$SU(3)$ relations and the CP asymmetries in $b \to s\bar{s}s$ decays

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- Motivation

  ... What have we learned? How to test CKM?

- 2-body: ϕK_S, $\eta' K_S$, and $\ldots$

  ... $SU(3)$ — how far can we get with minimal assumptions?

- 3-body: $K^+K^-K_S$

  ... CP decomposition, I-spin, U-spin

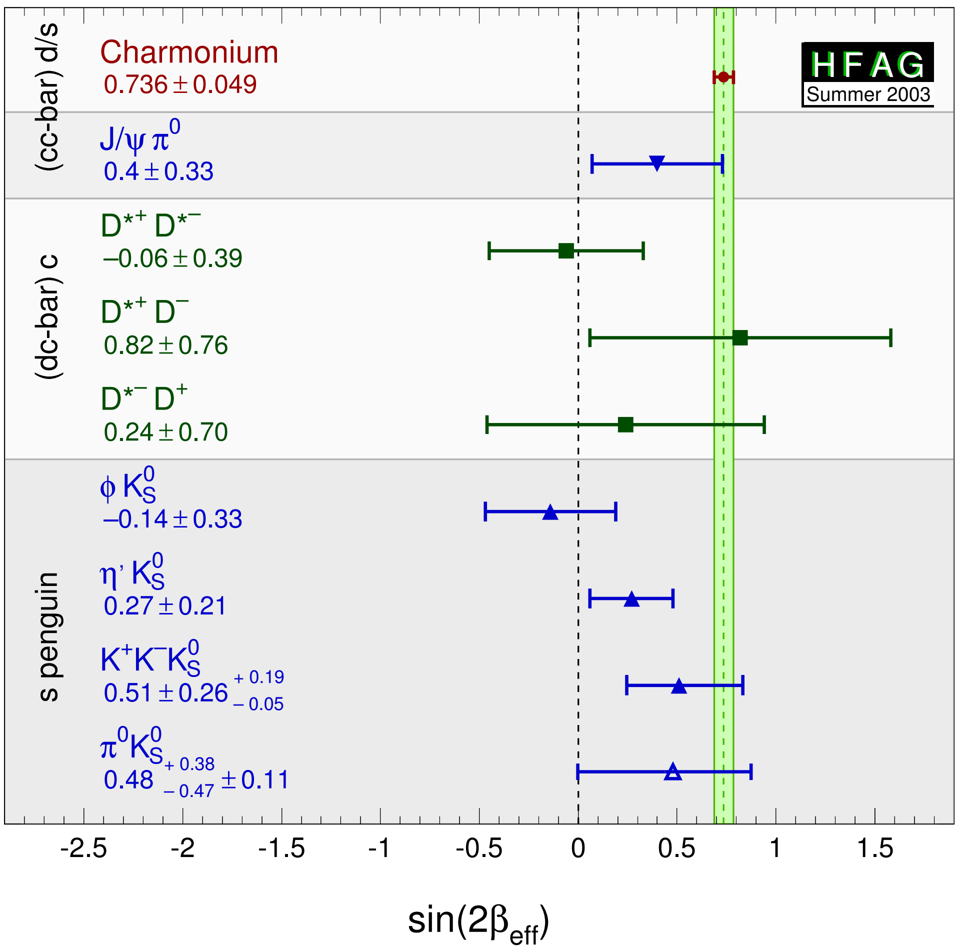
- Conclusions

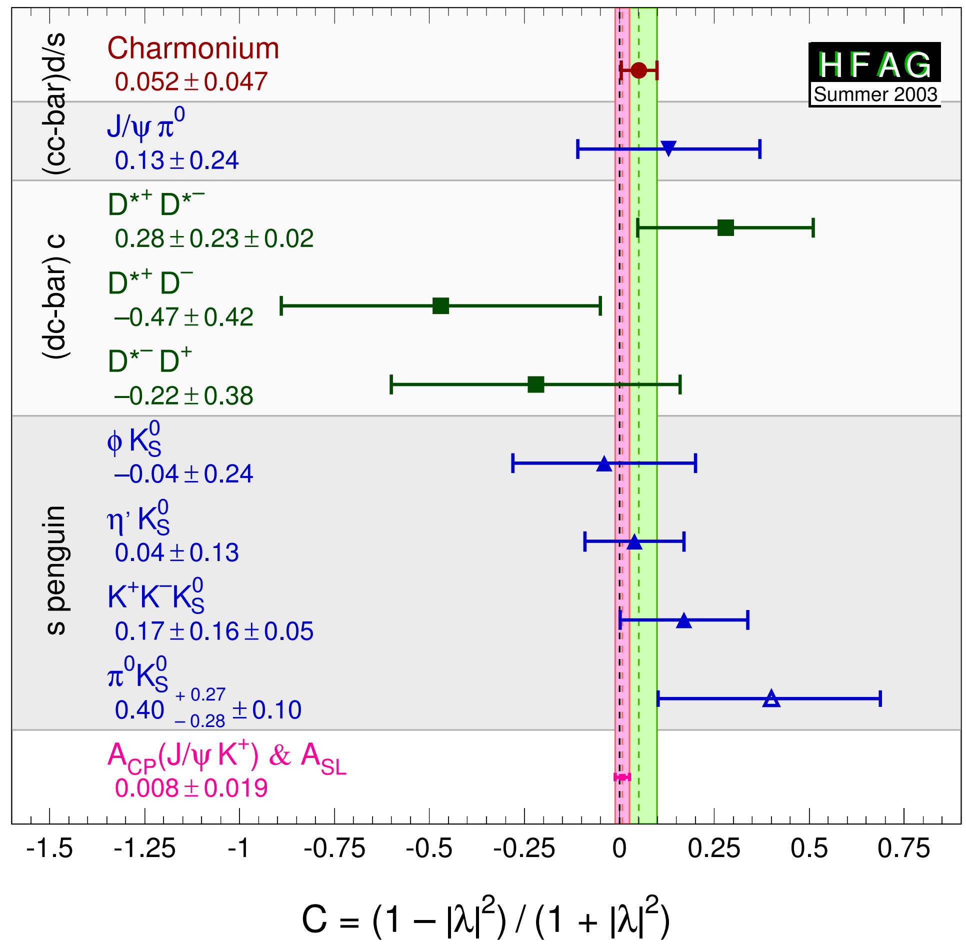
based on: Grossman, ZL, Nir and Quinn, PRD **68** (2003) 015004 [hep-ph/0303171]

related work: Gronau, Rosner, hep-ph/0304178; Chiang, Gronau, Rosner, hep-ph/0306021

Lots of new data!

- CP violation in $b \to c\bar{c}s,\ c\bar{c}d,\ sg$: in SM all are $\sin 2\phi_1 +$ "small corrections"





$$S = -\eta_f \sin 2\beta_{\rm eff}\ , \qquad (C \equiv -A)$$

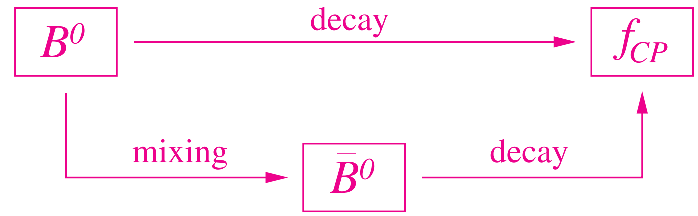
Why is flavor physics and CPV interesting?

- Almost all extensions of the SM contain new sources of CP and flavor violation
  (e.g., 43 new CPV phases in SUSY [must see superpartners to discover it])
- A major constraint for model building
  (flavor structure: universality, heavy squarks, squark-quark alignment, ...)
- May help to distinguish between different models
  (mechanism of SUSY breaking: gauge-, gravity-, anomaly-mediation, ...)
- The observed baryon asymmetry of the Universe requires CPV beyond the SM
  (not necessarily in flavor changing processes in the quark sector)

There is no “standard” new physics scenario in flavor sector...

CPV in interference between decay and mixing

- Especially interesting if both B^0 and $\overline{B}^0$ can decay to same final state, e.g., $|f\rangle = |f_{CP}\rangle$:



$$\lambda_{f_{CP}} = \frac{q}{p}\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}}\frac{q}{p}\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}$$

$$a_{f_{CP}} = \frac{\Gamma[\overline{B}^0(t)\to f] - \Gamma[B^0(t)\to f]}{\Gamma[\overline{B}^0(t)\to f] + \Gamma[B^0(t)\to f]} = \underbrace{\frac{2\,\mathrm{Im}\,\lambda_f}{1+|\lambda_f|^2}}_{S_f}\sin(\Delta m\,t) - \underbrace{\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}}_{C_f(\equiv -A_f)}\cos(\Delta m\,t)$$

CP violation: $|\lambda_f| \neq 1 \;\Rightarrow$ CPV in mixing and/or decay

$\mathrm{Im}\,\lambda_f \neq 0 \;\Rightarrow$ CPV in interference

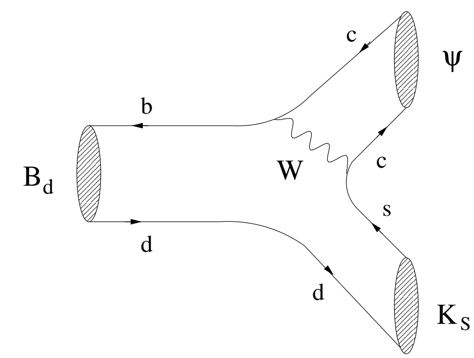
- If amplitudes with one weak phase dominate a decay then the CP asymmetry measures a phase in the Lagrangian theoretically cleanly

(Then $|\lambda_f| \simeq 1$, since $|q/p| - 1 < \mathcal{O}(10^{-2})$ in $B_{d,s}$ mixing)

$$a_{f_{CP}} = \mathrm{Im}\,\lambda_f \sin(\Delta m\,t)$$

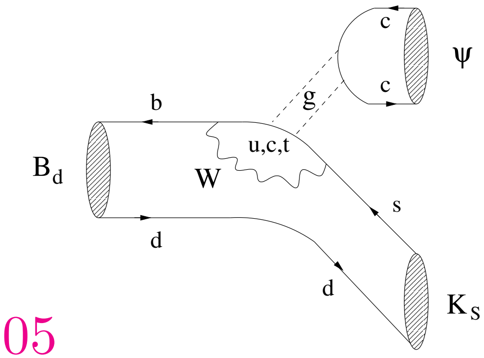
The cleanest case: $B \to \psi K_{S,L}$

- Several contributions:



“Tree” ($b \to c\bar{c}s$): $\overline{A}_T = \overset{[\lambda^2]}{V_{cb}V_{cs}^*}\, A_{c\bar{c}s}$

“Penguin”: $\overline{A}_P = \overset{[\lambda^2]}{V_{tb}V_{ts}^*}\, P_t + \overset{[\lambda^2]}{V_{cb}V_{cs}^*}\, P_c + \overset{[\lambda^4]}{V_{ub}V_{us}^*}\, P_u$



Write sum as:

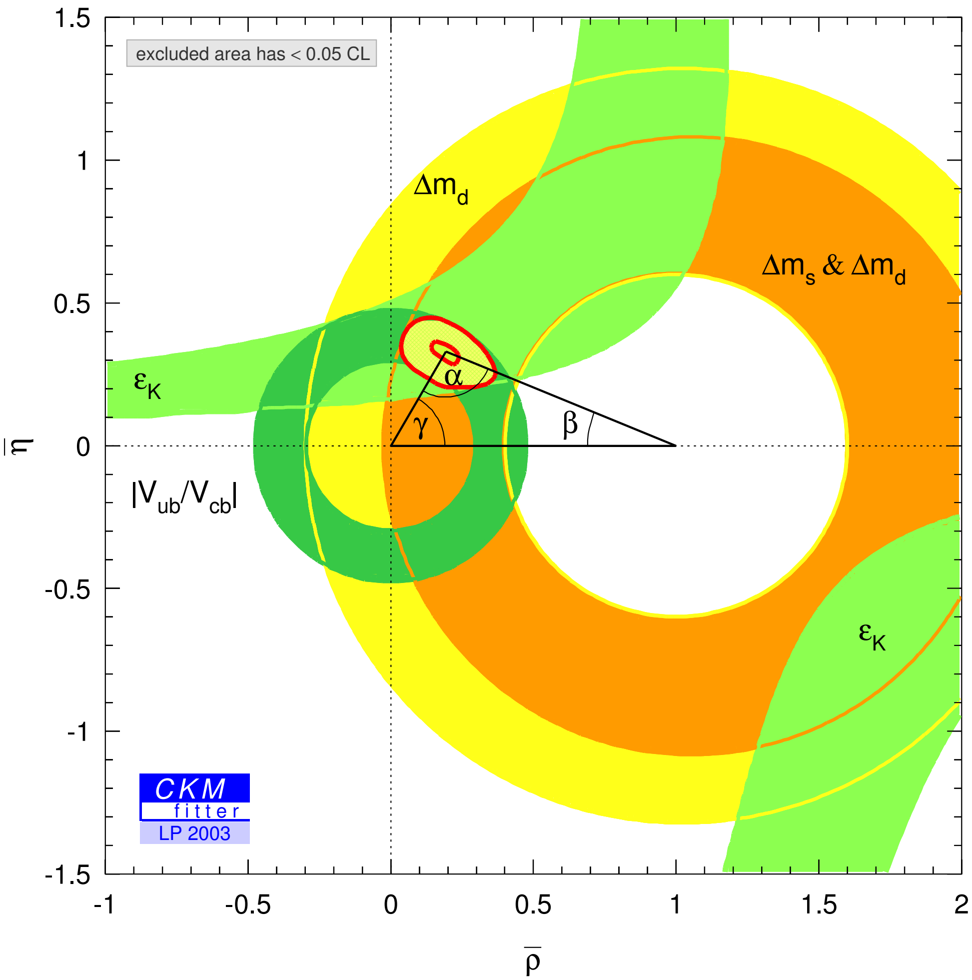
$$\overline{A}_{\psi K_S} = \underbrace{\overset{[\lambda^2]}{V_{cb}V_{cs}^*}}_{\text{“Tree” phase}} [A_{c\bar{c}s} + P_c - P_t] + \underbrace{\overset{[\lambda^4]}{V_{ub}V_{us}^*}}_{\text{suppressed by } \lambda^2 \sim 0.05} [P_u - P_t]$$

- The $V_{cb}V_{cs}^*$ term dominates $\Rightarrow$ theoretically very clean

$$\lambda_{\psi K_{S,L}} = \mp \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) \left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right) \left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}\right) = \mp e^{-2i\phi_1} \quad \Rightarrow \quad \mathrm{Im}\lambda_{\psi K_{S,L}} = \pm \sin 2\phi_1$$

Present knowledge of $(\bar{\rho}, \bar{\eta})$

Standard model fit without $\sin 2\phi_1$



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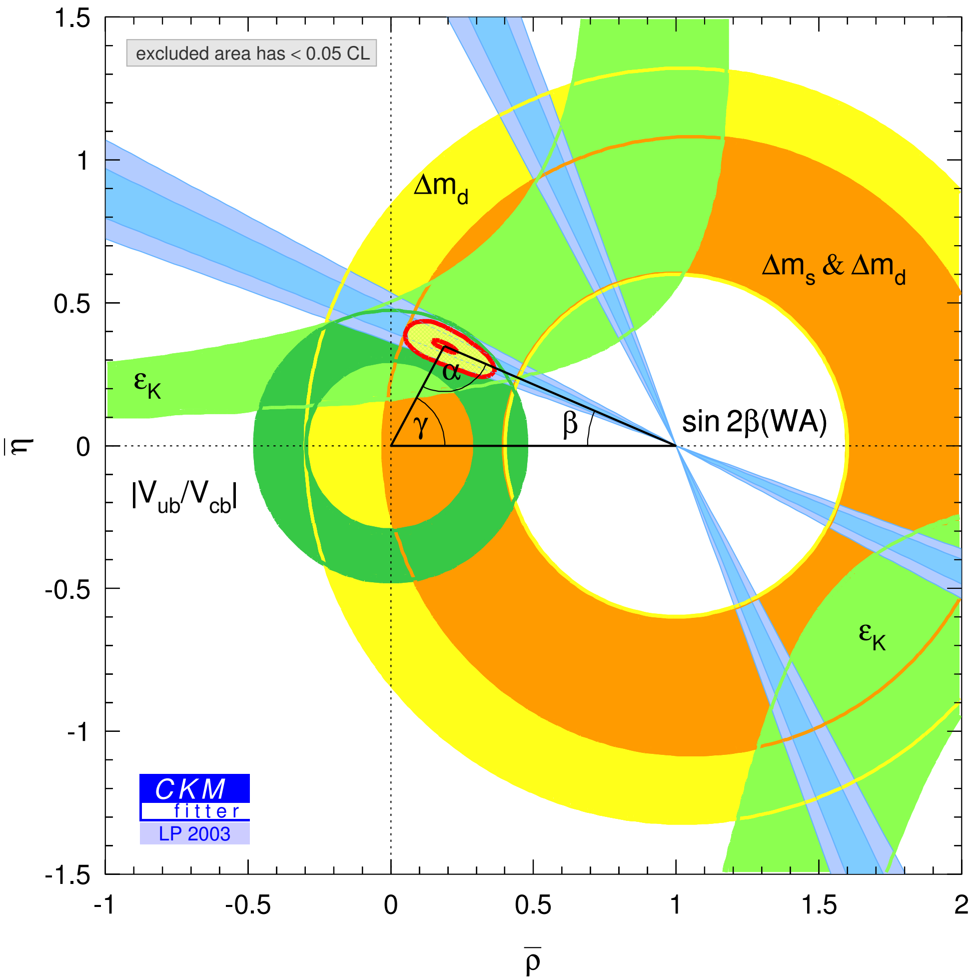

The CKM picture passed its first real test

Approximate CP (in the sense that all CPV phases are small) is excluded

New CPV phases that enter $B_d - \overline{B}_d$ mixing at $\geq 20\%$ level disfavored

Paradigm change: look for corrections, rather than alternatives to CKM

Is the SM the only source of CPV?

Does the SM fully explain flavor physics?

$\Rightarrow$ Need detailed tests (Δm_{B_s}, $S_{s\bar{s}s}$, ...)

How to test CKM?

How to find new physics?

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i) In most NP models $\phi_1 + \phi_2 + \phi_3 = \pi$

ii) Even if $\phi_1 + \phi_2 + \phi_3 \neq \pi$, probably an easier test will show NP first

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Testing, e.g., $S_{\psi K_S} = S_{\phi K_S}$ is easier than checking $\phi_1 + \phi_2 + \phi_3 = \pi$, and more sensitive to NP

What are we after?

- Within SM: Only V_{ub} and V_{td} have large phases (in usual parameterization) any large interference type CPV is a function of these

ϕ_1 is “easy” to measure, second can be called: $\phi_2,\ \phi_3,\ \phi_1+\phi_3,\ 2\phi_1+\phi_3$ ...
but this does not make any difference

$\Rightarrow$ Independent measurements are cross-checks

- Beyond SM: Many phases can be large and different ($B_{d,s}$ mixing, decays)

“$\phi_1,\ \phi_2,\ \phi_3$” is only a language: measurements that relate to the same parameter in SM can be sensitive to different NP

$\Rightarrow$ Independent measurements (which have clean interpretation) search for NP

Corrections to SM may still be large in Δm_{B_s}, $S_{s\bar{s}s}$, CP asymmetries in B_s decay

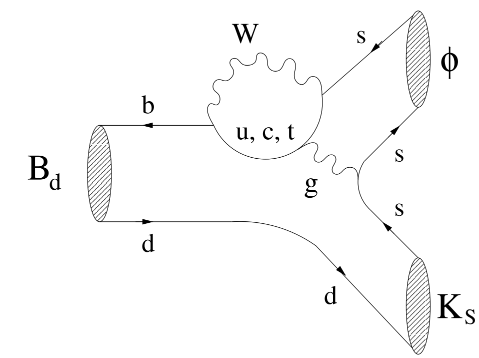
Two-body decays

ϕK_S and $\eta' K_S$

CP violation in $b \to s\bar{s}s$

- Amplitudes with one weak phase expected to dominate:

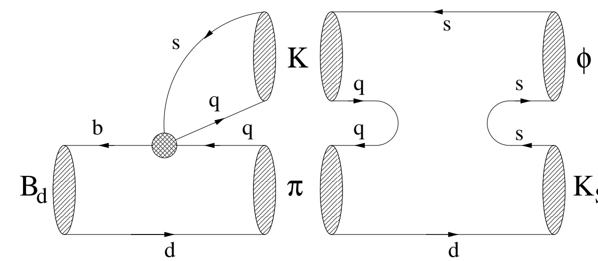
$$\overline{A} = \underbrace{\overset{[\lambda^2]}{V_{cb}V_{cs}^*}}_{\text{dominant contribution}} [P_c - P_t + T_{c\bar{c}s}] + \underbrace{\overset{[\lambda^4]}{V_{ub}V_{us}^*}}_{\text{suppressed by } \lambda^2} [P_u - P_t + T_{u\bar{u}s}]$$



Within SM: dominated by a single phase $\Rightarrow\ C \approx 0$

expect $S_{s\bar{s}s} \approx S_{\psi K}$ at $\mathcal{O}(\lambda^2) \sim 5\%$ level

With NP: $S_{s\bar{s}s} \neq S_{\psi K}$ and $C_{s\bar{s}s} \neq 0$ possible

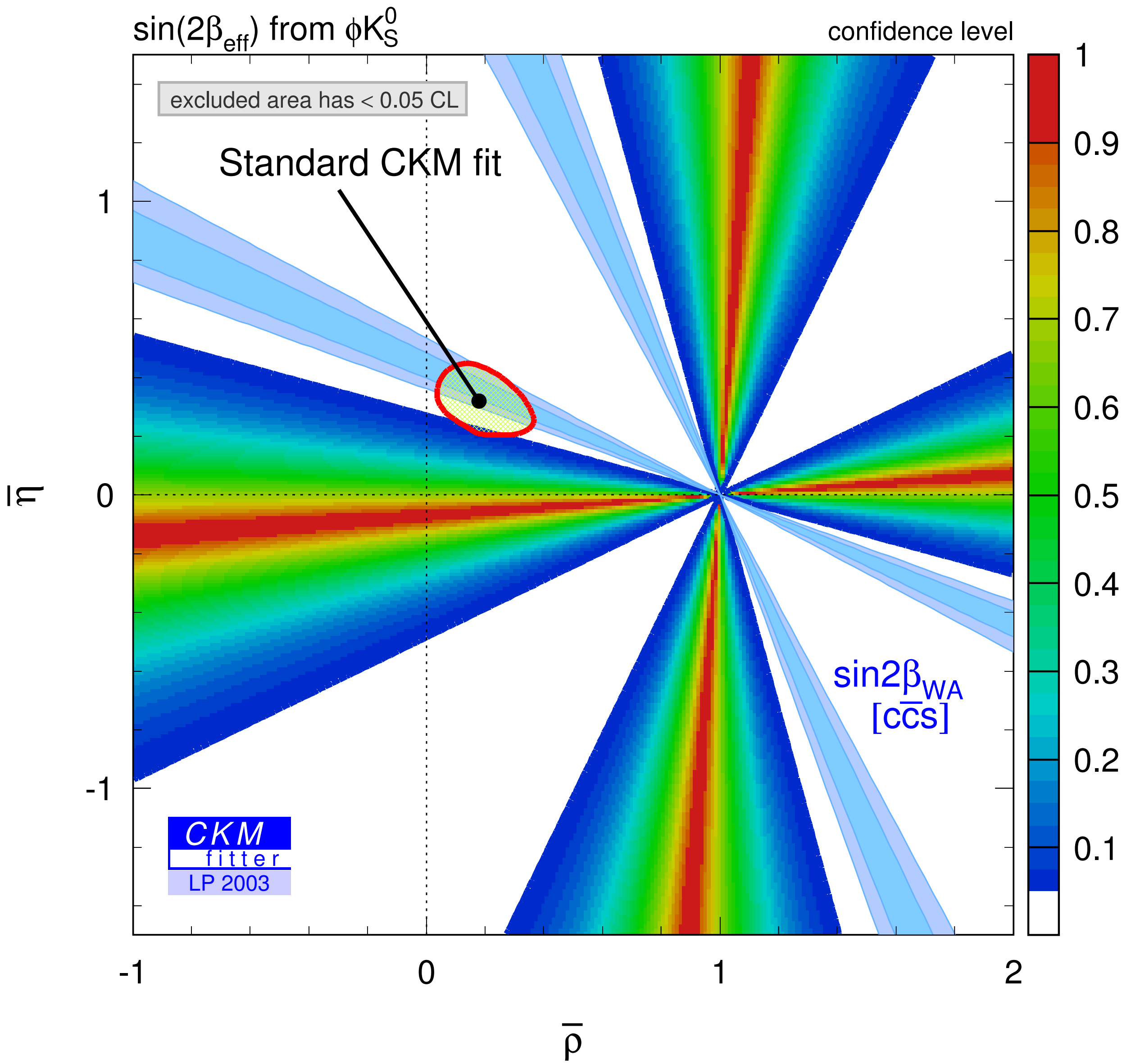


ψK_S: NP could enter through mainly q/p

ϕK_S: NP could enter through both q/p and $\overline{A}/A$

- Measuring same angle in decays sensitive to different short distance physics may be the key to finding deviations from the SM!

$B \to \phi K_S$ — present status



BABAR and BELLE:

$$S_{\psi K}^{(\mathrm{WA})} = 0.739 \pm 0.048$$

$$S_{\phi K}^{(\mathrm{BABAR})} = 0.45 \pm 0.43$$

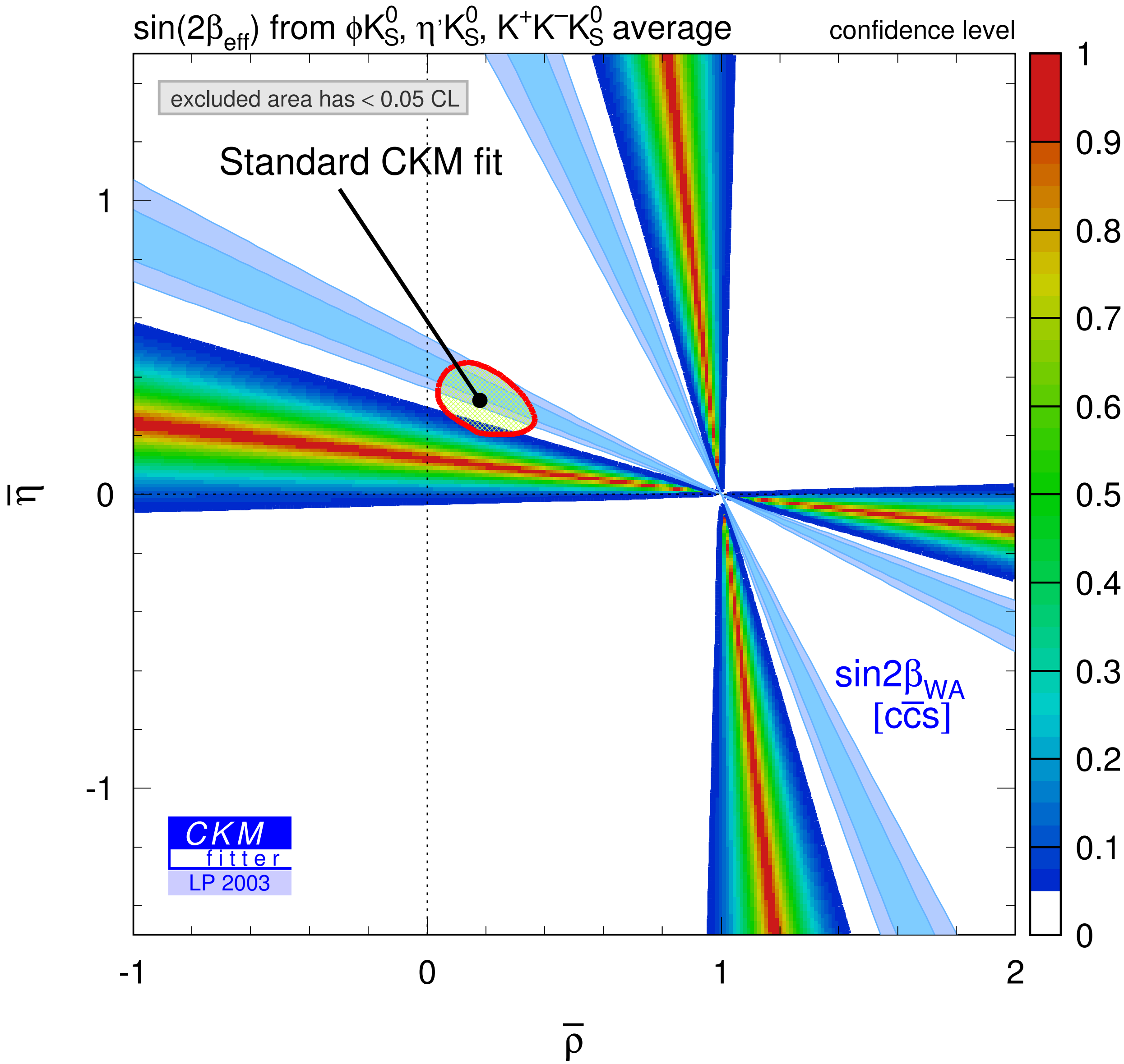
$$S_{\phi K}^{(\mathrm{BELLE})} = -0.96 \pm 0.51$$

$$S_{\phi K}^{(\mathrm{WA})} = -0.14 \pm 0.33$$

$$C_{\phi K}^{(\mathrm{WA})} = 0.09 \pm 0.23$$

$\Rightarrow$ S terms differ by $\sim 2.7\sigma$

$B \to \phi K_S, \eta' K_S, K^+K^-K_S$



BELLE:

$$S_{\eta' K_S} = 0.43 \pm 0.27$$

$$S_{K^+K^-K_S} = 0.51 \pm 0.26^{+0.18}_{-0.00}$$

BABAR:

$$S_{\eta' K_S} = 0.02 \pm 0.34$$

C terms consistent with 0

$\Rightarrow$ There is still a lot to be learned from future measurements!

The question

- How large should $S_{s\bar{s}s} - S_{\psi K}$ be, so that it is definitively due to new physics?

Disclaimer: The following bounds are NOT my best estimatetes of $S_{s\bar{s}s} - S_{\psi K}$ (That is not the question we were interested in)

- The successes of the SM are impressive:

  Any of $\sin 2\phi_1$, $B \to X_s\gamma$, Δm_B, Δm_K, $\epsilon_K^{(\prime)}$, ... could have shown new physics

- $\Rightarrow$ Only truly convincing deviations are likely to be interesting

The question

- How large should $S_{s\bar{s}s} - S_{\psi K}$ be, so that it is definitively due to new physics?

In the SM there is a second, CKM suppressed, term:

$$A_f \equiv A(B^0 \to f) = V_{cb}^* V_{cs}\, a_f^c + V_{ub}^* V_{us}\, a_f^u = V_{cb}^* V_{cs}\, a_f^c\, (1 + \xi_f)$$

$$\xi_f \equiv \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}} \frac{a_f^u}{a_f^c}\,, \qquad \left|\frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\right| = \mathcal{O}(\lambda^2)\,, \qquad \delta_f = \arg \frac{a_f^u}{a_f^c}$$

$$\Rightarrow\ -\eta_f S_f - \sin 2\phi_1 = 2\cos 2\phi_1\, \sin\phi_3\, \cos\delta_f\, |\xi_f|$$

$$C_f = -2\sin\phi_3\, \sin\delta_f\, |\xi_f|$$

$$C_f^2 + [(\eta_f S_f + \sin 2\phi_1)/\cos 2\phi_1]^2 = 4\sin^2\phi_3\, |\xi_f|^2$$

How large can $\xi_{\eta' K_S}$ be?

- $\mathcal{O}(0.04)$ [CKM suppression]
- $SU(3)$ relations [GLNQ]
- Quark model [London & Soni, hep-ph/9704277: ~ 0.02]
- BBNS [Beneke & Neubert, hep-ph/0210085: ~ 0.07]

The strategy

- For $b \to q\bar{q}s$ transitions:

  $$A_f = V_{cb}^* V_{cs}\, a_f^c + V_{ub}^* V_{us}\, a_f^u = V_{cb}^* V_{cs}\, a_f^c\, (1 + \xi_f)$$

  For $b \to q\bar{q}d$ transitions:

  $$A_{f'} = V_{cb}^* V_{cd}\, b_{f'}^c + V_{ub}^* V_{ud}\, b_{f'}^u = V_{ub}^* V_{ud}\, b_{f'}^u\, (1 + \lambda^2 \xi_{f'}^{-1})$$

- $SU(3)$ gives relations among a_f^q and $b_{f'}^q$: $a_f^u = \sum_{f'} x_{f'}\, b_{f'}^u$

  The branching ratios $\mathcal{B}(f)$ constrain a_f^c and $b_{f'}^u$: $\left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cs}} \frac{b_{f'}^u}{a_f^c} \right| \sim \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}$

- Combining $SU(3)$ and experimental data gives, conservatively:

  $$|\xi_f| \equiv \left| \frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}} \frac{a_f^u}{a_f^c} \right| < \left| \frac{V_{us}}{V_{ud}} \right| \sum_{f'} |x_{f'}| \sqrt{\frac{\mathcal{B}(f')}{\mathcal{B}(f)}}$$

  ... actually, the constraint is on $\left| \frac{\xi_f + (V_{us} V_{cd})/(V_{ud} V_{cs})}{1+\xi_f} \right|$, small difference if $\lambda^2 \ll \xi_f < 1$

$SU(3)$ relations for $B \to P_8 P_8$

$f^{(\prime)}$	A^{27}_{15}	A^{8}_{15}	$A^{8}_{\bar{6}}$	A^{8}_{3}	A^{1}_{3}
$\eta_8 K^0$	$4\sqrt{6}/5$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$-1/\sqrt{6}$	0
$K^0\pi^0$	$12\sqrt{2}/5$	$1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	0
$K^+\pi^-$	$16/5$	-1	1	1	0
$\eta_8 K^+$	$8\sqrt{6}/5$	$-\sqrt{3/2}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	0
$K^+\pi^0$	$16\sqrt{2}/5$	$3/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$	0
$K^0\pi^+$	$-8/5$	3	-1	1	0
$\eta_8\pi^0$	0	$5/\sqrt{3}$	$1/\sqrt{3}$	$-1/\sqrt{3}$	0
$\pi^0\pi^0$	$-13\sqrt{2}/5$	$1/\sqrt{2}$	$1/\sqrt{2}$	$1/(3\sqrt{2})$	$\sqrt{2}$
$\eta_8\eta_8$	$3\sqrt{2}/5$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/(3\sqrt{2})$	$\sqrt{2}$
$\pi^-\pi^+$	$14/5$	1	1	$1/3$	2
K^-K^+	$-2/5$	2	0	$-2/3$	2
$K^0\overline{K^0}$	$-2/5$	-3	-1	$1/3$	2
$\eta_8\pi^+$	$4\sqrt{6}/5$	$\sqrt{6}$	$-\sqrt{2/3}$	$\sqrt{2/3}$	0
$\pi^+\pi^0$	$4\sqrt{2}$	0	0	0	0
$K^+\overline{K^0}$	$-8/5$	3	-1	1	0

$H \sim (\bar{b}\, q_i)(\bar{q}_j q_k)$ transforms as

$$3 \times 3 \times \bar{3} = 15 + \bar{6} + 3 + 3$$

$$8 \times 8 = 27 + 10 + \overline{10} + 8_S + 8_A + 1$$

5 amplitudes describe 15 final states when $SU(3)$ breaking is neglected

For $\eta^{(\prime)}$ (singlet part), 3 more $B \to P_8 P_1$ matrix elements

$\Rightarrow$ Relations among the matrix elements

Decomposition of a^u_f and $b^u_{f'}$, identical with that of a^c_f and $b^c_{f'}$, although the matrix elements are independent $\Rightarrow$ use: $a(f) \equiv a^{u,c}_f$ and $b(f') \equiv b^{u,c}_{f'}$

The answer

- Best bound at present comes from: $(s \equiv \sin\theta_{\eta\eta'},\, c \equiv \cos\theta_{\eta\eta'})$

$$a(\eta' K^0) = \frac{s^2 - 2c^2}{2\sqrt{2}} b(\eta'\pi^0) - \frac{3sc}{2\sqrt{2}} b(\eta\pi^0) + \frac{\sqrt{3}s}{4} b(\pi^0\pi^0) - \frac{\sqrt{3}s(s^2+4c^2)}{4} b(\eta'\eta') + \frac{3\sqrt{3}sc^2}{4} b(\eta\eta) + \frac{\sqrt{3}c(2c^2-s^2)}{2\sqrt{2}} b(\eta\eta')$$

$$|\xi_{\eta' K_S}| < \left|\frac{V_{us}}{V_{ud}}\right| \left[0.59\sqrt{\frac{\mathcal{B}(\eta'\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.33\sqrt{\frac{\mathcal{B}(\eta\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.14\sqrt{\frac{\mathcal{B}(\pi^0\pi^0)}{\mathcal{B}(\eta' K^0)}} + 0.53\sqrt{\frac{\mathcal{B}(\eta'\eta')}{\mathcal{B}(\eta' K^0)}} + 0.38\sqrt{\frac{\mathcal{B}(\eta\eta)}{\mathcal{B}(\eta' K^0)}} + 0.96\sqrt{\frac{\mathcal{B}(\eta\eta')}{\mathcal{B}(\eta' K^0)}}\right]$$

$$\Rightarrow\ |\xi_{\eta' K_S}| < 0.36$$

Using bounds for charged modes

- Similar relations hold for the charged modes ($x =$ free param.)

$$a(\eta' K^+) = \frac{(3-x)cs}{2}\, b(\eta\pi^+) + \frac{(x-1)s^2+2c^2}{2}\, b(\eta'\pi^+)$$
$$+\frac{(x-3)s}{2\sqrt{3}}\, b(\pi^+\pi^0) + \frac{xs}{\sqrt{6}}\, b(\overline{K^0}K^+)$$

Using experimental data $\Rightarrow\ |\xi_{\eta' K^+}| < 0.09$

- We have $a^c_{\eta' K^0} = a^c_{\eta' K^+}$, but $a^u_{\eta' K^0} \neq a^u_{\eta' K^+}$

$a^u_{\eta' K^+}$ has a color-allowed tree diagram contribution
$a^u_{\eta' K^0}$ only arises from a color-suppressed tree diagram or penguins

Assumption: $|a^u_{\eta' K^0}| \not> |a^u_{\eta' K^+}|$ (large-N_c predicts r.h.s. larger)

$\Rightarrow\ |\xi_{\eta' K_S}| < 0.09$

Bounds for $B \to \phi K_S$

- For PV final state, more matrix elements... more complicated relations:

$$a(\phi K^0) = \frac{1}{2}\left[b(\overline{K^{*0}}K^0) - b(K^{*0}\overline{K^0})\right] + \frac{1}{2}\sqrt{\frac{3}{2}}\left[cb(\phi\eta) - sb(\phi\eta')\right]$$
$$+ \frac{\sqrt{3}}{4}\left[cb(\omega\eta) - sb(\omega\eta')\right] - \frac{\sqrt{3}}{4}\left[cb(\rho^0\eta) - sb(\rho^0\eta')\right]$$
$$+ \frac{1}{4}b(\rho^0\pi^0) - \frac{1}{4}b(\omega\pi^0) - \frac{1}{2\sqrt{2}}b(\phi\pi^0)$$

$\Rightarrow$ No bound on $\xi_{\phi K_S}$ with present data using only $SU(3)$

- Charged modes: $a(\phi K^+) = b(\phi\pi^+) + b(\overline{K^{*0}}K^+)$ (Grossman, Isidori, Worah, hep-ph/9708305)

Contrary to $\eta' K_S$, now $a^u_{\phi K^0}$ and $a^u_{\phi K^+}$ are of same order in large-N_c

Dynamical assumption: $|a^u_{\phi K^0}| \not> |a^u_{\phi K^+}|$ $\Rightarrow$ $|\xi_{\phi K_S}| < 0.25$

“We missed”

“We missed”: $B \to \pi^0 K_S$

- Dominantly $b \to d\bar{d}s$ instead of $b \to s\bar{s}s$, but same physics (bound $b \to u\bar{u}s$ tree)

BABAR (113/fb): $S_{\pi^0 K_S} = 0.48^{+0.38}_{-0.47} \pm 0.11$,

$$C_{\pi^0 K_S} = 0.40^{+0.27}_{-0.28} \pm 0.10$$

- Amplitude relation:

$$a(\pi^0 K_S) = \frac{1}{\sqrt{2}} b(K^+ K^-) - b(\pi^0 \pi^0)$$

Follows from table shown 4 pages earlier... not noticed until asked by Babarians

$\Rightarrow \; |\xi_{\pi^0 K_S}| < 0.15$ (Gronau, Grossman, Rosner, to appear)

Yet another very interesting mode!

Three-body decays

$$K^+K^-K_S$$

CP decomposition — BELLE

- $K^+K^-K_S$ does not have definite CP, so more complicated than 2-body decays

Consider only $b \to sg$ penguin diagrams: $I=0$; initial $B \in \frac{1}{2} \;\Rightarrow\; K^I \overline{K}^J K^L \in \frac{1}{2}$
Only 2 isospin amplitudes [$(K^I K^L) \in \{0,\,1\}$], no interference in total rates

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Denote: $A_{IJL}(p_1,p_2,p_3) \equiv A[B \to K^I(p_1)\overline{K}^J(p_2)K^L(p_3)] \qquad I,J,L=\{+,-,0,S\}$

Then: $\quad A_{00+} = A_{+-0}\,, \qquad A_{+00} = A_{0-+}\,, \qquad A_{000} = A_{+-+}$

- Can write H as: $H \propto (B^i K_i)\left[x\,(K^j K_j)_{l=\mathrm{even}} + \sqrt{1-x^2}\,(K^j K_j)_{l=\mathrm{odd}}\right]$

$B \to K^+K^-K^0$: $CP(K^+K^-) = +1 \;\Rightarrow\; CP(K^+K^-K_S) = (-1)^l$
$\Rightarrow\; x^2$ gives the CP-even fraction

$B \to K^0\overline{K^0}K^+$: $l=$ even is $K_SK_S + K_LK_L$, $l=$ odd is K_SK_L
$\Rightarrow\; x^2 = 2\Gamma_{+SS}/\Gamma_{+00} = 2\Gamma_{+SS}/\Gamma_{+-0} = 0.97 \pm 0.15 \pm 0.07$

BELLE analysis (cont'd)

- Once we know the CP-even fraction, x^2, and it is near 1, we're in good shape:

$$S_{KKK} = \frac{S_{KKK}^{\rm exp}}{2\,x^2 - 1}$$

... S_{KKK} is the would-be S, if $K^+K^-K_S$ had a definite CP $\Rightarrow$ DONE!

- N.B.: predictions of isospin symmetry that enter this analysis are not yet tested

$$\mathcal{B}_{+-+} = (3.30 \pm 0.18 \pm 0.32) \times 10^{-5}$$

$$\mathcal{B}_{+-0} = (2.93 \pm 0.34 \pm 0.41) \times 10^{-5}$$

$$\mathcal{B}_{+SS} = (1.34 \pm 0.19 \pm 0.15) \times 10^{-5}$$

$$\mathcal{B}_{SSS} = (0.43^{+0.16}_{-0.14} \pm 0.75) \times 10^{-5}$$

Isospin does not imply $\mathcal{B}_{+-+} \approx \mathcal{B}_{+-0}$; a test requires measuring a rate with K_L, e.g.: $\mathcal{B}_{+-S} = \frac{1}{2}\mathcal{B}_{+SL} + \mathcal{B}_{+SS}$

Full isospin analysis

- When the $b \to u\bar{u}s$ part of H is included, $H \in \{0,\, 1\}$

  With $K^I \overline{K}^J K^L \in \{\frac{1}{2},\, \frac{3}{2}\}$ — 5 independent isospin amplitudes [$(K^I K^L) \in \{0,\, 1\}$]

Only a single amplitude relation remains:

$$A_{000} + A_{+-+} + A_{+00} + A_{00+} + A_{+-0} + A_{0-+} = 0$$

Both relations used before:

$$\Gamma_{+-S}(l = \text{even})/\Gamma_{+-S} = \Gamma_{+00}(l = \text{even})/\Gamma_{+00}$$

$$\Gamma_{+00} = \Gamma_{+-0}$$

are corrected by terms proportional to the ratio of $I \in 1$ and $I \in 0$ contributions

... At present no constraint on these from data

N.B.: Large isospin violation: $\mathcal{B}(\phi \to K^+K^-) \approx 49\%$ and $\mathcal{B}(\phi \to K_S K_L) \approx 34\%$ can be understood arising due to phase space; contribution to x^2 is only $\mathcal{O}(4\%)$

U-spin analysis

- Even if the $I=1$ part in H were negligible, so determination of CP-even fraction in $K^+K^-K_S$ very precise, it would not imply $-S_{KKK}=\sin 2\phi_1$ to same precision

  The $b \to u\bar{u}s$ tree has $I=0$ part, which would not affect extraction of CP-even fraction, but would shift S_{KKK} from $\sin 2\phi_1$

- U-spin ($d \leftrightarrow s$) relates $B^+ \to h_i^+ h_j^- h_k^+$ modes: B^+ is singlet, (K^+, π^+) is doublet

  $b \to (\bar{u}u + \bar{d}d + \bar{s}s)q$ penguin and $b \to u\bar{u}q$ tree amplitudes ($q = d, s$) are $\Delta U = 1/2$

  $\Rightarrow$ 2 U-spin amplitudes for $B^+ \to h_i^+ h_j^- h_k^+$

  U-spin relation: $a(K^+K^-K^+) = b(\pi^+\pi^-\pi^+)$ [same accuracy as $SU(3)$]

  Experimental data: $\Rightarrow |\xi_{KKK}| = \left|\frac{V_{us}}{V_{ud}}\right| \sqrt{\frac{\mathcal{B}(B^+ \to \pi^+\pi^-\pi^+)}{\mathcal{B}(B^+ \to K^+K^-K^+)}} \approx 0.13\,(\pm 0.06?)$

Conclusions

Summary of results

$$|\xi_{\eta' K_S}| < \begin{cases} 0.36 & SU(3) \\ 0.09 & SU(3) \text{ + leading } N_c \text{ assumption} \end{cases}$$

$$|\xi_{\phi K_S}| < \quad 0.25 \quad SU(3) \text{ + non-cancellation assumption}$$

$$|\xi_{K^+ K^- K_S}| \sim \quad 0.13 \quad U\text{-spin}$$

These bounds also constrain $|C_f|$: $C_f = -2 \sin\phi_3 \, \sin\delta_f \, |\xi_f|$

Conclusions

- These bounds are weaker than estimates based on explicit calculations, but have the advantage of being model independent
- $SU(3)$ breaking effects could be significant, but the bounds are probably still very conservative
- The same bounds also apply in MFV models
- With more data, the bounds will improve

If experiments find deviations larger than our bounds
$\Rightarrow$ A convincing case for new physics