# Spin Correlations of Proton Pairs as Tests of Bell's and Wigner Inequalities

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- Introduction
- Bell’s inequality
- Wigner’s inequality
- Experiment
- First results
- Future prospects

# Do Physical theories reveal physical reality ?

*As every physicist knows, or is supposed to have been taught, physics <u>does not</u> deal with physical reality. Physics deals with mathematically describable patterns in our observations*

**Henry P. Stapp** ***LBL-28087***

# Let's ask a philosopher

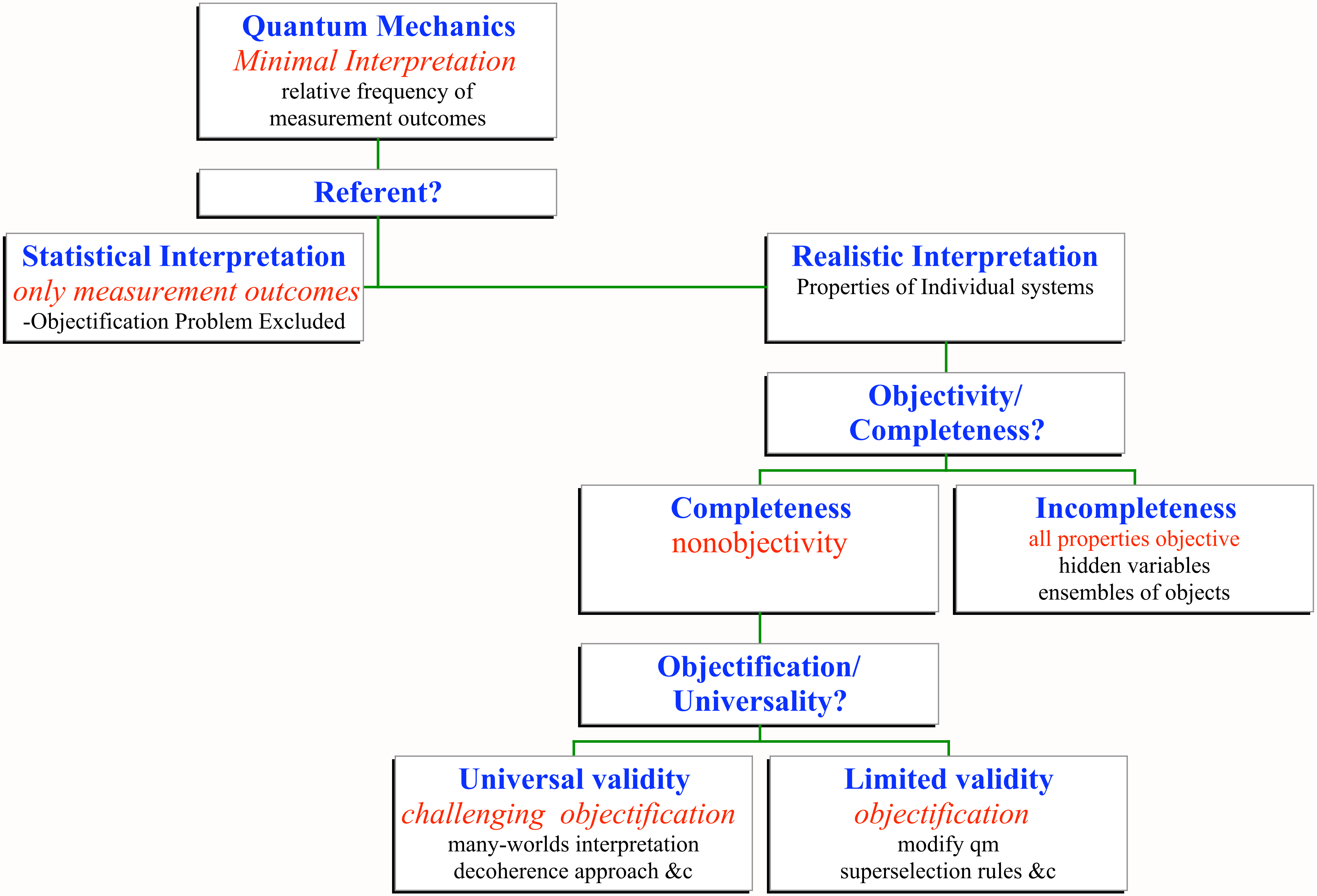
- *the only test permitting us to judge a physical theory and pronounce it good or bad is the comparison between the consequences of this theory and the experimental laws it has to represent and classify* **Pierre Duhem**

***Cushing***

*Philosophical concepts in physics*

- Let’s talk to a Mathematician

*Paul Busch*, Univ. of Hull, UK


Quantum Mechanics
Minimal Interpretation
relative frequency of
measurement outcomes
Referent?
Statistical Interpretation
only measurement outcomes
-Objectification Problem Excluded
Realistic Interpretation
Properties of Individual systems
Objectivity/
Completeness?
Completeness
nonobjectivity
Incompleteness
all properties objective
hidden variables
ensembles of objects
Objectification/
Universality?
Universal validity
challenging objectification
many-worlds interpretation
decoherence approach &c
Limited validity
objectification
modify qm
superselection rules &c

# Measurement

- For each measurement there is a QM operator
- Two operators constitute measurement operators if and only if they commute
- If they do not commute, at least one of them is a transition operator

# Angular Momentum operators

$[J^2, J_i] = 0$ $\qquad$ $[J_i, J_j] = \Box_{ijk} J_k$

i.e. Two perpendicular components cannot simultaneously be canonical momenta -- *Classical Mechanics*

***Only one among x, y and z components qualifies as a measurement operator.***

$J_x = (J_+ + J_-)/2$

Operating on an eigenstate of $J_z$, $J_x$ rotates the vector.

Tells us nothing about x-component.

## *Can Quantum Mechanical Description of Physical Reality be considered complete ?*

Einstein Podolsky Rosen *P.R. 47(1935) 777*

- **Completeness of Quantum Mechanics**
- **Physical Reality:**
  - *If, without in any way disturbing a system, we can predict with certainty*

  *(i.e. with probability equal to unity)*

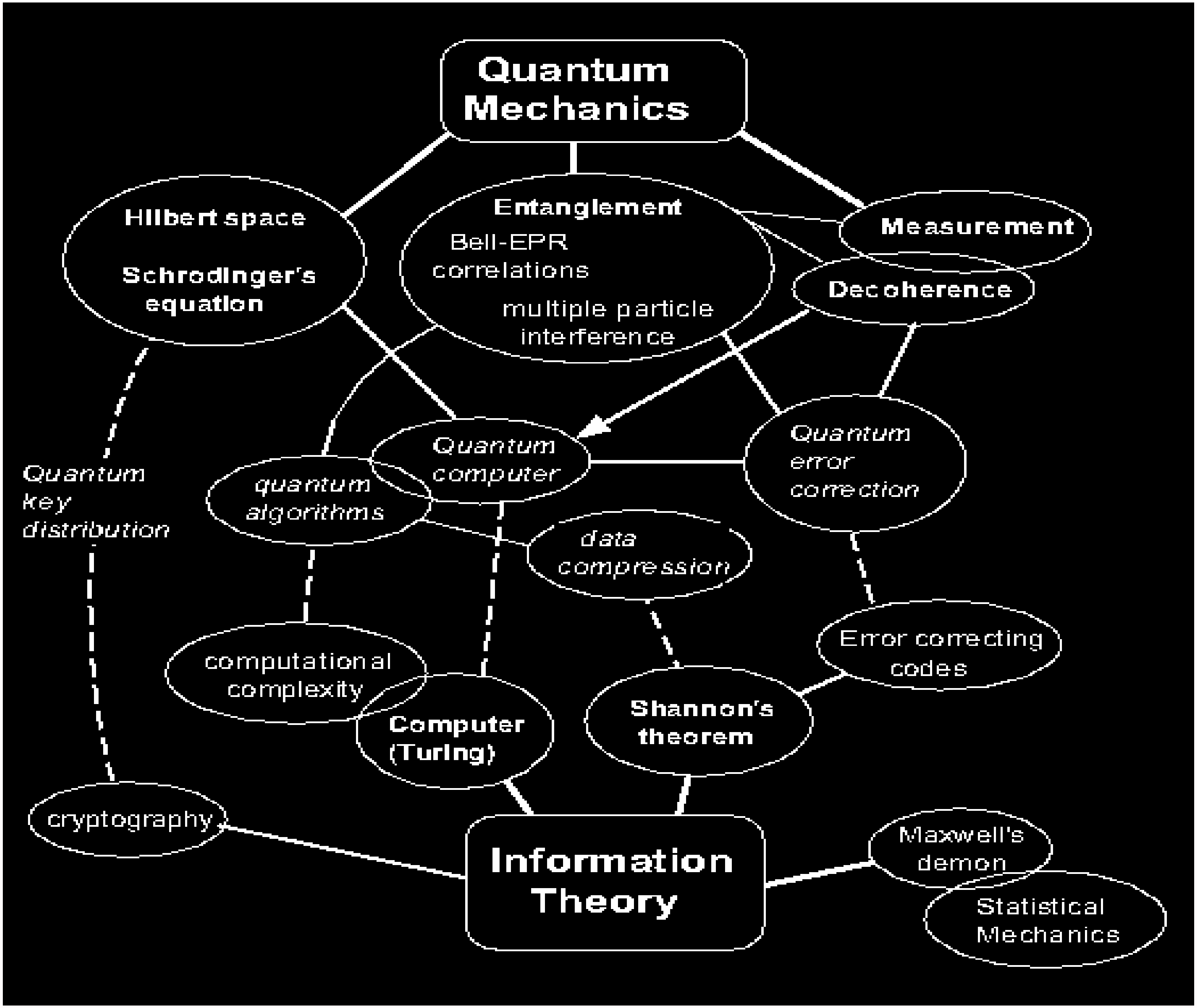
  *the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*

- **EPR argument for continuous conjugate variables** *Gedanken experiments*

- **Bohm's introduction of discrete**

**Eigen values - spin measurements (1952)**

**Bell's inequalities to test local hidden variable theories. (1964)**


Quantum Mechanics
Hilbert space
Schrodinger's equation
Entanglement
Bell-EPR correlations
multiple particle interference
Measurement
Decoherence
Quantum key distribution
quantum algorithms
Quantum computer
Quantum error correction
data compression
computational complexity
Computer (Turing)
Shannon's theorem
Error correcting codes
cryptography
Information Theory
Maxwell's demon
Statistical Mechanics

General Spin Projection Operator:

$$\hat{\sigma} \cdot \hat{n} = \begin{pmatrix} \cos\theta & \sin\theta e^{-\imath\phi} \\ \sin\theta e^{\imath\phi} & -\cos\theta \end{pmatrix}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -\imath \\ \imath & 0 \end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Commutation Relation:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} \neq 0 \Longrightarrow \text{Non-Compatible Observables}$$

$$[\sigma_i, \sigma_j] = \imath\hbar\epsilon_{ijk}\sigma_k \neq 0 \text{ where } i, j, k \in \{x, y, z\}$$

A Realistic or Epistemic Problem? Nature or Theory?

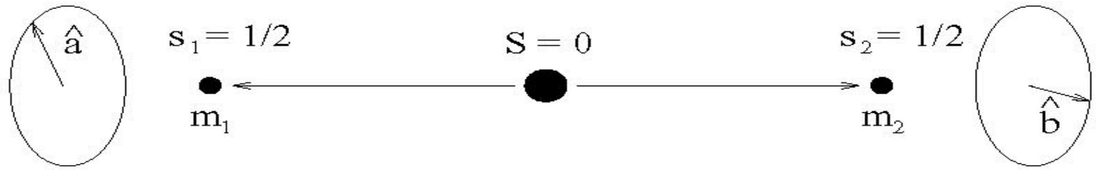
$m_{x1}$ and $m_{y1}$

$m_{x2}$ and $m_{y2}$

- Error in principles of angular momentum conservation?
- Super-luminal communication between protons?
- Extra parameters unknown to quantum theory?

2. Reality of physical observables whose values can be predicted with certainty, without the system being disturbed (governed by hidden parameters)
3. Local nature of quantum interactions and effects
4. Conservation of angular momentum

In light of EPRB, these cannot all simultaneously hold true.




$$A(\hat{a}) \equiv \sigma \cdot \hat{a}$$

$$B(\hat{b}) \equiv \sigma \cdot \hat{b}$$

$$A(\hat{a})B(\hat{b}) \equiv (\sigma \cdot \hat{a})_1(\sigma \cdot \hat{b})_2$$

Assume Hidden Parameters, $\lambda$, with distribution $\rho(\lambda)$, such that:

$$A(\hat{a}) = A(\hat{a}, \lambda)$$

$$B(\hat{b}) = B(\hat{b}, \lambda)$$

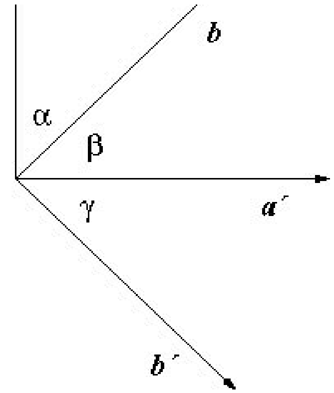



Figure 2.1: Orientation of unit vectors along which spin projections are measured for Bell's inequality, with arbitrary angles between the vectors.

illustrated in Figure 2.1. Then one has

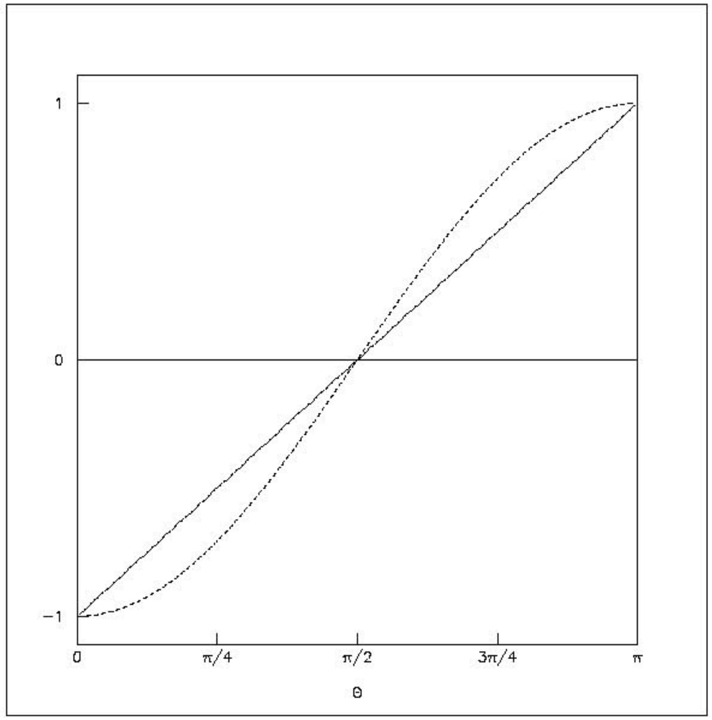
$$\begin{aligned}\Delta_{QM} &= P^{QM}(\alpha) - P^{QM}(\gamma) + P^{QM}(\beta-\alpha) + P^{QM}(\gamma-\beta) \\ &= -\cos(\alpha) + \cos(\gamma) - \cos(\beta-\alpha) - \cos(\gamma-\beta). \qquad (2.30)\end{aligned}$$

Choosing the angles to be $\alpha = \pi/4$, $\beta = \pi/2$, and $\gamma = 3\pi/4$ (ie. equal inter-angular separations of $\pi/4$), one gets

$$\begin{aligned}\Delta_{QM} &= -\cos(\frac{\pi}{4}) + \cos(\frac{3\pi}{4}) - \cos(\frac{\pi}{4}) - \cos(\frac{\pi}{4}) \\ &= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ &= -2\sqrt{2} \\ &\leq -2 \qquad (2.31)\end{aligned}$$

Violated by quantum expectation value for certain choices of angles.

Lamehi-Rachti Mittig (1976):



# *Lamehi-Rahti and Mittig*

**(Saclay-1976)**

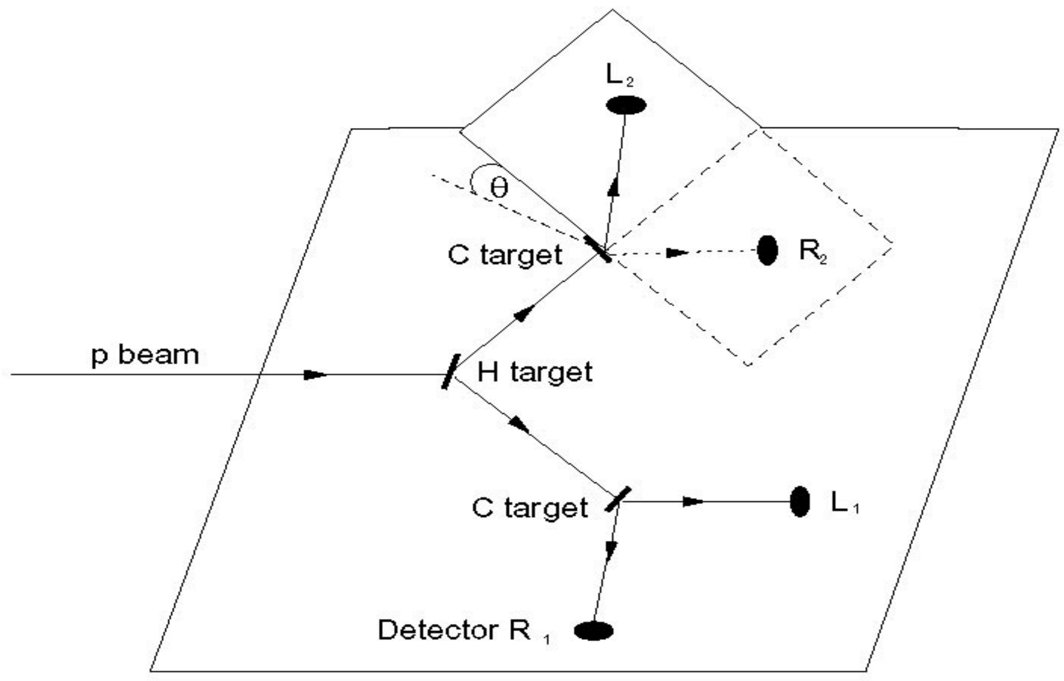



Figure 3.1: Experimental set-up for LRM proton-proton scattering.

# LRM result

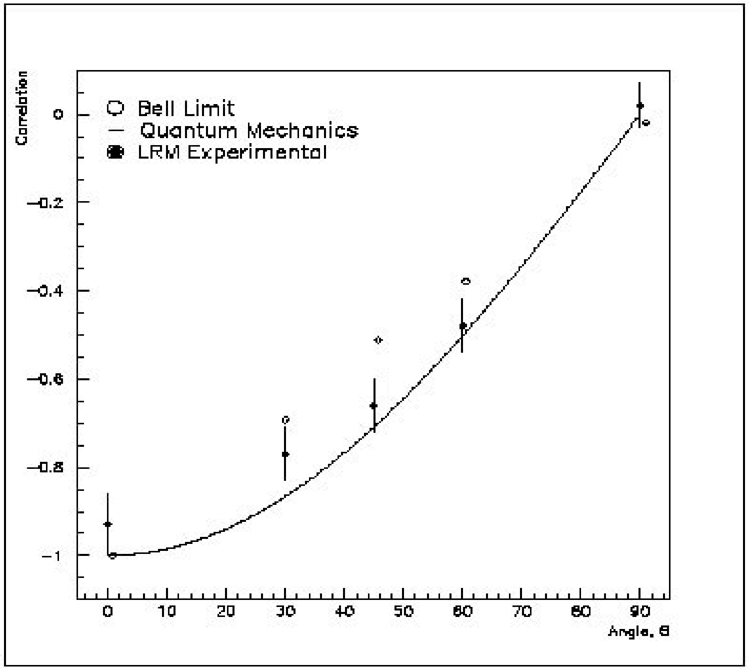



Figure 3.2: Experimental results for the correlation function found in the LRM study as compared to the (adjusted) bell limit and the predictions of quantum mechanics for the particular apparatus.

Wigner Inequality:

$$(P(\uparrow\uparrow)_{a,c} + -P(\uparrow\uparrow)_{a,b} - P(\uparrow\uparrow)_{b,c} \leq ($$

Violated by quantum expectation value for co-planar and the condition:

$$\cos^{-1}(\hat{a}\cdot\hat{b}) = \cos^{-1}(\hat{b}\cdot\hat{c}) = \frac{\cos^{-1}(\hat{a}\cdot}{2}$$

# State Preparation

- $^1S_0$ state of identical particles

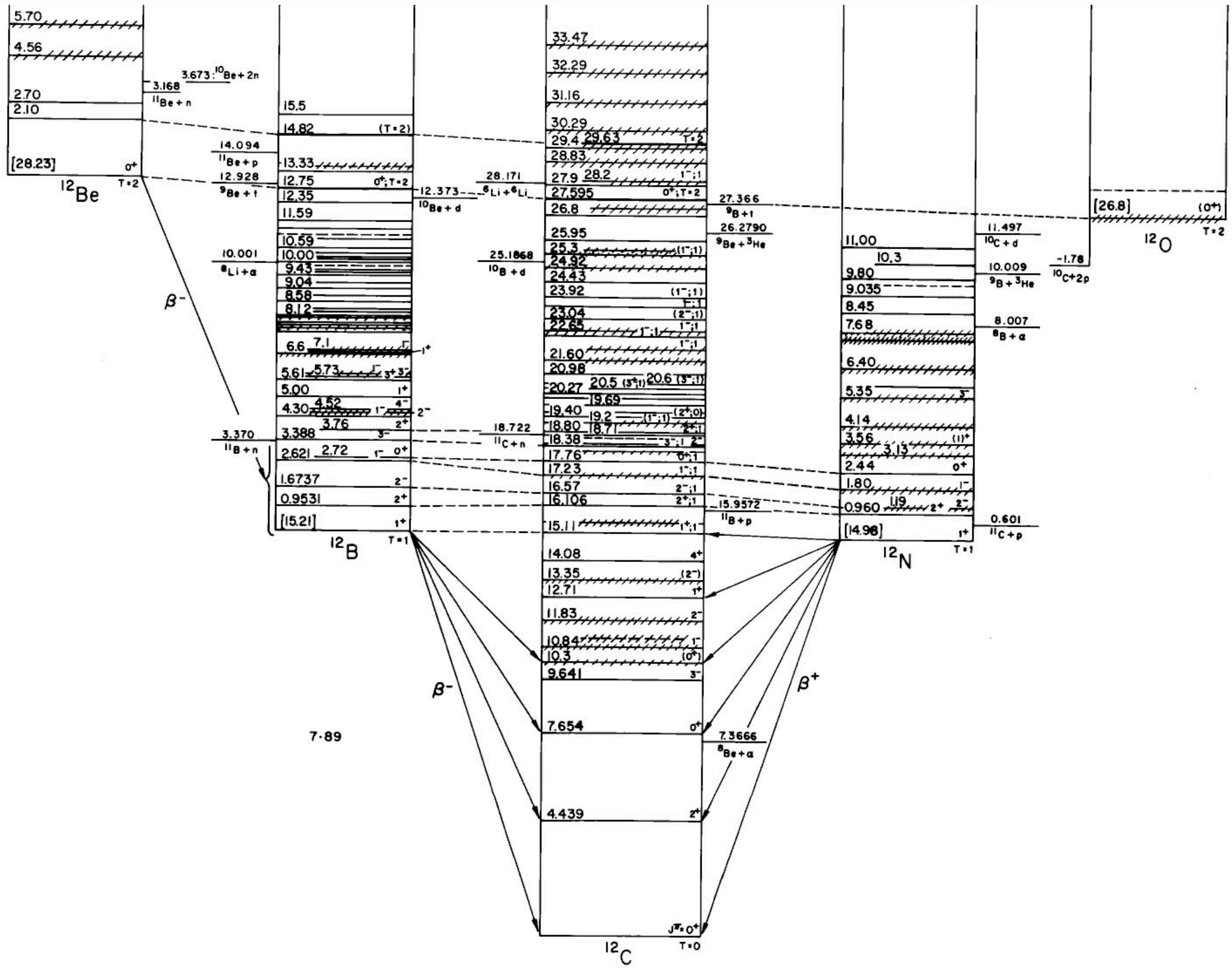
**Two-nucleon system (I=1)**

$^1S_0$, $^1D_2$, $^1G_4$..... $^3P_{0,1,2}$ $^3F_{2,3,4}$..

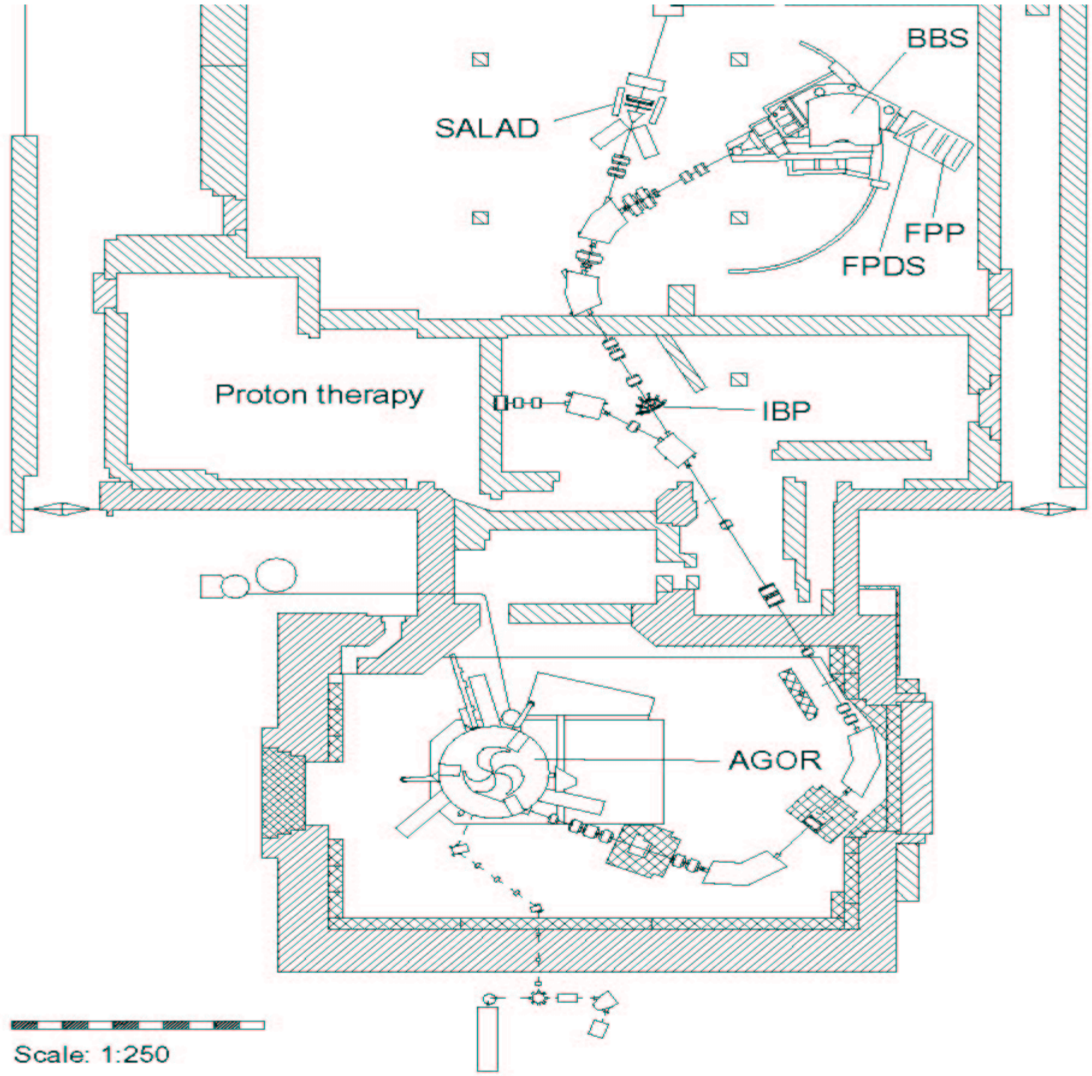
$^1S_0$ resonant state is known in deuteron

$_Z A(d, {}^2He)_{Z-1} A^*$ reaction

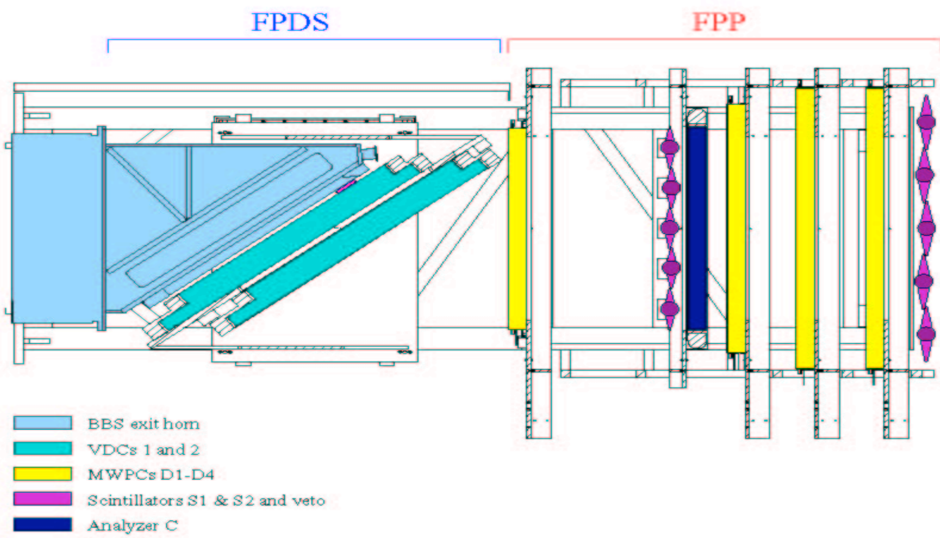
Work with carbon target

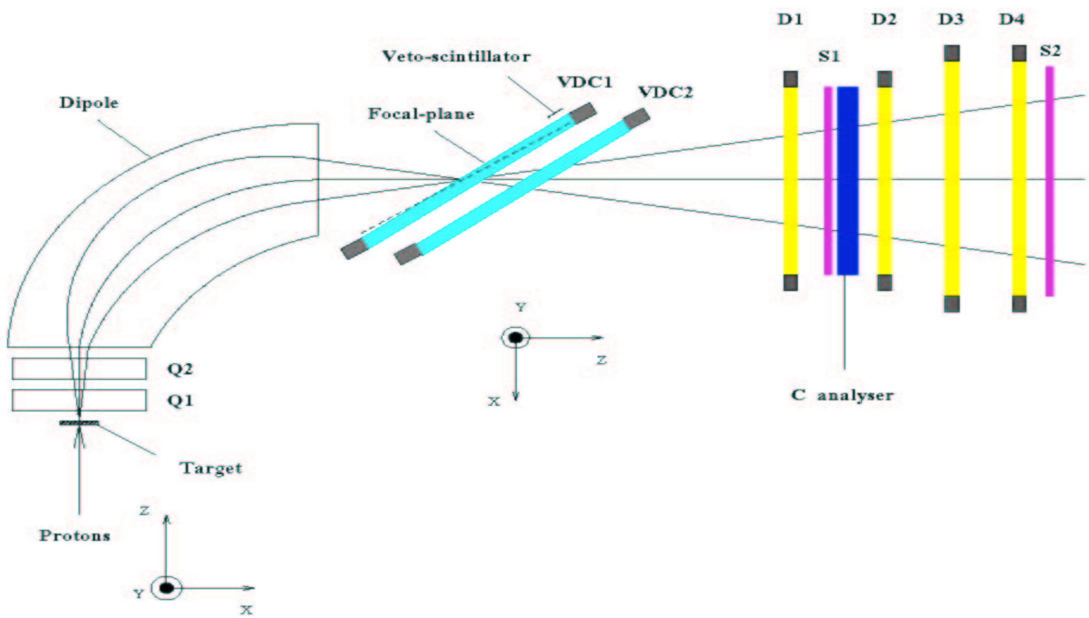


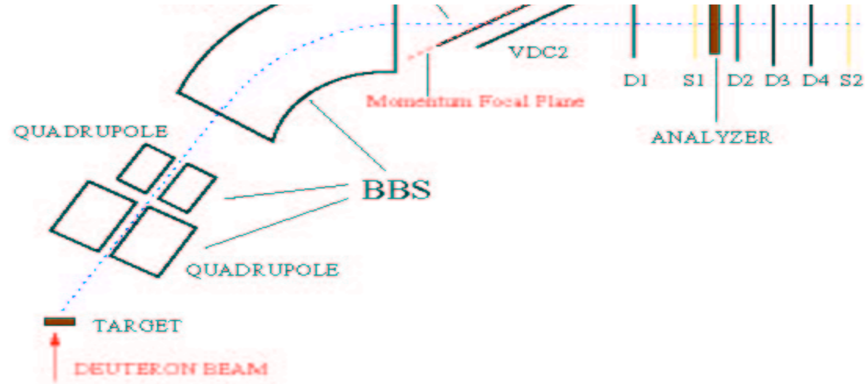
5.70
4.56
2.70
2.10
[28.23]
0+
T=2
12Be
3.168
11Be + n
3.673 : 10Be + 2n
β−
15.5
14.82 (T=2)
14.094
11Be + p
13.33
12.928
9Be + t
12.75 0+; T=2
12.35
12.373
10Be + d
11.59
10.59
10.001
8Li + α
10.00
9.43
9.04
8.58
8.12
6.6 7.1 1+
5.61 5.73 3+ 3,2
5.00 1+
4.30 4.52 1− 4− 2−
3.76 2+
3.370
11B + n
3.388 3−
2.621 2.72 1− 0+
1.6737 2−
0.9531 2+
[15.21] 1+
12B
T=1
β−
7-89
33.47
32.29
31.16
30.29
29.4 29.63 T=2
28.83
28.171
6Li + 6Li
27.9 28.2 1−;1
27.595 0+; T=2
26.8
25.95
25.1868
10B + d
25.3 (1−;1)
24.92
24.43
23.92 (1−;1)
23.04 (2−;1)
22.65 1−;1
21.60 1−;1
20.98
20.27 20.5 (3+;1) 20.6 (3−;1)
19.69
19.40 19.2 (1−;1) (2+;0)
18.722
11C + n
18.80 18.71 2+;1
18.38 3−;1 2−
17.76 0+;1
17.23 1−;1
16.57 2−;1
16.106 2+;1
15.11 1+;1
14.08 4+
13.35 (2−)
12.71 1+
11.83 2−
10.84 1−
10.3 (0+)
9.641 3−
7.654 0+
4.439 2+
Jπ=0+
T=0
12C
27.366
9B + t
26.2790
9Be + 3He
15.9572
11B + p
7.3666
8Be + α
β+
11.00
10.3
9.80
9.035
8.45
7.68
6.40
5.35 3−
4.14
3.56 (1)+
3.13
2.44 0+
1.80 1−
0.960 1.19 2+ 2−
[14.98] 1+
12N
T=1
11.497
10C + d
10.009
9B + 3He
8.007
8B + α
0.601
11C + p
−1.78
10C + 2p
[26.8] (0+)
12O
T=2




KVI


FPDS
FPP
BBS exit horn
VDCs 1 and 2
MWPCs D1-D4
Scintillators S1 & S2 and veto
Analyzer C


D1
D2
D3
D4
S1
S2
Veto-scintillator
VDC1
VDC2
Focal-plane
Dipole
Q2
Q1
Target
Protons
C analyser
Y
Z
X




| $^{12}C(d,^2He)^{12}B^*$ | $p(d,^2He)n$ |
|---|---|
| Kinematics: | Kinematics: |
| 77.9 MeV $\leq T_1 \leq$ 86.5 MeV | 83.9 MeV $\leq T_1 \leq$ 93.0 MeV |
| 69.0 MeV $\leq T_2 \leq$ 77.8 MeV | 74.8 MeV $\leq T_2 \leq$ 83.9 MeV |
| $T_1 + T_2 =$ 155.6 MeV | $T_1 + T_2 =$ 167.3 MeV |

- Mixture of background and singlet events
- $2\pi$ polarimeter acceptance

- Proposal - fall 2000
- Three days of running
  summer 2001

  deuteron beams = 170 MeV, 1nA

  carbon analyzer 10mg/cm$^2$

Several months of data analyses

# loopholes..

- Conscious Observer dependence
- Counterfactuality

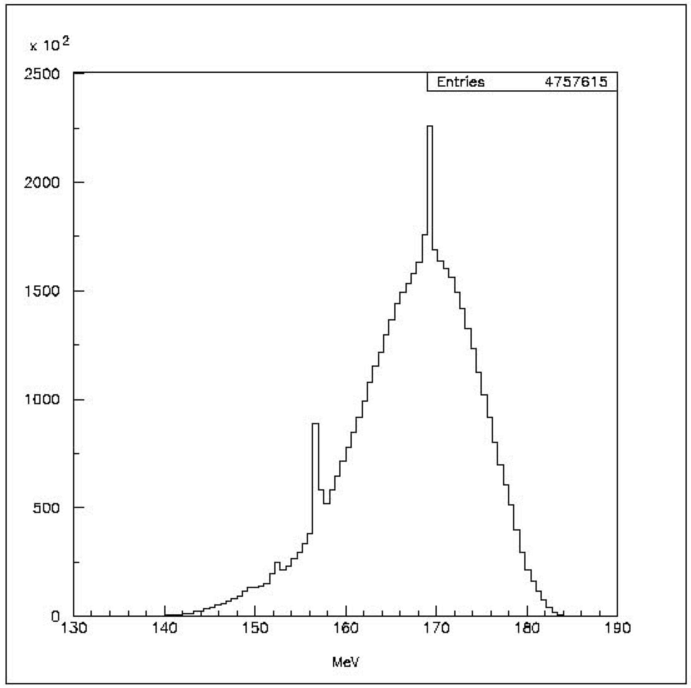
  due to small acceptance detectors

- Communication between the particles in pair

  space-like separation

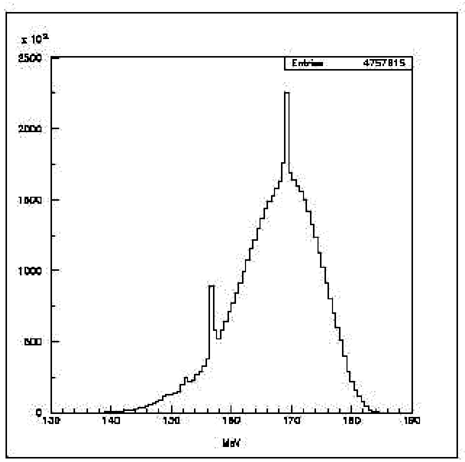
3. Bell inequality

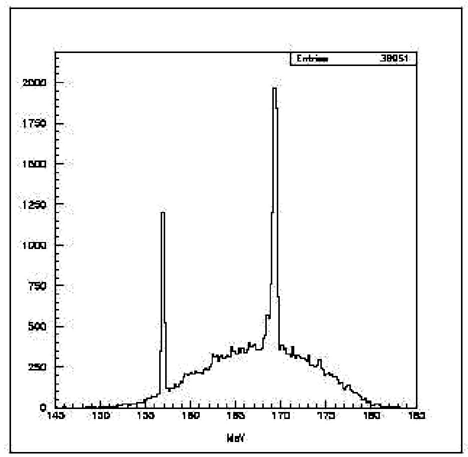
4. Wigner inequality




x

1. Relative proton time delay
2. Proton-pair internal energy
3. Non-reconstructable events
4. Non-scattering events
5. Unresolved events
6. Indistinguishable proton events




Total Analyzable Events Under Singlet Peaks: 33,322
(0.7% of raw events)

Physics of the interactions suggest a factor that can account for the analyzing power and any hypothetical systematic biases.

Singlet state property: $\vec{s_1} = -\vec{s_2} \Longrightarrow$ scattering in the analyzer to opposite hemispheres.

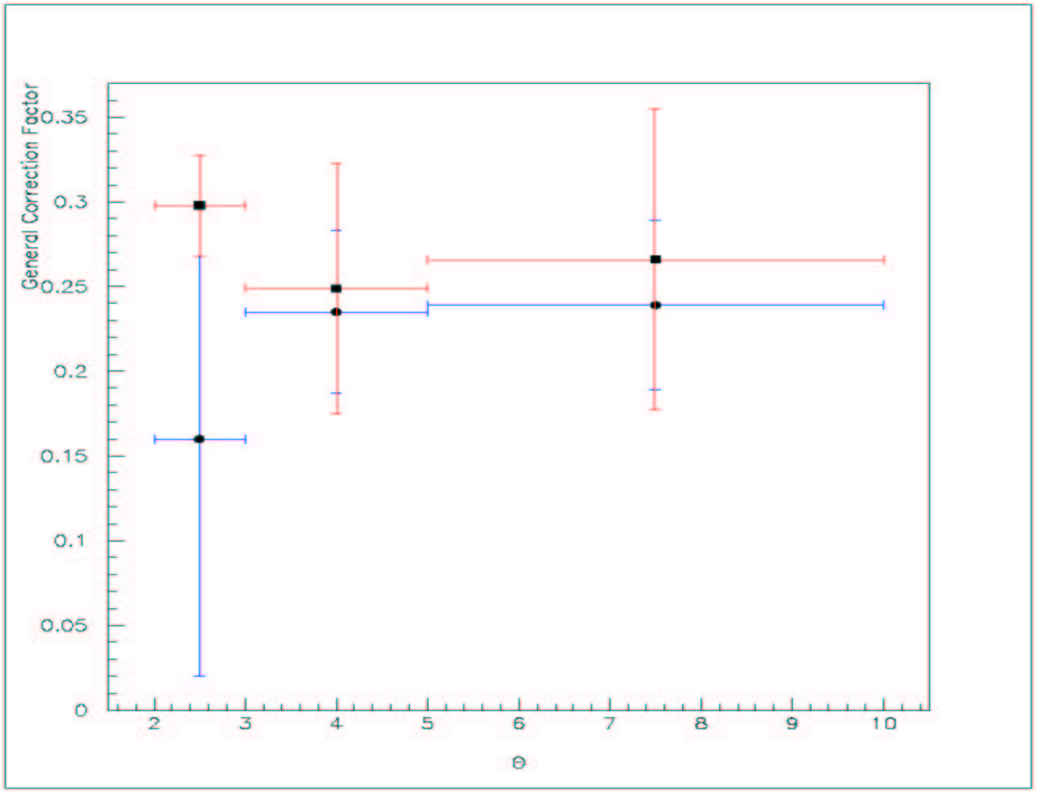
For polar scattering range $\Delta\theta$ and proton KE's $T_1, T_2$:

$$M_{\text{ideal}}(\Delta\theta, \Delta\theta, T_1, T_2) = \frac{N_s - N_d}{N_s + N_d} = -1$$

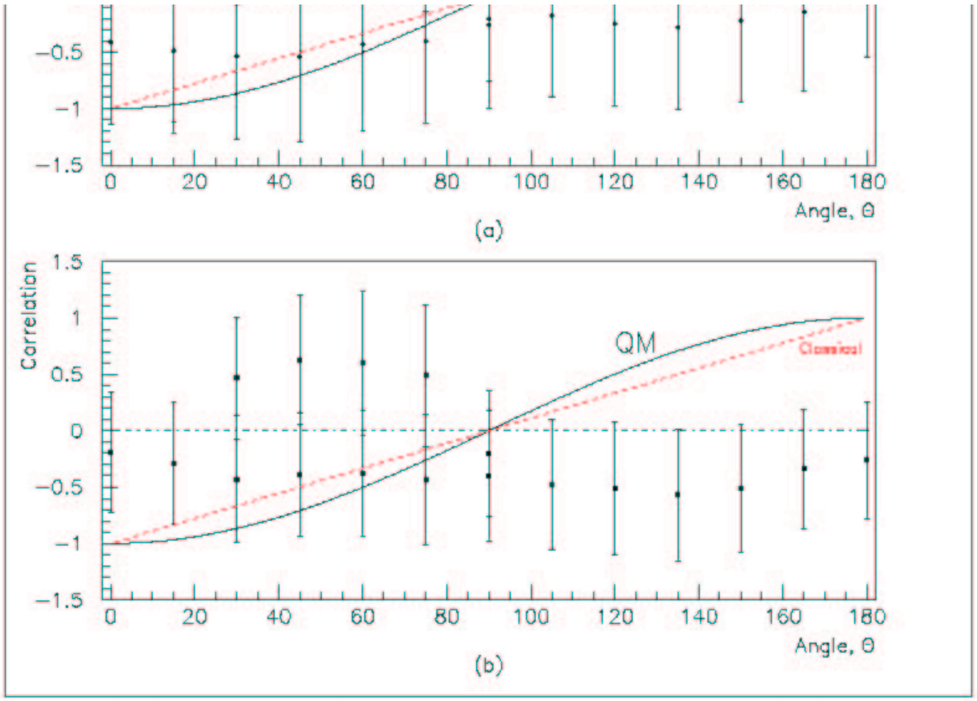
Actual measured value:

$$\begin{aligned} M_{\text{measured}}(\Delta\theta, \Delta\theta, T_1, T_2) &= M_{\text{ideal}}(\Delta\theta, \Delta\theta, T_1, T_2)\mathcal{A}'(\Delta\theta, T_1)\mathcal{A}'(\Delta\theta, T_2) \\ &= -\mathcal{A}'(\Delta\theta, T_1)\mathcal{A}'(\Delta\theta, T_1) \end{aligned}$$

$$P(\theta_1, \theta_2, T_1, T_2) = \frac{(N_s - N_d)/(N_s + N_d)}{\mathcal{A}'(\Delta\theta_1, T_1)\mathcal{A}'(\Delta\theta_2, T_2)}$$

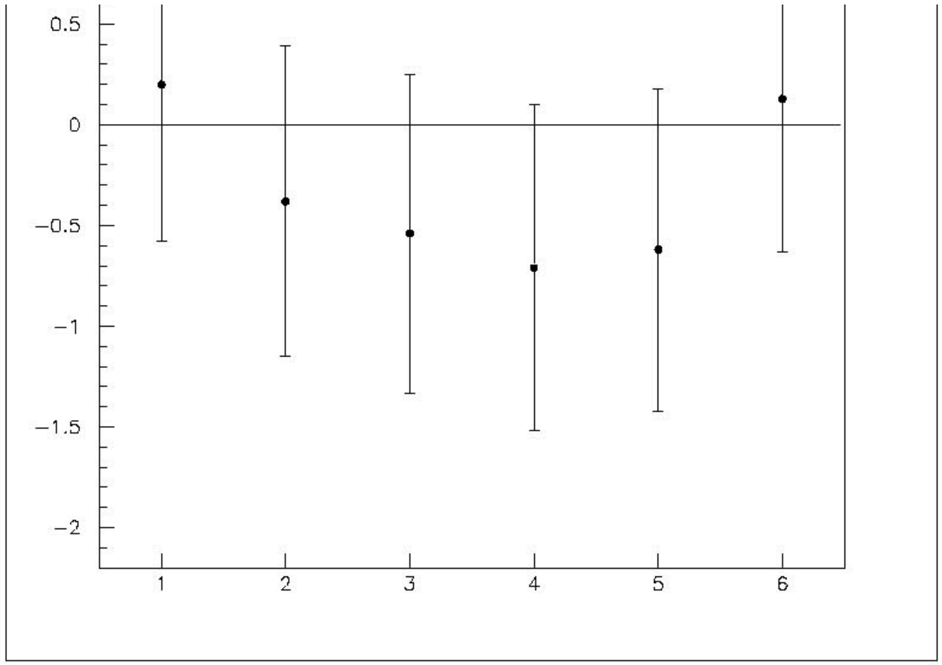



- Consistency of Results Between Singlet Peaks
- Good Agreement with Previous Estimate of 0.2




$$\begin{aligned}
\Delta[{}^{12}C(d,{}^{2}He){}^{12}B^{*}] &= P(0^\circ,45^\circ)-P(0^\circ,135^\circ)+P(0^\circ,45^\circ)+P(0^\circ,45^\circ)\\
&= -1.34\pm 1.51\\
\Delta[p(d,{}^{2}He)n] &= P(0^\circ,45^\circ)-P(0^\circ,135^\circ)+P(0^\circ,45^\circ)+P(0^\circ,45^\circ)\\
&= -0.60\pm 1.12
\end{aligned}$$

$$(\Delta_{\mathrm{QM}}=-2.83,\ -2\le\Delta_\lambda\le+2)$$




- Large experimental error
- Tendency toward disagreement with quantum completeness condition
- Fair consistency between two reaction data sets

# Where do we go from here?

- More statistics

  ……. A lot more

- p-p elastic scattering

  *polarized beam  and polarized target*

# Relative Merits

| $(d,^2He)$ | $(p,p)_{pol}$ |
|---|---|
| Pure singlet + | Polarization varied at will + |
| Low cross section – | High cross section + |
| Large background – | low background + |
| One spectrometer + | Need two specs. – |
| One polarimeter + | Two polarimeters – |

# Conclusion

- **Di-proton systems offer promise to test QM**

- **The quest continues**