resolution



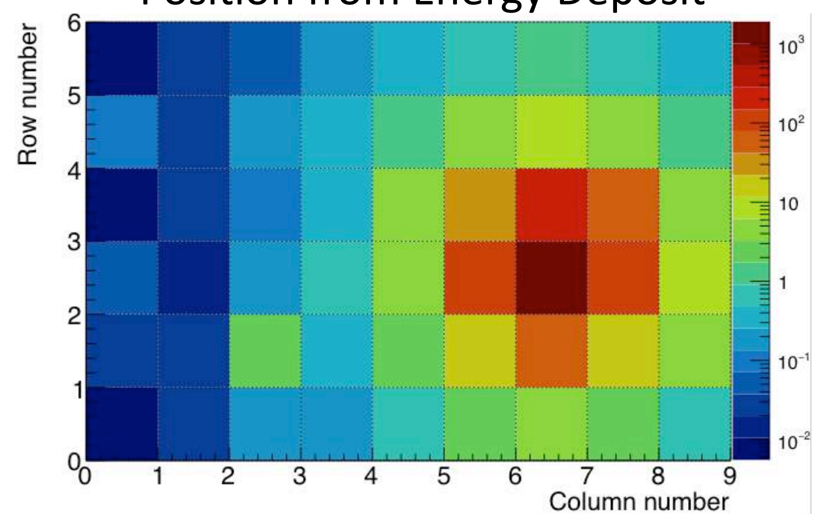
Timing Resolution



Electron pile-up



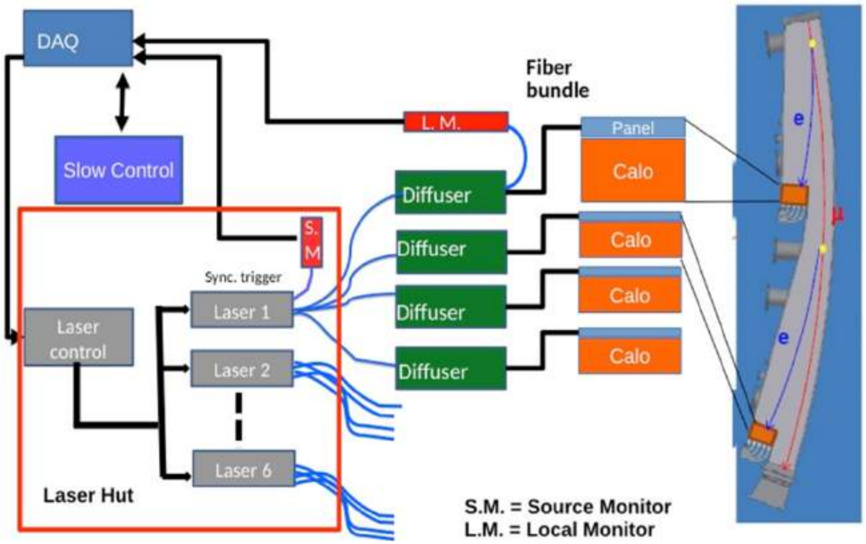
Position from Energy Deposit

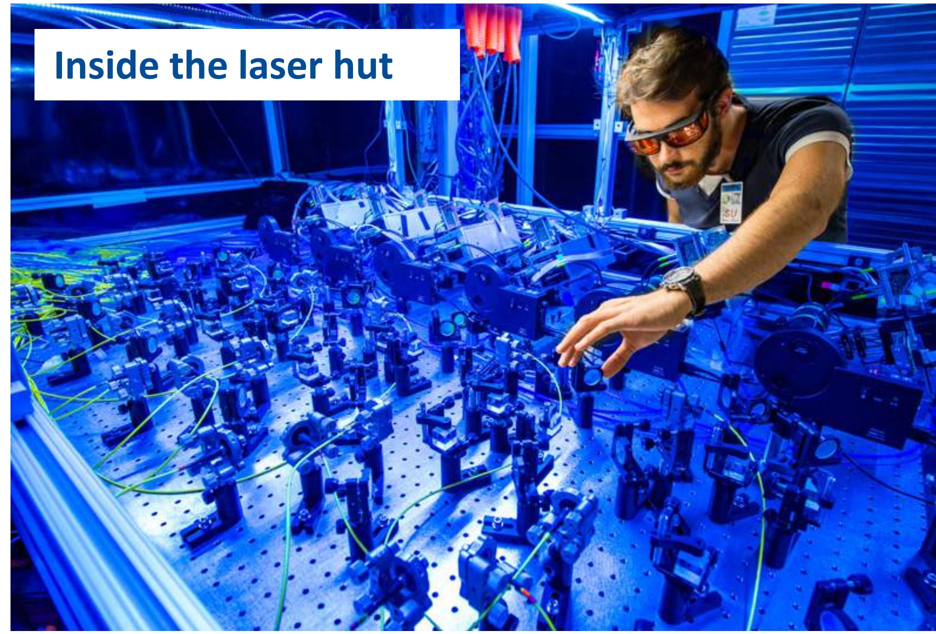


See [NIM A 783 (2015), pp 12–21] for details

Gain stability

State-of-the-art Laser-based calibration system also allows for pseudo data runs for DAQ





Contents lists available at ScienceDirect

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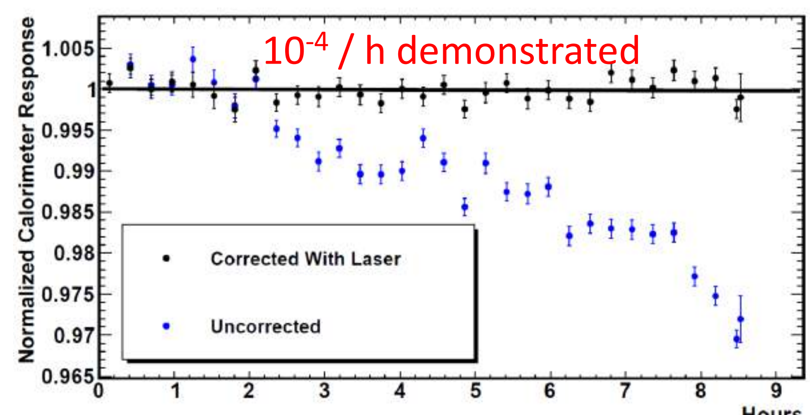
Test of candidate light distributors for the muon (g − 2) laser calibration system

A. Anastasi[a,c], D. Babusci[a], F. Baffigi[b], G. Cantatore[d,g], D. Cauz[d,i], G. Corradi[a], S. Dabagov[a], G. Di Sciascio[f], R. Di Stefano[e,j], C. Ferrari[a,b], A.T. Fienberg[l], A. Fioretti[a,b], L. Fulgentini[b], C. Gabbanini[a,b,*], L.A. Gizzi[b], D. Hampai[a], D.W. Hertzog[l], M. Iacovacci[e,h], M. Karuza[d,k], J. Kaspar[l], P. Koester[b], L. Labate[b], S. Mastroianni[l], D. Moricciani[f], G. Pauletta[d,i], L. Santi[d,i], G. Venanzoni[a]

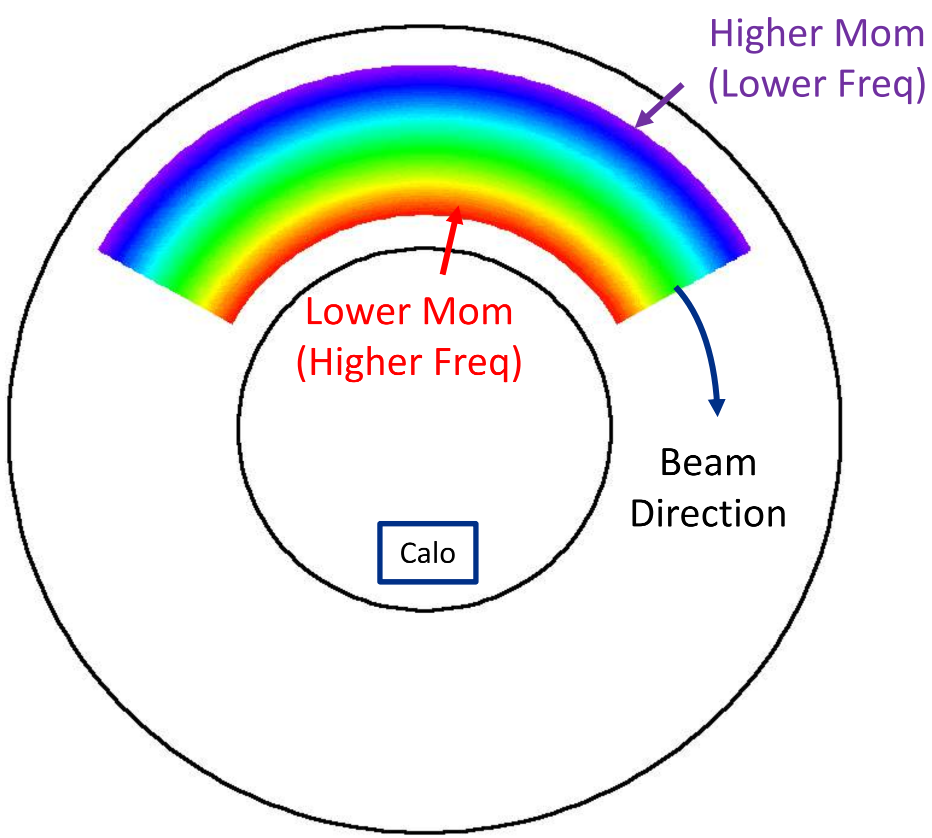


Determining the E-field correction

An Electric-field correction accounts for those muons **not at the magic radius**

→ This is achieved via a 'Fast Rotation' analysis of the stored beam de-bunching.

→ Over time, lower momentum will catch up with higher momentum…

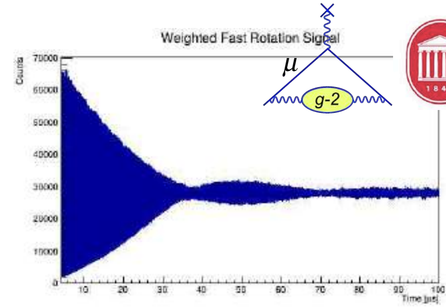


The way that the gaps between bunches are filled is related to the momentum distribution of the stored beam.

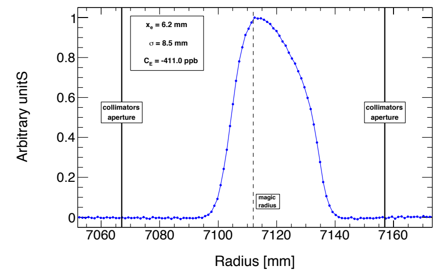
The fast rotation analysis (FRA)

What's so important about the fast rotation?

Fast rotation analysis



Radial Distribution



Muon equilibrium radius $\longleftrightarrow$ x_e

Electric (E) field correction $\longleftrightarrow$ $C_E = -2\langle n\rangle(1-\langle n\rangle)\beta^2 \dfrac{\langle x_e^2\rangle}{R_0^2}$

There are two approaches to the FRA:

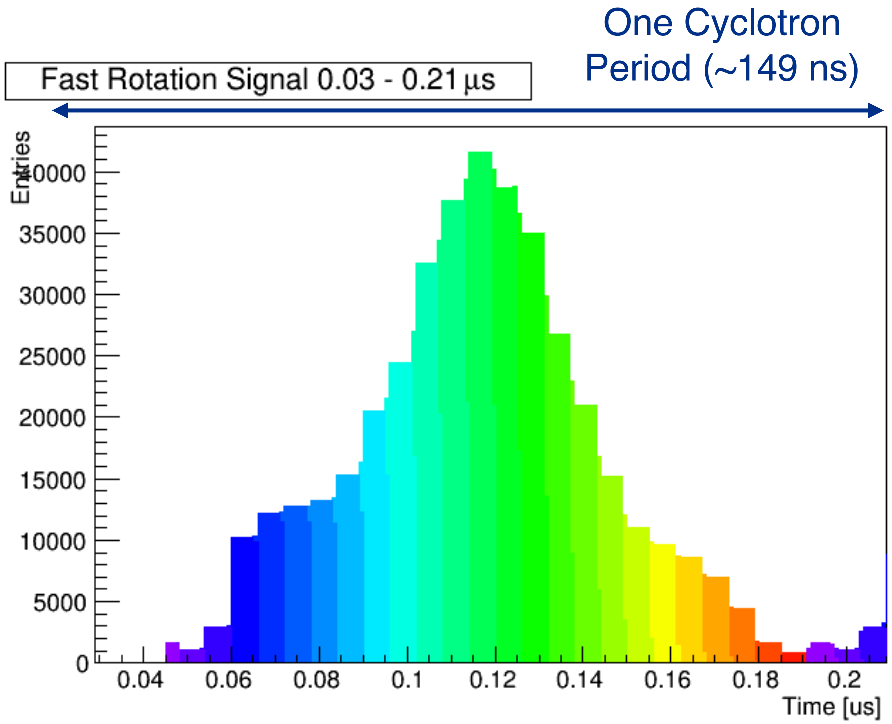
1. The Fourier transform method
2. The CERN III method

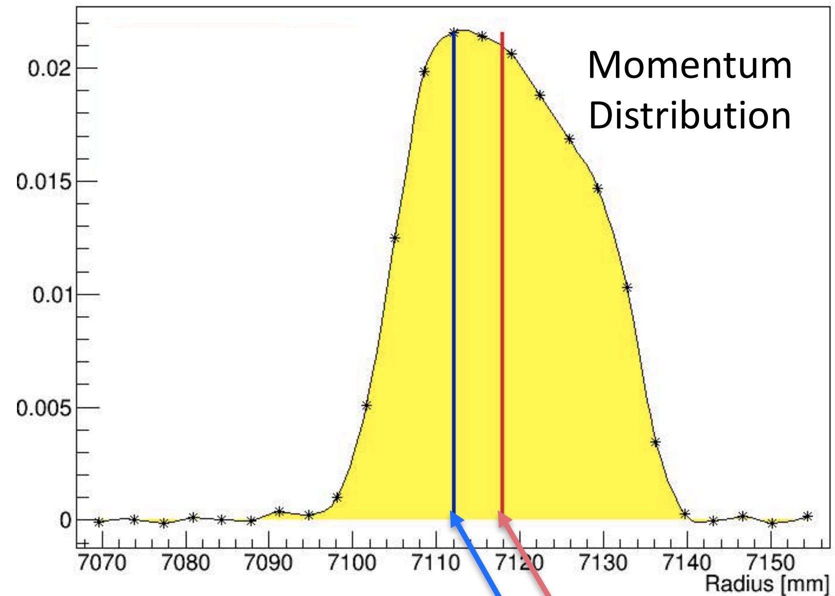
Should be in agreement to achieve E989 E and pitch correction goal of < 30ppb

Determining the E-field correction

The E-field correction accounts for those muons **not at the magic radius**

Use either an iterative χ^2 minimization or Fourier analysis to determine stored beam's time profile and momentum distribution

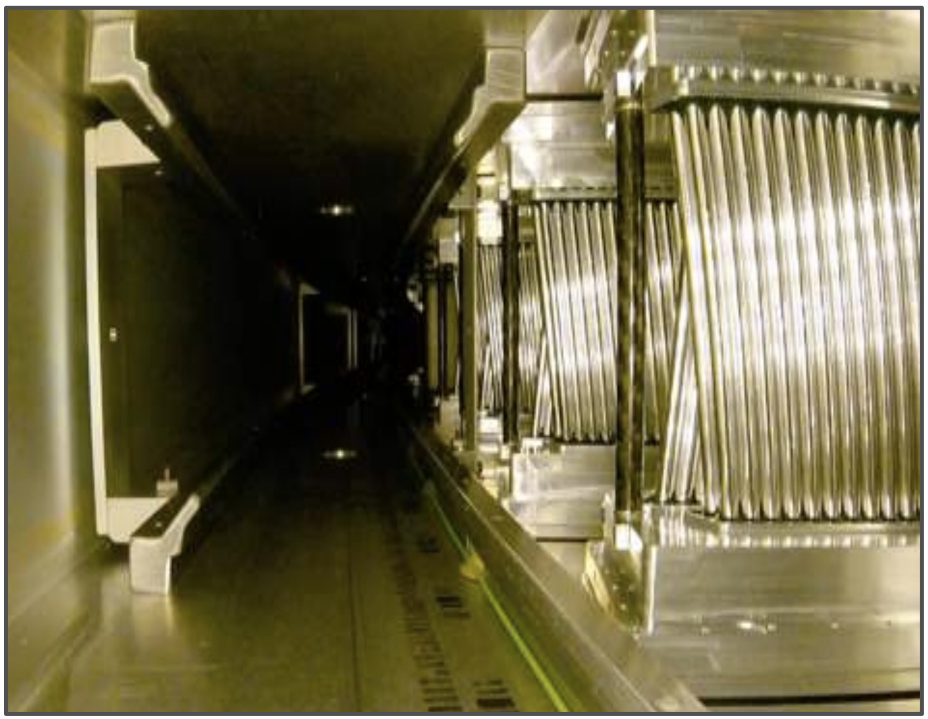


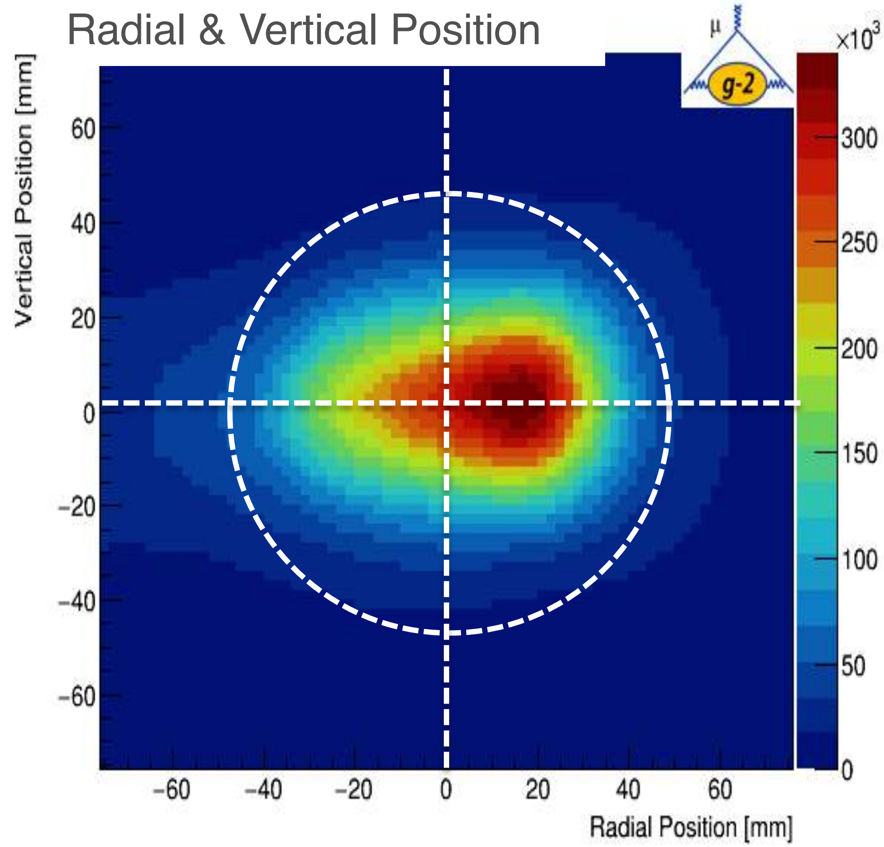


E-field correction: $$C_E = -2\langle n\rangle(1-\langle n\rangle)\beta^2 \frac{\langle x_e^2\rangle}{R_0^2}$$

Trackers mapping the muon beam motion

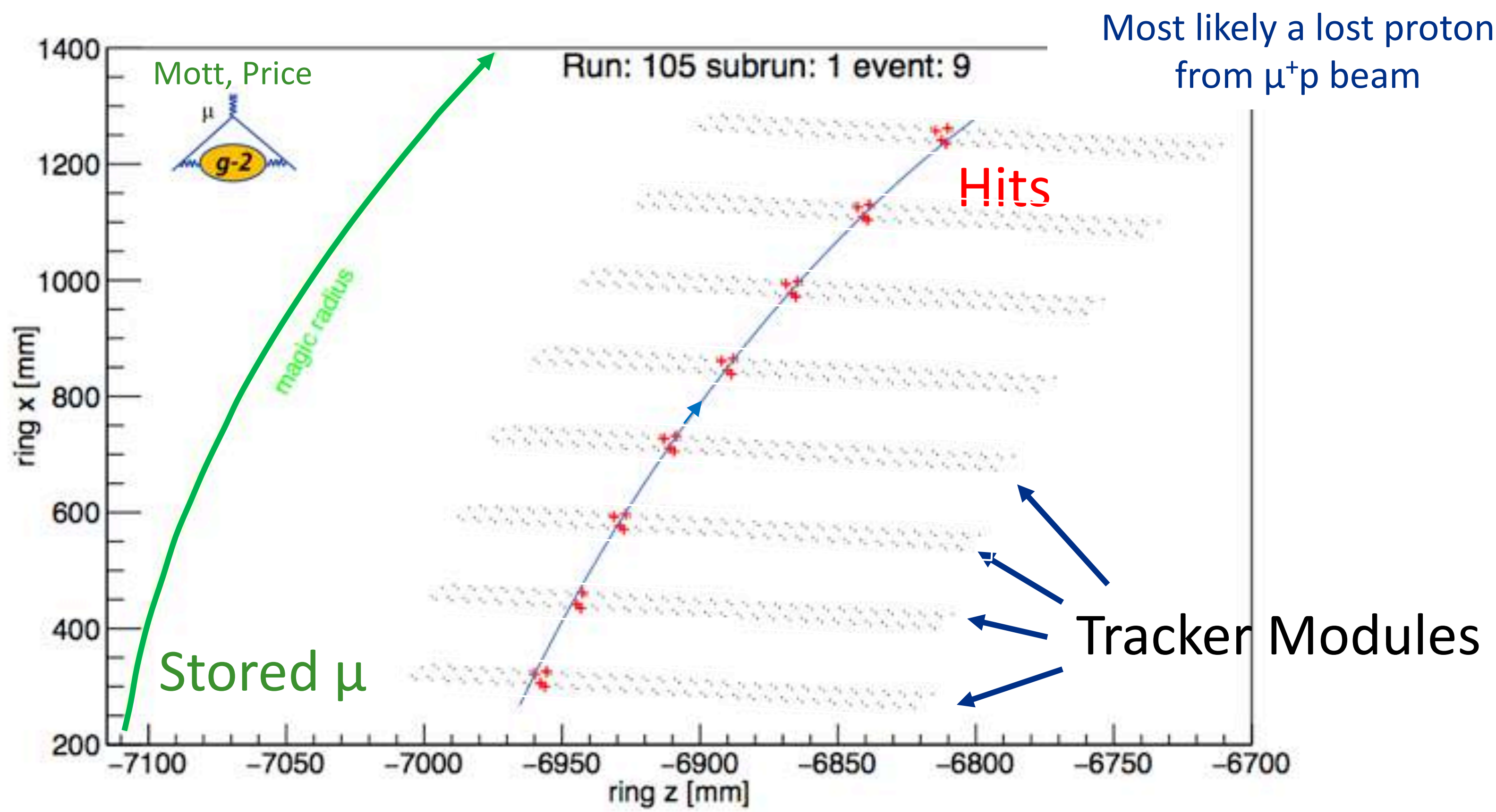
Cannot have detectors directly in the beam but instead we measure trajectory of decay e^+ and do an extrapolation back…





What does a track look like?

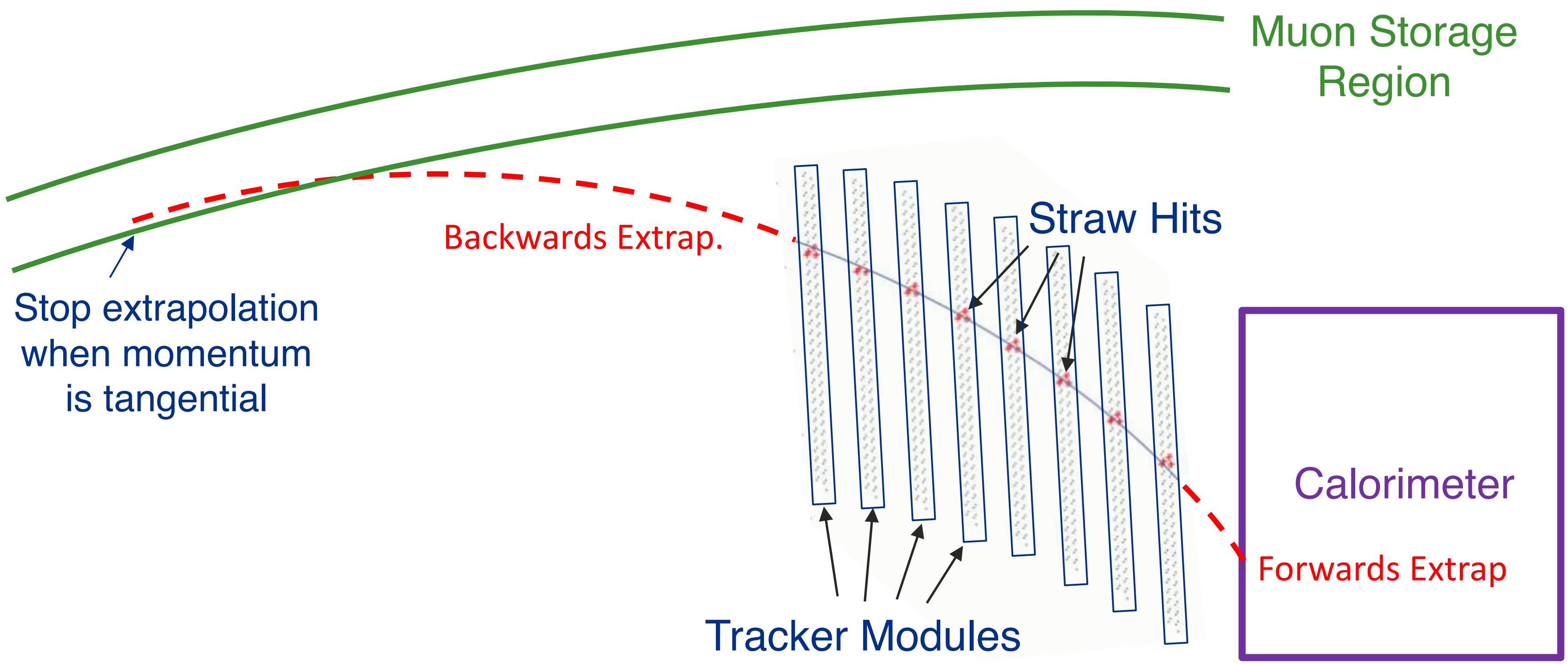
- First track seen at start of engineering run (June 2017)



- Track-fitting algorithm is a global χ^2 minimisation using Geant4 for particle propagation

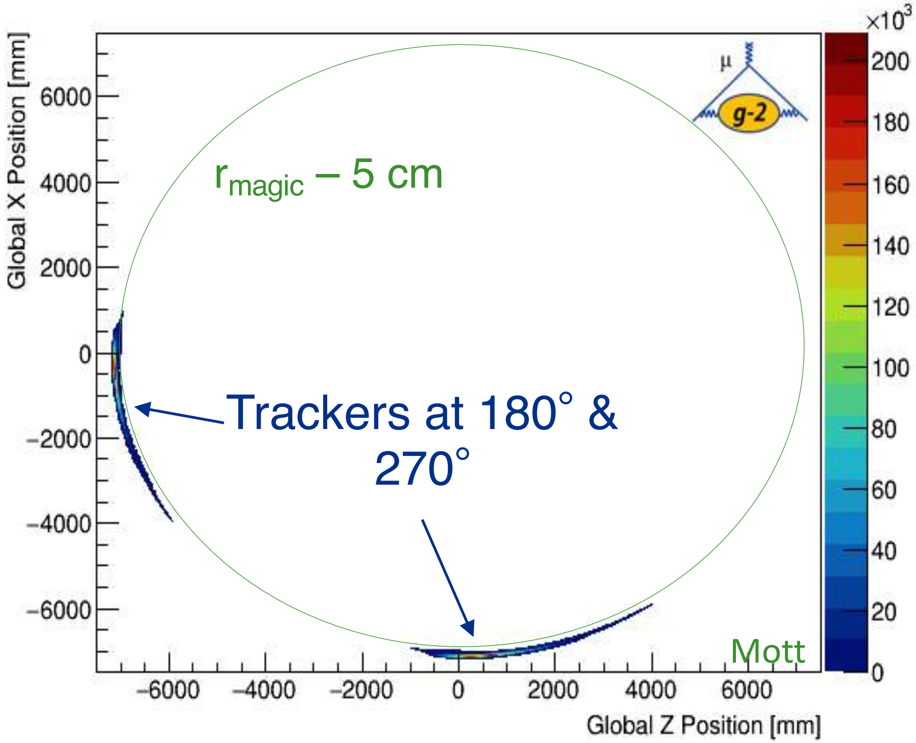
Track extrapolation

- We extrapolate tracks backwards to decay point and forwards to calorimeter:

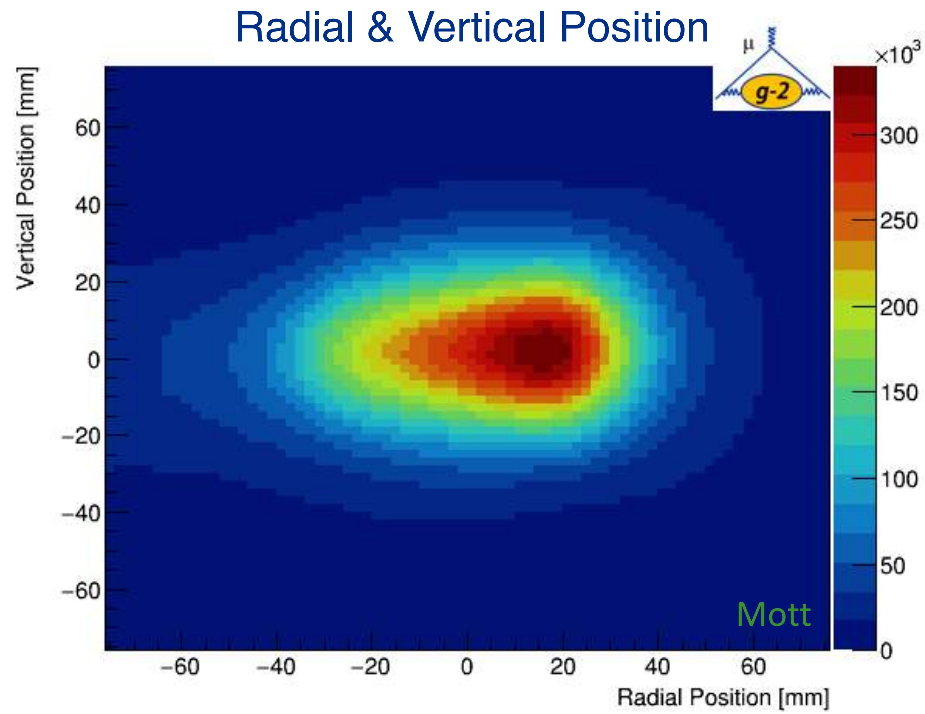


Beam distribution

- Extrapolate tracks to where they are tangential to magic radius:



Top-down view of decay vertices

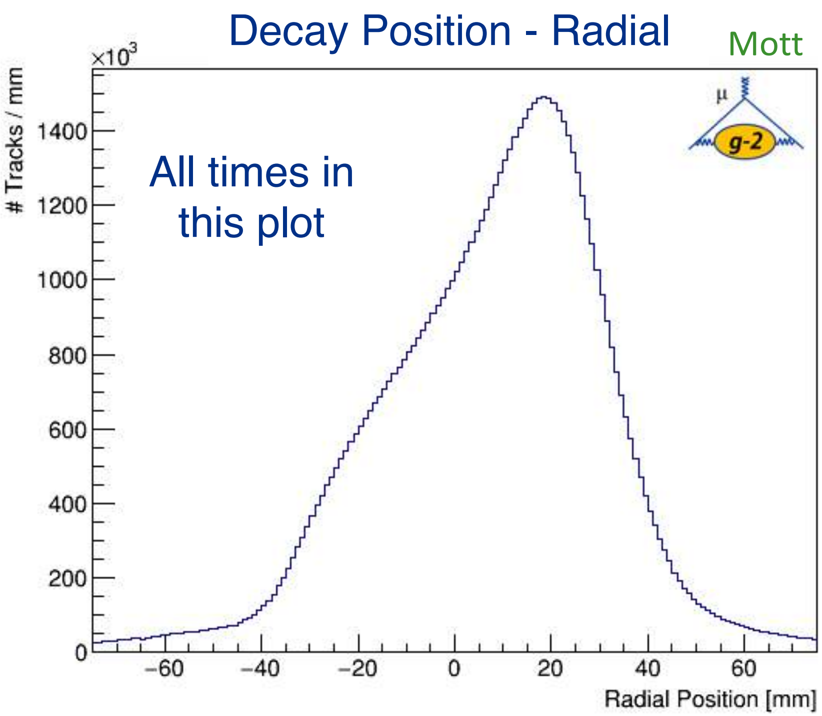


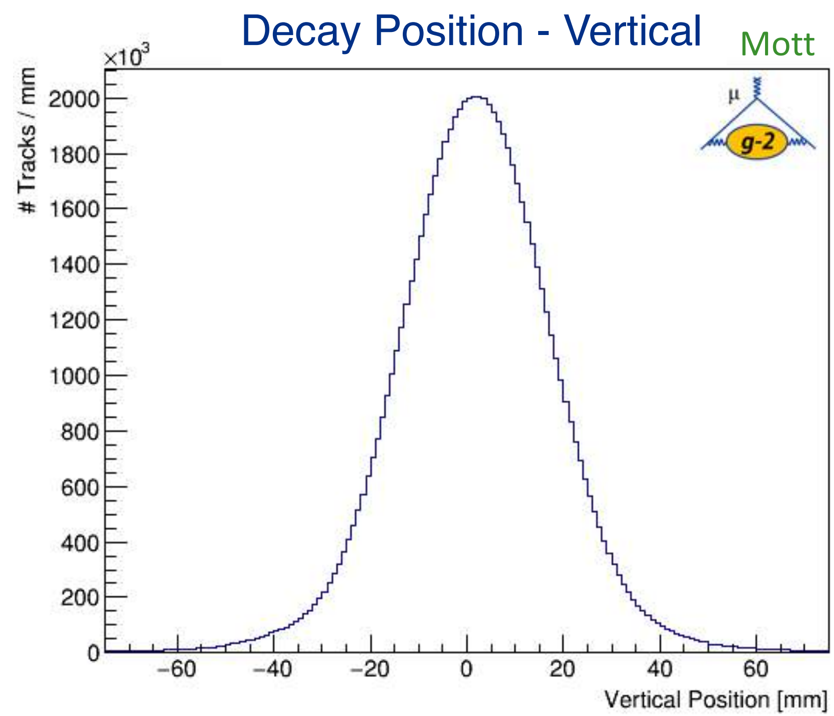
Projection of beam onto radial slice

- Use these distributions to get the effective field seen by the muons $B \circledast M_\mu$

Beam distribution

- Projections of 2D beam spot from previous slide onto radial and vertical directions:

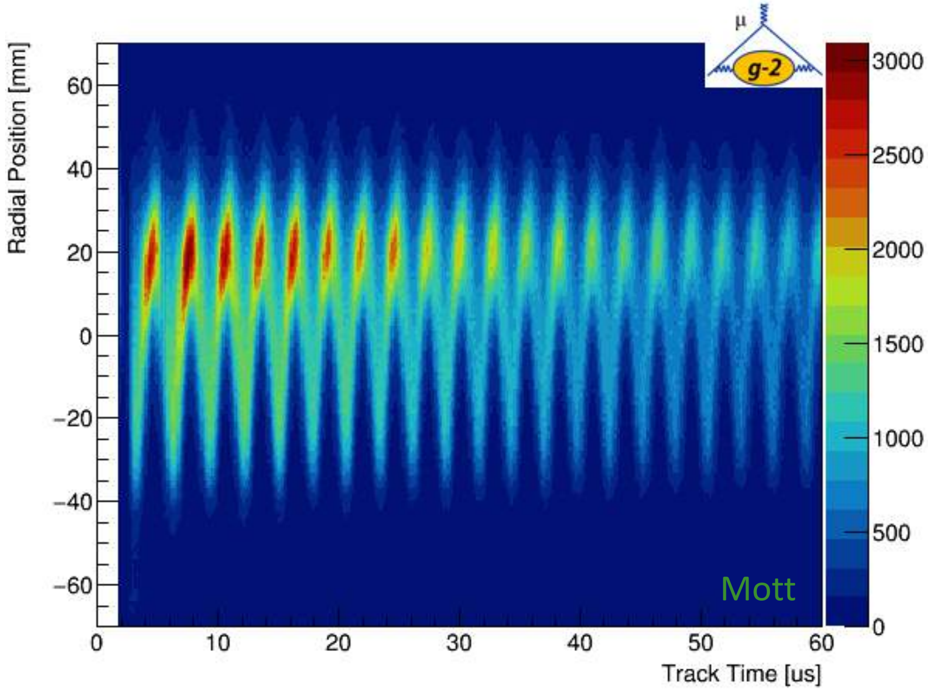




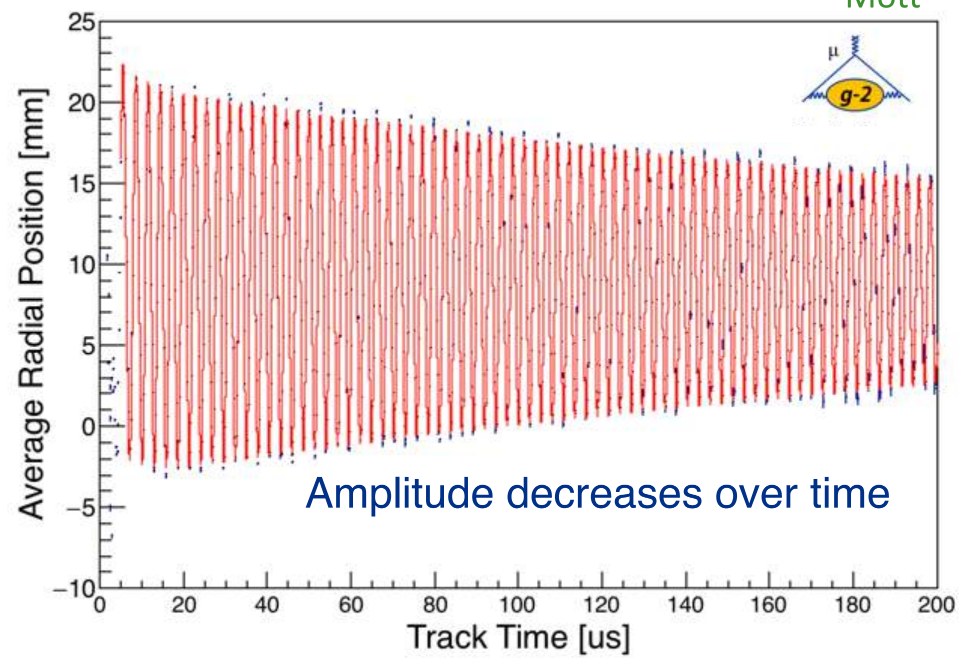
- Distributions are wider because the beam is oscillating
- We can also look at them in individual time slices…

Beam radial oscillation: amplitude

- Amplitude of radial oscillation decreases as beam spreads out:



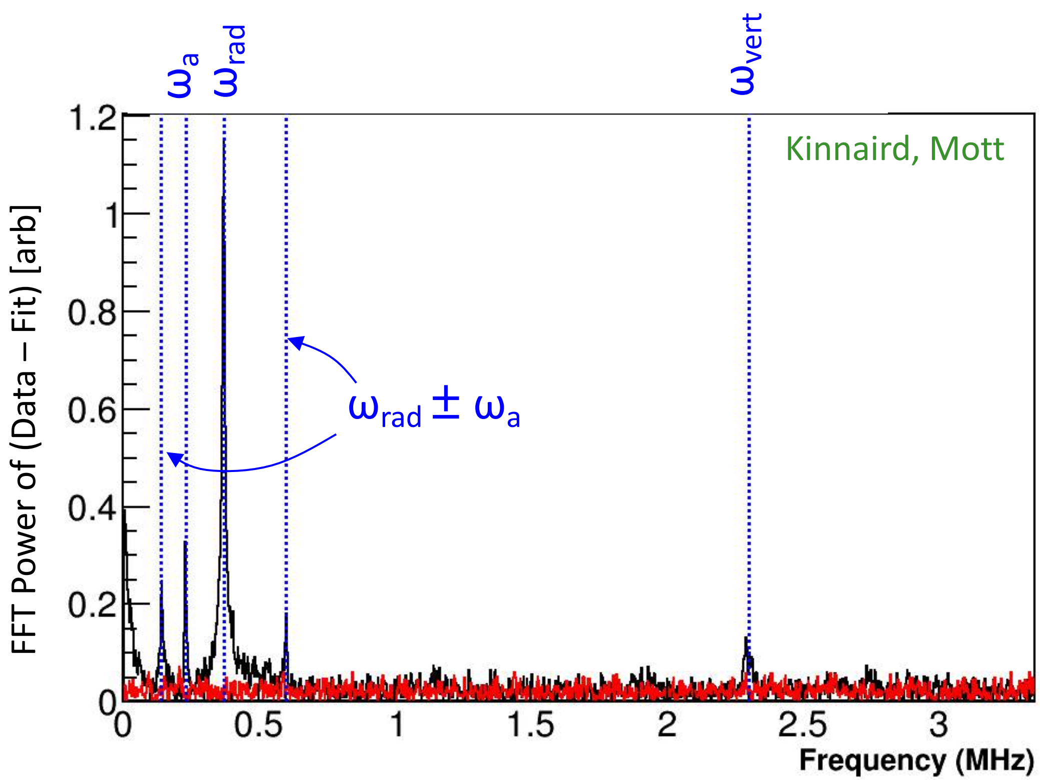
Mean of radial distribution

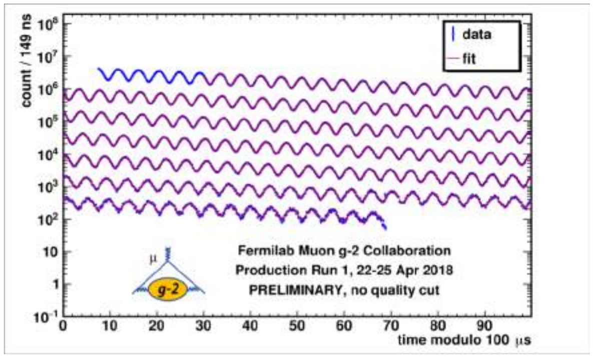


- Tracker measurements essential for calorimeter ω_a analysis:
  - Amplitude shape and lifetime
  - Oscillation frequency change

Beam radial oscillations and ω_a

- Beam oscillations couple to acceptance – change number of e^+ detected with time
- Oscillation frequencies in fit residuals which are removed by modifying fit function:





$$N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi))$$

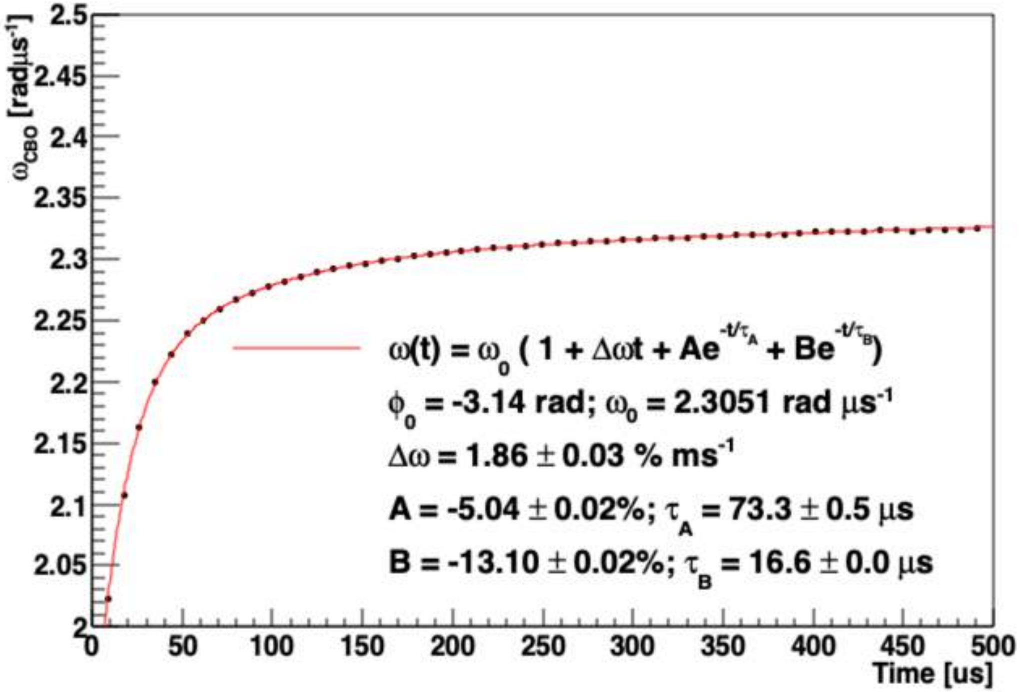
χ^2 / ndf = 7006 / 3148

Fit* including beam oscillations

χ^2 / ndf = 3052 / 3145

Beam radial oscillations: frequency

- We expected the oscillation frequency to be constant but I found that it was changing over time:

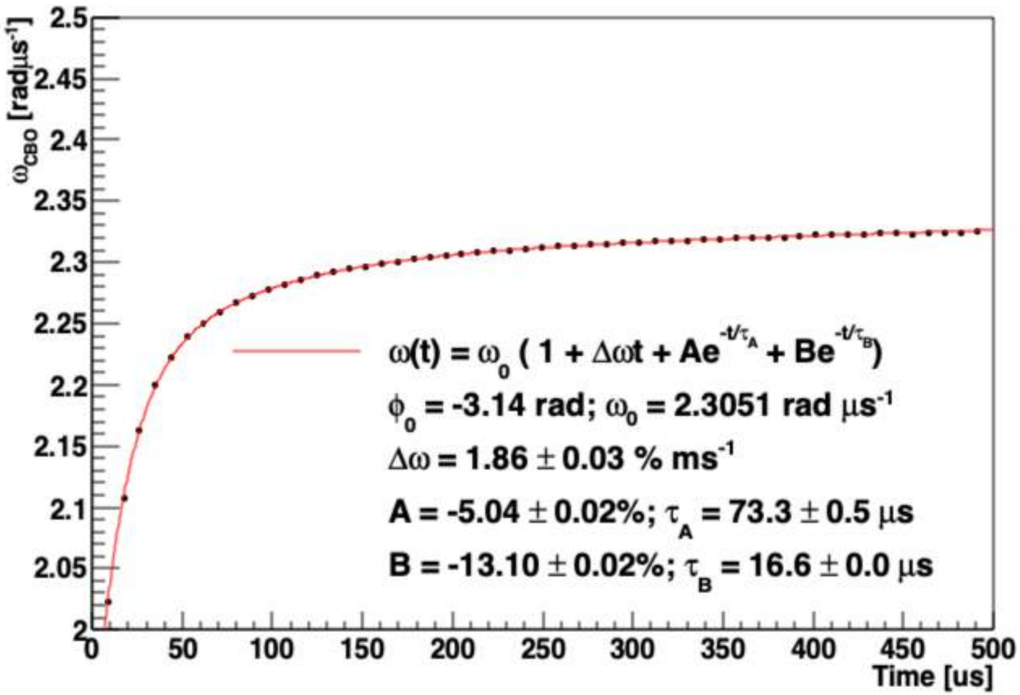


- Helped us to eventually locate the problem as faulty resistors in the electrostatic quadrupole system.

*Ratio fit

Beam radial oscillations: frequency

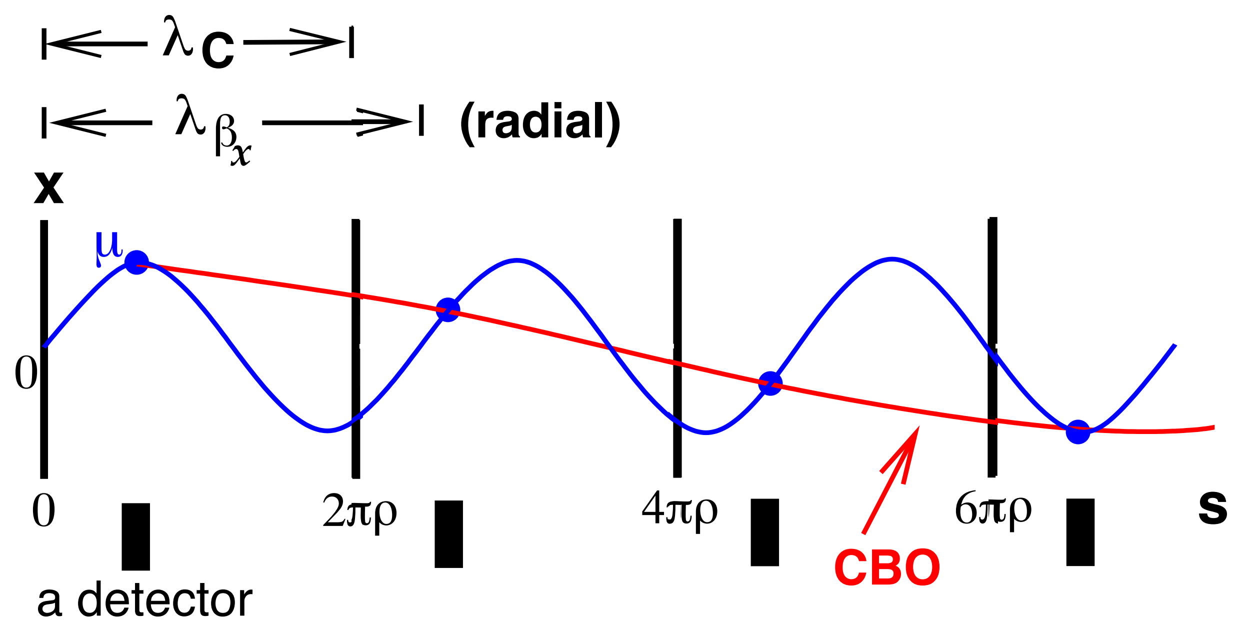
- We expected the oscillation frequency to be constant but I found that it was changing over time:



- Helped us to eventually locate the problem as faulty resistors in the electrostatic quadrupole system.

*Ratio fit

Coherent betatron oscillations



$$f_{\mathrm{CBO}} = f_C - f_x = (1 - \sqrt{1-n}) f_c$$

$$\lambda_{\mathrm{CBO}} \simeq \ 14 \text{ turns}$$

- Detector acceptance depends on the radial coordinate x.
- The CBO amplitude modulates the signal in the detectors

Weak focusing betatron

$$\text{Field index}: n = \frac{R_0}{\beta B_0}\frac{dE_r}{dr} \simeq 0.135$$

$$\text{radial}: f_x = f_C\sqrt{1-n} \simeq 0.929 f_c$$

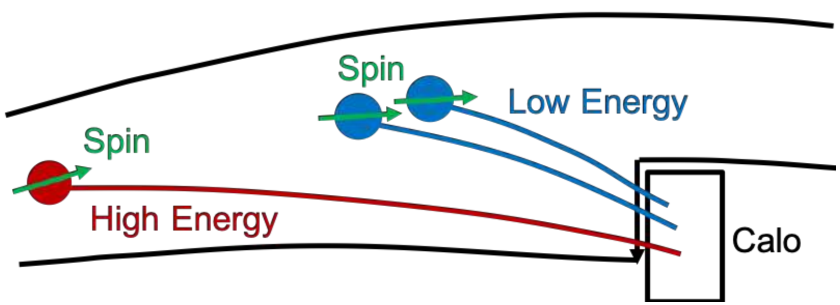
$$\text{vertical}: f_y = f_C\sqrt{n} \simeq 0.37 f_C$$

- The beam moves coherently radially relative to a detector with the "Coherent Betatron Frequency (CBO)

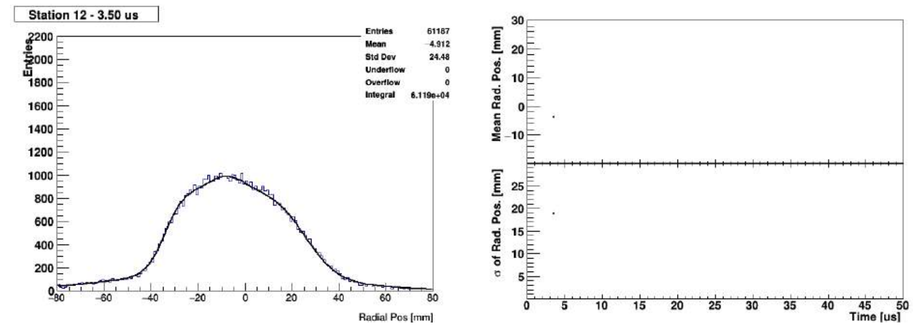
$$f_{\text{CBO}} = f_C - f_x = (1-\sqrt{1-n})f_C$$

Main systematic issues

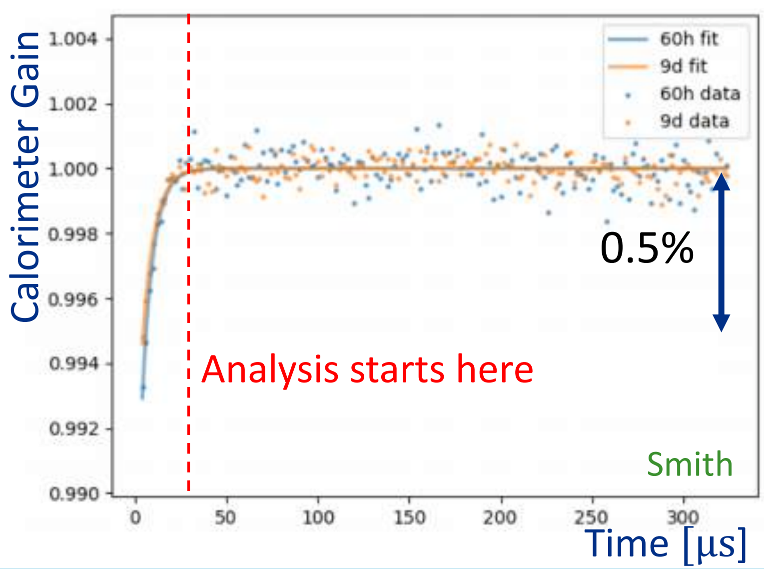
Pile Up



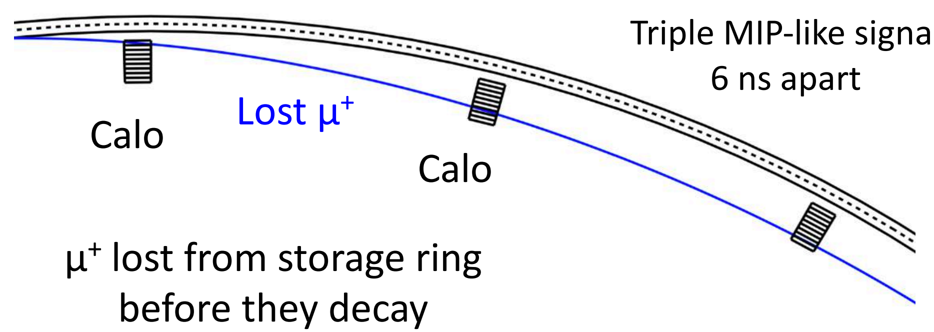
Beam Oscillations



Gain Change

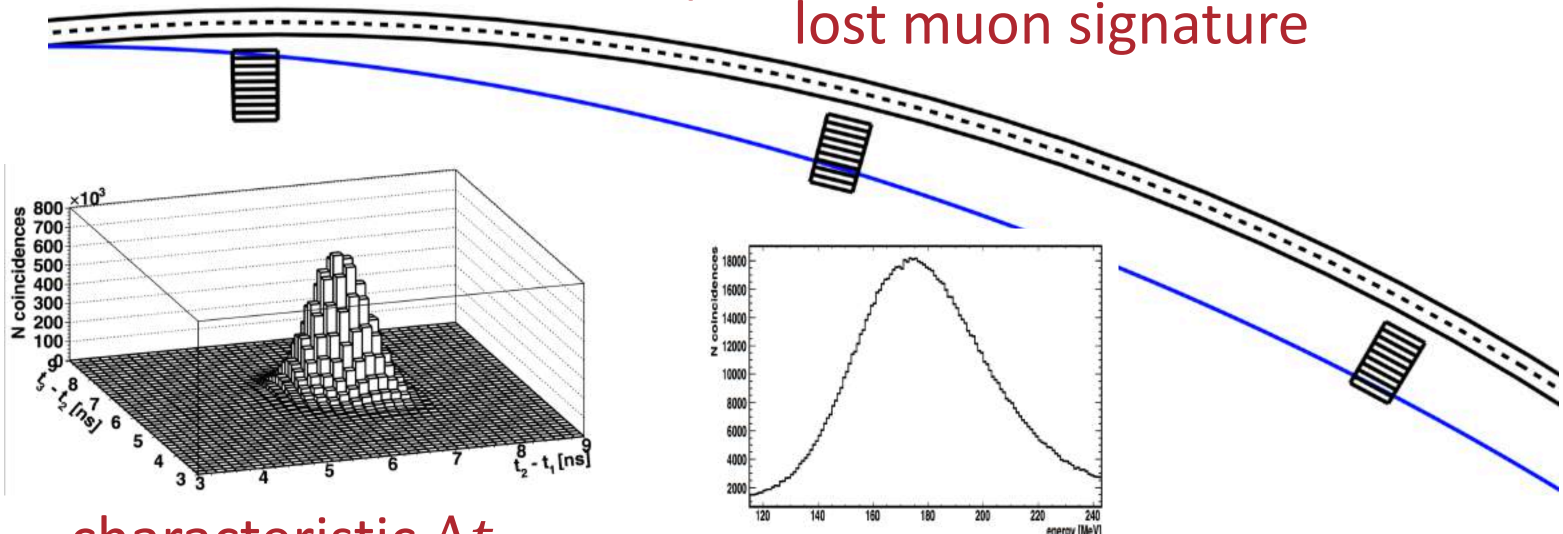


Lost Muons

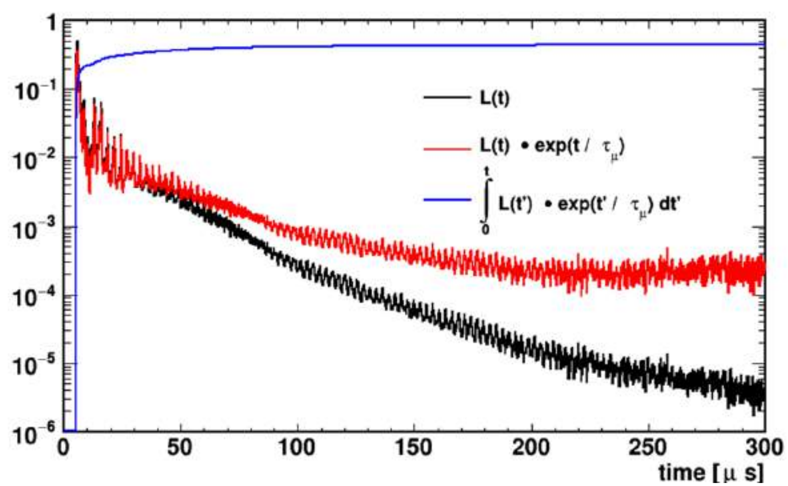


Lost muons

triple coincidence
lost muon signature



characteristic Δt

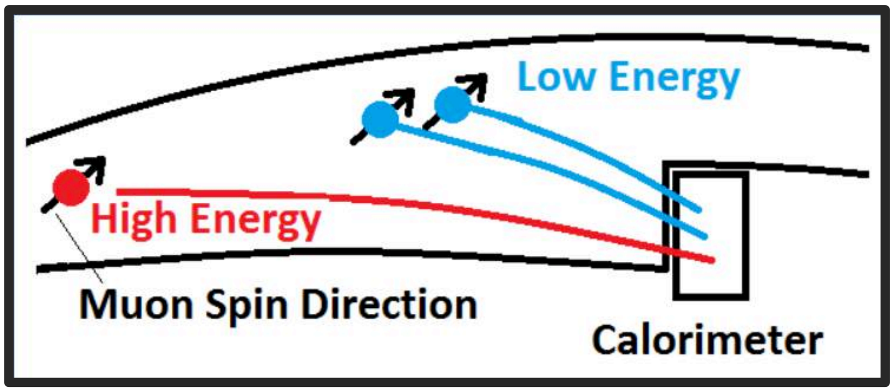


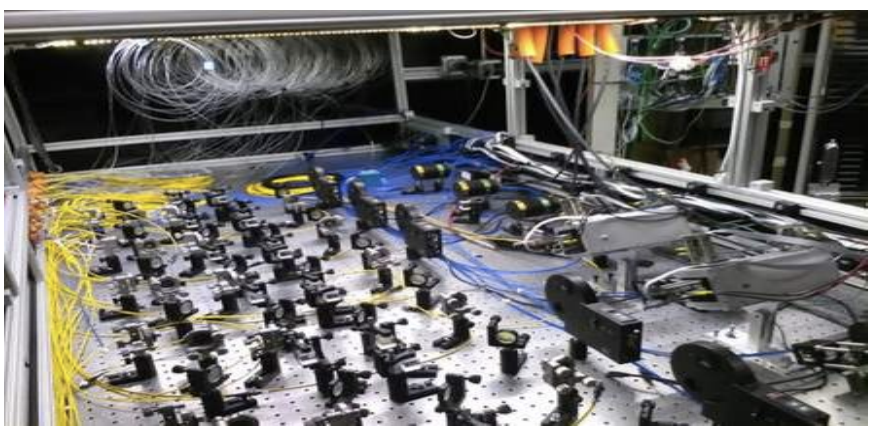
MIP-like energy deposition

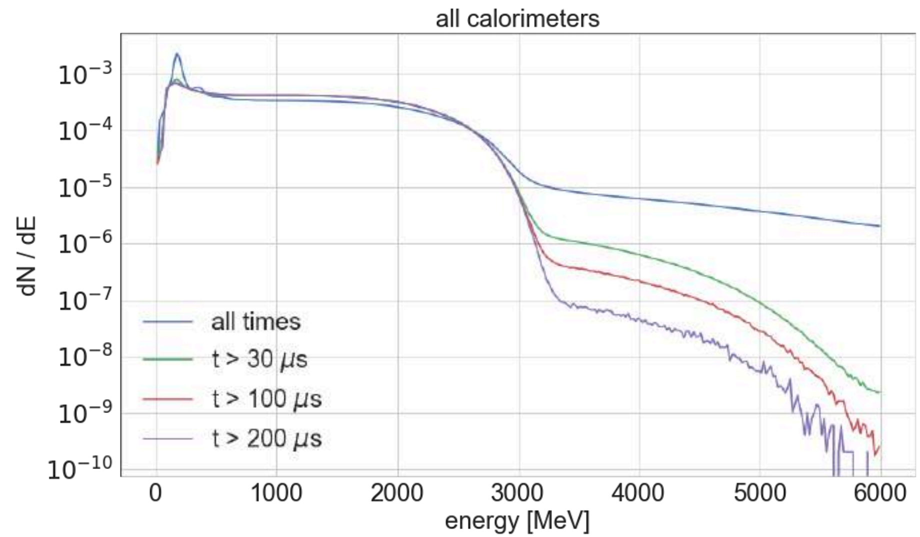
Describes the rate of muons escaping the ring; not decaying

Pileup and energy calibration

Direction of muon spin depends on energy of e^+
- need to track variations in energy calibration (laser system)
- correct for when two low energy e^+ fake one high energy (pileup)

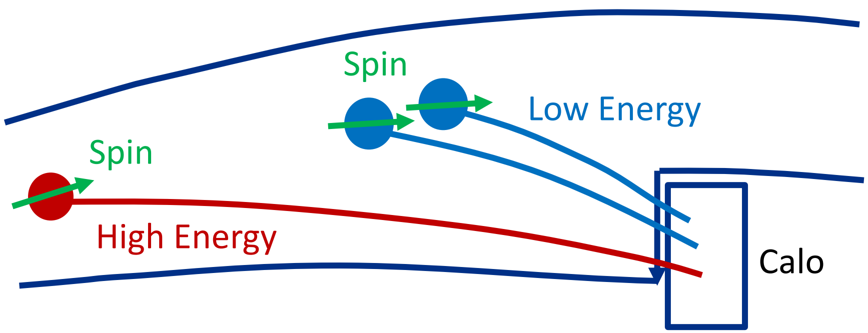




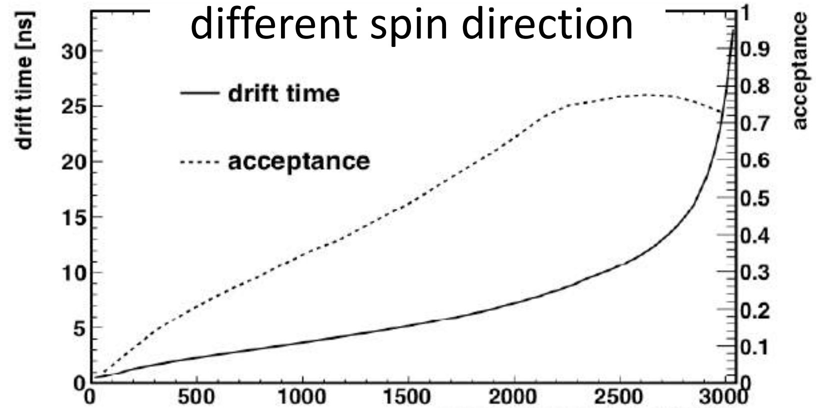


Pileup

Two low E e^+ can look like one high energy e^+



Different travel time means different spin direction



- Pile up happens less often as the muons decay so phase changes with time and we get ω_a wrong

- Derive a pile up correction from data and check validity above 3.1 GeV