The muon g-2: a new data-analysis

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Physics & Theory Seminar at KEK

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Based on
A. Keshavarzi, DN and T. Teubner **(KNT)**
arXiv:1802.02995 (Phys. Rev. D97 (2018) 114025) **(KNT18)**
arXiv:1911.00367 (submitted to Phys. Rev. D) **(KNT19)**

Muon g-2 in KEK Annual Report 2018

The **KNT18** paper is chosen as a Research Highlight in the KEK Annual Report 2018:

4.5

Muon g – 2 enigma: an omen of new physics?

The anomalous magnetic moment a_μ of the muon, also known as the muon g – 2, is one of the most precisely measured quantities in particle physics. In Ref. [1], we calculated the Standard Model (SM) prediction for the muon g – 2. Our main conclusion is that there is a 3.7 σ discrepancy between the experimental value and SM prediction of the muon g – 2 (see Fig. 1). This discrepancy could be a hint of new physics beyond the SM, and hence has drawn the intense attention of many physicists. In view of its importance, Ref. [1] has been selected as an "Editors' Suggestion" for Physical Review D. Ref.[1] is also featured in "Physics", which is an online magazine from the American Physical Society to report highlights of papers from the "Physical Review" journals.

Table 1 presents a breakdown of the SM prediction for the muon g – 2 together with the current experimental value. The SM prediction can conveniently be separated into three pieces: the QED, electroweak (EW), and hadronic contributions.

The QED and EW contributions are perturbatively calculable, and both are known with sufficient precision.

The most problematic contributions are the hadronic contributions. The leading-order (LO) hadronic vacuum polarization (VP) contribution and the hadronic light-by-light (LbL) contribution are the main sources of the uncertainty of the total SM prediction. To obtain a precise SM prediction, it is crucial to evaluate these contributions with sufficient precision.

The VP contributions can be computed more reliably with the help of dispersion relations, using experimental data of $e^+e^- \to$ hadrons as input. These data have been published in papers, and can be found, e.g., in the Inspire high-energy physics literature database (https://inspirehep.net). The LO hadronic VP contribution $a_\mu^{\text{had, LO VP}}$ can be written as an integral over the total cross-section $\sigma_{\text{had}}(s)$ of the reaction $e^+e^- \to$ hadrons at the center-of-mass energy $\sqrt{s}$:

$$a_\mu^{\text{had, LO VP}} = \frac{m_\mu^2}{12\pi^3} \int_{m_\pi^2}^{\infty} \frac{ds}{s} \hat{K}(s) \sigma_{\text{had}}(s) \, ,$$

where $\hat{K}(s)$ is a monotonically increasing function with $\hat{K}(m_\pi^2) = 0.40$ and $\hat{K}(s) \to 1$ for $s \to \infty$. Because the weight factor in the integrand emphasizes the lowest energy region, good input data at low energies are vital to obtain an accurat cise prediction for $a_\mu^{\text{had, LO VP}}$. In particular approximately 73% of the integral is fro

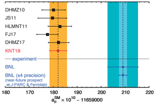


Fig. 1. SM prediction for the muon g – 2 from Ref. [1], shown by the error bar labeled as "KNT18", compared with the experimental value, shown by the wider blue band labeled "BNL", and a near-future prospect of the experimental value (the narrower blue band) expected from experiments at J-PARC and Fermilab. Currently there is a 3.7 σ discrepancy between the experimental value and SM prediction. Other recent SM predictions are also shown. See Ref. [1] for further details.

Table 1. Breakdown of the SM prediction for the muon g – 2 compared with the experimental value. The numbers are taken from Ref. [1], and are given in units of 10^{-10}. "VP" and "LbL" in the table stand for "vacuum polarization" and "light-by-light", respectively. The abbreviations PRD, PRL, and PLB in the last column represent the journals Phys. Rev. D, Phys. Rev. Lett., and Phys. Lett. B, respectively.

QED contributions	11 658 471.8971 (0.007)	Aoyama et al., PRD 97, 030001 (2018)
EW contributions	15.36 (0.10)	Gnendiger et al., PRD 88, 053005 (2013)
hadronic contributions		
LO hadronic VP contributions	693.27 (2.46)	Ref. [1]
NLO hadronic VP contributions	–9.82 (0.04)	Ref. [1]
NNLO hadronic VP contributions	1.24 (0.01)	Kurz et al., PLB 734, 144 (2014)
hadronic LbL contributions	9.8 (2.6)	Nyffeler, PRD 94, 053006 (2016)
hadronic LbL NLO contributions	0.3 (0.2)	Colangelo et al., PLB 735, 90 (2014)
Standard Model prediction, a_μ(SM)	11 659 182.05 (3.56)	Ref. [1]
experimental value, a_μ(exp)	11 659 209.1 (6.3)	Bennett et al., PRL 92, 161802 (2004)
difference, Δa_μ ($\equiv a_\mu$(exp) – a_μ(SM))	27.05 (7.26)	Ref. [1]

state $\pi^+\pi^-$, the $e^+e^- \to \pi^+\pi^-$ data are by far the most important.

To evaluate the integral, we employ data of exclusive measurements over all the hadronic final states for $\sqrt{s} \leq 2$ GeV, and of inclusive measurements at $2 \leq \sqrt{s} \leq 11$ GeV. Above ~11 GeV we employ perturbative QCD.

It is far from easy to evaluate the integral. First, we have to take lots of data into account. In the case of Ref. [1], we have summed over approximately 30 exclusive modes and newly added data from approximately 30 papers to our analysis compared to our previous study. We now consider data from over 100 papers in total.

A further complexity of the analysis lies in the combination of the experimental data. Because there are many data points for each hadronic final state, we have to take a 'weighted average' of these. To do so, we construct and minimize a χ^2 function. This may appear straightforward, but is not so simple. If one naively carries this out when the normalization errors of the data are non-negligible (as in our case), it is known that one ends up with a biased result. To avoid potential biases, we newly adopt an iterative method, originally utilized (by other researchers) for different purposes, such as the deations of uncertainties associated with radiative corrections to the data.

To evaluate the hadronic LbL contributions, to some extent we must rely on models of hadronic interactions, which always introduce some degree of model dependence.

Our conclusion is that there is a 3.7 σ discrepancy between the experimental observation and SM prediction. Some people have attempted to explain the discrepancy in terms of new physics such as axion-like light particles. Others have attempted to cross-check our result using lattice QCD.

There are two ongoing experiments to measure the muon g – 2 : one at J-PARC and the other at Fermilab. These are aiming to reduce the experimental error by a factor of four. Once achieved, the new result may provide further insights into our microscopic world through the muon sector.

Reference

[1] A. Keshavarzi, D. Nomura and T. Teubner, Phys. Rev. D97, 114025 (2018).

From KEK Annual Report 2018 (to appear very soon)

Thank you for your support!

Muon g-2: introduction

Lepton magnetic moment $\vec{\mu}$: $\mathcal{H} = -\vec{\mu} \cdot \vec{B}$

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad \left(\vec{s} = \frac{1}{2}\vec{\sigma}\ \text{(spin)}, \qquad g = 2 + 2F_2(0)\right)$$

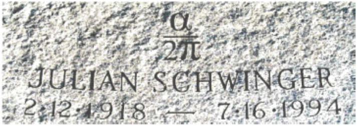
where

$$\overline{u}(p+q)\Gamma^{\mu}u(p) = \overline{u}(p+q)\left(\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

★ One of the **most precisely measured** quantities:

$$\boxed{a_{\mu}^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}}\ [0.5\text{ppm}]$$

(Bennett *et al*)

★ **Extremely useful** in **probing/constraining physics beyond the SM**

Why Muon g-2?

- $\gtrsim$ 3.5 σ Anomaly Observed
  Long standing anomaly ($\sim$ 20 yrs), in spite of careful studies on every aspect.
  (→ Major theoretical blunder unlikely.)
  Hint of New Physics beyond the Standard Model?
- No new physics at the LHC so far
  Intensity frontier: more and more important
- Long history of research
  1st $(g-2)_\mu$ exp.: Garwin, Lederman & Weinrich (1957)
  Well-established place to search for new physics
- Leptonic observable
  Experimentally and theoretically clean

Muon g-2: previous exp. (after 1960)

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]	Sensitivity
CERN I	1961	μ^+	11 450 000(220 000)	4300	2-loop QED contrib. (3600 ppm)
CERN II	1962-1968	μ^+	11 661 600(3100)	270	3-loop QED contrib. (260 ppm)
CERN III	1974-1976	μ^+	11 659 100(110)	10	hadronic vacuum polarization contrib. (60 ppm)
CERN III	1975-1976	μ^-	11 659 360(120)	10	
BNL	1997	μ^+	11 659 251(150)	13	
BNL	1998	μ^+	11 659 191(59)	5	4-loop QED contrib. (3.3 ppm)
BNL	1999	μ^+	11 659 202(15)	1.3	electroweak contrib. (1.3 ppm)
BNL	2000	μ^+	11 659 204(9)	0.73	hadronic light-by-light contrib. (0.86 ppm)
BNL	2001	μ^-	11 659 214(9)	0.72	hadronic NLO vacuum pol. contrib. (-0.85 ppm)
Average			11 659 208.0(6.3)	0.54	

Table from BNL-E821 final report, Phys. Rev. D 73 (2006) 072003

History of muon g-2 exp. is a history of SM tests.
This is not the whole story: the history still goes on.

Muon g-2 vs New Physics

Basically, any new particle which couples to the muon gives a non-zero contribution to the muon g-2:

- SUSY particles ($\widetilde{\mu}, \widetilde{W}^{\pm}, \widetilde{Z}^0, \widetilde{B}^0, \ldots$)
- extra Higgses ($H^{\pm}, A^0, H^{\pm\pm}, \ldots$)
- Kaluza-Klein excitations of μ and γ
- extra Z-like particle (Z', “dark Z”, …)
- extra γ-/axion- like light particle (“dark photon”, ...)
- leptoquarks
- $\vdots$

In many cases, the mass and couplings of these new particles are free parameters. By tuning them, one can explain the muon g-2 anomaly. But it is often non-trivial to explain why Nature chooses such a parameter set.

Allowed region in the $(\tilde{g}, g')$ plane:

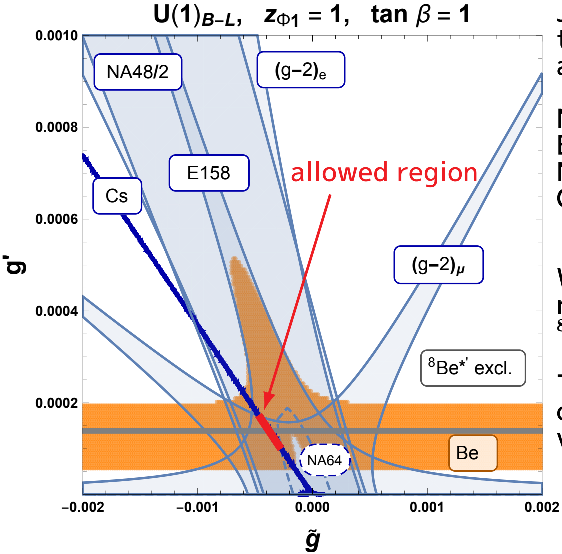


Fig. from L. Delle Rose et al, arXiv:1812.05497

Just an example when the $U(1)'$ charges = B-L and $\tan\beta = 1$

NA48/2: $\pi^0 \to Z'\gamma$ searches
E158: Møller scattering
NA64: e^- beam dump exp.
Cs: atomic parity violation in Cs atom

White region is excluded by non-observation of Z' in $^8\text{Be}^{*\prime} \to {}^8\text{Be}$ transition

The strongest constraint comes from atomic parity violation in Cs.

Viable parameter region still exists.

Breakdown of SM prediction for muon g-2

	2011		2018		2019
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]	
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]	
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) (	9.34 (2.92)
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]	
	HLMNT11		**KNT18**		**KNT19**
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work	692.78 (2.42)
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work	-9.83 (0.04)
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]	
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work···	181.08 (3.78)
Experiment			11659209.10 (6.33)	world avg	
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work	28.0 (7.4)
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work	3.8σ

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

(Numbers taken from KNT18 and from KNT19)

QED contribution

QED contribution:

$$a_\mu(\text{QED}) = \frac{\alpha}{2\pi} + 0.765857425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.05050996(32)\left(\frac{\alpha}{\pi}\right)^3 + 130.8796(63)\left(\frac{\alpha}{\pi}\right)^4 + 753.3(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

$$= 11658471.895(0.008) \times 10^{-10}\,,$$ (numbers from PDG 2018)

where the uncertainty is dominated by that of α.

- 5-loop calculation! (Aoyama, Hayakawa, Kinoshita & Nio)
- The 4-loop corrections $\simeq 38 \times 10^{-10} \simeq \mathcal{O}(a_\mu(\text{exp}) - a_\mu(\text{SM}))$.
- The 4-loop contribution now fully cross-checked by another group. Mass-independent part by S. Laporta (Phys.Lett. **B772** (2017) 232), and mass-dependent part by A. Kurz et al (Nucl. Phys. **B879** (2014) 1; Phys. Rev. **D92** (2015) 073019; ibid. **D93** (2016) 053017)
- The 5-loop contribution very small ($\simeq 0.5 \times 10^{-10} \ll a_\mu(\text{exp}) - a_\mu(\text{SM})$)

Electroweak Contribution

Electroweak (EW) contribution:

$$a_\mu(\text{EW}) = \underbrace{19.48 \times 10^{-10}}_{\text{1-loop}} + \underbrace{(-4.12(10) \times 10^{-10})}_{\text{2-loop}} + \underbrace{\mathcal{O}(10^{-12})}_{\text{leading log 3-loop}}$$

$$= 15.36(10) \times 10^{-10}\,, \qquad \text{(Number taken from PDG 2018)}$$

where the uncertainty mainly comes from quark loops.

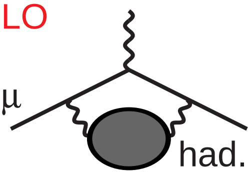
- 1-loop result published by many groups (Bardeen-Gastmans-Lautrup, Altarelli-Cabibbo-Maiani, Jackiw-Weinberg, Bars-Yoshimura, Fujikawa-Lee-Sanda) in 1972, and now a textbook exercise (Peskin & Schroeder's textbook, Problems 6.3 (Higgs) and 21.1 (W, Z))
- 2-loop contribution ($\sim$ 1700 diagrams in the 't Hooft-Feynman gauge) enhanced by $\ln(m_Z/m_\mu)$ and also by a factor of $\mathcal{O}(10)$,

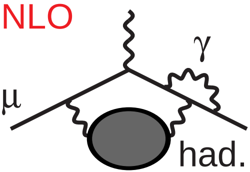
$$a_\mu(\text{EW, 2-loop}) \simeq -10 \left(\frac{\alpha}{\pi}\right) a_\mu(\text{EW, 1-loop}) \left(\ln \frac{m_Z}{m_\mu} + 1\right)\,,$$

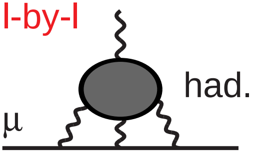
where the factor of 10 appears since many "order one" diagrams accidentally add up coherently.

Hadronic Contributions

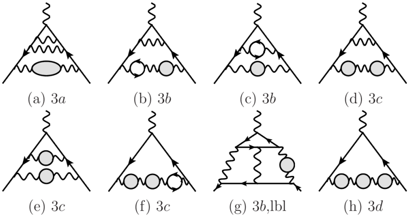
There are several hadronic contributions:



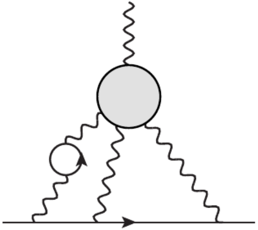




LO: Leading Order (or Vacuum Polarization) Hadronic Contribution
NLO: Next-to-Leading Order Hadronic Contribution
l-by-l: Hadronic light-by-light Contribution



NNLO Hadronic Contributions

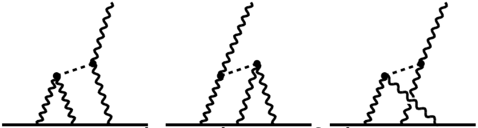


Hadronic l-by-l NLO Contrib.

Modern evaluation of l-by-l contribution

(Melnikov & Vainshtein)

1. First, use the large N_C expansion to find that the leading contribution is the pion pole contribution.

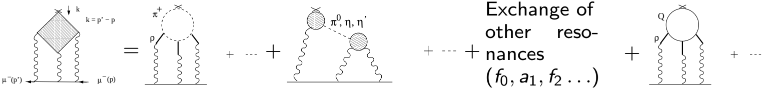


2. Choose the momentum-dependence of the $\pi\gamma\gamma$ coupling (form factor) in such a way that it is consistent with a constraint from QCD (OPE) at the momentum region $q_1^2 \sim q_2^2 \gg q_3^2$. Integrate over the loop momenta.
3. Repeat the above for $\eta, \eta', a_1, \ldots$. Basically that's all for the LO in $1/N_C$.
4. As for NLO in $1/N_C$, it depends on authors which diagram is numerically important.

For example,

$$a_\mu^{\mathrm{lbyl}} = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{'Glasgow consensus', arXiv:0901.0306} \\ (9.8 \pm 2.6) \times 10^{-10} & \text{'G.c.' w/ correction by Nyffeler, PRD94(2016)053006} \\ (10.2 \pm 3.9) \times 10^{-10} & \text{Nyffeler, arXiv:1710.09742} \end{cases}$$

HLbL in muon $g-2$: summary of selected results (model calculations)



de Rafael '94:					
Chiral counting:		p^4	p^6	p^8	p^8
N_C-counting:		1	N_C	N_C	N_C
Contribution to $a_\mu \times 10^{11}$:					
BPP:	+83 (32)	-19 (13)	+85 (13)	-4 (3) [f_0, a_1]	+21 (3)
HKS:	+90 (15)	-5 (8)	+83 (6)	+1.7 (1.7) [a_1]	+10 (11)
KN:	+80 (40)		+83 (12)		
MV:	+136 (25)	0 (10)	+114 (10)	+22 (5) [a_1]	0
2007:	+110 (40)				
PdRV:	+105 (26)	-19 (19)	+114 (13)	+8 (12) [f_0, a_1]	+2.3 [c-quark]
N,JN:	+116 (39)	-19 (13)	+99 (16)	+15 (7) [f_0, a_1]	+21 (3)
		ud.: -45	ud.: $+\infty$		ud.: +60

ud. = undressed, i.e. point vertices without form factors

Pseudoscalars: numerically dominant contribution (according to most models !).

Recall (in units of 10^{-11}): $\delta a_\mu(\text{HVP}) \approx 40$; $\delta a_\mu(\text{exp [BNL]}) = 63$; $\delta a_\mu(\text{future exp}) = 16$

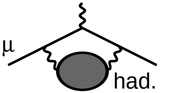
BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; "Glasgow consensus"); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

Recent reevaluations of axial vector contribution lead to much smaller estimates than in MV '04: $a_\mu^{\text{HLbL;axial}} = (8 \pm 3) \times 10^{-11}$ (Pauk, Vanderhaeghen '14; Jegerlehner '14, '15). Would shift central values of compilations downwards:

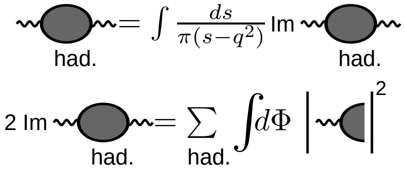
$a_\mu^{\text{HLbL}} = (98 \pm 26) \times 10^{-11}$ (PdRV) and $a_\mu^{\text{HLbL}} = (102 \pm 39) \times 10^{-11}$ (N, JN).

Slide by A. Nyffeler (Mainz) at 'Muon g-2 lbyl Workshop' at Connecticut, March 12-14, 2018

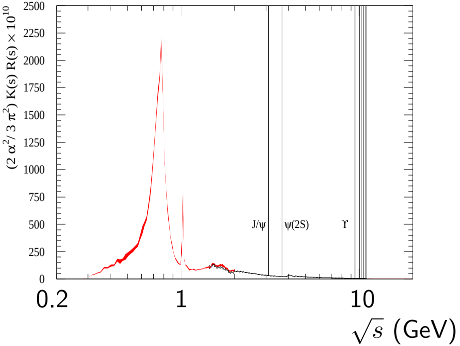
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$\boxed{a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)}$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
- Improvements in the estimates of uncertainties due to radiative corrections (Vacuum Polarization Radiative Corrections & Final State Radiations)
- Improvements in data-combination method

Main improvements between HLMNT11 and KNT18/19

- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
- Improvements in the estimates of uncertainties due to radiative corrections (Vacuum Polarization Radiative Corrections & Final State Radiations)
- Improvements in data-combination method

Channel	Energy range [GeV]	$a_\mu^{\rm had, LO\,VP} \times 10^{10}$	$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \times 10^4$	New data
		Chiral perturbation theory (ChPT) threshold contributions		
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	$\cdots$
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	$\cdots$
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	$\cdots$
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	$\cdots$
		Data based channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{{\rm no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	$\cdots$
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{{\rm no}\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	$\cdots$
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{{\rm no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	$\cdots$
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	$\cdots$
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	$\cdots$
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to {\rm npp})K\bar{K}_{{\rm no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
		Estimated contributions ($\sqrt{s} \le 1.937$ GeV)		
$(\pi^+\pi^-3\pi^0)_{{\rm no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	$\cdots$
$(\pi^+\pi^-4\pi^0)_{{\rm no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	$\cdots$
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	$\cdots$
$\omega(\to {\rm npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	$\cdots$
$\omega(\to {\rm npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	$\cdots$
$\omega(\to {\rm npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	$\cdots$
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	$\cdots$
		Other contributions ($\sqrt{s} > 1.937$ GeV)		
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	$\cdots$	6.26 ± 0.19	7.07 ± 0.22	$\cdots$
ψ'	$\cdots$	1.58 ± 0.04	2.51 ± 0.06	$\cdots$
$\Upsilon(1S-4S)$	$\cdots$	0.09 ± 0.00	1.06 ± 0.02	$\cdots$
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	$\cdots$
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	$\cdots$

Table from KNT18, Phys. Rev. D97 (2018) 114025

Breakdown of contributions to a_μ(had, LO VP) from various hadronic final states

We have included new data sets from $\sim$ 30 papers, in addition to those included in the HLMNT11 analysis

We have included $\sim$ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

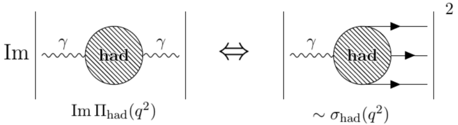
At higher energies $\gtrsim 11$ GeV, we use pQCD

Main improvements between HLMNT11 and KNT18/19

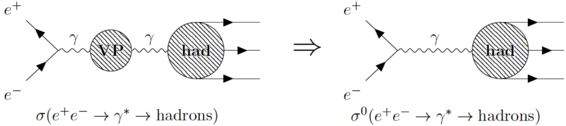
- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
- Improvements in the estimates of uncertainties due to radiative corrections (Vacuum Polarization Radiative Corrections & Final State Radiations)
- Improvements in data-combination method

Vacuum Polarization Corrections to $\sigma(e^+e^- \to \text{hadrons})$

Optical Theorem:



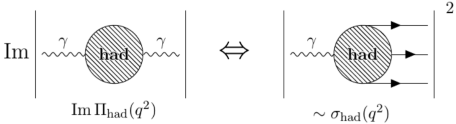
Experimentally observed cross section:



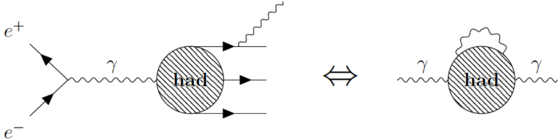
To evaluate $a_\mu^{\text{LO, had}}$, we need to subtract the vacuum polarization (VP) contribution.

It is straightforward to subtract the leptonic part of the VP, but the hadronic part is non-trivial: we need to do this recursively by using hadronic data. (We did this in the KNT18 paper.)

Optical Theorem:



To evaluate $a_\mu^{\text{LO, had}}$, by definition, we use the hadronic cross sections which include all the Final State Radiations (FSR).



In real experiments, people often impose cuts on the final state photons and/or miss photons in the final states. So we have to add back those missed photons, which introduces uncertainties.

In KNT18, we revisited the FSR corrections in the K^+K^- and $K^0_S K^0_L$ final states, and found smaller FSR uncertainties than our previous papers.

Main improvements between HLMNT11 and KNT18/19

- Lots of new input $\sigma(e^+e^- \to \text{hadrons})$ data
- Improvements in the estimates of uncertainties due to radiative corrections (Vacuum Polarization Radiative Corrections & Final State Radiations)
- Improvements in data-combination method

Data Combination

To evaluate the vacuum polarization contribution, we have to combine lots of experimental data.

To do so, we usually construct a χ^2 function and find the value of $R(s)$ at each bin which minimizes χ^2.

Naively, the χ^2 function defined as

$$\chi^2(\{\overline{R}_i\}) \equiv \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_{\text{bin}}} \sum_{j=1}^{N_{\text{bin}}} (R_i^{(n)} - \overline{R}_i)(V_n^{-1})_{ij}(R_j^{(n)} - \overline{R}_j) \ ,$$

where V_n is the cov. matrix of the n-th exp.,

$$V_{n,ij} = \begin{cases} (\delta R_{i,\text{stat}}^{(n)})^2 + (\delta R_{i,\text{sys}}^{(n)})^2 & (\text{for } i = j) \\ (\delta R_{i,\text{sys}}^{(n)})(\delta R_{j,\text{sys}}^{(n)}) & (\text{for } i \neq j) \end{cases}$$

may seem OK, but when there are non-negligible normalization uncertainties in the data, we have to be more careful.

χ^2 vs normalization error: d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

We first consider an observable x whose true value is 1. Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%.
Now, assume that this experiment measured x twice:

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ .$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + 0.09^2 & 0.09 \cdot 0.11 \\ 0.09 \cdot 0.11 & 0.1^2 + 0.11^2 \end{pmatrix} ,$$

$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} .$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? In the previous page,

$$\text{1st result:} \quad 0.9 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ ,$$
$$\text{2nd result:} \quad 1.1 \pm 0.1_{\text{stat}} \pm 10\%_{\text{syst}} \ .$$

we took the syst. errors 0.09 and 0.11, respectively, which made the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$(\text{cov.}) = \begin{pmatrix} 0.1^2 + (0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2 + (0.1\bar{x})^2 \end{pmatrix} ,$$

$$\chi^2 = \begin{pmatrix} x - 0.9 & x - 1.1 \end{pmatrix} (\text{cov.})^{-1} \begin{pmatrix} x - 0.9 \\ x - 1.1 \end{pmatrix} .$$

χ^2 takes its minimum at $x = 1.00$: Unbiased!

In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization. R.D.Ball et al, JHEP 1005 (2010) 075.

$\sigma(e^+e^- \to \pi^+\pi^-)$ data

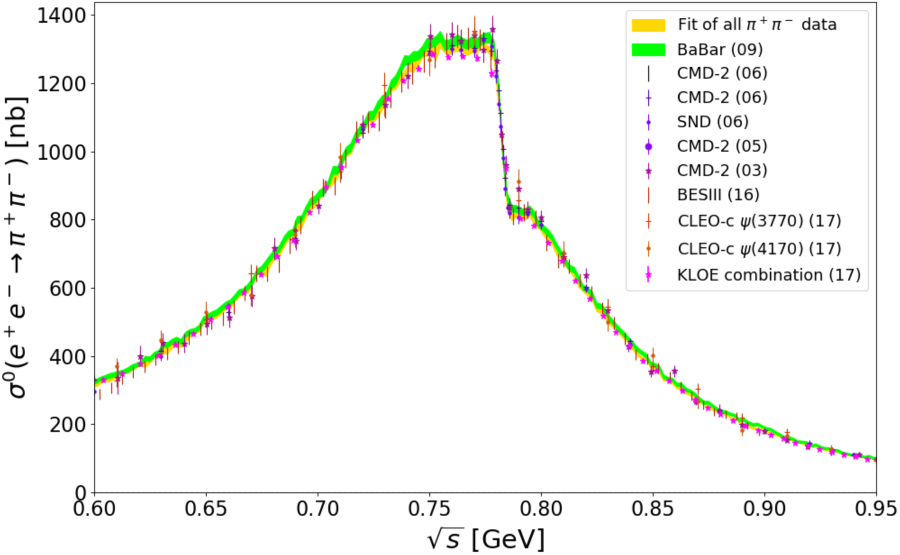


Fig. from KNT19, arXiv:1911.00367

$\sigma(e^+e^- \to \pi^+\pi^-)$: ρ-ω interference region

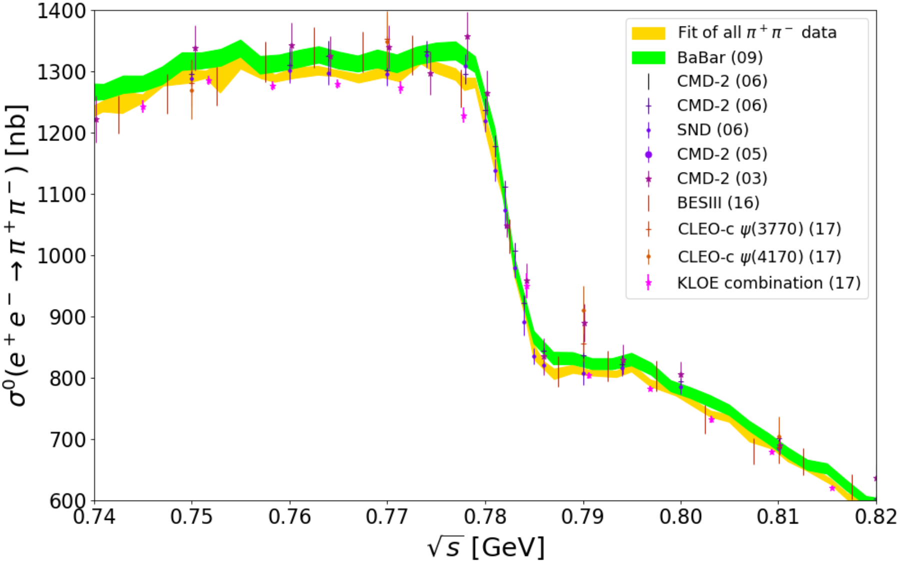


Fig. from KNT19, arXiv:1911.00367

$\sigma(e^+e^- \to \pi^+\pi^-)$: relative differences

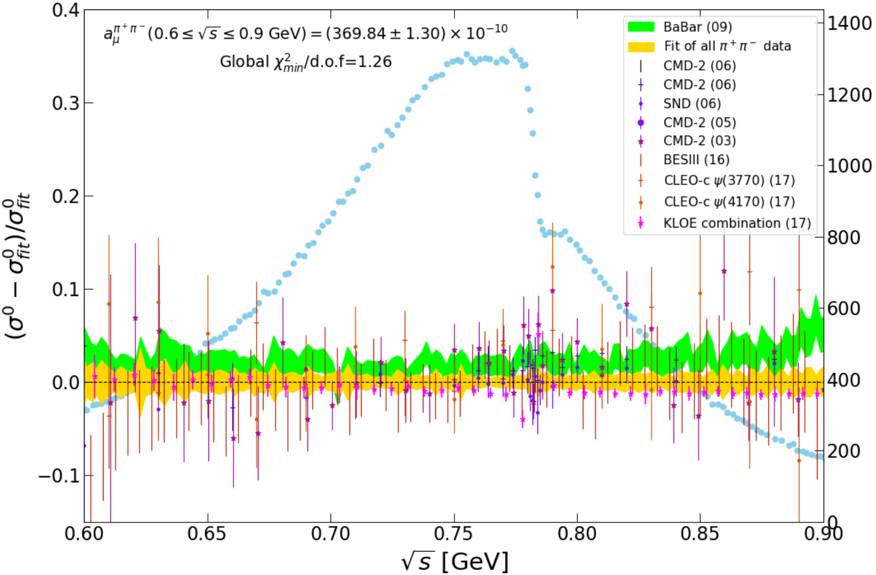


Fig. from KNT19, arXiv:1911.00367

Contribution to $(g-2)_\mu$ from $\pi^+\pi^-$ channel

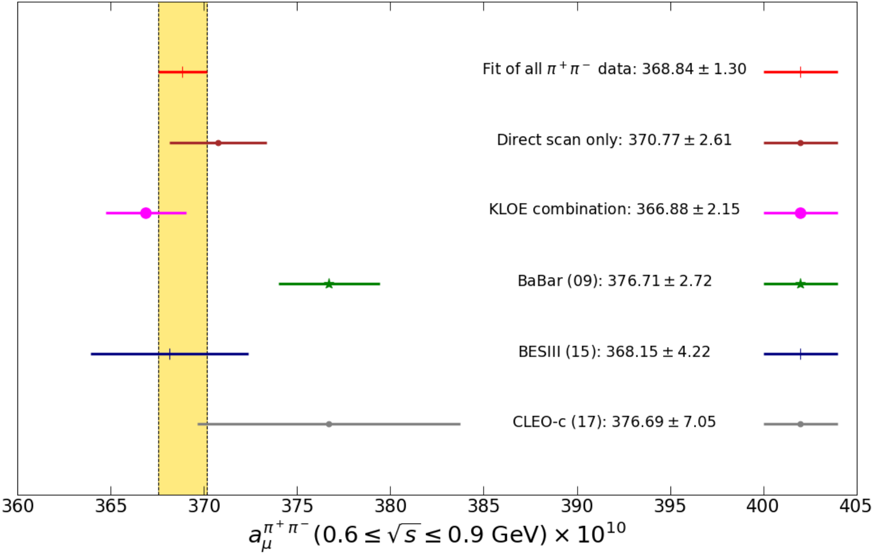
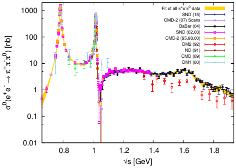


Fig. from KNT19, arXiv:1911.00367

Other notable exclusive channels [KNT18: arXiv:1802.02995, PRD (in press)]

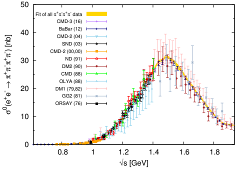
$\pi^+\pi^-\pi^0$



HLMNT11: 47.51 ± 0.99

KNT18: 47.92 ± 0.89

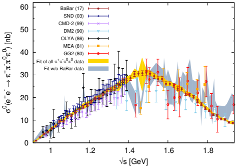
$\pi^+\pi^-\pi^+\pi^-$



HLMNT11: 14.65 ± 0.47

KNT18: 14.87 ± 0.20

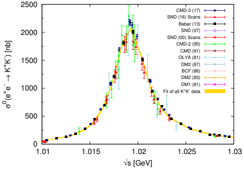
$\pi^+\pi^-\pi^0\pi^0$



HLMNT11: 20.37 ± 1.26

KNT18: 19.39 ± 0.78

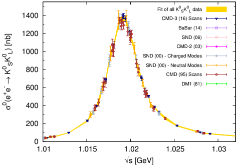
K^+K^-



HLMNT11: 22.15 ± 0.46

KNT18: 23.03 ± 0.22

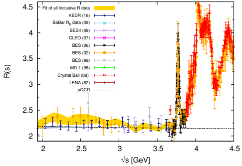
$K^0_S K^0_L$



HLMNT11: 13.33 ± 0.16

KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)



HLMNT11: 41.40 ± 0.87

KNT18: 41.27 ± 0.62

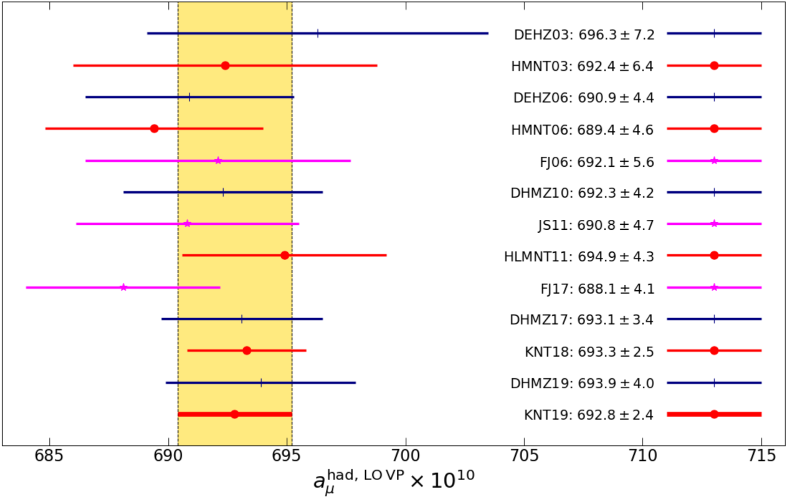
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

Hadronic VP Contributions: comparison

Adding up all the channels, pQCD & narrow resonances contributions, we get

$a_\mu^{\text{had, LO VP}}(\text{KNT19}) = (692.8 \pm 2.4) \times 10^{-10}$ (KNT18: $(693.3 \pm 2.5) \times 10^{-10}$)

$a_\mu^{\text{had, NLO VP}}(\text{KNT19}) = (-9.83 \pm 0.04) \times 10^{-10}$ (KNT18: $(-9.82 \pm 0.04) \times 10^{-10}$)



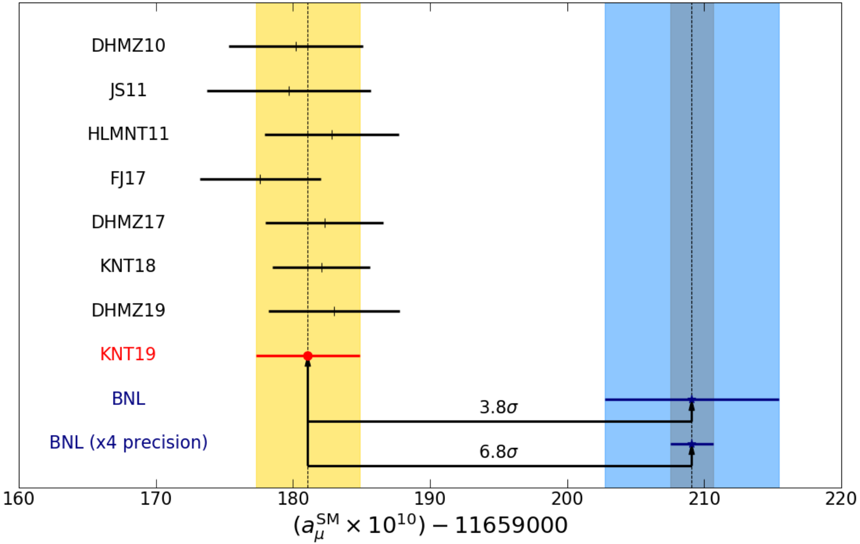
Breakdown of SM prediction for muon g-2

	2011		2018		2019
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]	
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]	
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) (	9.34 (2.92)
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]	
	HLMNT11		**KNT18**		**KNT19**
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work	692.78 (2.42)
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work	-9.83 (0.04)
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]	
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work $\cdots$	181.08 (3.78)
Experiment			11659209.10 (6.33)	world avg	
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work	28.0 (7.4)
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work	3.8σ

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

(Numbers taken from KNT18 and from KNT19)

Exp. value of muon g-2 vs SM prediction



Comparison with Other Work

Contributions from major channels to a_μ(LO,had) for $\sqrt{s} < 1.8$GeV:

channel	KNT18	DHMZ19	diff
$\pi^+\pi^-$	503.74 ± 1.96	507.80 ± 3.35	-4.06
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
K^+K^-	23.00 ± 0.22	23.08 ± 0.44	-0.08
$\pi^+\pi^-2\pi^0$	18.15 ± 0.74	18.01 ± 0.55	0.14
$2\pi^+2\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$K^0_S K^0_L$	13.04 ± 0.19	12.82 ± 0.24	0.22
$\pi^0\gamma$	4.58 ± 0.10	4.29 ± 0.10	0.29
$\vdots$	$\vdots$	$\vdots$	$\vdots$

"DHMZ19"= M. Davier et al, arXiv:1908.00921

Difference in the $\pi^+\pi^-$ channel is mainly from the way to combine the data sets.

KNT18: Global χ^2 minimization

DHMZ19: Takes the average of "all but KLOE" and "all but BaBar" as the mean value, and counts the half of the diff of the two as an additional systematic uncertainty.

Comparison with Lattice Results

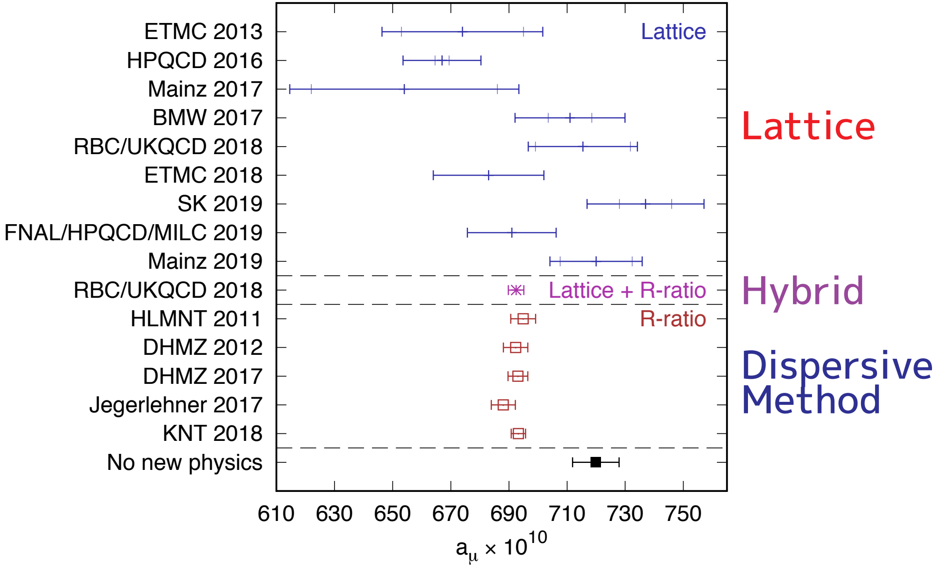


Fig. by C. Lehner (BNL), talk at Lattice 2019

Muon g-2 Theory Initiative

Steering Committee:

G. Colangelo	(Hadron Theory)	HVP and HLbL
M. Davier	(e^+e^- exp. (BaBar))	HVP
S. Eidelman	(e^+e^- exp. (CMD-2, CMD-3 & SND))	HVP
A. El-Khadra	(Lattice QCD)	HVP
C. Lehner	(Lattice QCD)	HVP and HLbL
T. Mibe	(J-PARC g-2 exp.)	
A. Nyffeler	(Hadron Theory)	HLbL
B. L. Roberts	(Fermilab g-2 exp.)	
T. Teubner	(Hadron Theory)	HVP

HVP: Hadronic Vacuum Polarization

HLbL: Hadronic Light-by-Light

Muon g-2 Theory Initiative: Goals

- theory support to the Fermilab and J-PARC experiments to maximize their impact
  ➟ need theoretical predictions of the hadronic corrections with reduced and reliably estimated uncertainties
- summarize the theoretical calculations of the hadronic corrections to the muon g-2
  ➟ comparisons of intermediate quantities between the different approaches. For example, lattice vs experiment
  ➟ assess reliability of uncertainty estimates
- combine to provide theory predictions for a_μ^{HVP} and a_μ^{HLbL} and write a report **before** the Fermilab and J-PARC experiments announce their first results.

slide by A. El-Khadra at Phipsi17, June 26-29, 2017 (Underlines by DN)

Muon g-2 Theory Initiative: Workshops

- 1st plenary workshop: near Fermilab, June 2017
- Hadronic Vacuum Polarization workshop: KEK, February 2018
- Hadronic Light-by-Light workshop: Connecticut, March 2018
- 2nd plenary workshop: Mainz, June 2018
- 3rd plenary workshop: Seattle, September 2019
- 4th plenary workshop: KEK, June 2020

We have discussed a lot about the White Paper:
In particular, how to come up with a single theory prediction to be compared with the exp. result.

Timeline for the White Paper

- Earliest possible release date for Fermilab g-2 measurement:
  15-20 December 2019
- Post the WP on arXiv by:
  1 Dec. 2019
- Deadline for finalizing individual WP chapters:
  1 Nov 2019
  At this date the Overleaf chapters will be frozen.
- Editorial board will release complete WP to authors for feedback on:
  15 Nov. 2019
  will need to receive feedback from authors within a week
- Experimental and theoretical inputs used in WP must be published by:
  15 Oct 2019
  To make sure to be included in WP discussion, a paper to be posted in arXiv by same date.

Note: The WP will be posted on arXiv in December, even if the Fermilab experiment's release date is delayed.

slide by A. El-Khadra at the Seattle muon g-2 workshop, September 9-13, 2019

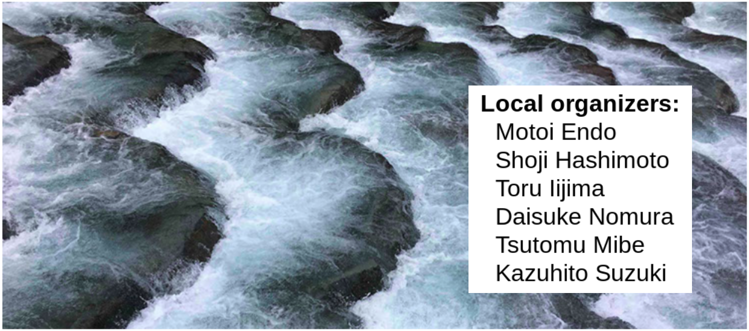
White Paper Outline

- Executive Summary
- Introduction
- Chapter 1: data-driven HVP
- Chapter 2: lattice HVP
- Chapter 3: data-driven HLbL
- Chapter 4: lattice HLbL
- Chapter 5: QED + EW
  T. Aoyama, T. Kinoshita, M. Nio
  D. Stöckinger, H. Stöckinger-Kim
- Summary, Conclusions, and Outlook

slide by A. El-Khadra at the Seattle muon g-2 workshop, September 9-13, 2019 https://www-conf.kek.jp/muong-2theory/

Muon g-2 theory initiative workshop

June 1-5, 2020
KEK, Tsukuba, Japan



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- Program
- Remote connection
- Links
- Contacts

About

The muon g-2 is arguably one of the most important observables in contemporary particle physics. The long-standing anomaly at the level of more than 3 standard deviations between the experimental value

June 1-5, 2020 at KEK

an activity of the Muon g-2 Theory Initiative

Summary

- Standard Model prediction for $(g-2)_\mu$: $\gtrsim 3.5\sigma$ deviation from measured value $\Longrightarrow$ New Physics?
- Recent data-driven evaluations of hadronic vacuum polarization contributions seem convergent
- To better establish the $g-2$ anomaly, better data for $e^+e^- \to \pi^+\pi^-$ welcome (from CMD-3, SND, Belle II, ...)
- Lattice calculations still suffer from large uncertainties (but a hybrid approach is useful)
- New exp. at Fermilab and J-PARC expected to reduce the uncertainty of $(g-2)_\mu$ by a factor of 4