Update of *CP* Violation Study in $B^0 \to \pi^+\pi^-$ decay

KEK seminar

Jan 20, 2004

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for

the Belle Collaboration

Outline

1. Introduction
2. Experimental Apparatus (**KEKB** & **Belle**)
3. Analysis procedure
   - Event and time reconstruction for $B^0 \to \pi^+\pi^-$ events
   - Time-dependent analyses
4. Results
   - *CP*-asymmetry parameters $A_{\pi\pi}$ and $S_{\pi\pi}$
   - Constraints on the CKM angle ϕ_2
5. Prospects
6. Conclusion

Brand new result !

Paper has just been submitted to

Phys. Rev. Lett !

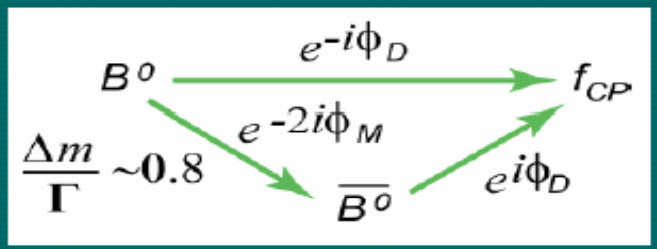
hep-ex/0401029

Belle preprint 2004-1

KEK preprint 2003-110

1. Introduction

CP violation in $B^0\overline{B}^0$ system



$$A_{CP}(t) \equiv \frac{\Gamma(\overline{B^0} \to f_{CP}; t) - \Gamma(B^0 \to f_{CP}; t)}{\Gamma(\overline{B^0} \to f_{CP}; t) + \Gamma(B^0 \to f_{CP}; t)} = A_f \cos(\Delta m t) + S_f \sin(\Delta m t)$$

A_f, S_f: CP violating parameters

$$A_f = \frac{|\lambda_f|^2 - 1}{|\lambda_f|^2 + 1} \qquad S_f = \frac{2Im(\lambda_f)}{|\lambda_f|^2 + 1}$$

$$\lambda_f \equiv e^{-2i\phi_M} \frac{A(\overline{B} \to \overline{f})}{A(B \to f)}$$

Standard model predictions

	$b \to c\bar{c}s$	$b \to c\bar{c}d$	$b \to s\bar{s}s$	$b \to u\bar{u}d$
example	$J/\psi Ks$	$J/\psi \pi^0$	ϕKs	$\pi^+\pi^-$
A_f	0	0	small	$\neq 0$
S_f	$\sin 2\phi_1$	$\sin 2\phi_1$	$\sin 2\phi_1$	"$\sin 2\phi_2$"

Note: $A_f \neq 0 \Rightarrow \Gamma(B \to f) \neq \Gamma(\overline{B} \to \overline{f}) \Rightarrow$ direct CP violation.

CPV : $A_f \neq 0$, $BS_f \neq 0$

time integrated case : sin ***term*** (***with*** $S_{\pi\pi}$) ***vanishes***

cos ***term*** (***with*** $A_{\pi\pi}$) ***remains***

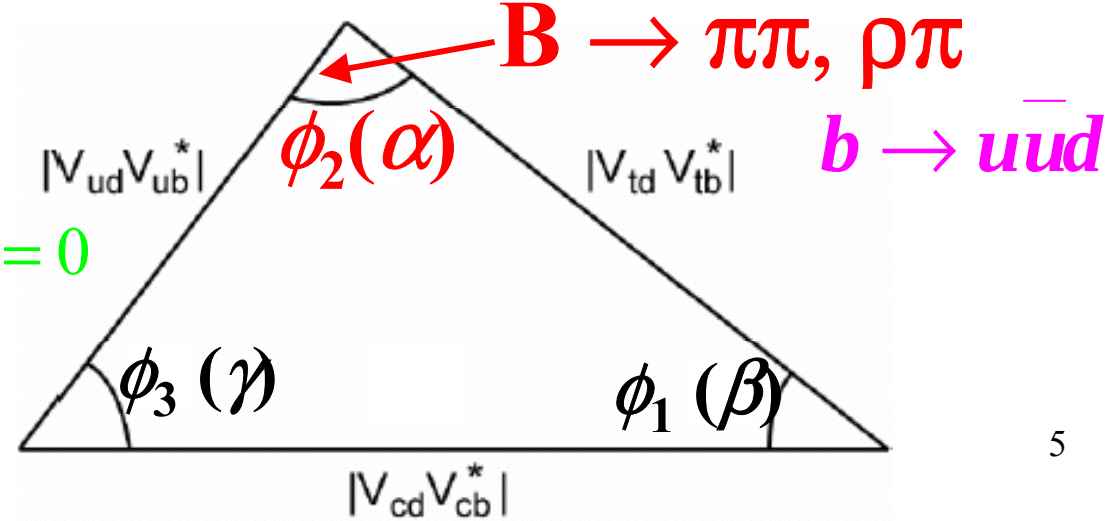
Quark mixing is described by the 3x3 Cabibbo-Kobayashi-Maskawa (CKM) matrix. (Wolfenstein parametrization)

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

One irreducible **complex phase** derives ***CP*** violation .

Unitarity triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



CKM angle ϕ_1

In 2001 (*Phys. Rev. Lett. 87, 091802 (2001)*)
Observation of large CP violation in the Neutral *B* meson system

$b \to c\bar{c}s$ **modes** ($B^0 \to J/\psi K_S$ etc.)

$$\sin 2\phi_1 = 0.99 \pm 0.14 \pm 0.06 \quad \text{(Belle)}$$

Latest* sin2ϕ_1 *results (HFAG Summer 2003)

Belle	$0.733 \pm 0.057 \pm 0.028$
BaBar	$0.741 \pm 0.067 \pm 0.034$
Ave.	0.736 ± 0.049

Points of $B^0 \to \pi^+\pi^-$ decay

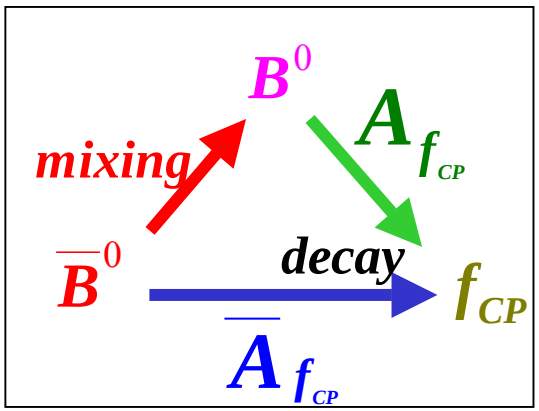
- sensitive to the measurement of CKM angle ϕ_2

- **direct *CP* violation** may also occur
  because of interference between
  the $b \to u$ ***tree*** amplitude and
  the $b \to d$ ***penguin*** amplitude

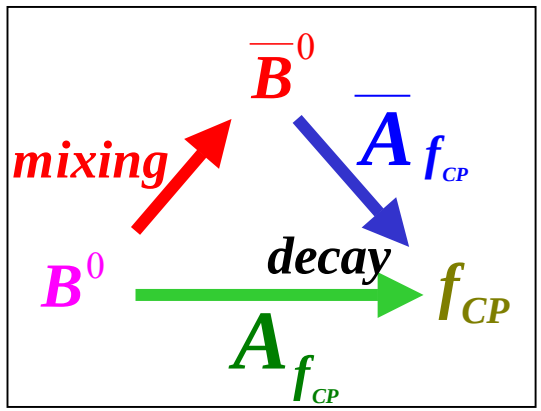
$$\left|\frac{P}{T}\right| \sim 0.3$$

- It's very important to check the KM matrix by other measurements.

CP violation in mixing and decay



$\neq$



$$\lambda_{f_{CP}} = \frac{q}{p} \cdot \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}}$$

$$f_{CP} = \pi^+\pi^-$$

***CP* violation** in **neutral *B* meson** results from
the **interference** **between** **decays** **with** and **without** **mixing.**

(Sππ)

Time-dependent CP asymmetries in $B^0 \to \pi^+\pi^-$ -- the best way to access the CKM angle ϕ_2

$$P^{q}_{\pi\pi}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}[1 + q \cdot \{S_{\pi\pi}\sin(\Delta m_d \Delta t) + A_{\pi\pi}\cos(\Delta m_d \Delta t)\}]$$

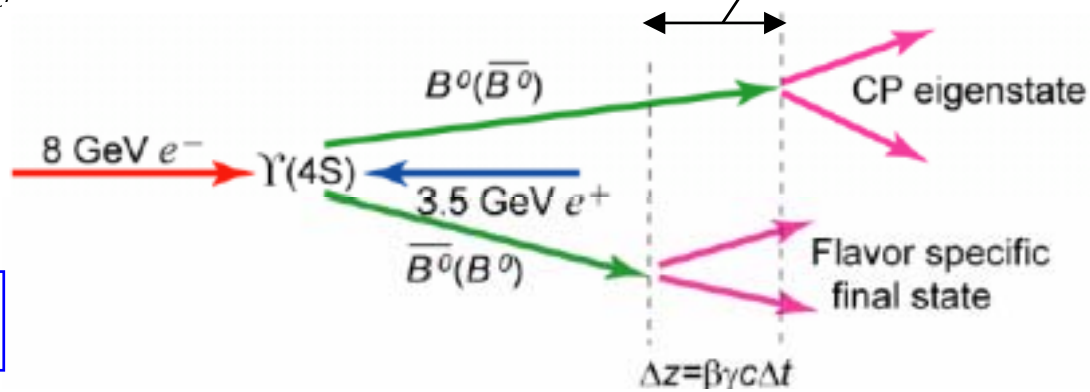
$q = \begin{cases} +1 & \text{for } B^0 \text{ tag} \\ -1 & \text{for } \overline{B}^0 \text{ tag} \end{cases}$

$\Delta t = \Delta z/(\beta\gamma c)$

$$A_{\pi\pi} = \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1}, \quad S_{\pi\pi} = \frac{2Im\lambda}{|\lambda|^2 + 1},$$

$$\lambda \equiv e^{-2i\phi_M}\frac{A(\overline{B} \to \overline{f})}{A(B \to f)}$$

direct CP asymmetry



$$S_{\pi\pi} = \sqrt{1 - A_{\pi\pi}^2}\,\sin 2\phi_{2eff}$$

$$= \sqrt{1 - A_{\pi\pi}^2}\,\sin 2(\phi_2 + \theta)$$

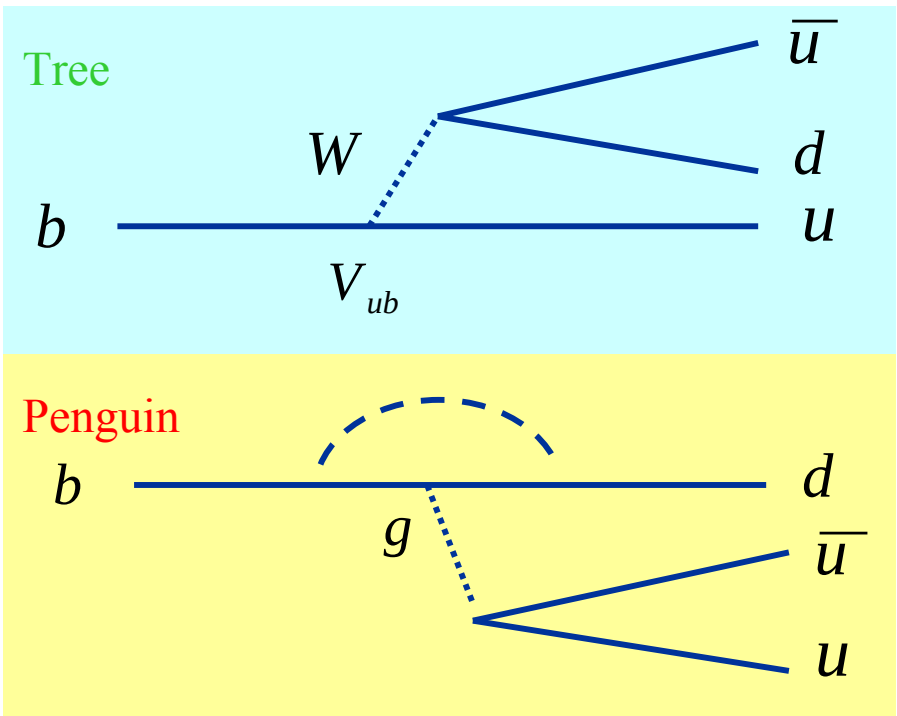
$A_{\pi\pi} = 0,\ S_{\pi\pi} = \sin 2\phi_2$

if "penguin" is negligible

Direct *CP* violation in $B^0 \to \pi^+\pi^-$

Penguin diagram is not negligible at all.

$|P/T| \sim 0.3$ (theoretical preference)



Nasty "pollution" for precise ϕ_2 measurements

Wonderful "contribution" for the first observation of direct *CP* violation in *B*-meson decays !

Previous results at Belle

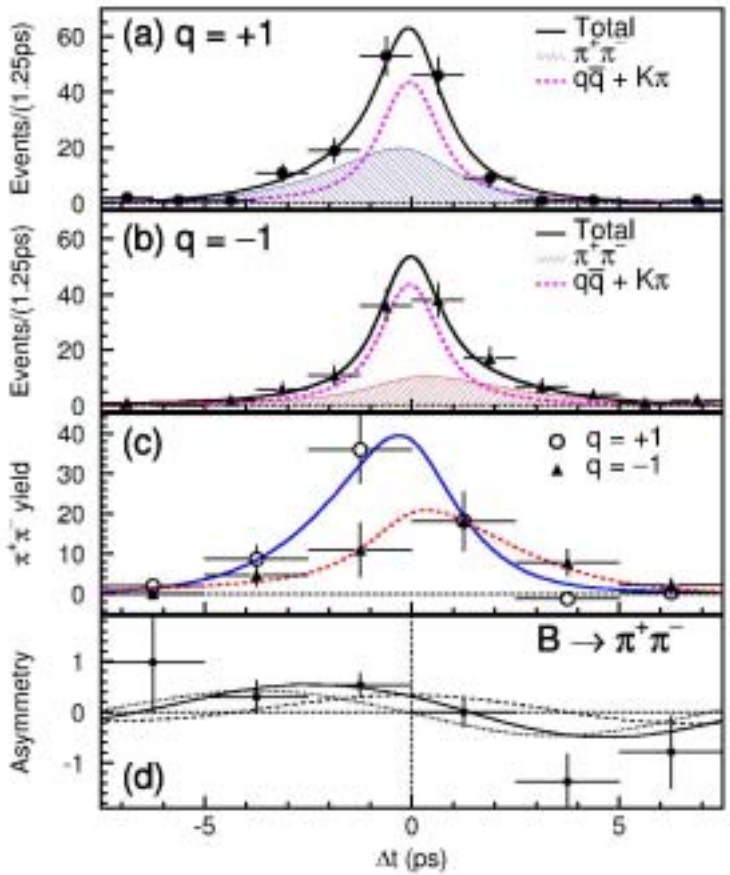
PRD **68**, 012001 (2003)

- 85 million B-meson pairs ($78 fb^{-1}$)
- 760 $B^0 \to \pi^+\pi^-$ candidates

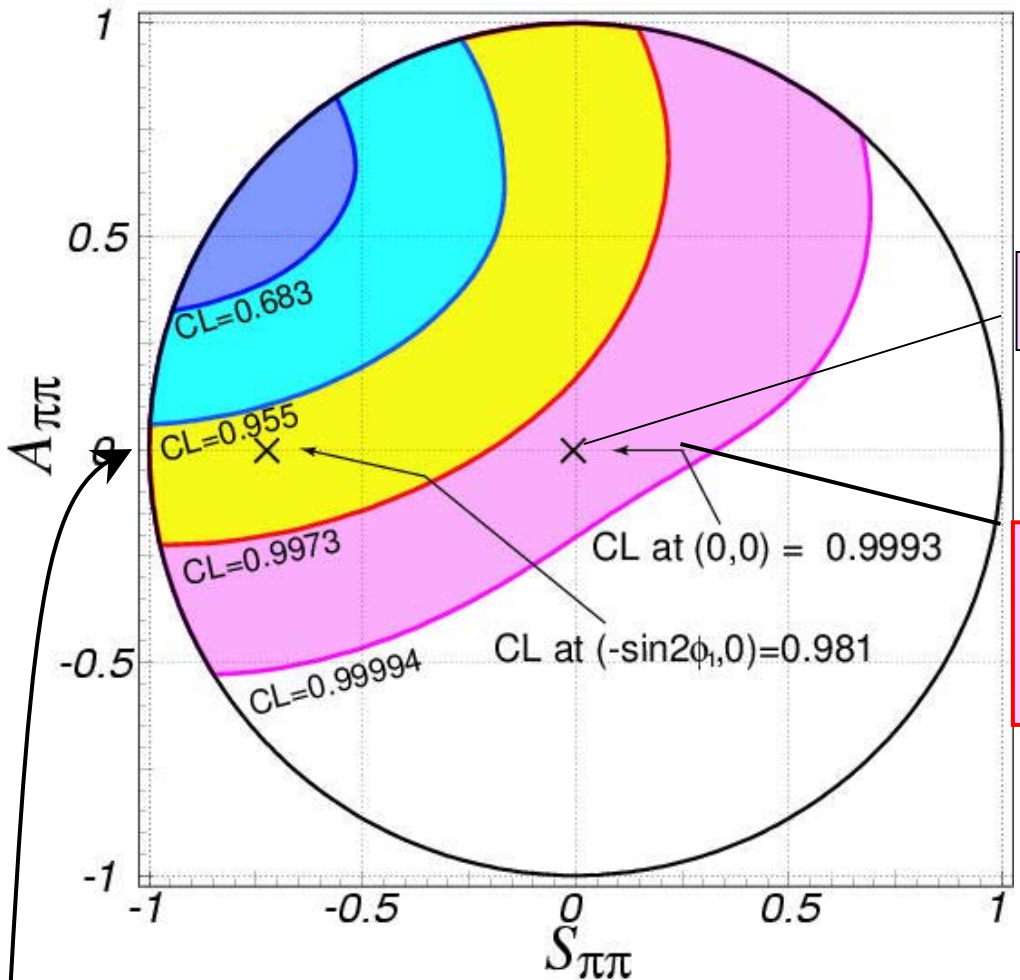
$$S_{\pi\pi} = -1.23 \pm 0.41 \,{}^{+0.08}_{-0.07}$$

$$(stat.)\ (syst.)$$

$$A_{\pi\pi} = +0.77 \pm 0.27 \pm 0.08$$



Previous results at Belle : Confidence Regions



CP conserving case (0,0)

1) Evidence for CP violation in $B^0 \to \pi^+\pi^-$ (**3.4σ**)

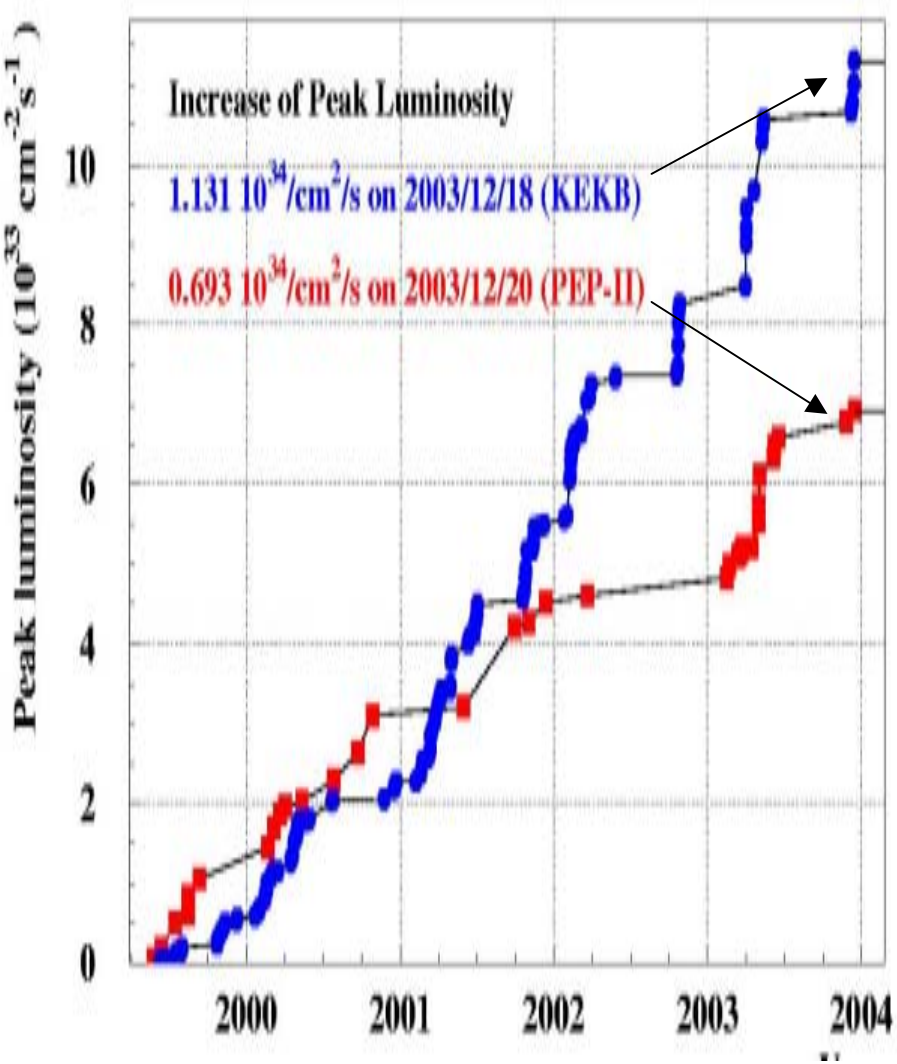
2) "Indication" of **direct CP Violation** ($A_{\pi\pi} \neq 0$) (>2.2σ)

Analyzed data

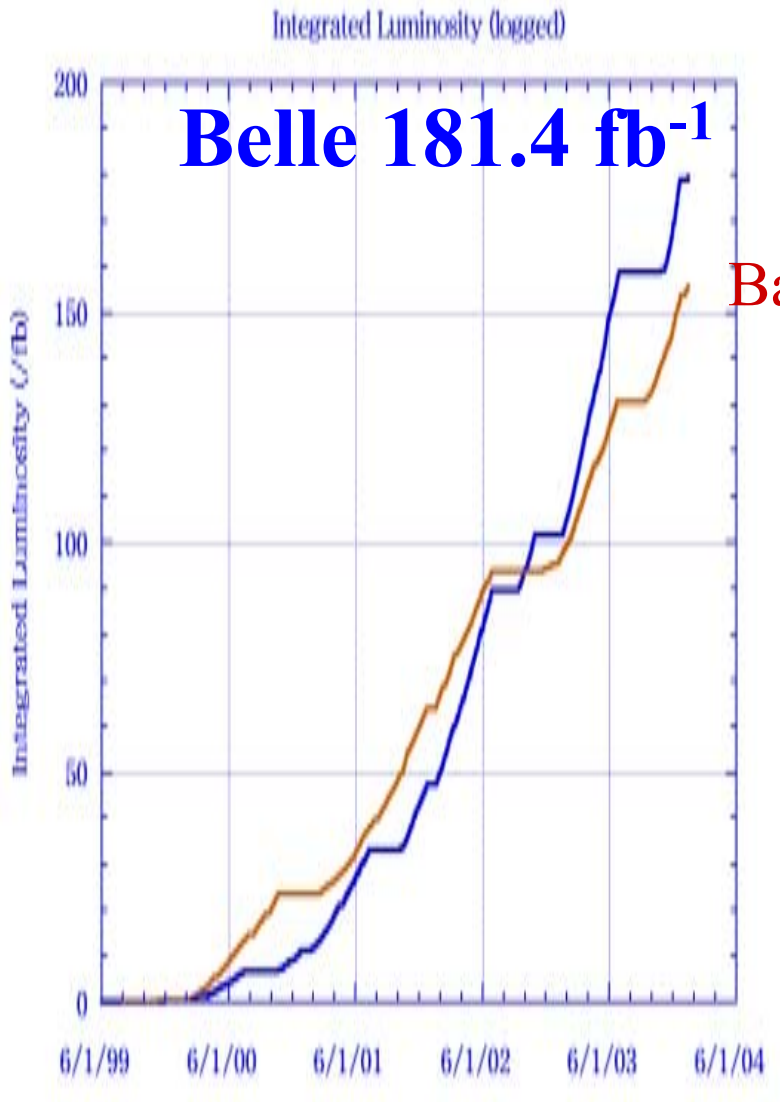
152 million $B\bar{B}$ events (**$140fb^{-1}$**)
at the **Y(4S)** resonance
recorded by the **Belle**
before the summer 2003
are used in this analysis.

2. Experimental apparatus
KEKB & Belle

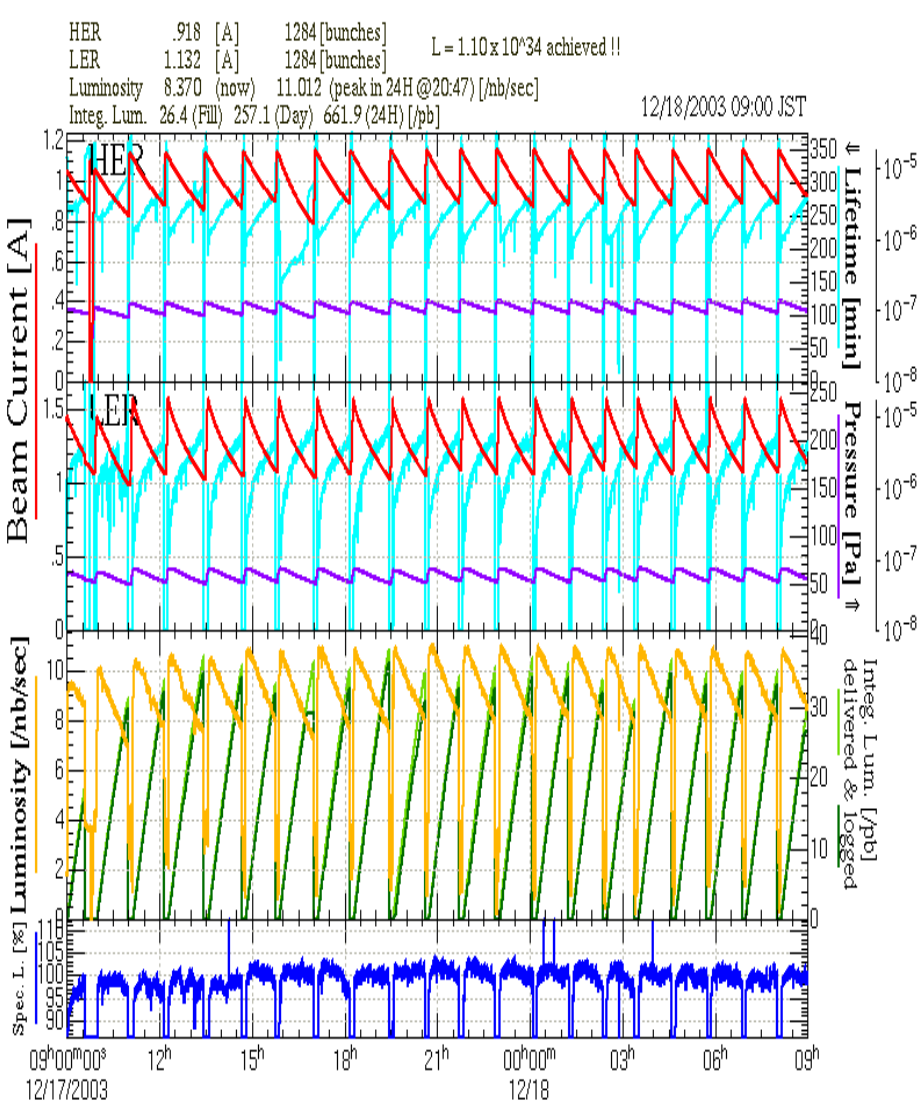
Peak Luminosity



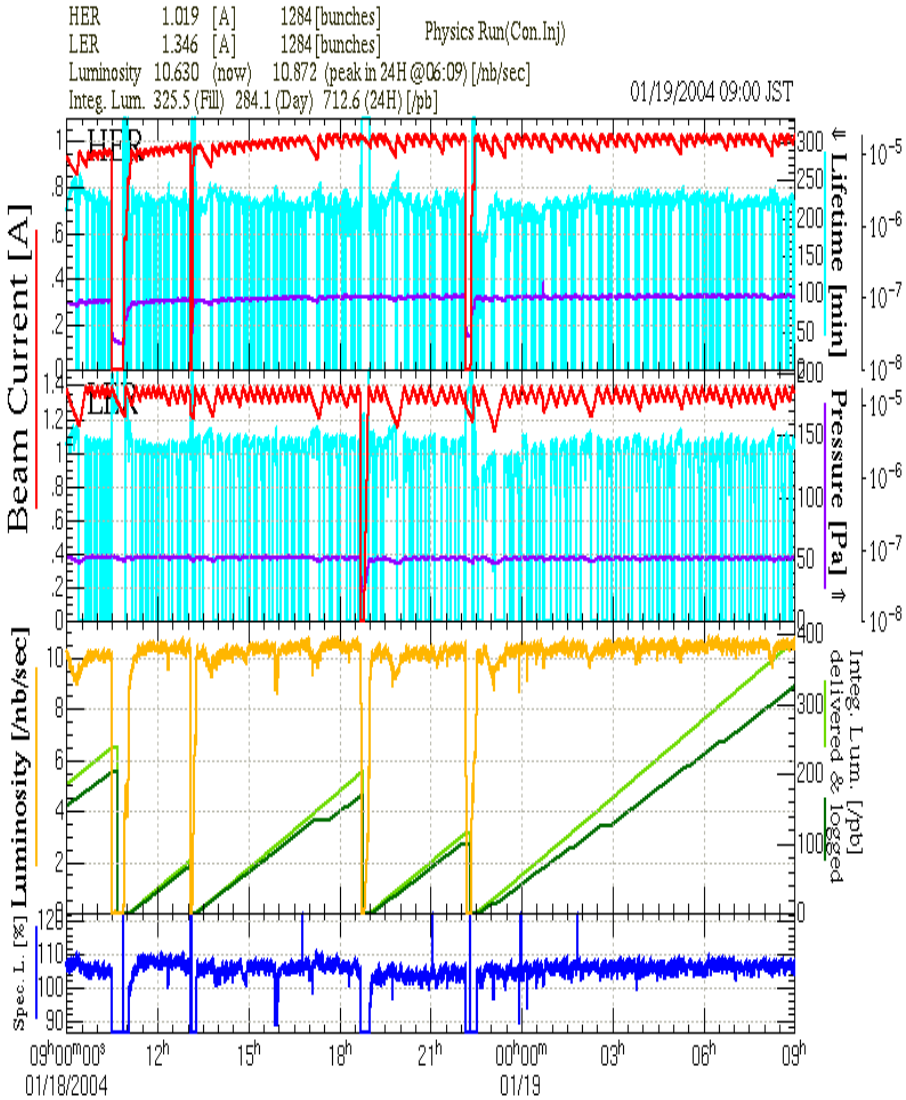
Integrated Luminosity



Normal injection

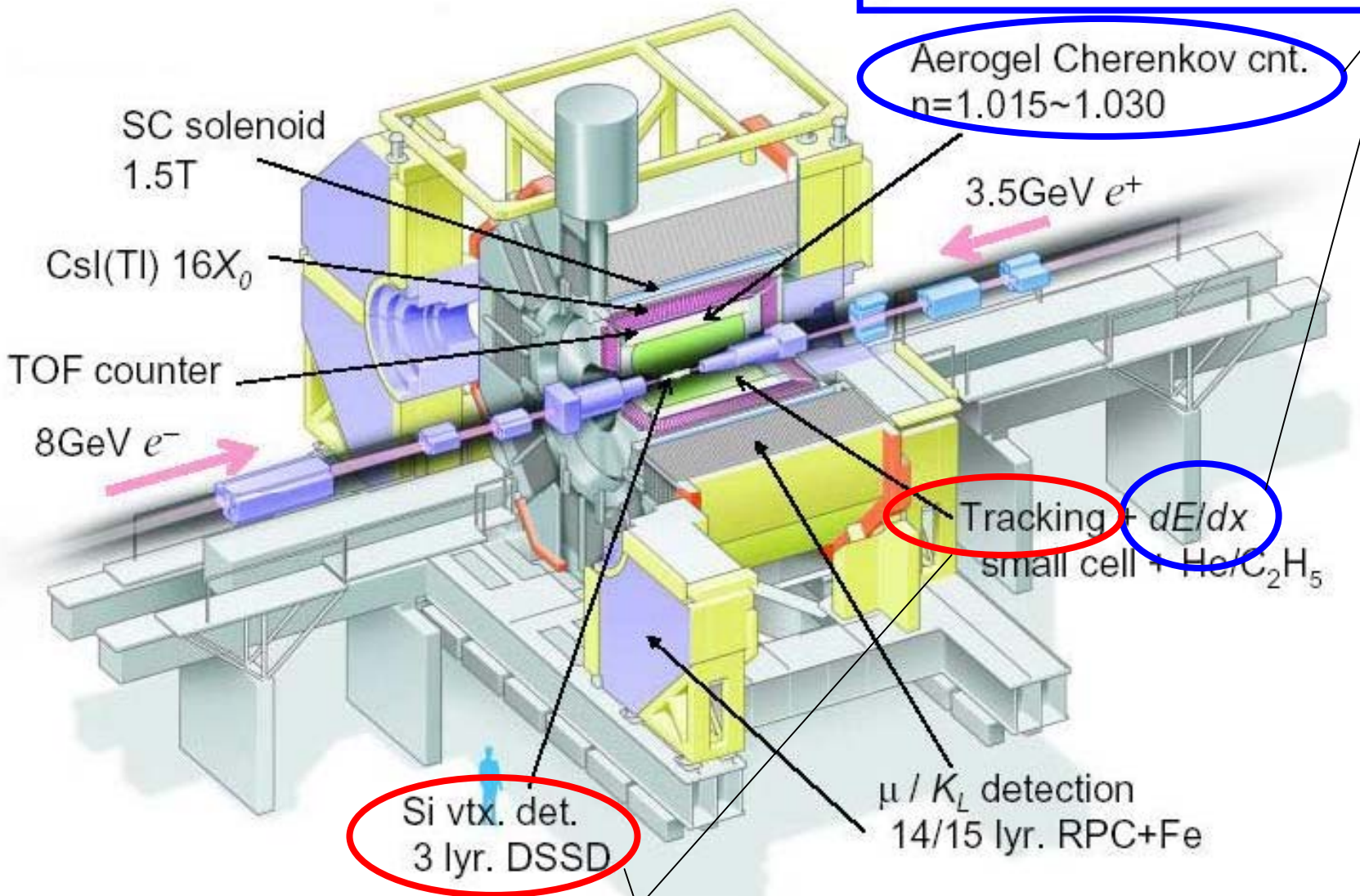


Continuous beam injection



Belle detector

K/π identification for high momentum tracks



Vertex resolutions: ($\sigma(z_{cp}) = 75\mu m$; $\sigma(z_{tag}) = 140\mu m$)

Changes in the new analysis

- More data ! [152×10^6 B pairs (140 fb^{-1})]
  (previous 78 fb^{-1} + new 62 fb^{-1} sample)
- Improvements in the analysis
  - **Better signal yield extraction**
    - **1D fit in ΔE $\rightarrow$ 2D fit in ΔE-Mbc plane**
  - **Improved continuum background rejection**

3. Analysis procedure

Event reconstruction

Flow: $B^0 \to \pi^+\pi^-$ Selection → Flavor tagging → Continuum suppression → Vertex and Δt

Kinematics

Beam-constrained mass

$$M_{bc} \equiv \sqrt{E^{*}_{beam}{}^{2} - p^{*}_{B}{}^{2}}$$

3σ requirements (σ = resolution)

signal region

$$5.271 < M_{bc} < 5.287\ GeV/c^2$$

Energy difference

$$\Delta E \equiv E^{*}_{B} - E^{*}_{beam}$$

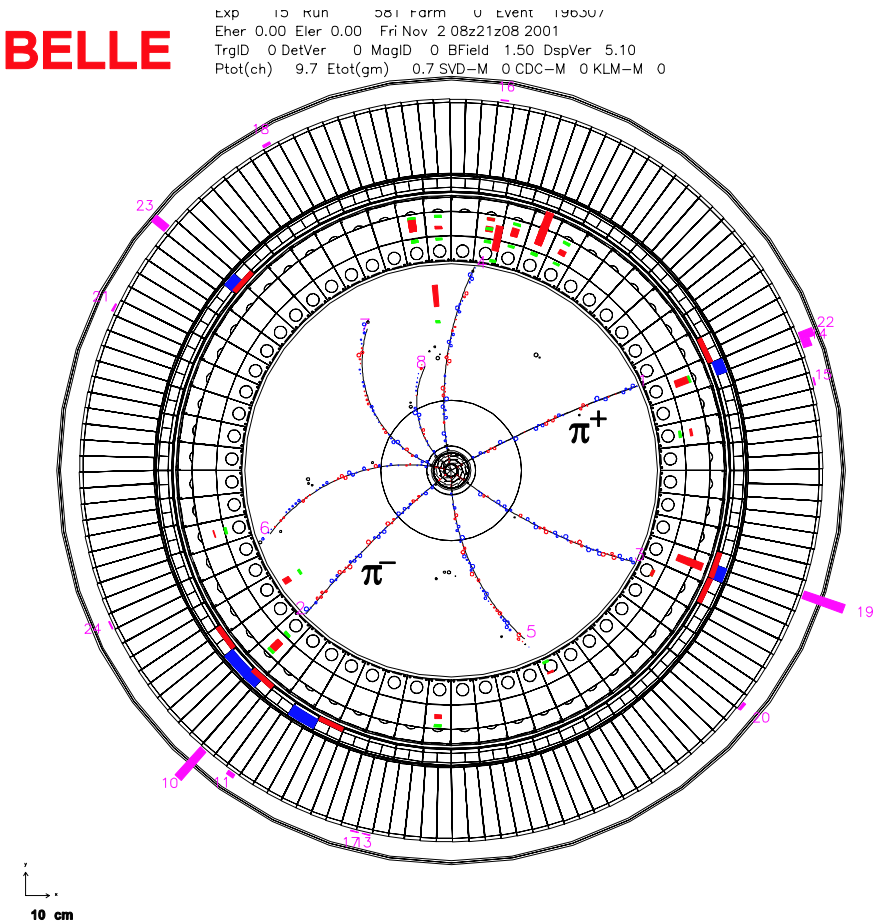
$$|\Delta E| < 0.064\ GeV$$

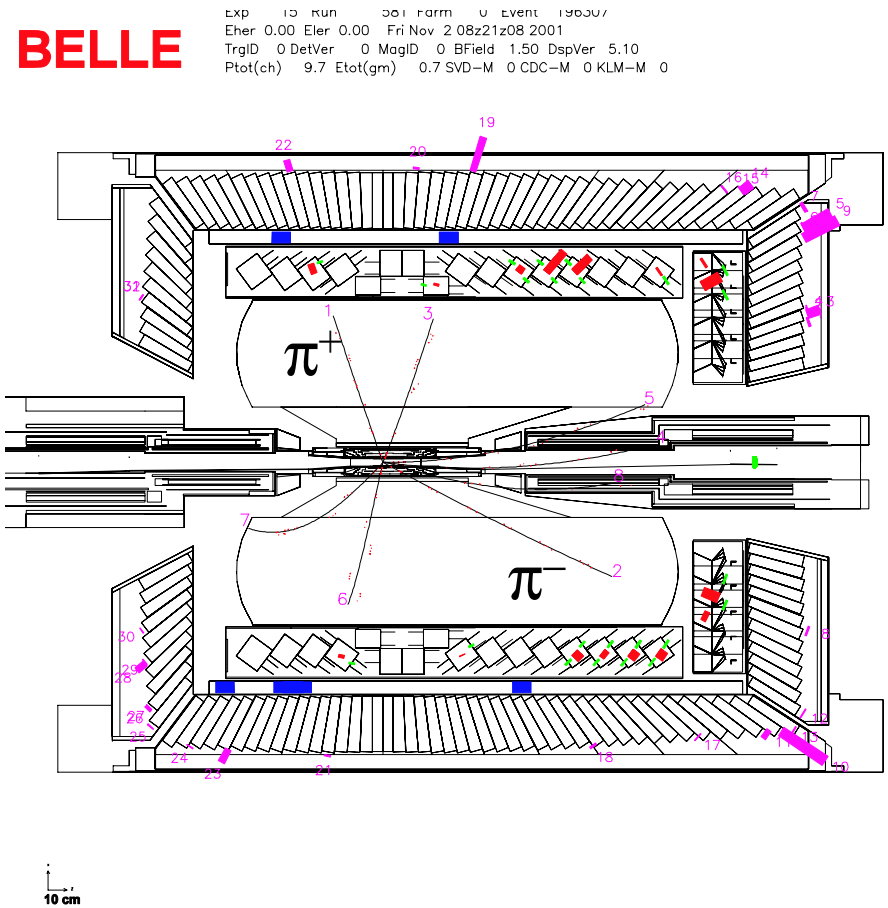
E^{*}_{beam} : *cms beam energy*

E^{*}_{B} : *cms energy of B candidate*

p^{*}_{B} : *cms momentum of B candidate*

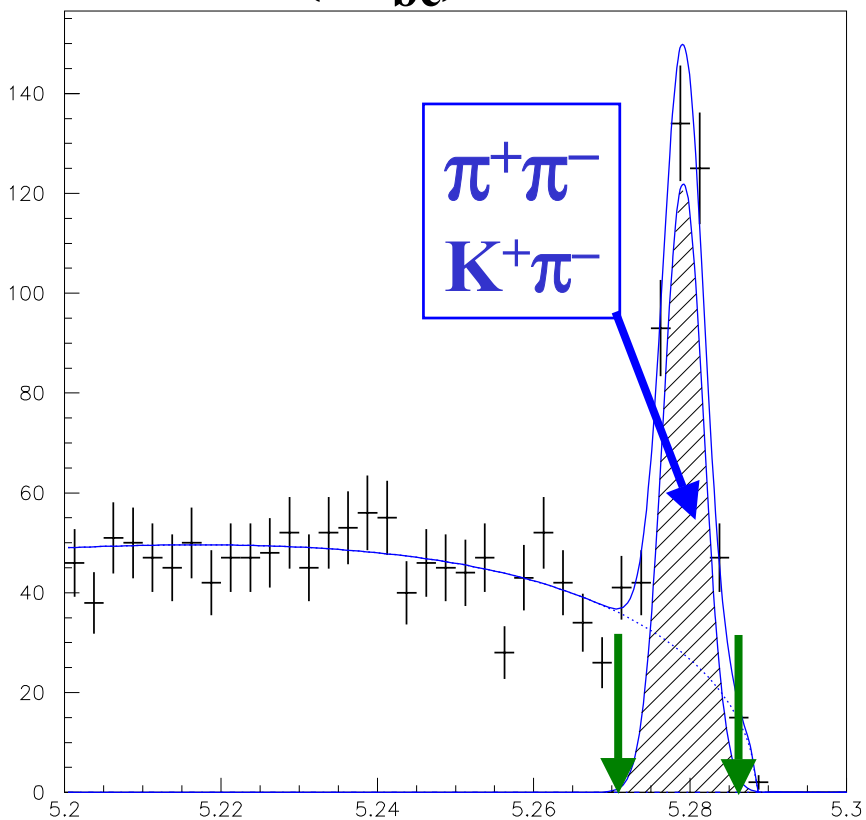
$B^0 \to \pi^+\pi^-$ example



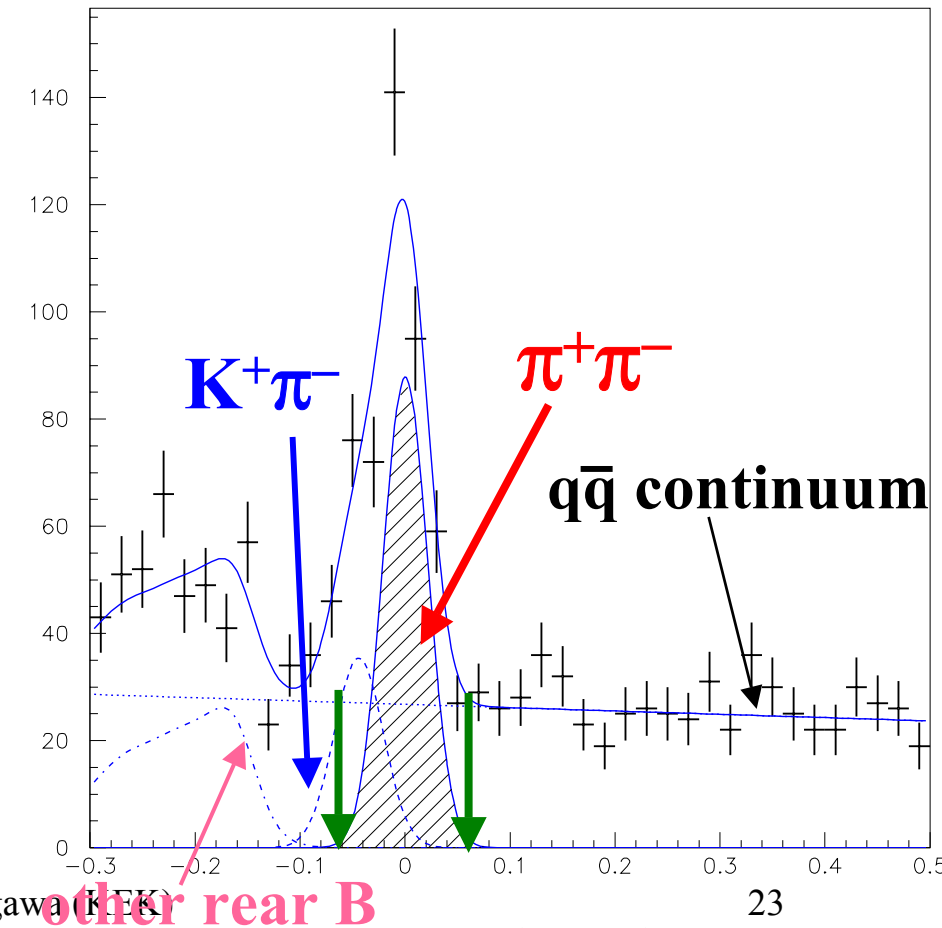


Event reconstruction

Beam-constrained mass (M_{bc})

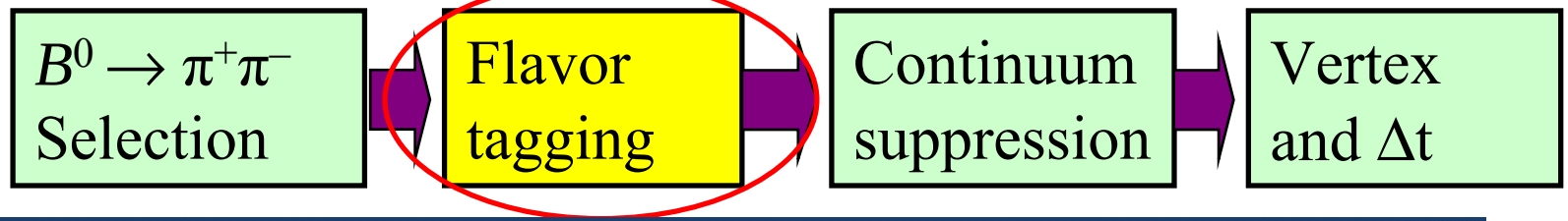


energy difference (ΔE)



Event and time reconstruction (2)

Flow



Identify B^0/ $\overline{B^0}$ by the charges of the inclusive decay products.

Use flavor specific properties and correlations

- Inclusive leptons:
  - high p l^- $\quad b \to c\, l^- \nu$
  - intermed. p l^+ $\quad \hookrightarrow s\, l^+ \nu$
- Inclusive hadrons:
  - high p π^+ $\quad B^0 \to D^{(*)-}\pi^+$, $D^{(*)-}\rho^+$, etc.
  - intermed. p K^+ $\quad \hookrightarrow K^+ X$, $\quad \hookrightarrow \pi^+\pi^0$
  - low p π^- $\quad \hookrightarrow \overline{D^0}\,\pi^-$

Event and time reconstruction (2)

Flow

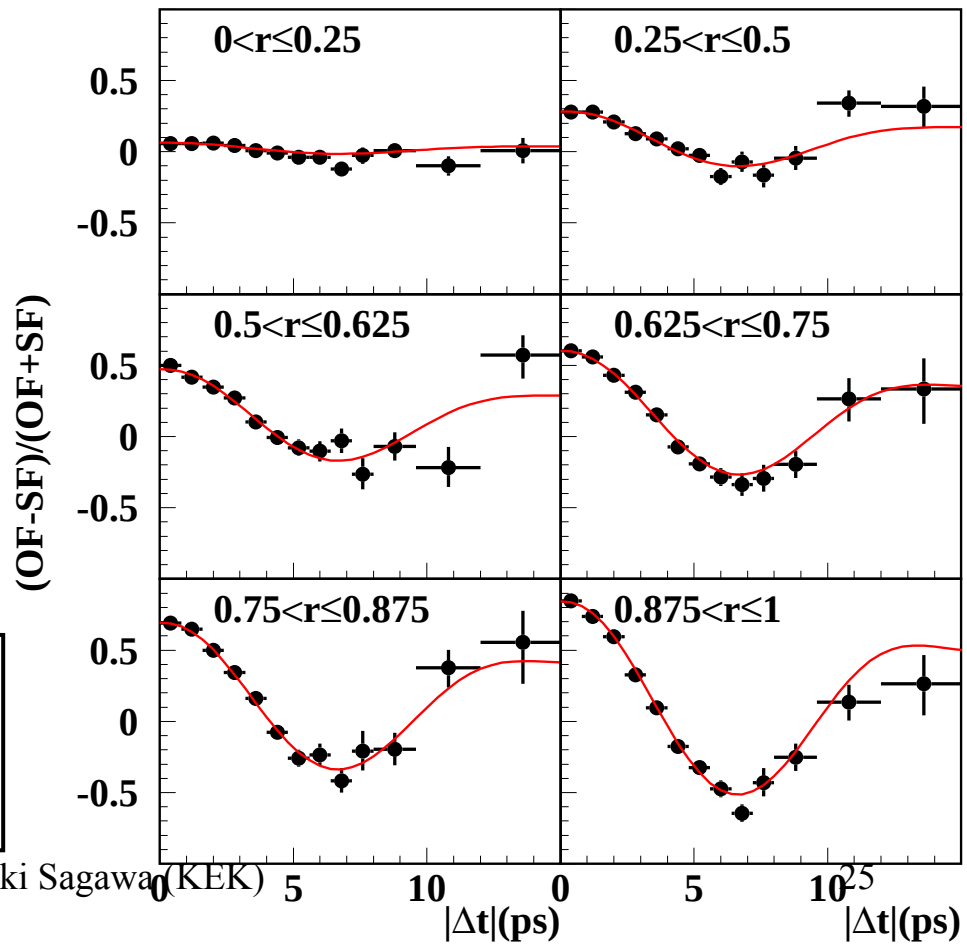
$B^0 \to \pi^+\pi^-$ Selection → **Flavor tagging** → Continuum suppression → Vertex and Δt

Classify events based on expected dilution

Flavor tag quality **r**

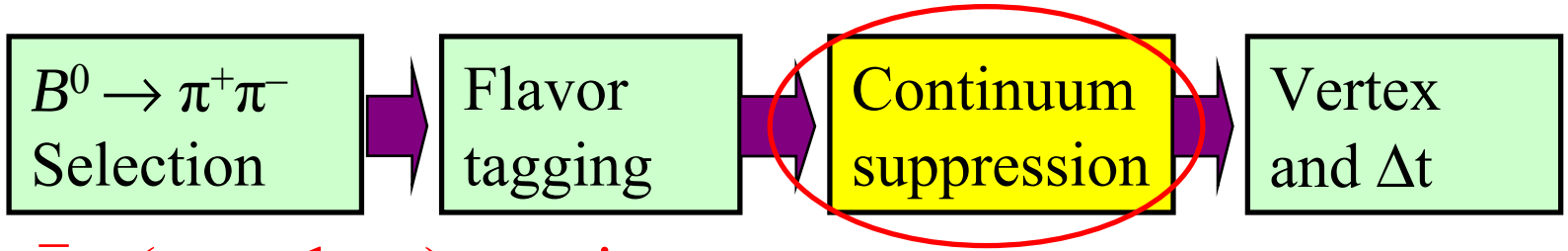
r = 0: no flavor discrimination
r = 1:unambigous flavor assignment

$B\bar{B}$-mixing fit → flavor tag performance
(6 r intervals used)

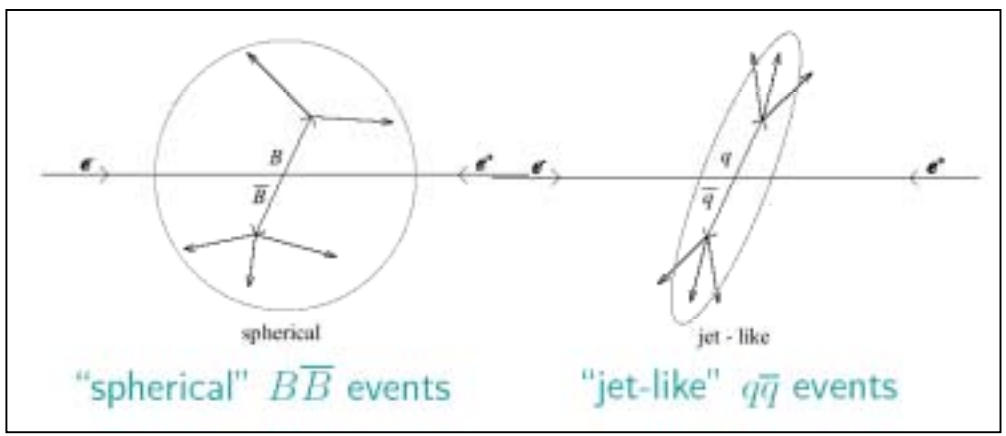


Event and time reconstruction (3)

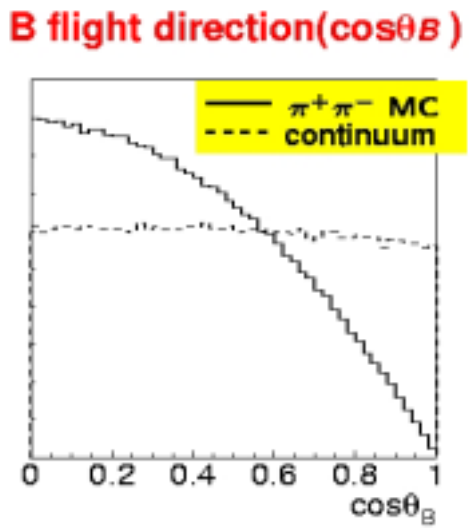
Flow



- $e^+e^- \to \bar{q}q$ (q=u,d,s,c) continuum background suppression
  - **Event topology** **&** **Angular distribution**

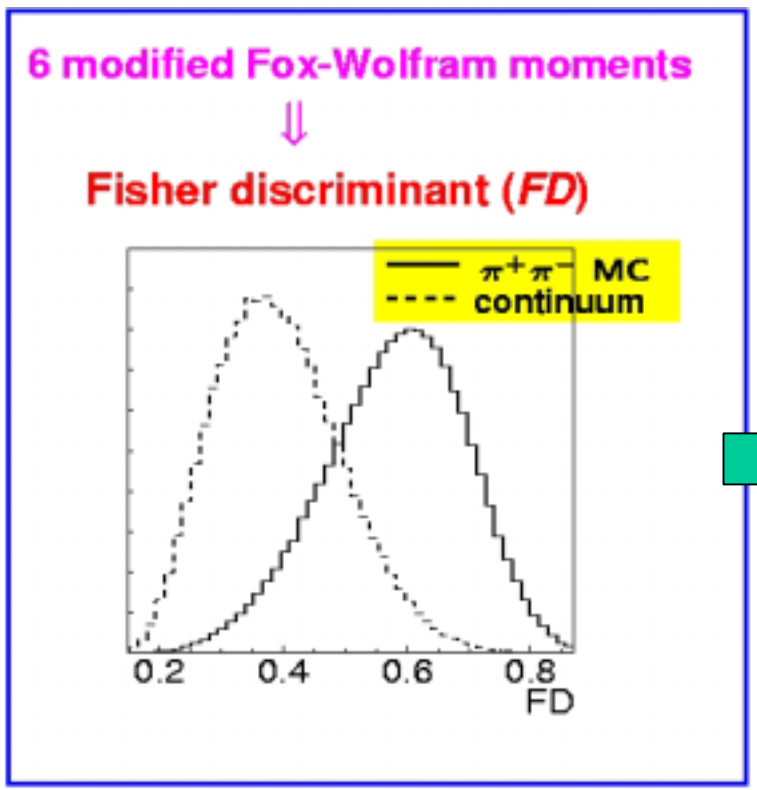


B flight direction

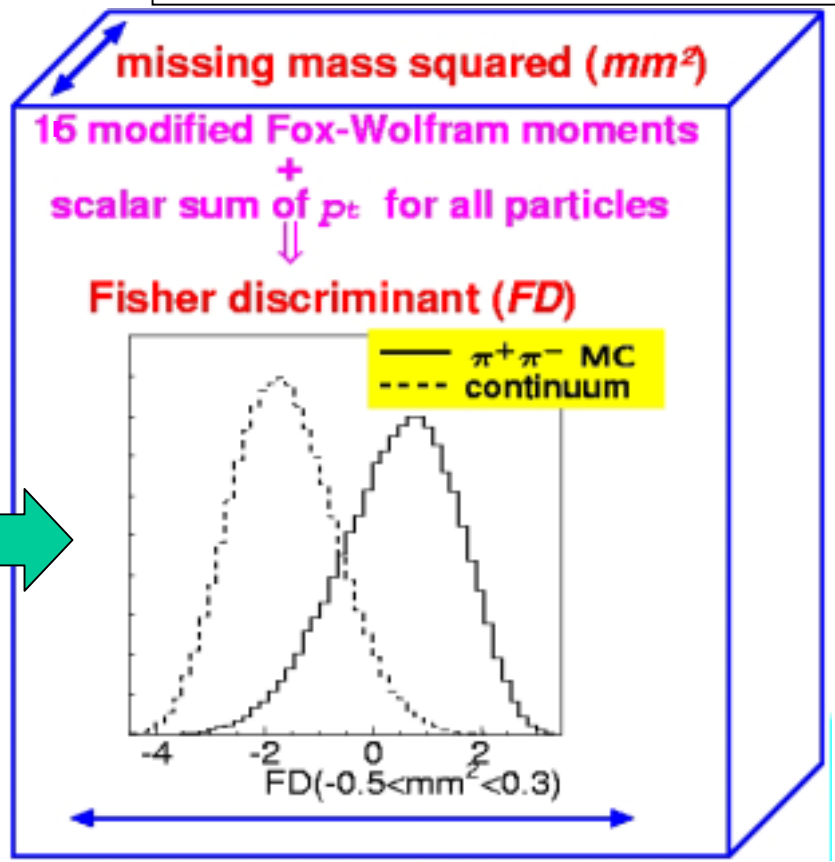


Improved method of continuum suppression

Previous method



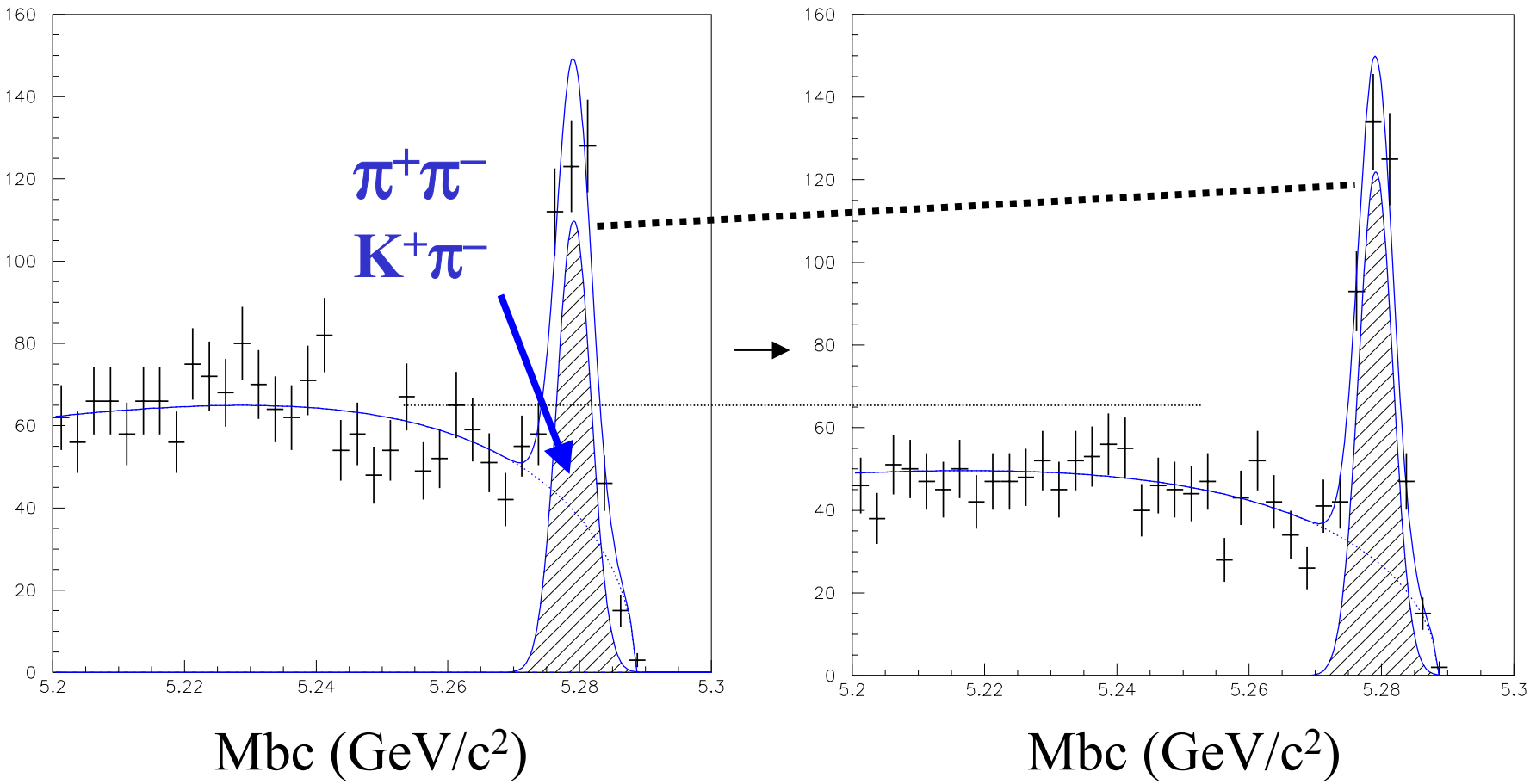
Improved method



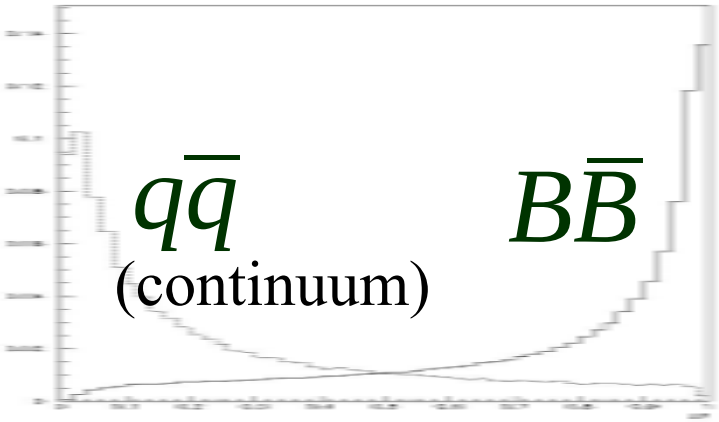
More information discriminates more between $B\bar{B}$ and $q\bar{q}$.

Continuum background rejection

By the previous method **By the improved method**

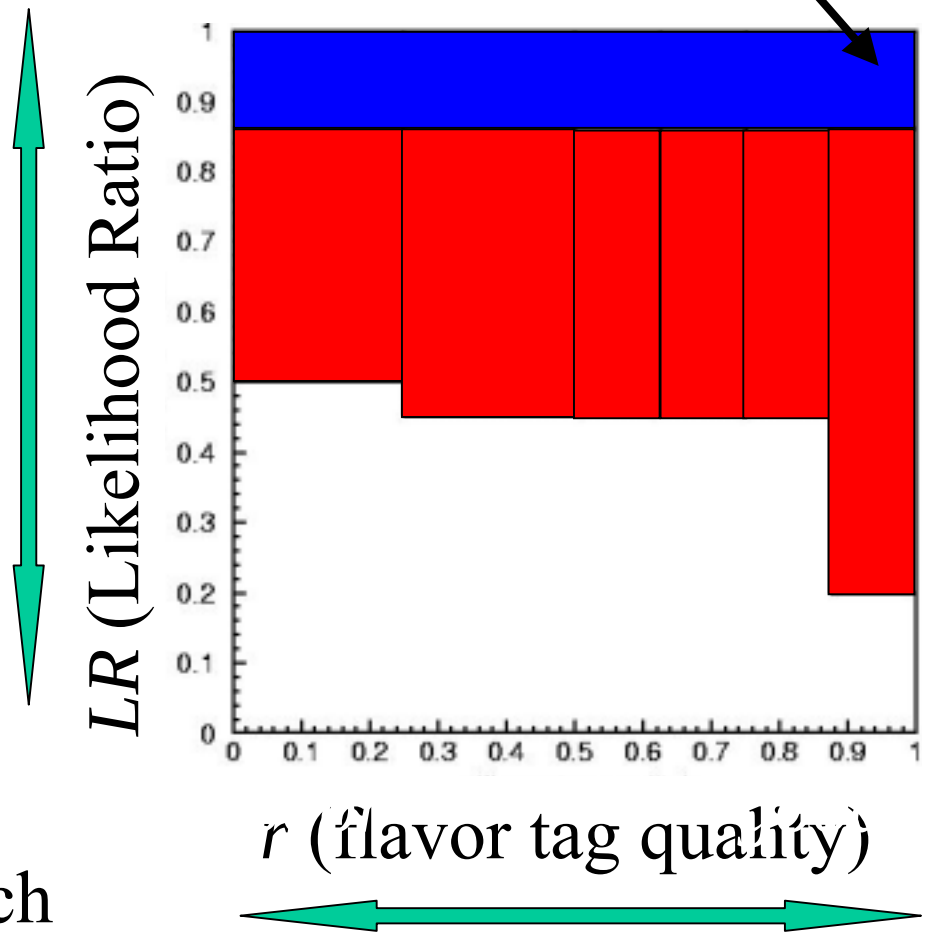


Continuu background rejection



$$LR = \frac{\mathcal{L}_{B\bar{B}}}{\mathcal{L}_{B\bar{B}} + \mathcal{L}_{q\bar{q}}}$$

$B\bar{B}$ purity is better.



Optimized selection for each region in the *LR*-*r* plane (12 regions in total)

Event and time reconstruction (4)

Flow

$B^0 \to \pi^+\pi^-$ Selection → Flavor tagging → Continuum suppression → Vertex and Δt

Vertex reconstruction

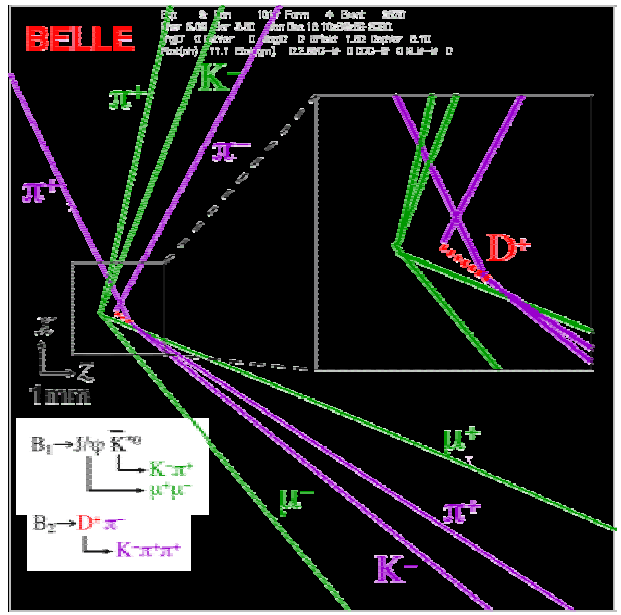
- The same algorithm as that used for $\sin 2\phi_1$ meas.
- Resolution mostly determined by the tag-side vtx.
- B lifetime demonstration with 152 million B pairs

$B^0 \to D^+\pi^-$, $D^{*+}\pi^-$, $D^{*+}\rho^-$, $J/\psi K_S$ and $J/\psi K^{*0}$

B^0 lifetime
1.533±0.008(stat) ps

(consist. with PGD03)

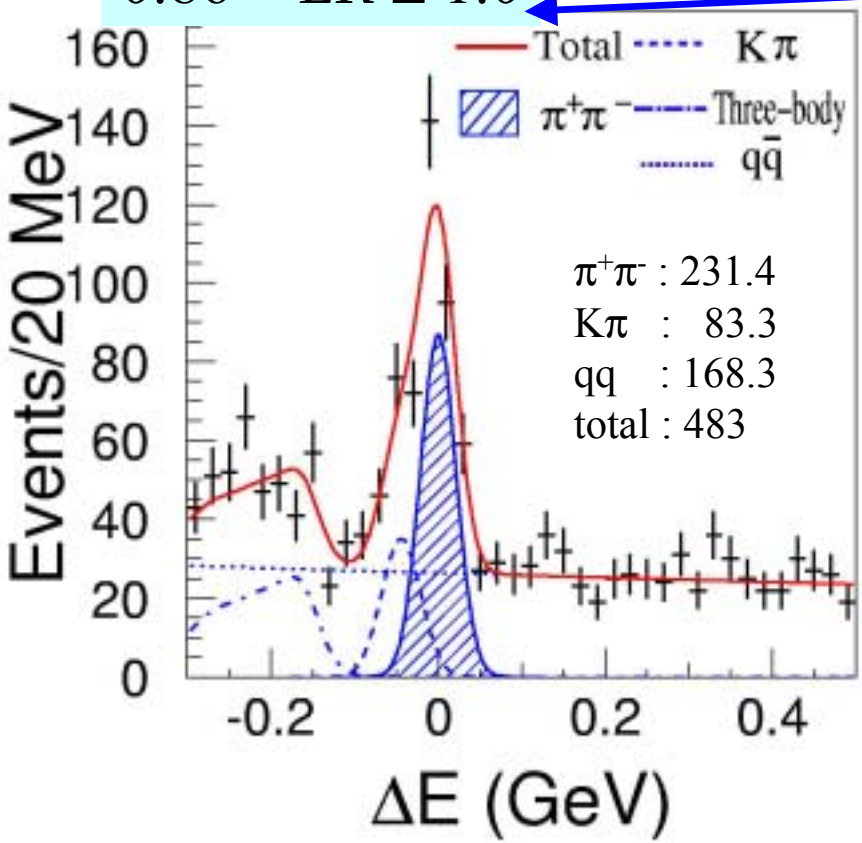
Time resolution (rms)
1.43ps

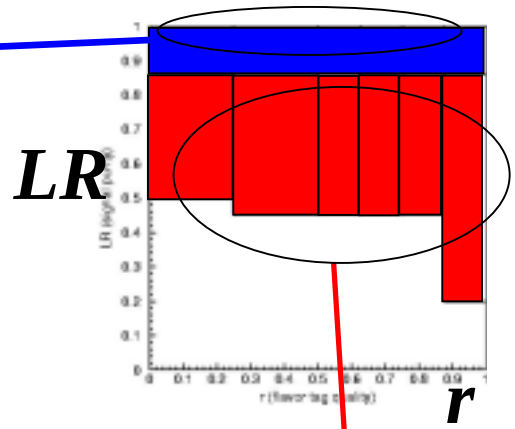


Example vertices

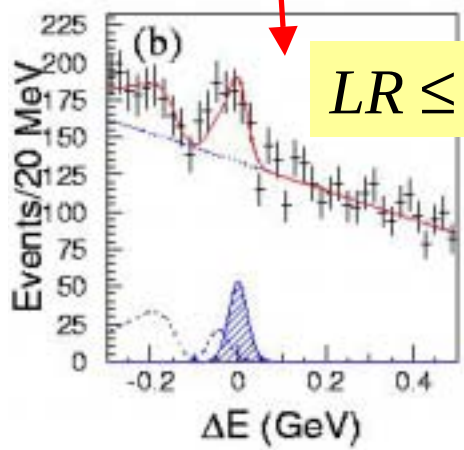
$B^0 \to \pi^+\pi^-$ candidates (Belle 140fb^{-1})

$0.86 < LR \leq 1.0$





$LR \leq 0.86$



1529 candidates (801 B^0- and 728 B^0-tags) containing (372 ± 32) $\pi^+\pi^-$ signal events

Time-dependent analyses

Unbinned maximum-likelihood fit

2 free parameters ($A_{\pi\pi}$, $S_{\pi\pi}$) in the final fit

ΔE-M_{bc} dist.

$$L = \Pi_i P_i$$

$$P_i = (1 - f_{ol}) \int_{-\infty}^{+\infty} \{ (f_{\pi\pi}^{m} P_{\pi\pi}^{q} + f_{K\pi}^{m} P_{K\pi}) \cdot R_{sig} + f_{q\bar{q}}^{m} P_{q\bar{q}} \cdot R_{q\bar{q}} \} d\Delta t'$$

$$+ f_{ol} \cdot P_{ol}(\Delta t_i)$$

(single Gaussian outlier)

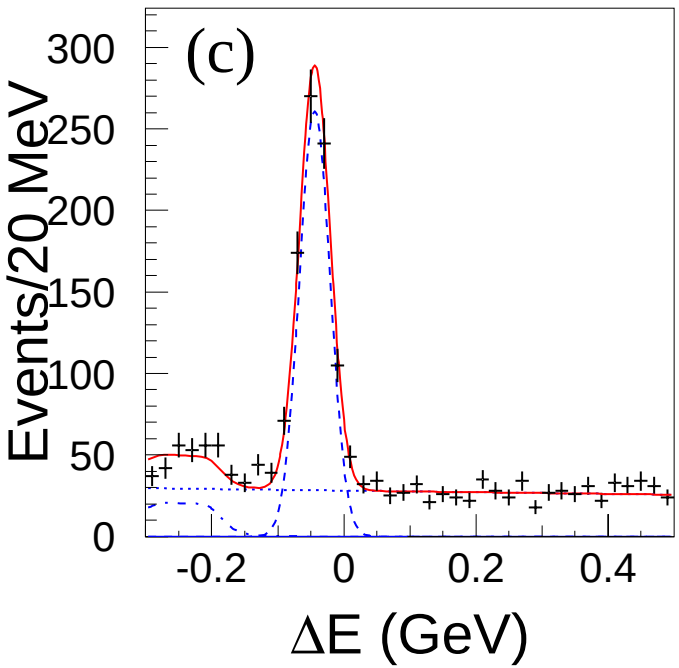
- **Now we are able to obtain $A_{\pi\pi}$ and $S_{\pi\pi}$.**

- **But let's go through several crosschecks before opening the box.**
  - **Lifetime measurement**
    - **$B^0 \to \pi^+\pi^-$, $B^0 \to K^+\pi^-$**
  - **Δt asymmetry**
    - **Sideband ($q\bar{q}$), $B^0 \to K^+\pi^-$**
  - **MC pseduo-experiments**

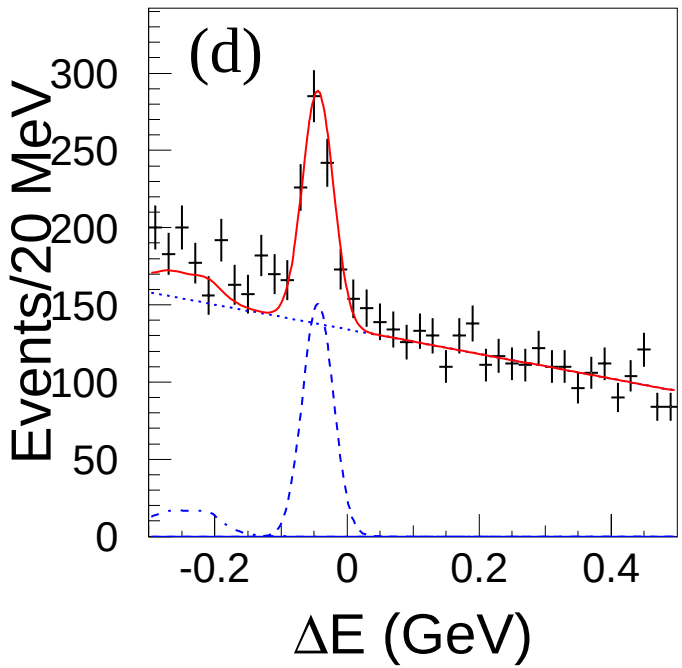
$B^0 \to K^+\pi^-$ control sample

Positively-identified kaons (opposite use of PID)

LR > 0.86

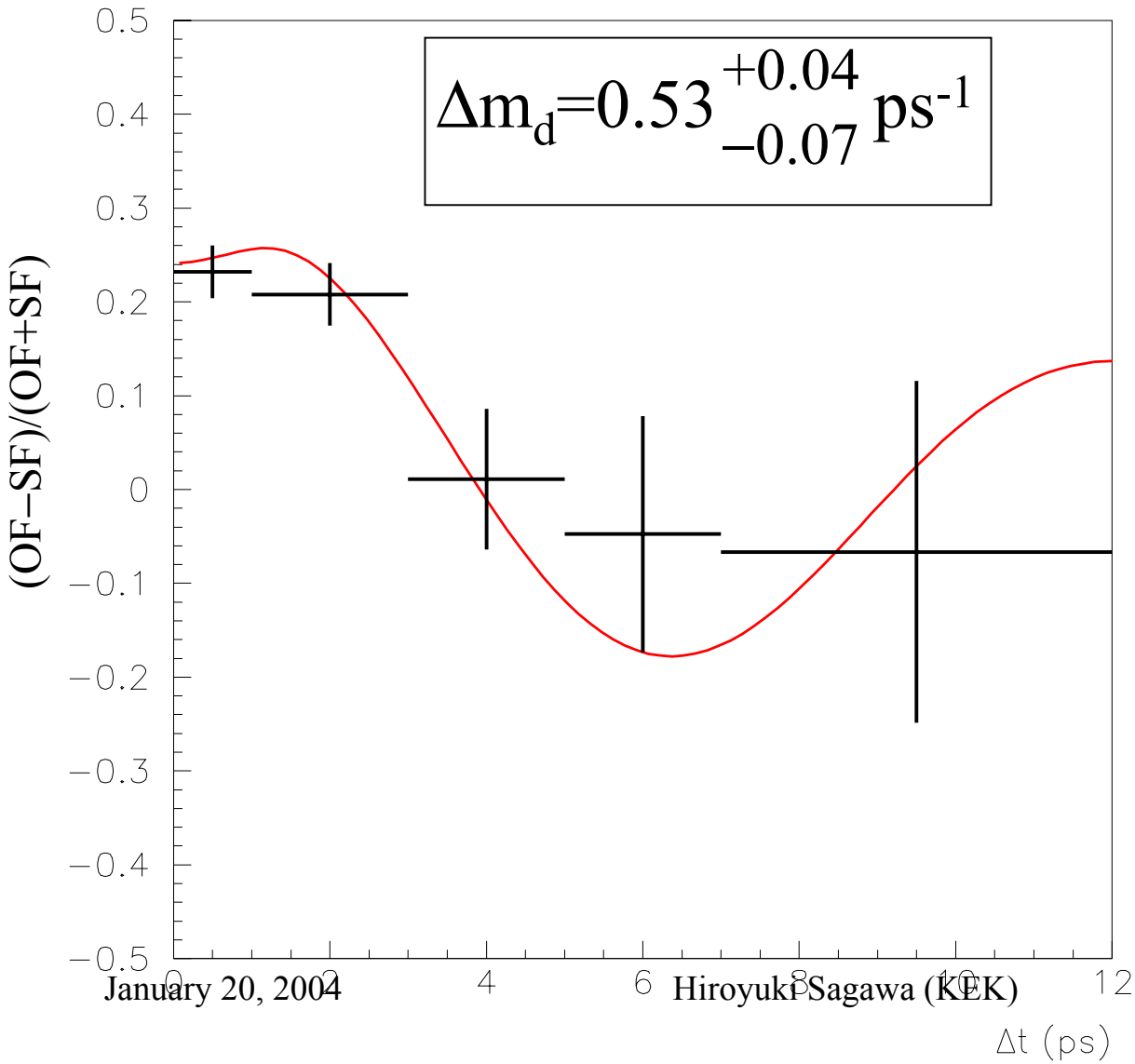


LRmin < LR ≤ 0.86



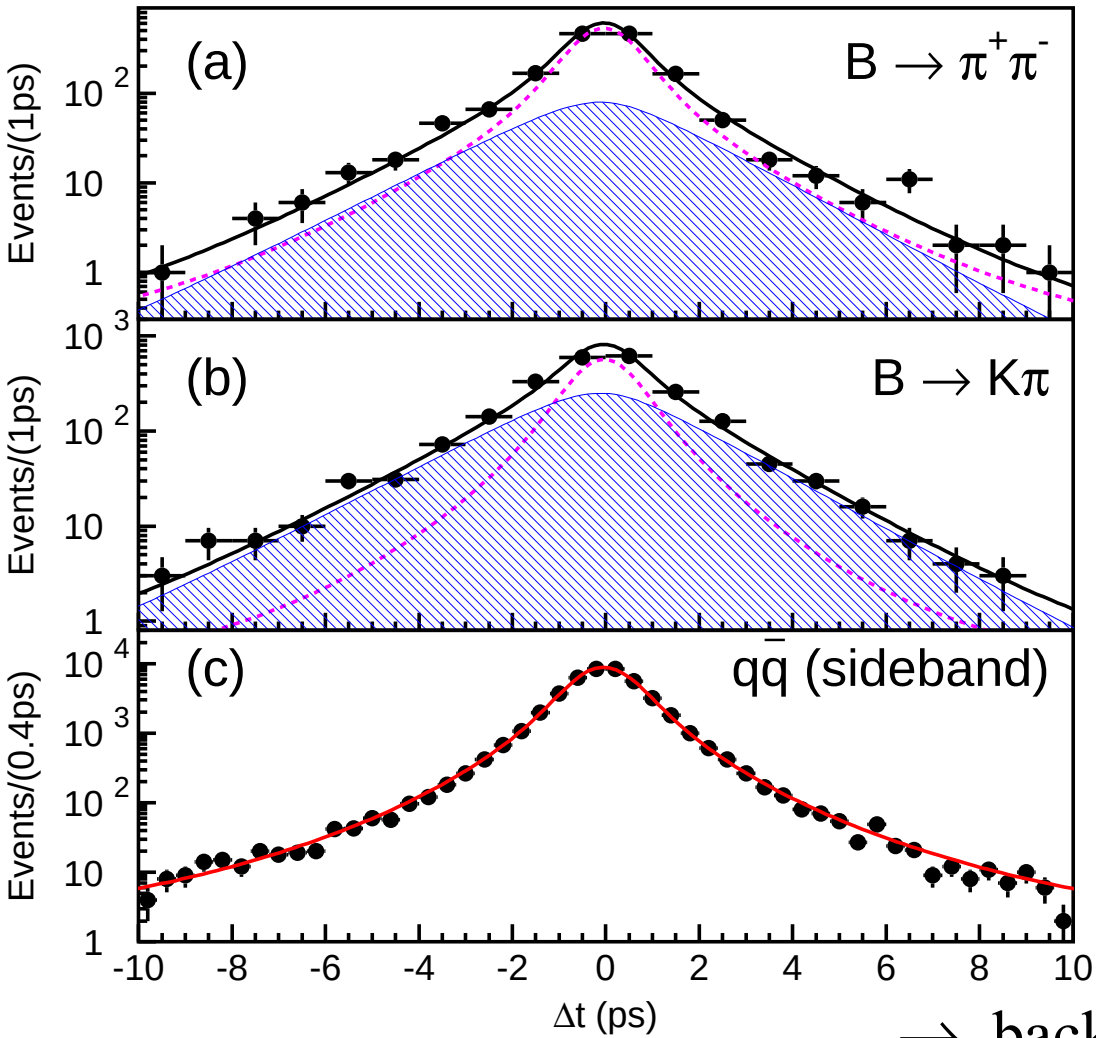
2358 candidates

Mixing fit using $B^0 \to K^+\pi^-$: OK !



Consistent with the world average (0.502 ± 0.007) ps^{-1} PDG2003

Lifetime measurements: also good !



world average (PDG2003)
(1.537 ± 0.015) ps

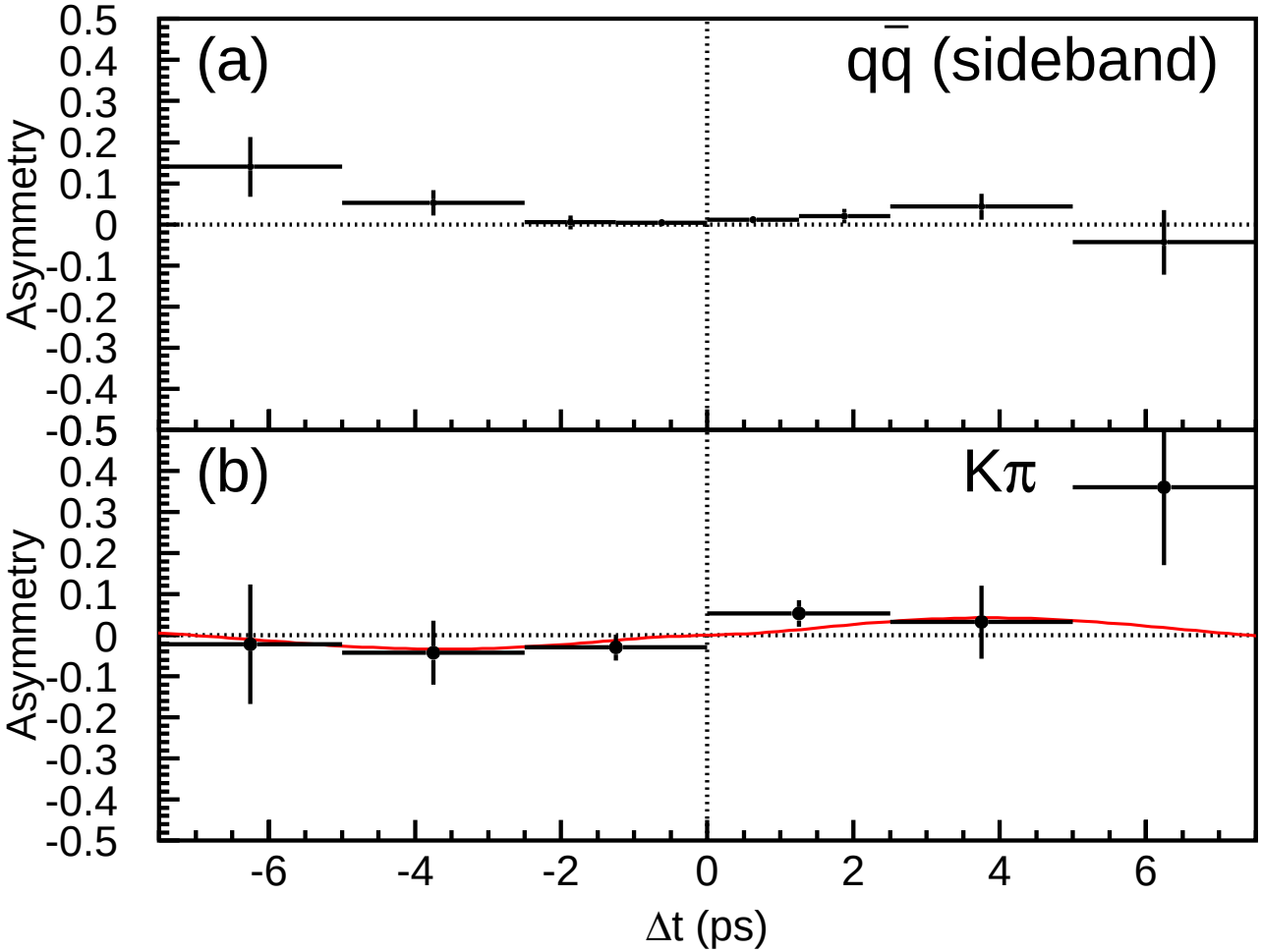
$\pi\pi$: τ_B=(1.47 ± 0.09) ps

$K\pi$: τ_B=(1.52 ± 0.06) ps

BG shape fit

→ background treatment is correct !

Null asymmetry tests: OK !



Null asymmetry

$S_{K\pi} = \quad 0.14 \pm 0.11$

$A_{K\pi} = -0.02 \pm 0.08$

(consistent with counting analysis)

$A_{K\pi} = -0.09 \pm 0.03$

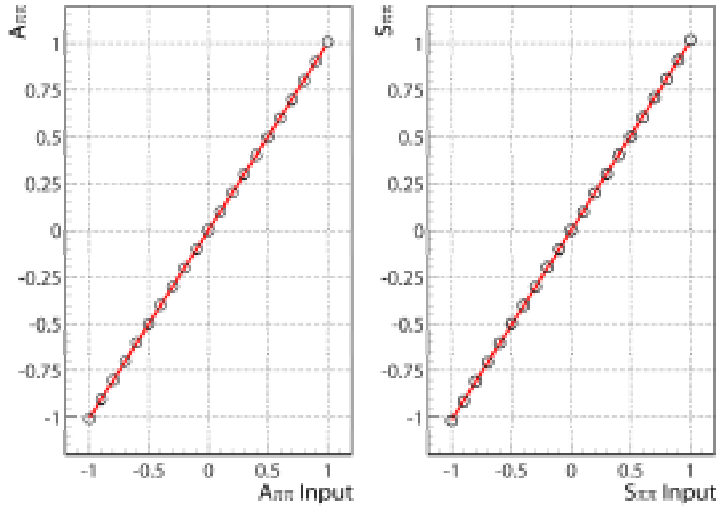
$B^0 \to K^+\pi^-$

$\bar{B}^0 \to K^-\pi^+$

MC pseudo-experiments

Generate events according to the PDF used for the fit →

Good linearity over the entire range



- Feldman-Cousins approach for **Aππ** and **Sππ** confidence regions.
- We quote the **rms** values of the $A_{\pi\pi}$ and $S_{\pi\pi}$ distributions in the MC pseudo-experiments as the **statistical errors** of $A_{\pi\pi}$ and $S_{\pi\pi}$.

140fb^{-1}

Aππ error: **±0.15** (± 0.27@78fb-1)
Sππ error: **±0.15** (± 0.41@78fb-1)

Similar to errors defined by log-likelihood curves now

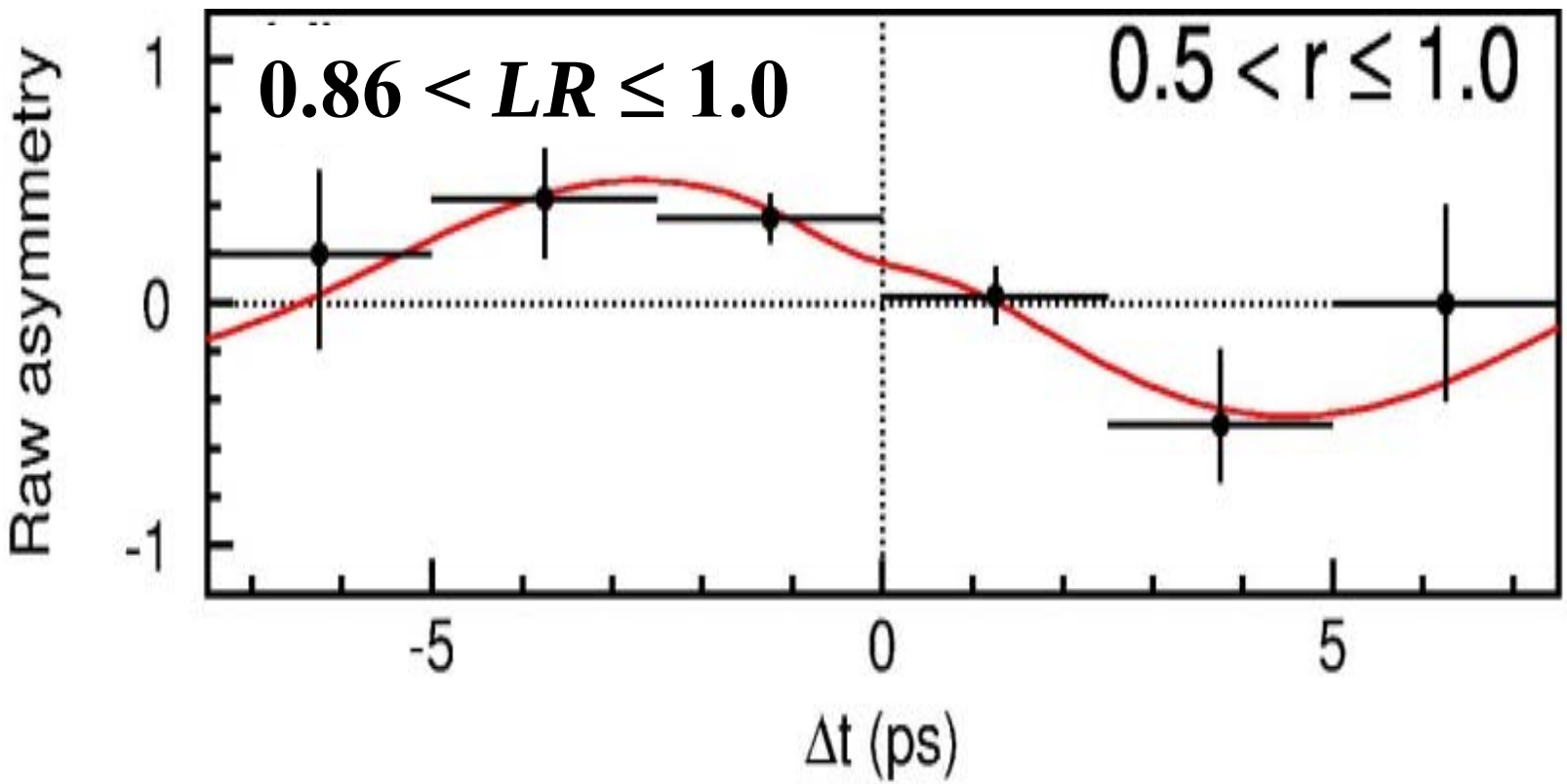
- **PDF is based on data** (control samples, sideband) → MC pseudo-experiments are free from possible systematics in Geant-based MC.

3. Results

Fit results ($140 fb^{-1}$)

$$A_{\pi\pi} = +0.58 \pm 0.15(\text{stat}) \pm 0.07(\text{syst})$$
$$S_{\pi\pi} = -1.00 \pm 0.21(\text{stat}) \pm 0.07(\text{syst})$$

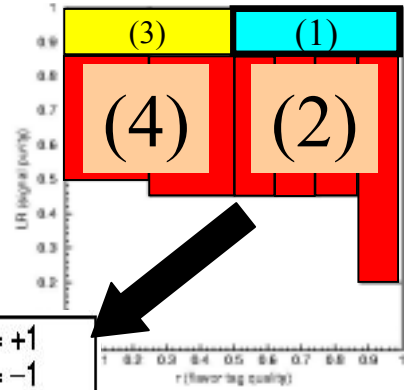
1529 ev. (all *LR*-*r* regions)

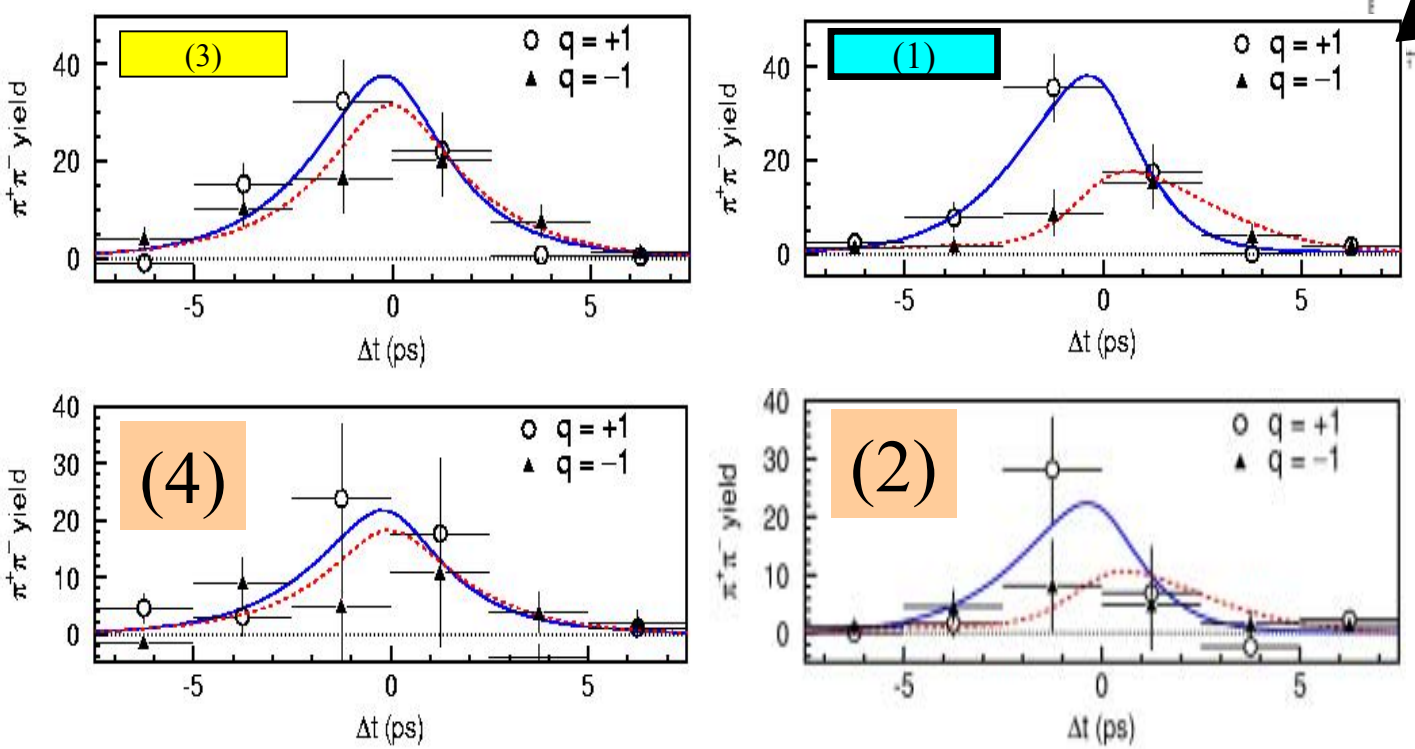


Δt distributions (background subtracted)

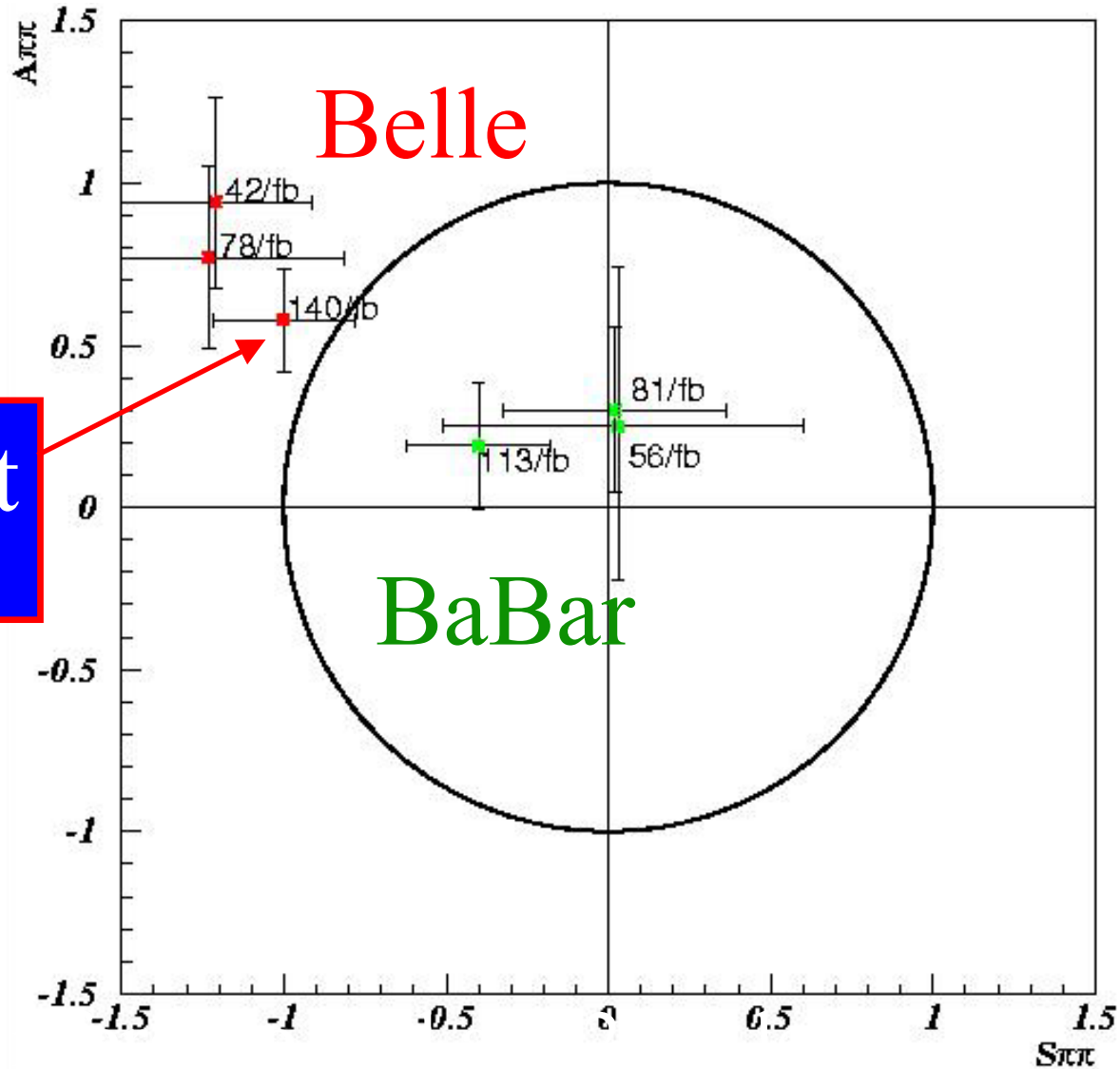
$A_{\pi\pi} = +0.58 \pm 0.15(\text{stat}) \pm 0.07(\text{syst})$

$S_{\pi\pi} = -1.00 \pm 0.21(\text{stat}) \pm 0.07(\text{syst})$





History of $A_{\pi\pi}$ and $S_{\pi\pi}$



This result

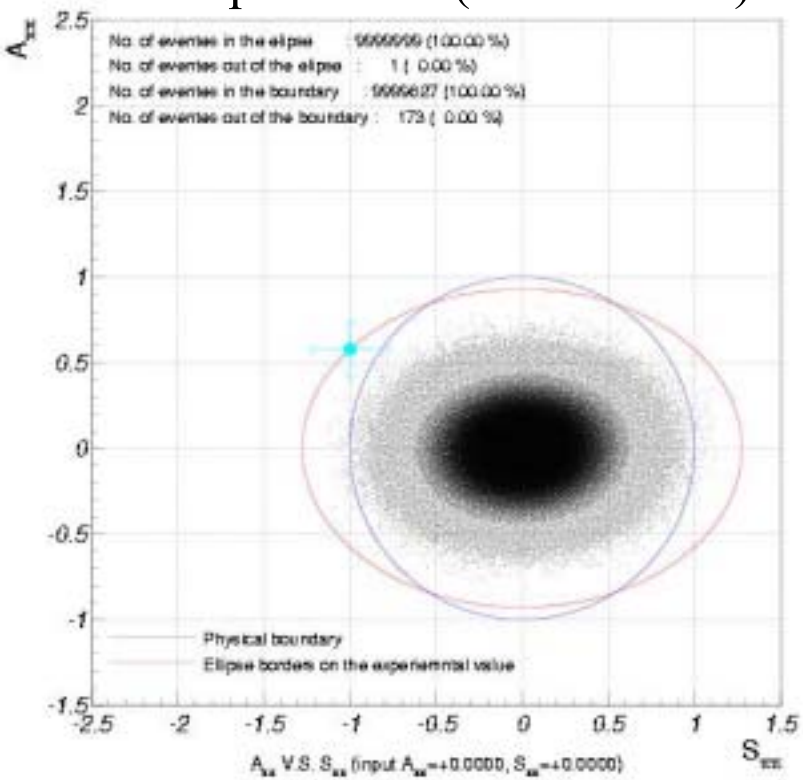
Belle 140fb^{-1}

MC pseudo-experiments

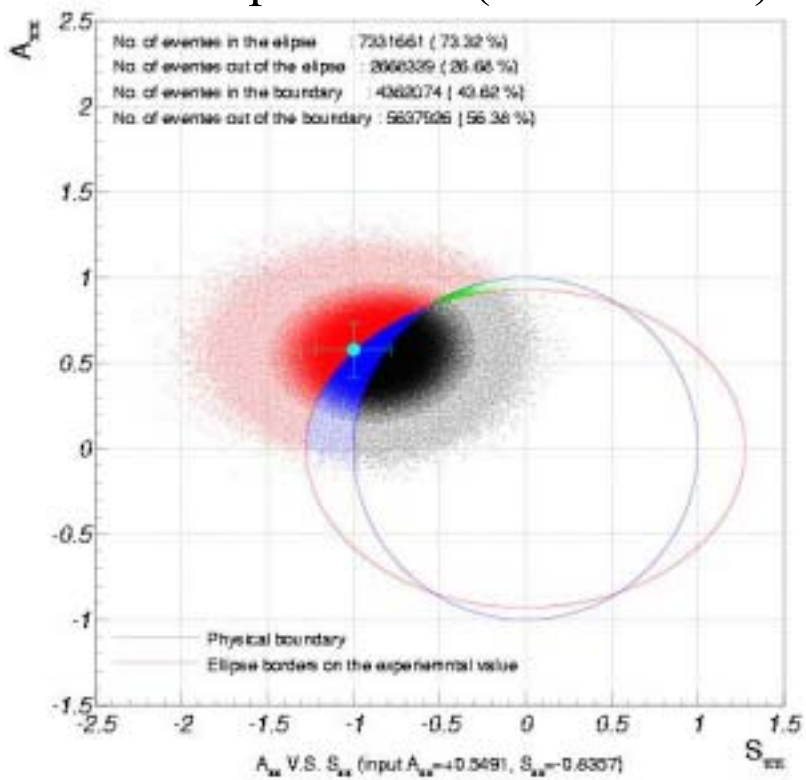
Input: $(A_{\pi\pi}, S_{\pi\pi}) = (0,0)$
Belle result almost impossible
probability **~ 0.1 ppm**

Input: $(A_{\pi\pi}, S_{\pi\pi})$ = maximum likelihood point in the physical region
Belle result no problem
probability **~ 27%**

10^7 experiments ($140fb^{-1}$ each)



10^7 experiments ($140fb^{-1}$ each)

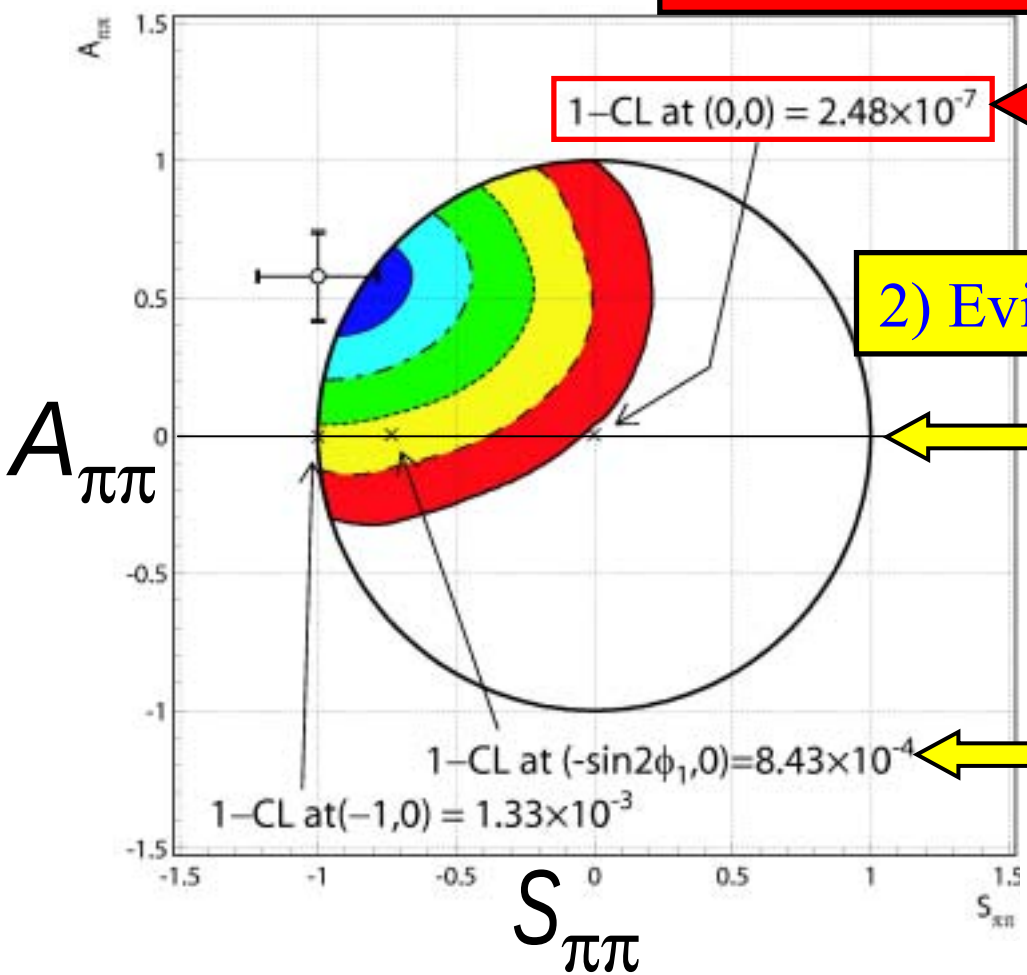


Systematic Uncertainties	$A_{\pi\pi}$	$S_{\pi\pi}$
Wrong tag fraction	±0.009	±0.009
Physics (τ_B, Δm_d, $A_{k\pi}$)	±0.024	±0.007
Resolution function	±0.010	±0.020
Background shape	±0.014	±0.021
Background fractions	±0.028	±0.025
Fit bias	±0.018	±0.023
Vertexing	±0.039	±0.045
Tag side interference	±0.027	±0.011
Total	**±0.066**	**±0.066**

Systematic error < Statistical error

Significance

Feldman-Cousins Analysis



1) Observation of *CP* violation

(5.2σ)

2) Evidence for direct *CP* violation

≥3.2σ

for $A_{\pi\pi}$=**0** and any $S_{\pi\pi}$

3.3σ

for "**superweak**" case

Constraints on the CKM angle ϕ_2

$$A(B^0 \to \pi^+\pi^-) = -(|T|e^{i\delta_T}e^{i\phi_3}) + |P|e^{i\delta_P},$$
$$A(\overline{B}^0 \to \pi^+\pi^-) = -(|T|e^{i\delta_T}e^{-i\phi_3}) + |P|e^{i\delta_P},$$

convention taken from
M.Gronau and J.L.Rosner
Phys. Rev. D65, 093012 (2002)

$$\lambda_{\pi\pi} = e^{i\phi_2}\frac{1+|P/T|e^{i(\delta+\phi_3)}}{1+|P/T|e^{i(\delta-\phi_3)}}$$

4 parameters

$$S_{\pi\pi} = [\sin 2\phi_2 + 2|P/T|\sin(\phi_1 - \phi_2)\cos\delta - |P/T|^2 \sin 2\phi_1]/R_{\pi\pi},$$
$$A_{\pi\pi} = -[2|P/T|\sin(\phi_1+\phi_2)\sin\delta]/R_{\pi\pi},$$
$$R_{\pi\pi} = 1 - 2|P/T|\cos(\phi_1+\phi_2)\cos\delta + |P/T|^2$$

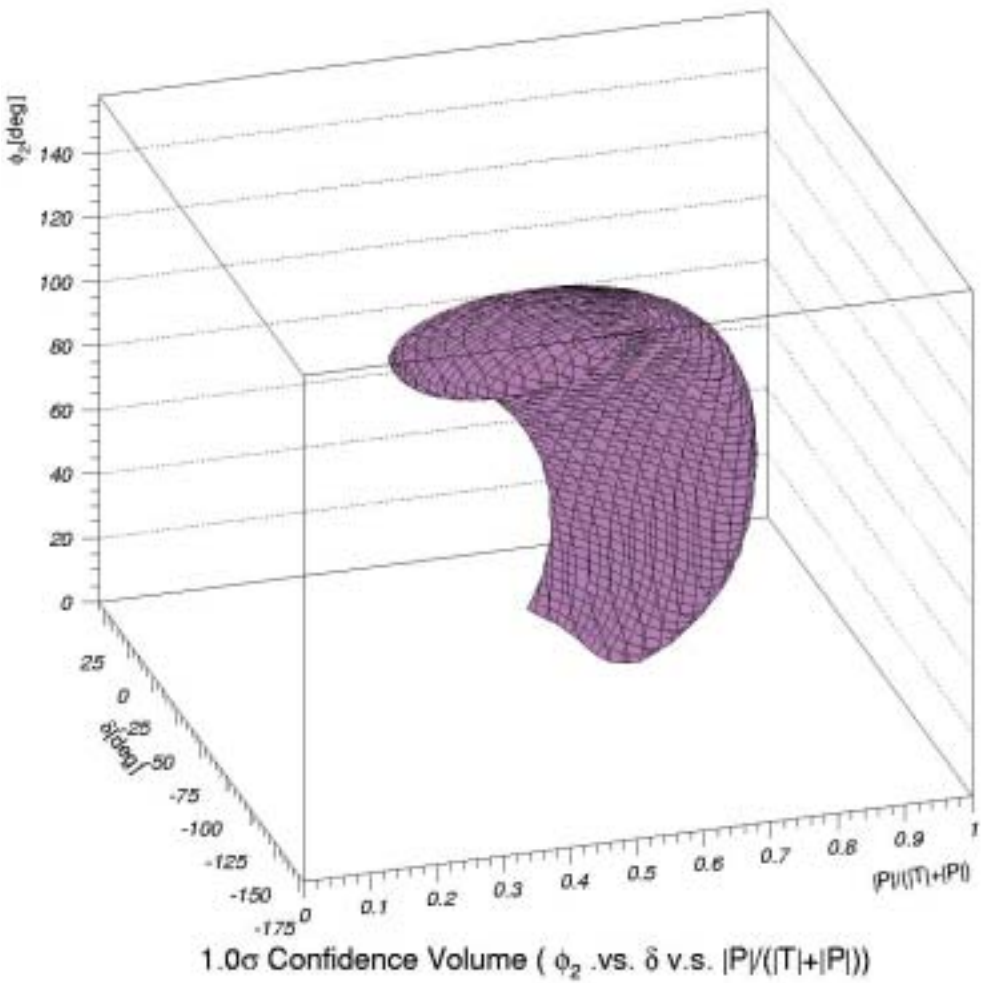
$\delta \equiv \delta_P - \delta_T$
Strong phase difference

$\|P/T\|$	0.15-0.45 (representative) Theory ~0.3
ϕ_1	23.7 deg (Belle & BaBar combined)

Model-independent constraint on ϕ_2, δ, $|P/T|$

$\phi 2$ (deg)

δ (deg)



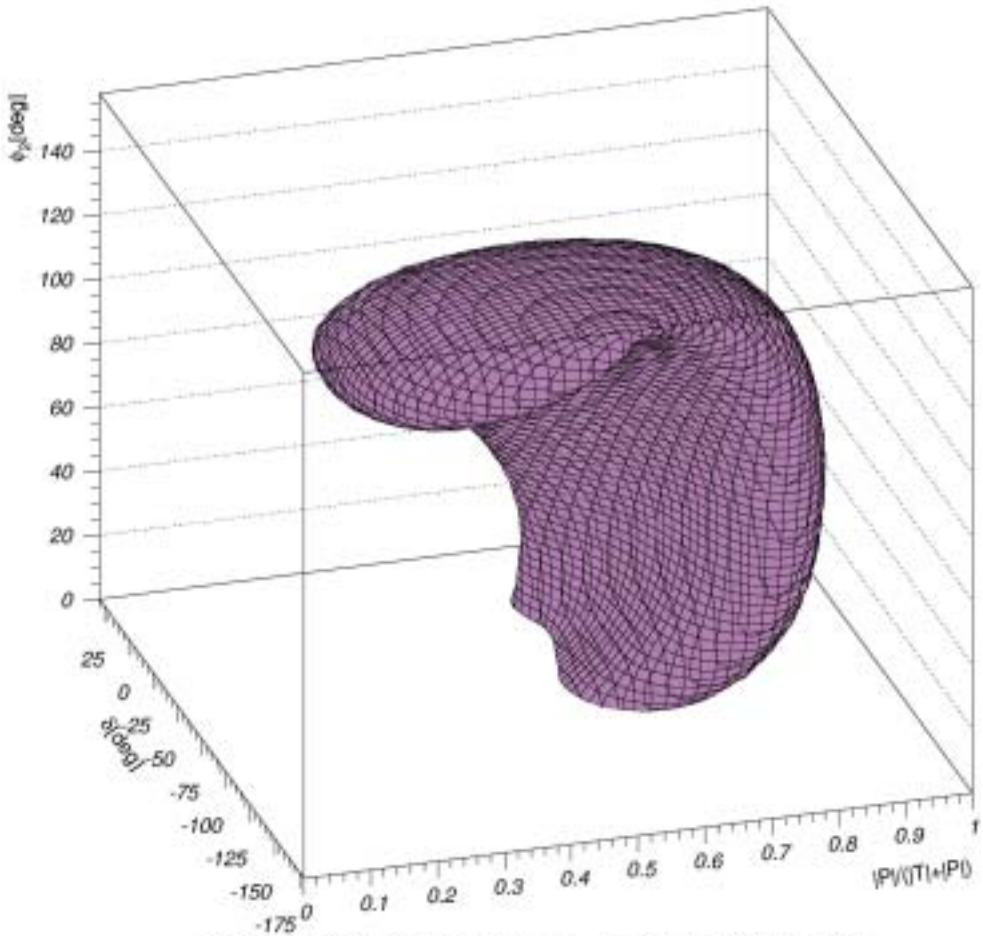
$|P|/(|T|+|P|)$

$\sin 2\phi_1 = 0.736$

68.3% C.L. volume

Model-independent constraint on ϕ_2, δ, |P/T|

$\phi 2$ (deg)



δ (deg)

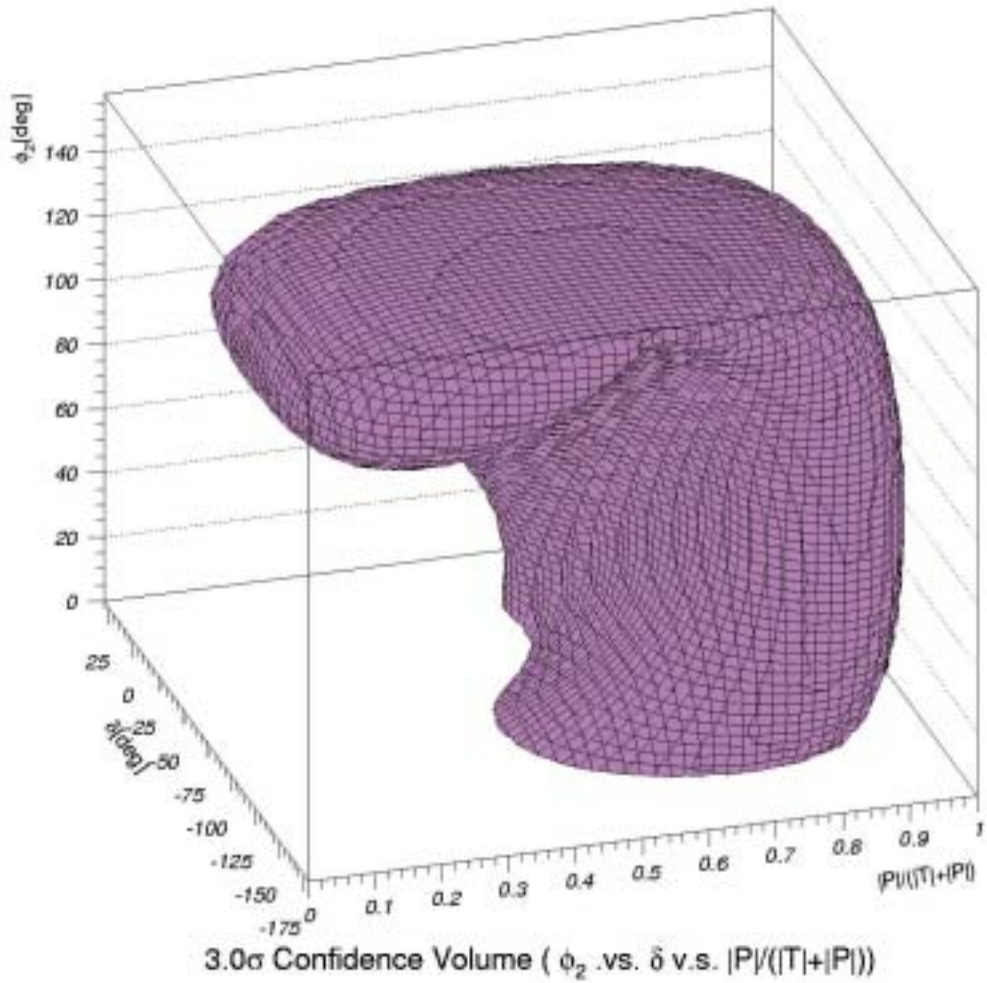
$\sin 2\phi_1 = 0.736$

95.5% C.L. volume

|P|/(|T|+|P|)

Model-independent constraint on ϕ_2, δ, |P/T|

$\phi 2$ (deg)



δ (deg)

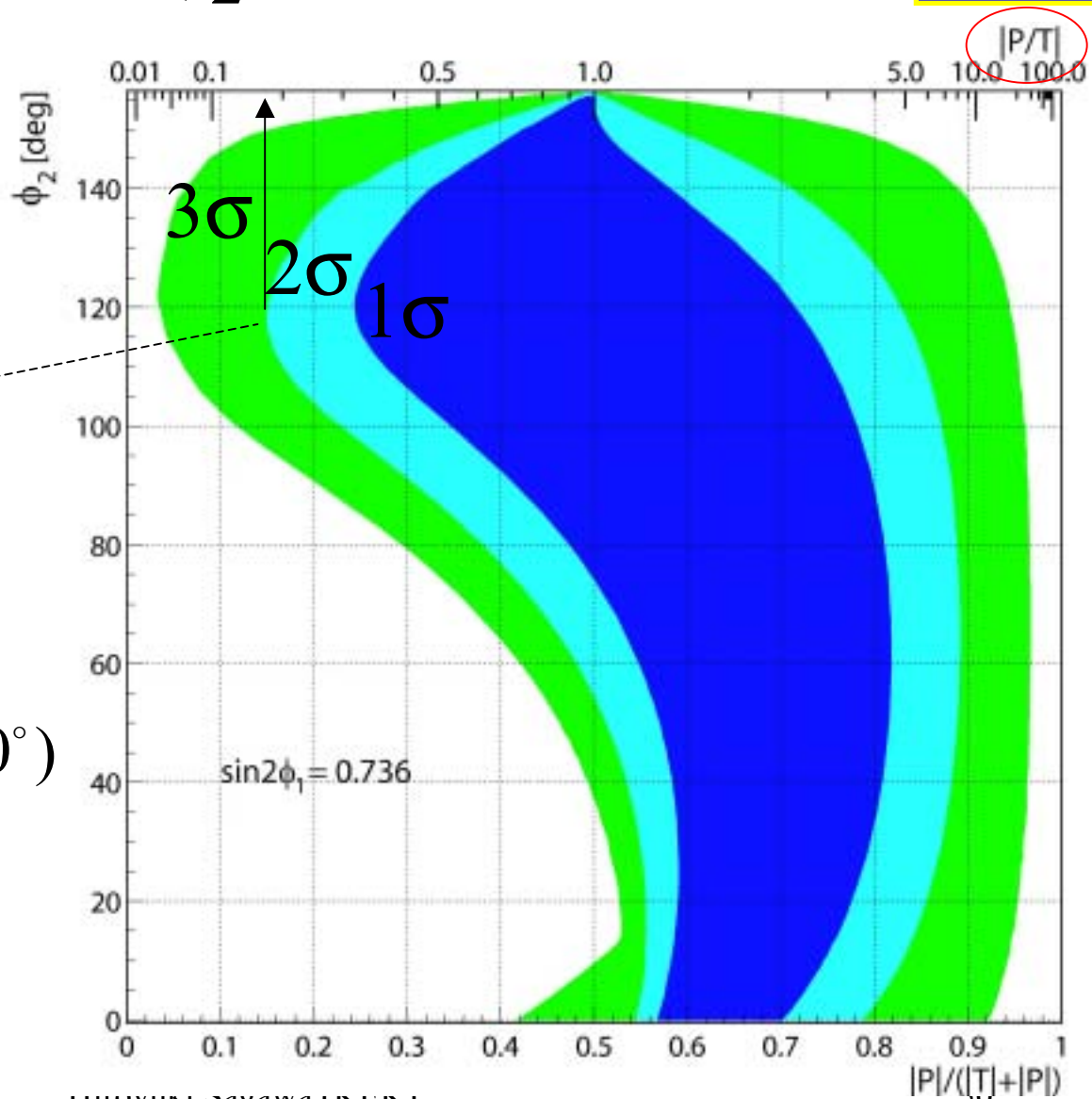
$\sin 2\phi_1 = 0.736$

99.7% C.L. volume

|P|/(|T|+|P|)

Constraint on ϕ_2 and |P/T|

Belle
140fb^{-1}



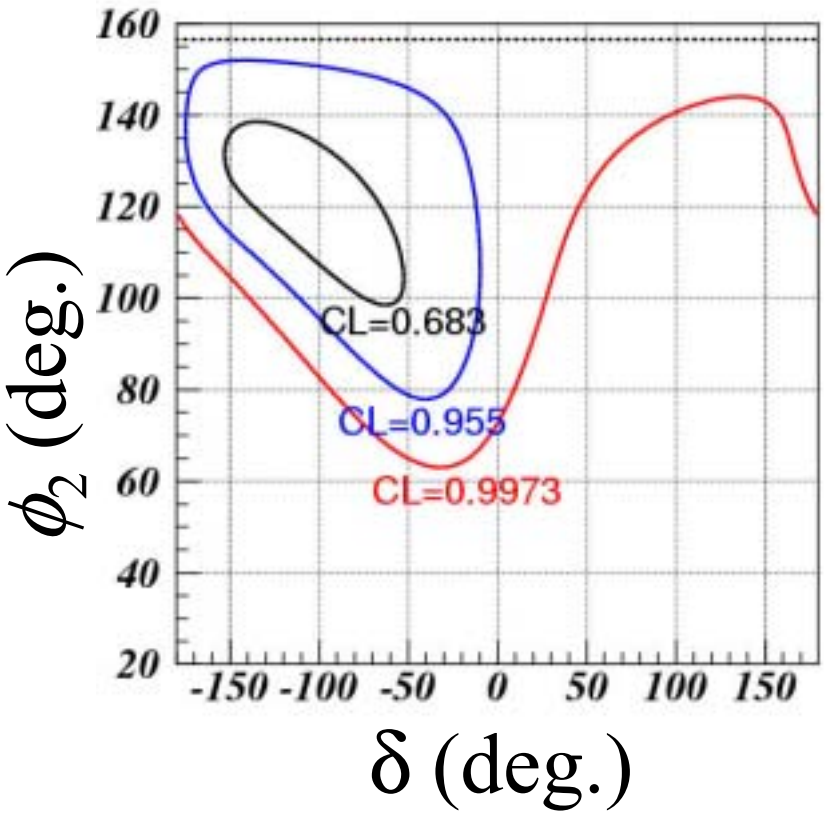
|P/T| > 0.17
[95.5% C.L. (→ 2σ)]

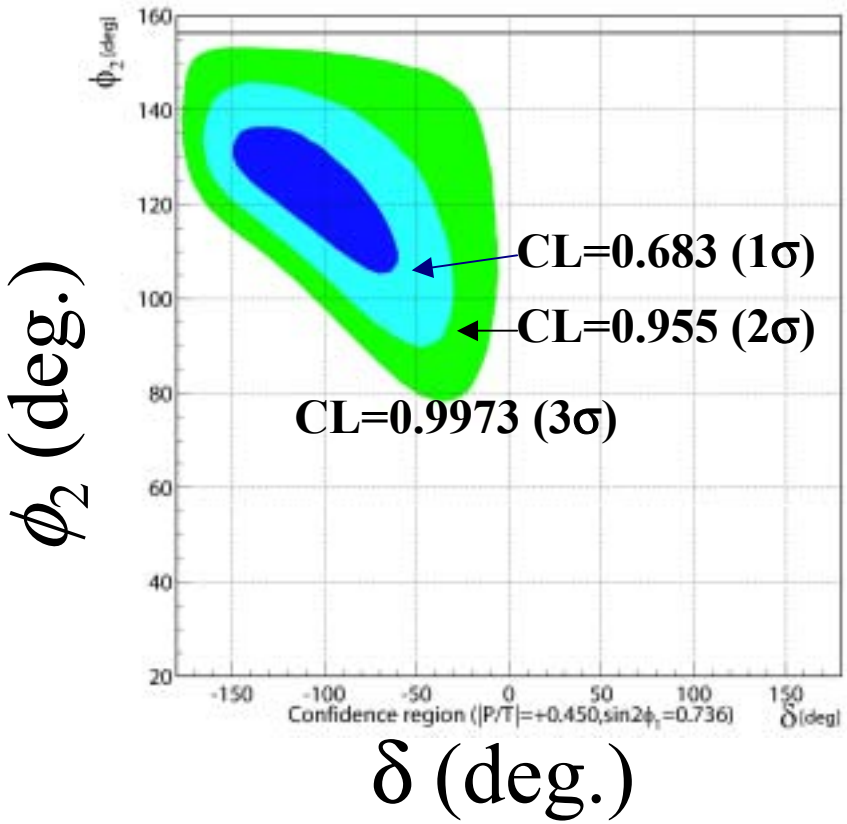
for any δ

$(-180^\circ \le \delta \le 180^\circ)$

Constraint on ϕ_2 and δ

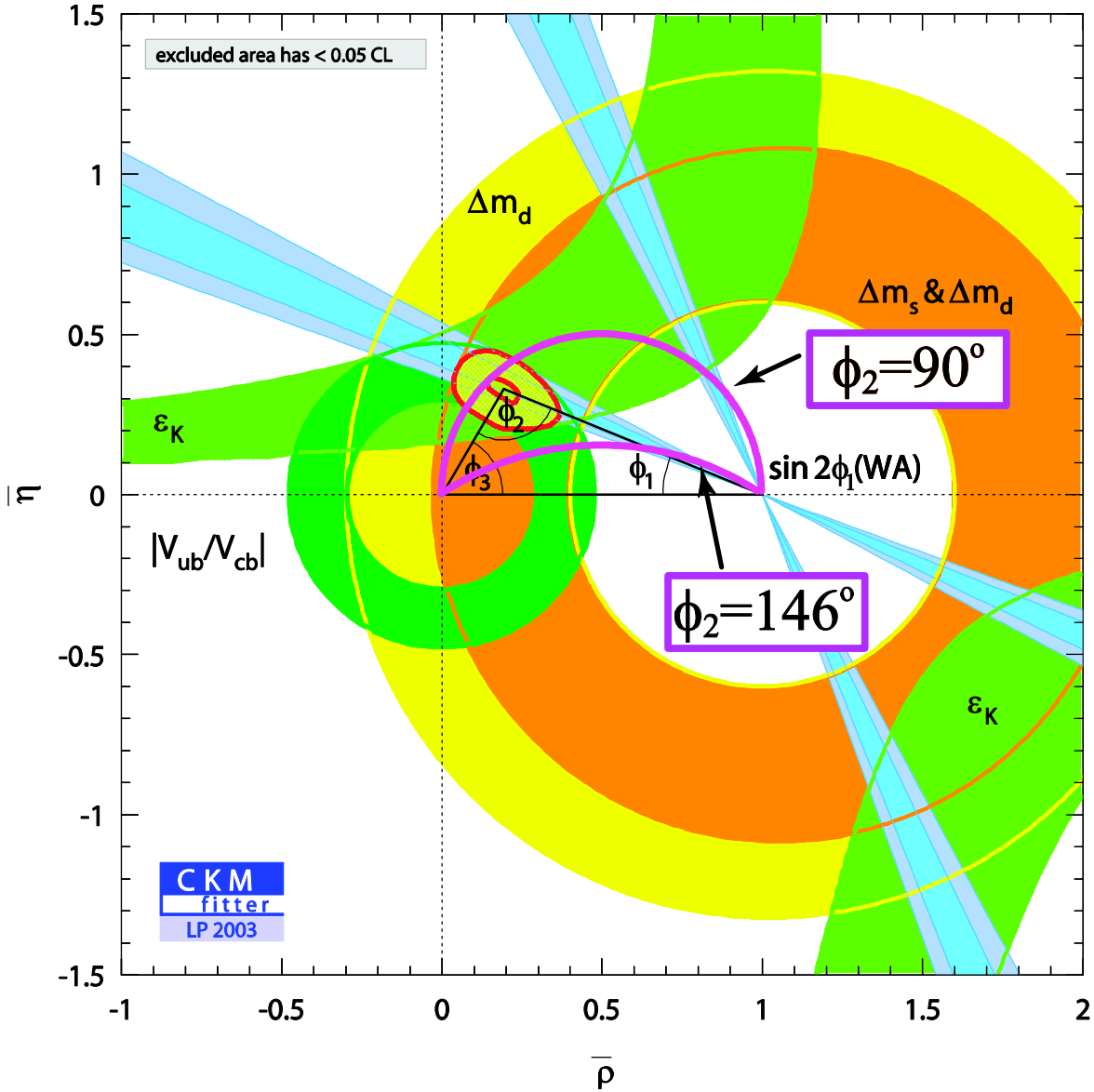
Belle
140fb^{-1}





$$90° \leq \phi_2 \leq 146° \text{ (95.5\% CL)}$$

Constraint on ρ-η



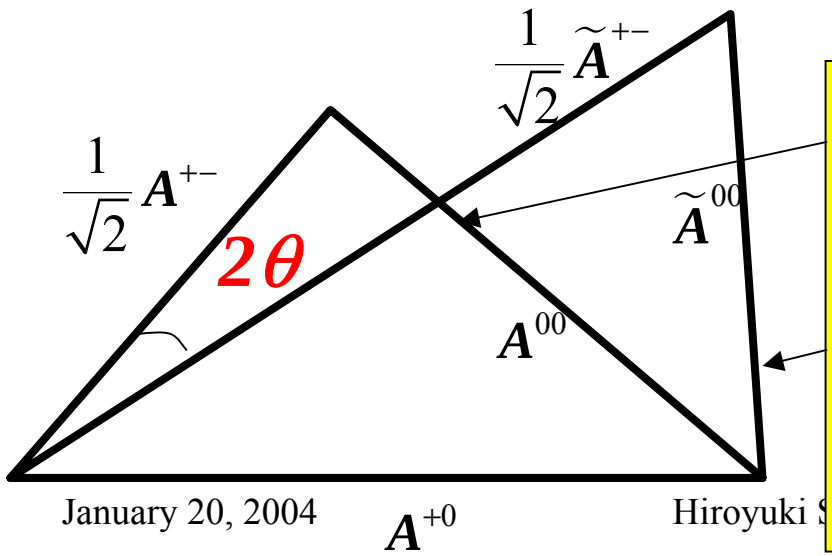
$\sin 2\phi_1$ (WA) and Belle's ϕ_2 are consistent with other measurements.

5. Prospects

$\pi\pi$ isospin analysis ($\rightarrow \theta = \phi_2 - \phi_{2eff}$)

$$S_{\pi\pi} = \sqrt{1 - A_{\pi\pi}^2}\,\sin 2\phi_{2eff}$$
$$= \sqrt{1 - A_{\pi\pi}^2}\,\sin 2(\phi_2 + \theta)$$

Isospin triangles



$$A\ \textit{amplitude}: \frac{1}{\sqrt{2}}A^{+-} + A^{oo} = A^{+0}$$

$$\bar{A}\ \textit{amplitude}: \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{oo} = \bar{A}^{+0}$$

$$\tilde{A}^{ij} \equiv e^{2i\phi_3}\bar{A}^{ij}$$

$\pi\pi$ isospin analysis ($\rightarrow 2\theta$) + S$\pi\pi$ $\rightarrow \phi_2$

We need following information.

Branching Ratios : $B^0 \rightarrow \pi^+\pi^-$,

$B^{\pm} \rightarrow \pi^{\pm}\pi^0$,

$B^0 \rightarrow \pi^0\pi^0$

CP asymmetries :

direct asymmetry : $A_{\pi\pi}$ (***for*** $\pi^+\pi^-$),

$A_{\pi^0\pi^0}$ (***for*** $\pi^0\pi^0$)

determine 2θ

$$S_{\pi\pi} = \sqrt{1 - A_{\pi\pi}^2}\,\sin 2\phi_{2eff}$$

$$= \sqrt{1 - A_{\pi\pi}^2}\,\sin 2(\phi_2 + \theta)$$

Branching ratios (WA), $A\pi\pi$, $S\pi\pi$

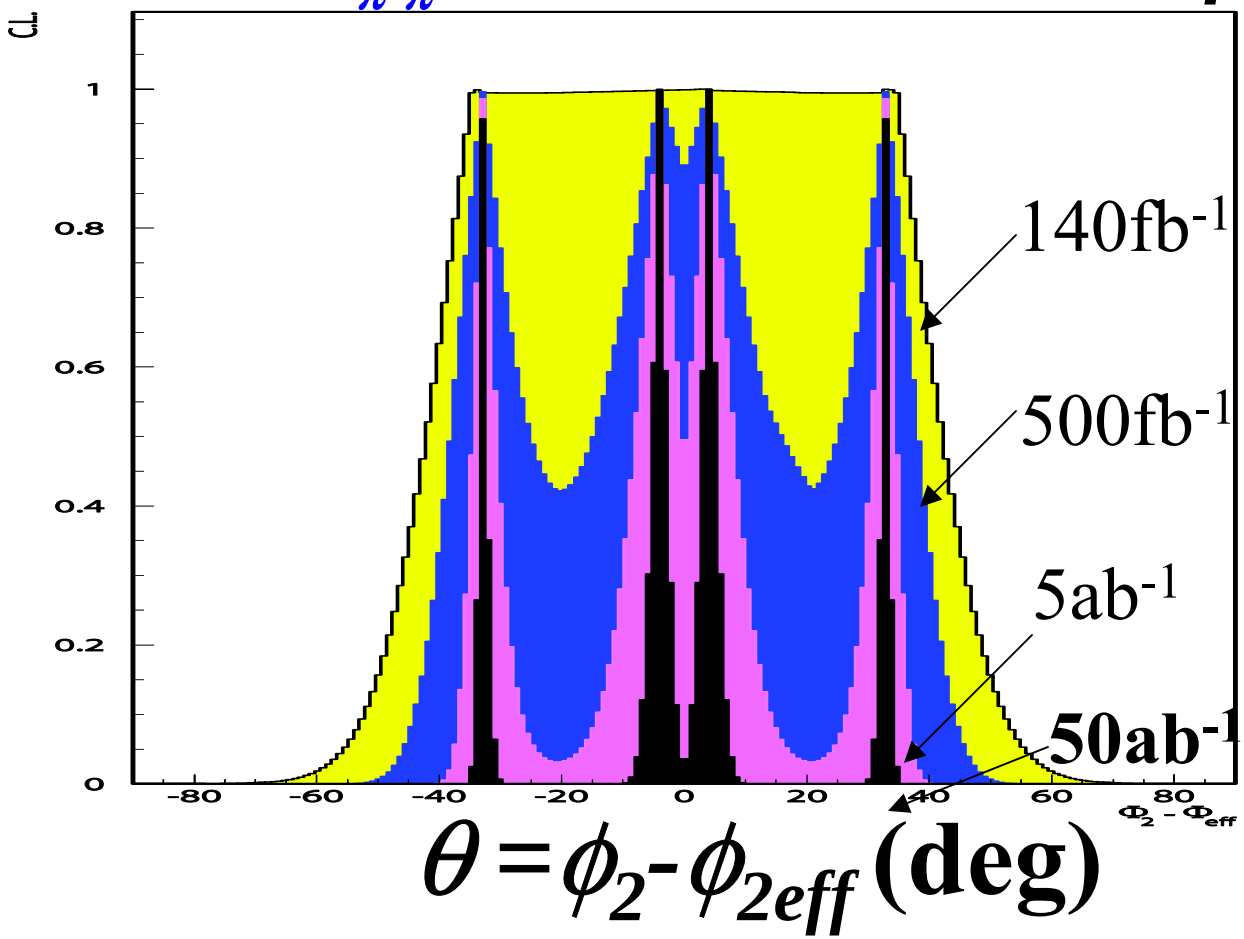
Table 1: The recent results of $BR(B \to \pi\pi)$ and $\mathcal{A}_{\pi\pi}$ and $\mathcal{S}_{\pi\pi}$ in $B^0 \to \pi^+\pi^-$ deca
The unit is 10^{-6}.

Mode	Belle	BaBar	CLEO	New Avg.
$\pi^+\pi^-$	$4.4 \pm 0.6 \pm 0.3$	$4.7 \pm 0.6 \pm 0.2$	$4.5^{+1.4+0.5}_{-1.2-0.4}$	4.55 ± 0.44
$\pi^+\pi^0$	$5.3 \pm 1.3 \pm 0.5$	$5.5^{+1.0}_{-0.9} \pm 0.6$	$4.6^{+1.8+0.6}_{-1.6-0.7}$	5.27 ± 0.79
$\pi^0\pi^0$	$1.7 \pm 0.6 \pm 0.2$	$2.1 \pm 0.6 \pm 0.3$	< 4.4	1.90 ± 0.47
$\mathcal{A}_{\pi\pi}$	$+0.77 \pm 0.27 \pm 0.08$	$+0.19 \pm 0.19 \pm 0.05$	–	$+0.38 \pm 0.16$
$\mathcal{S}_{\pi\pi}$	$-1.23 \pm 0.41^{+0.08}_{-0.07}$	$-0.40 \pm 0.20 \pm 0.03$	–	-0.58 ± 0.20

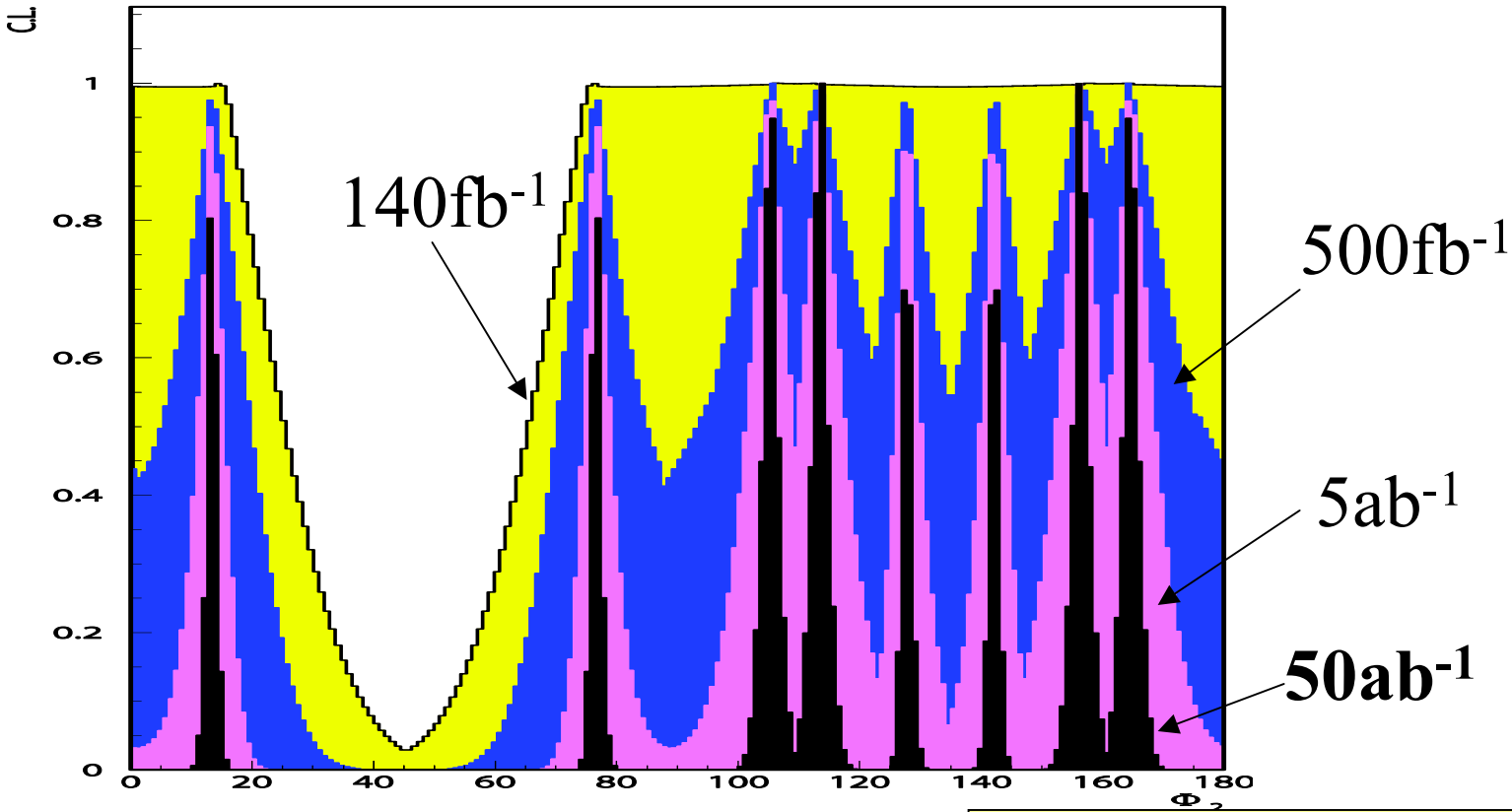
HFAG Summer 03

Prospect of $\theta = \phi_2 - \phi_{2eff}$

In the case that $A_{\pi^0\pi^0} = 0.0 \pm 1.0$ ***at*** $L = 140\ fb^{-1}$,



Prospect of ϕ_2



ϕ_2 **(deg)**

Typical ϕ_2 resolution
1 deg @ 50ab^{-1}

6. Conclusion

Belle 140 fb^{-1}

$$A_{\pi\pi} = +0.58 \pm 0.15(\text{stat}) \pm 0.07(\text{syst})$$

$$S_{\pi\pi} = -1.00 \pm 0.21(\text{stat}) \pm 0.07(\text{syst})$$

Observation of CP violation in $B^0 \to \pi^+\pi^-$! (5.2σ)

Evidence for direct CP violation in $B^0 \to \pi^+\pi^-$! ($\geq 3.2\sigma$ for any $S_{\pi\pi}$)