Soft-Collinear Factorization

& The Theory of $B \to X_s \gamma$ Decay

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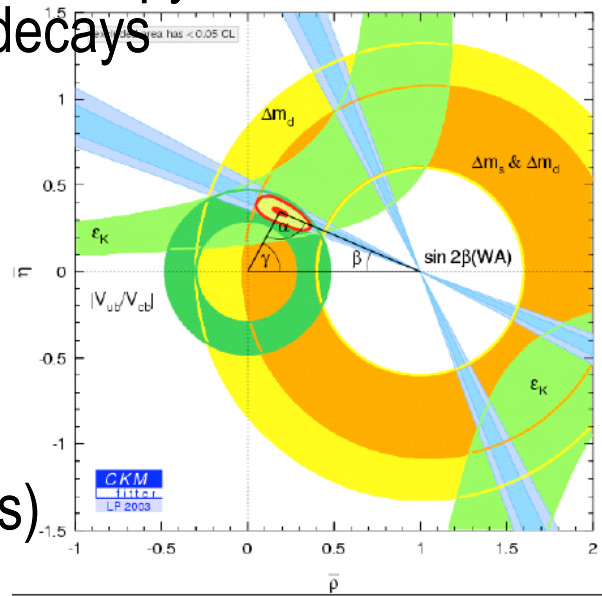
Outline

- Introduction
- Concepts and applications of soft-collinear effective theory (SCET)
- Factorization in $B \to X_s \gamma$ decay
- Outlook

Based on: hep-ph/0402094 (with S.Bosch, B.Lange, G.Paz)
& hep-ph/0408179

Introduction

- Heavy-quark expansions have become main theoretical tool to explore properties and decay processes of heavy (*b*) hadrons
  - Many applications to beauty & charm spectroscopy, exclusive $b\to cl\nu$ decays ($|V_{cb}|$), inclusive decays



- B-factory era (2000's)
  - Focus on $B\to$light processes:
    - $|V_{ub}|$ determinations (UT sides)
    - Rare decays (UT angles)
    - Searches for New Physics
  - Processes at large recoil (fast light particles)

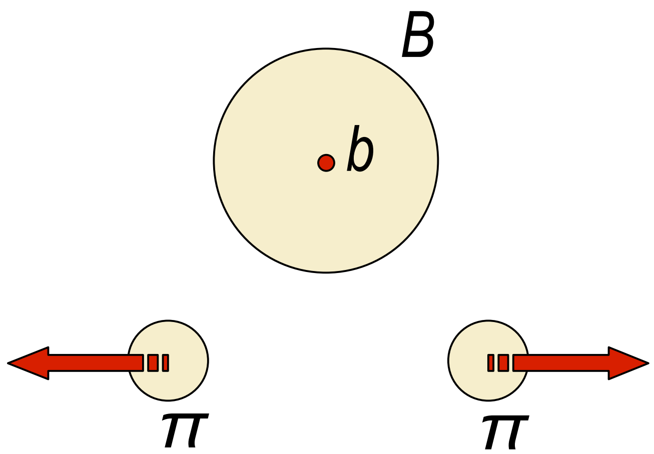
Challenge

- Construct heavy-quark expansions for processes involving both soft and energetic light partons

  – Soft: $p_{soft} \sim \Lambda_{QCD} << m_b$

  – Collinear: $p_{col}^2 << E_{col}^2$

  => p_{soft} ¢ p_{col}: semi-hard scale



- Systematic expansion ⇨ framework for studying power corrections (important for phenomenology)

Soft-Collinear Effective Theory

[Bauer, Pirjol, Stewart & Fleming, Luke]

- Define light-like vectors:

$$n^\mu = (1,0,0,1)\,, \quad \bar{n}^\mu = (1,0,0,-1)\,, \qquad n^2 = \bar{n}^2 = 0$$

- Expand 4-vectors:

$$p^\mu = (n\cdot p)\,\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\,\frac{n^\mu}{2} + p_\perp^\mu \equiv p_+^\mu + p_-^\mu + p_\perp^\mu$$

- Scaling of collinear momenta:

$$\bar{n}\cdot p \gg n\cdot p\,, \qquad \frac{n\cdot p}{\bar{n}\cdot p} \sim \lambda \quad (\text{or } \lambda^2)$$

Soft-Collinear Effective Theory

- Systematic power counting in $\lambda = \Lambda_{QCD}/E$
  - Momenta, coordinates, fields, operators
- Effective Lagrangians for strong and weak interactions (currents, 4-quark operators), expanded in powers of λ, e.g.:

$$\mathcal{L}_s^{(0)} = \bar{q}_s \, i\not{D}_s \, q_s + \bar{h} \, iv \cdot D_s \, h$$

$$\mathcal{L}_{hc}^{(0)} = \bar{\xi}_{hc} \frac{\not{\bar{n}}}{2} \left(in \cdot D_{hc+s} + i\not{D}_{hc\perp} \frac{1}{i\bar{n} \cdot D_{hc}} i\not{D}_{hc\perp} \right) \xi_{hc}$$

Non-local !

- Symmetries (gauge and reparameterization invariance)

Soft-Collinear Effective Theory

- Much more complicated than previous heavy-quark expansions
  - Many degrees of freedom (hard-collinear, collinear, soft, soft-collinear messengers)
  - Appearance of Wilson lines
  - Expansion in non-local string operators integrated over light-like field separation
  - Light-cone physics (not accessible to lattice QCD)

Different Versions of SCET

- Depending on kinematic situation, one distinguishes:
  - SCET-1: hard-collinear & soft *[Bauer, Pirjol, Stewart; Beneke, Feldmann et al.; Chay, Kim]*
    - e.g.: form-factor relations, inclusive $B \to X_s \gamma$ and $B \to X_u l \nu$ decays, jet physics, threshold production of unstable particles
  - SCET-2: collinear & soft & soft-collinear messengers *[Becher, Hill, MN]*
    - e.g.: exclusive $B \to \pi\pi$, $B \to K^* \gamma$ decays, $B \to$ light form factors
  - Often 2-step matching: QCD → SCET-1 → SCET-2
    a or: QCD → SCET-1 → HQET

Sample Applications

- Have entered an era in which many new results are being derived using SCET
  - Examples SCET-1:
    - Jet physics: enhanced power corrections to event shapes in $e^+e^- \to$ hadrons (α_s determination) *[Bauer, Lee, Manohar, Wise 03]*
    - Effective field theory for unstable particles *[Beneke, Chapovsky, Signer, Zanderighi 03]*
    - Inclusive B decays: first complete NLO predictions (RG-improved) for $B \to X_u l \nu$ spectra in shape-function region (precision determination of $|V_{ub}|$) *[Bauer, Manohar 03; Bosch, Lange, MN, Paz 04]*
    - RG-improved predictions for $B \to X_s \gamma$ *[this talk]*

Sample Applications

– Examples SCET-2:
  - QCD factorization proof and complete next-to-leading order Sudakov resummation for $B \to \gamma\, l\, \nu$ *[Lunghi, Pirjol, Wyler 02; Bosch, Hill, Lange, MN 03]*
  - Symmetry relations for color-suppressed $B \to D^0 \pi^0$ decays *[Mantry, Pirjol, Stewart 03]*
  - QCD factorization proof for $B \to K^* \gamma$ (parts for $B \to \pi\pi$) *[Becher, Hill, MN, Pecjak 03; Bauer, Pirjol, Stewart 04]*
  - Complete Sudakov resummation for $B \to$ light form factors *[Lange, MN 03; Becher, Hill, Lee, MN 04]*

- Relevance to *B*-factory program (γ from $B \to \pi\pi$, πK, $\rho\pi$; New Physics in $B \to \Phi K_s$ and πK_s "anomalies", etc.)

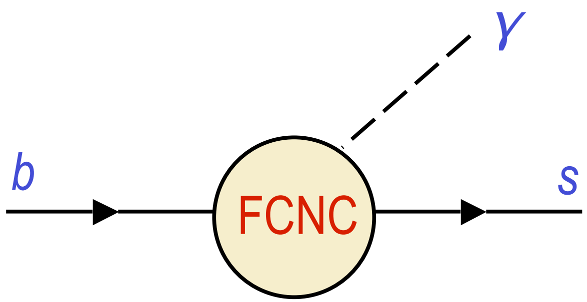
Factorization in Inclusive *B* Decays

Universal QCD factorization formula:

$$\Gamma = H\, J \otimes S$$

Importance of $B \to X_s \gamma$

- Prototype of all FCNC processes, with potentially large sensitivity to New Physics effects



- Crucial to have reliable prediction of inclusive rate in Standard Model and its extensions

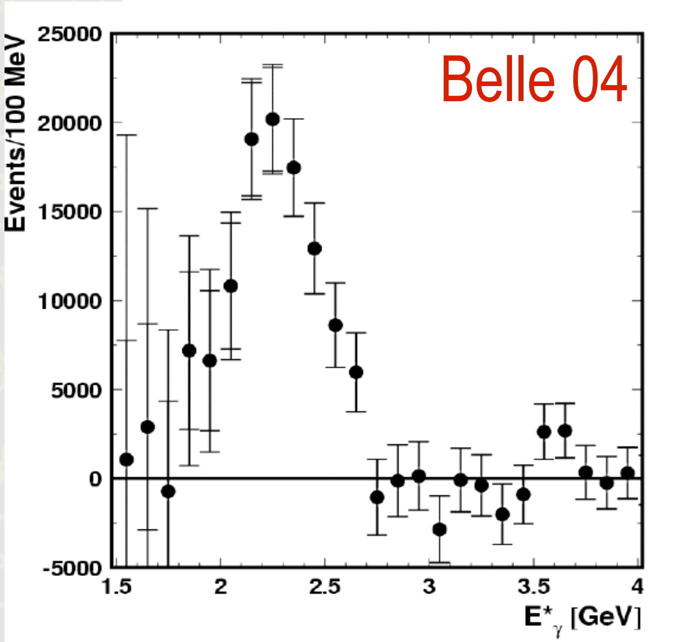
Present Status

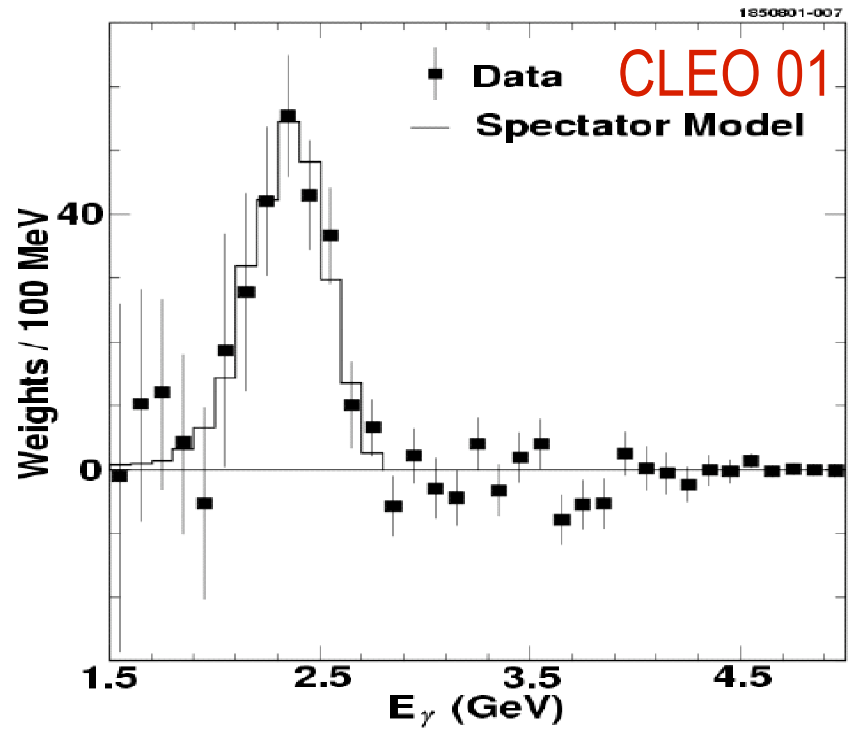
- Total rate known at NLO in RG-improved perturbation theory (claimed accuracy: $\delta\Gamma/\Gamma=10\%$)
- Heroic effort under way to compute the next order, requiring:
  - 3- and 4-loop anomalous dimensions
  - 2- and 3-loop matrix elements in the electroweak theory
  - One of most ambitious calculations in high-energy physics
- However, it is impossible to measure the total rate!

The Problem

- Experimental cutoff on photon energy:
  - CLEO: E_γ>2.0 GeV
  - Belle: E_γ>1.8 GeV

In both cases, “good” measurements exist down to just below 2 GeV !





The Problem

- Introduces sensitivity to scales below m_b, e.g.:

$$\Gamma \sim m_b^5 \left[1 - \frac{\alpha_s}{3\pi} \left(2 \ln^2 \frac{\Delta}{m_b} + 7 \ln \frac{\Delta}{m_b} + \ldots \right) \right] \; ; \quad \Delta = m_b - 2E_{\text{cut}}$$

Pole mass?

Large ratio!

- Relevant scales:
  - Hard: m_b
  - Hard-collinear: $\sqrt{m_b \Delta}$ ~ hadronic invariant mass of X_s
  - Soft: $\Delta = m_b - 2E_{cut}$
- Must disentangle physics at soft scale (nonperturbative) !
  - Main reason for RG analysis (not resummation of logs)

Scale Separation using EFT

- Two-stage matching of effective field theories:
  - QCD → SCET-1 → HQET
- QCD factorization formula:

$$\frac{d\Gamma}{dE_\gamma} = \frac{G_F^2\alpha}{2\pi^4}\,|V_{tb}V_{ts}^*|^2\,E_\gamma^3\,\overline{m}_b^2(\mu)$$

$$\times\,|H(\mu)|^2 \int_0^{M_B-2E_\gamma} d\hat{\omega}\,J(m_b(M_B-2E_\gamma-\hat{\omega}),\mu)\,S(\hat{\omega},\mu)+\dots$$

Hard-collinear contributions (jet function)

Hard contributions

Soft contributions (shape function)

No reference to pole mass!

Shape Function

- Definition:

$$S(\hat{\omega}) = \int \frac{dt}{2\pi}\, e^{i\omega t}\, \frac{\langle \bar{B}(v)|\, \bar{h}(0)\, [0, tn]\, h(tn)\, |\bar{B}(v)\rangle}{2M_B}$$

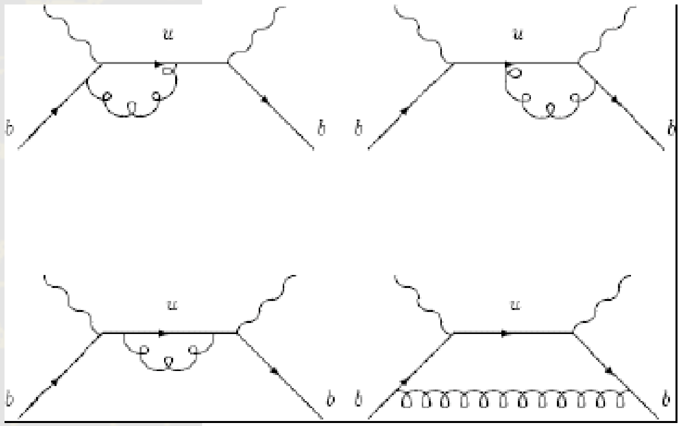
$$= \frac{\langle \bar{B}(v)|\, \bar{h}\, \delta(\omega - in\cdot D)\, h\, |\bar{B}(v)\rangle}{2M_B}$$

- Momentum distribution function (parton distribution), where $\hat{\omega} = \bar{\Lambda} - \omega$, and ω corresponds to the residual momentum of the heavy quark inside the *B* meson

[MN 93; Bigi, Shifman, Uraltsev, Vainshtein 94]

Matching 1: QCD $\rightarrow$ SCET

- QCD graphs:



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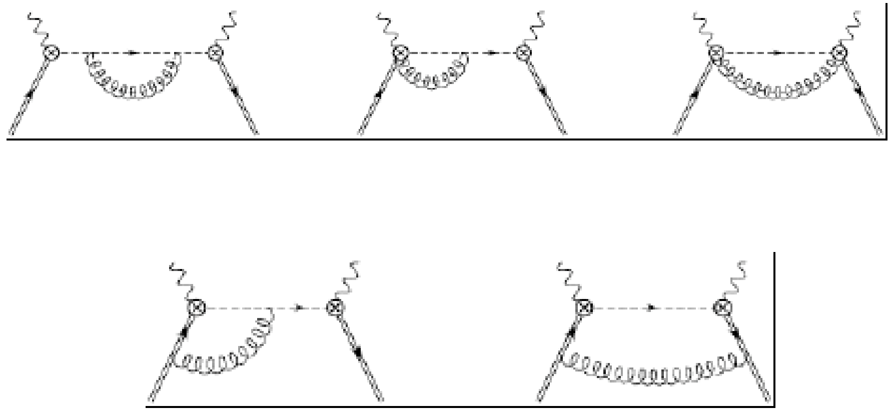
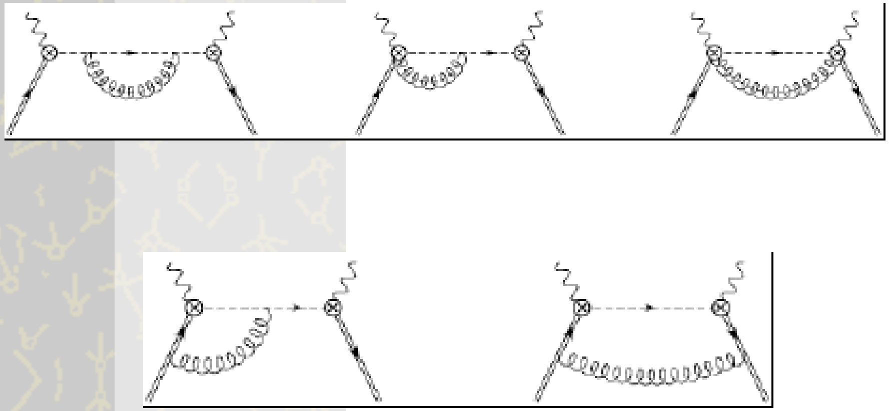
- SCET graphs:



⇨ determines hard function *H*

Matching 2: SCET $\to$ HQET

- SCET graphs:



- HQET graphs:

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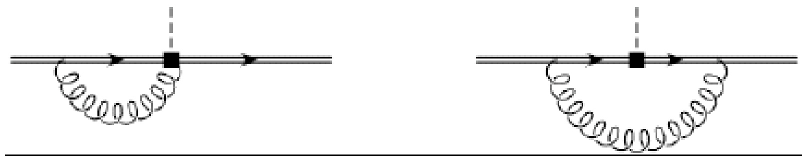


⇨ determines jet function *J*

RG Evolution

- Perturbative expansion of hard function is well behaved at scale $\mu_h \sim m_b$
- Perturbative expansion of jet function is well behaved at scale $\mu_i \sim \sqrt{m_b \Lambda_{QCD}}$
- Physics of shape function is associated with a hadronic scale $\mu_0 \sim \Lambda_{QCD}$
- Resum large logarithms (Sudakov double logarithms) using RG equations

Multi-Step Procedure

- Master formula for the rate:

$S(\mu_i)$: input

$$\Gamma \sim H(\mu_h) \; * \; U(\mu_h,\mu_i) \; * \; J(\mu_i) \; * \; U(\mu_i,\mu_0) \; * \; S(\mu_0)$$

QCD ⟶ SCET ⟶ (RG evolution) ⟶ HQET ⟶ (RG evolution)

Perturbation theory

Non-perturbative physics

Evolution Equations

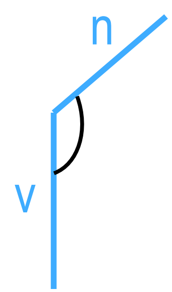
- RGE for hard functions:

$$\frac{d}{d\ln\mu} H(\mu) = 2\gamma_J(m_b,\mu)\, H(\mu)$$

- Anomalous dimension:

Cusp anomalous dimension

$$\gamma_J(m_b,\mu) = -\Gamma_{\text{cusp}}(\alpha_s)\, \ln\frac{\mu}{m_b} + \gamma'(\alpha_s)$$



- Can be solved using “standard” techniques (requires 3-loop anomalous dimension at NLO!)
  - One loop more than usual due to extra logarithm

Evolution Equations

- RGE for shape function (integro-differential equation):

$$\frac{d}{d\ln\mu} S(\hat\omega,\mu) = -\int d\hat\omega'\, \gamma_S(\hat\omega,\hat\omega',\mu)\, S(\hat\omega',\mu)$$

- Exact solution: *[Lange, MN 03; see also Mannel et al. 98]*

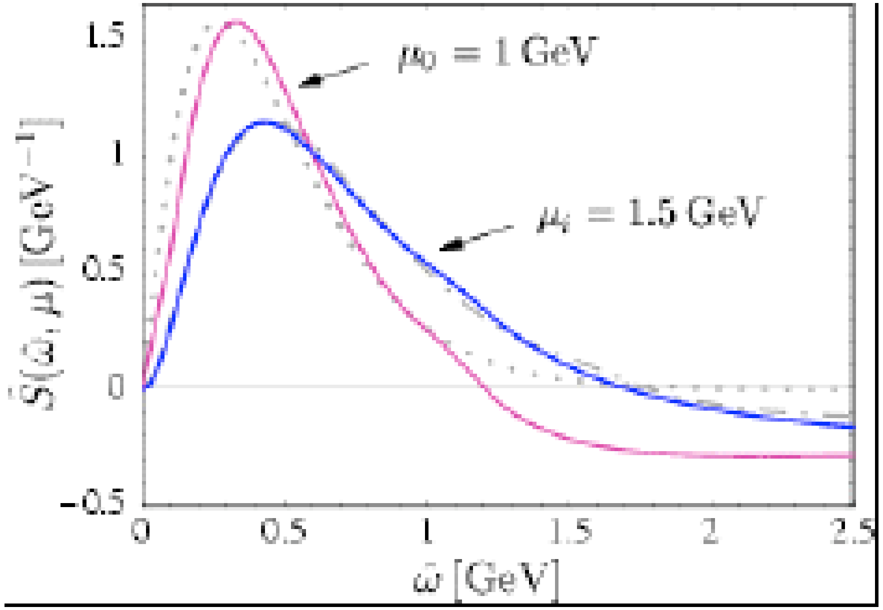
$$S(\hat\omega,\mu) = e^{V_S(\mu,\mu_0)}\,\frac{e^{-\eta\gamma_E}}{\Gamma(\eta)}\int_0^{\hat\omega} d\hat\omega'\,\frac{S(\hat\omega',\mu_0)}{\mu_0^\eta\,(\hat\omega-\hat\omega')^{1-\eta}}$$

where:

$$\eta = \eta(\mu,\mu_0) = 2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\,\Gamma_{\rm cusp}(\alpha)$$

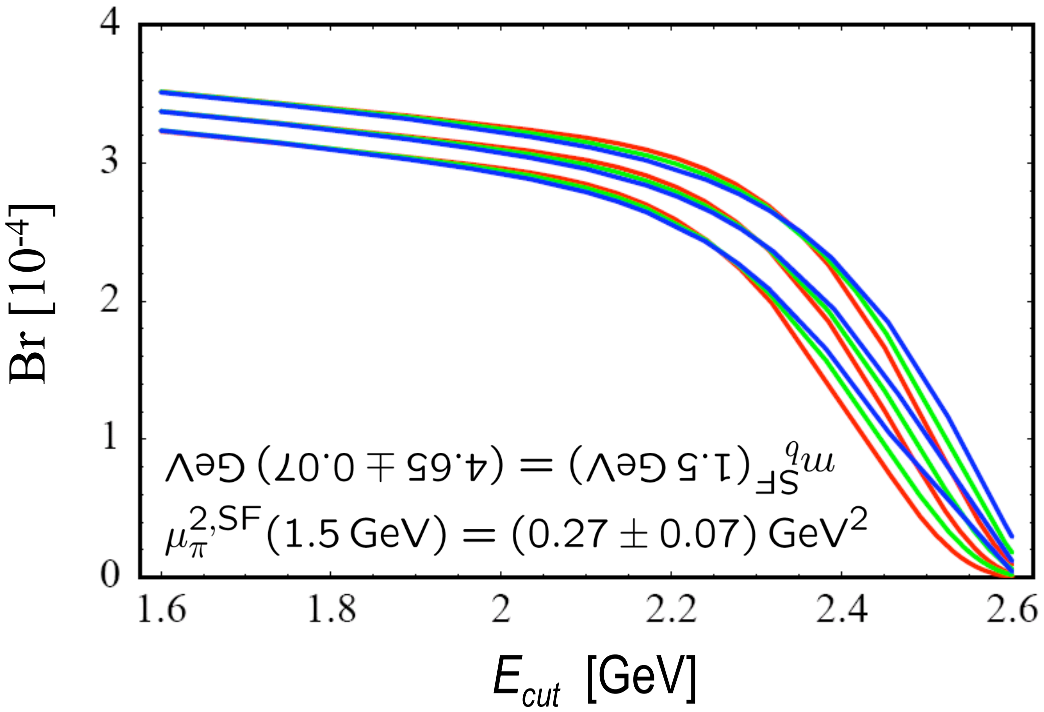
Shape Function Plots

- Evolution effects:



$B \to X_s \gamma$ Branching Ratio

- Branching ratio for $E_\gamma < E_{cut}$ in units of 10^{-4}, for different shape-function models (variation of m_b and μ_π^2):

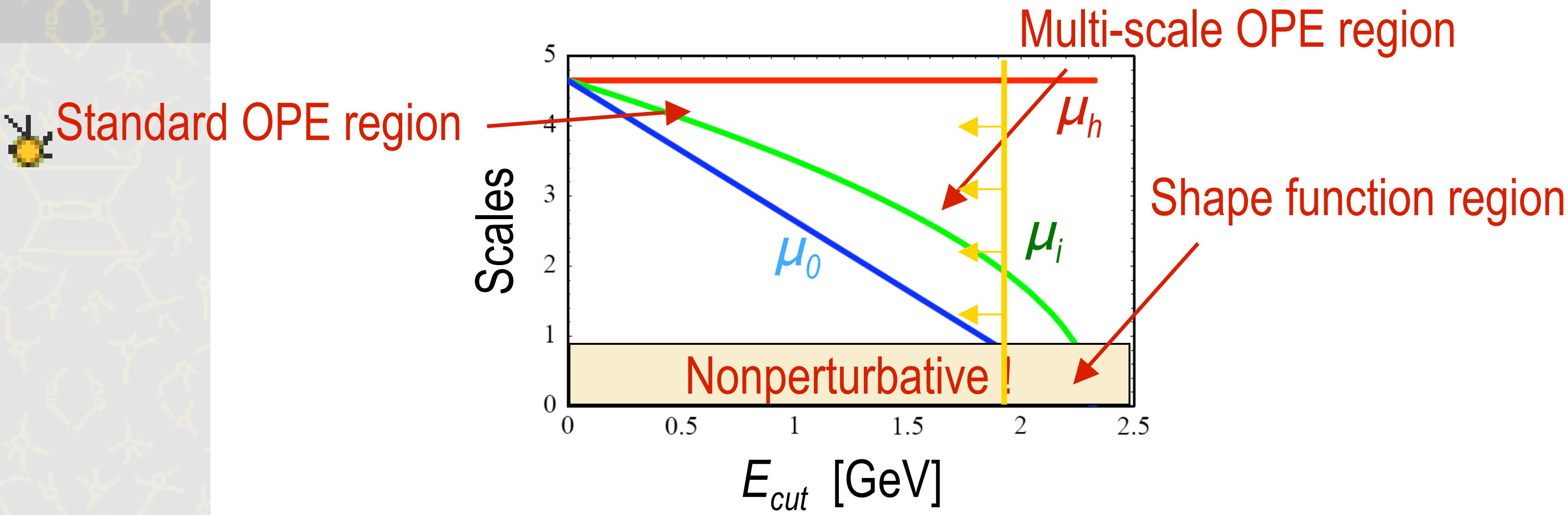


Folklore

- Spectral shape at large E_γ (2.1-2.6 GeV) very sensitive to hadronic physics (shape function) *[Kagan, MN 98]*
- Once cutoff is below 2 GeV, rate can be calculated using conventional heavy-quark expansion in $\alpha_s(m_b)$ and Λ_{QCD}/m_b
  - Assumes relevance of only 2 physical scales: m_b and Λ_{QCD}
  - Variation $m_b/2 < \mu < 2m_b$ yields small scale dependence

But this is wrong!

Smooth Transition:
Shape-Function Region to OPE Region



- For realistic values, $\mu_0 \sim \Delta = m_b - 2E_{cut} \sim 1$ GeV $\ll m_b$
- Scale separation needed, since $\alpha_s(m_b) \approx 0.2$ and $\alpha_s(\Delta) \approx 0.5$ are rather different, and power corrections in Λ_{QCD}/Δ can be larger than those in Δ/m_b
- Requires sophisticated *multi-scale OPE*

Scale Separation using SCET

- Master formula for the rate:

$$\Gamma \sim H(\mu_h) \;*\; U(\mu_h,\mu_i) \;*\; J(\mu_i) \;*\; U(\mu_i,\mu_0) \;*\; M(\mu_0)$$

QCD ⟶ SCET ⟶ (RG evolution) ⟶ HQET ⟶ (RG evolution) ⟶ local OPE

Perturbation theory

Non-perturbative physics

OPE for Shape-Function Integrals

[Bosch, Lange, MN, Paz hep-ph/0402094]

- Short-distance expansion of arbitrary shape-function integrals defined with a hard UV cutoff Λ_{UV} :

$$\int_0^{\Lambda_{UV}+\bar\Lambda} d\hat\omega\, f(\hat\omega)\, S(\hat\omega,\mu)$$

$$= K_0^{(f)}(\Lambda_{UV},\mu) + K_2^{(f)}(\Lambda_{UV},\mu)\cdot \underbrace{\frac{(-\lambda_1)}{3\Lambda_{UV}^2}}_{\approx \left(\frac{300\text{ MeV}}{\Lambda_{UV}}\right)^2} + \dots$$

Wilson coefficients

- Well-controlled connection with HQET parameters

Resummed Expression

(Leading term)

- Result after RG improvement at NNLO:

$$F_E(\Delta) = 1 + \frac{\alpha_s(m_b)}{3\pi}\left(-2\ln^2\frac{\Delta}{m_b} - 7\ln\frac{\Delta}{m_b}\right)$$

$$\to \frac{e^{-\eta\gamma_E}}{\Gamma(1+\eta)}\exp[2S(\mu_h,\mu_i)+2S(\mu_0,\mu_i)-2a_{\gamma'}(\mu_h,\mu_i)+2a_\gamma(\mu_0,\mu_i)]\left(\frac{m_b}{\mu_h}\right)^{-2a_\Gamma(\mu_h,\mu_i)}\left(\frac{\Delta}{\mu_0}\right)^{\eta}$$

$$\times\left\{1+\frac{\alpha_s(\mu_h)}{3\pi}\left[-4\ln^2\frac{m_b}{\mu_h}+10\ln\frac{m_b}{\mu_h}+\frac{7\pi^2}{6}-7\right]\right.$$

$$+\frac{\alpha_s(\mu_i)}{3\pi}\left[2\ln^2\frac{m_b\Delta}{\mu_i^2}-[4H(\eta)+3]\ln\frac{m_b\Delta}{\mu_i^2}+3H(\eta)+2H^2(\eta)-2\psi'(1+\eta)+7-\frac{2\pi^2}{3}\right]$$

$$\left.+\frac{\alpha_s(\mu_0)}{3\pi}\left[-4\ln^2\frac{\Delta}{\mu_0}+4[2H(\eta)-1]\ln\frac{\Delta}{\mu_0}+4H(\eta)-4H^2(\eta)+4\psi'(1+\eta)-\frac{5\pi^2}{6}\right]\right\}$$

where: $\Delta = m_b - 2E_{cut}$, and $\mu_h \sim m_b$, $\mu_i \sim \sqrt{m_b\Delta}$, $\mu_0 \sim \Delta$

Leading Power Corrections

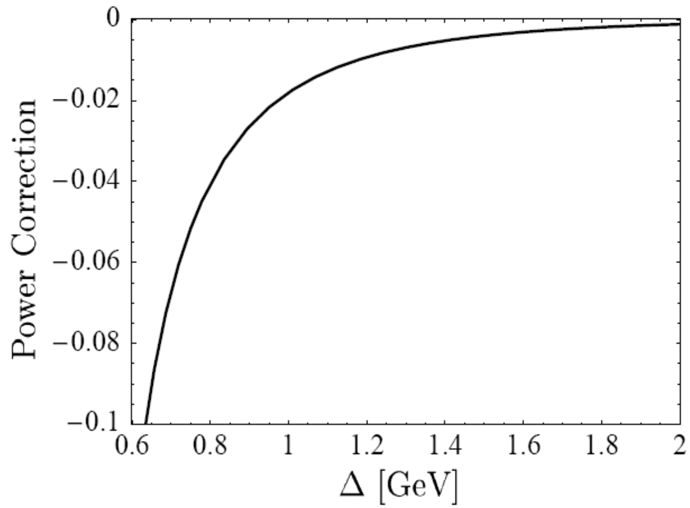
- Small effect of λ_1/Δ^2 corrections:
  - 

Figure 2: Size of the enhanced power correction proportinal to λ_1/Δ^2 in (25) relative to the leading term, as a function of $\Delta = m_b - 2E_0$.

- In many respects, similarity with R_τ ratio

Numerical Results

- Estimate perturbative uncertainty by varying the 3 matching scales $\mu_h \sim m_b$, $\mu_i \sim \sqrt{m_b \Delta}$, $\mu_0 \sim \Delta$ by ±50% about their central values
- Also estimate parameter uncertainties

⇨ *The dominant uncertainty!*

Branching Ratio

- Detailed error analysis:

Table 2: $B \to X_s \gamma$ branching ratio (in units of 10^{-4}) with estimates of perturbative uncertainties obtained by variation of the matching scales, for three variants of the shape-function scheme. See text for explanation.

E_0	Scheme	Br $[10^{-4}]$	μ_h	μ_i	μ_0	Sum	Power Cors.	Combined
1.8 GeV	RS 1	3.44	$^{+0.03}_{-0.00}$	$^{+0.28}_{-0.40}$	$^{+0.51}_{-0.02}$	$^{+0.58}_{-0.40}$	$^{+0.12}_{-0.07}$	± 0.53
	RS 2	3.44	$^{+0.03}_{-0.00}$	$^{+0.28}_{-0.40}$	$^{+0.14}_{-0.15}$	$^{+0.31}_{-0.42}$	$^{+0.12}_{-0.07}$	± 0.53
	RS 3	3.42	$^{+0.03}_{-0.00}$	$^{+0.28}_{-0.40}$	$^{+0.17}_{-0.15}$	$^{+0.33}_{-0.42}$	$^{+0.12}_{-0.07}$	± 0.53
1.6 GeV	RS 1	3.51	$^{+0.03}_{-0.00}$	$^{+0.31}_{-0.41}$	$^{+0.17}_{-0.01}$	$^{+0.35}_{-0.41}$	$^{+0.10}_{-0.05}$	± 0.55
	RS 2	3.52	$^{+0.03}_{-0.00}$	$^{+0.31}_{-0.41}$	$^{+0.13}_{-0.09}$	$^{+0.33}_{-0.42}$	$^{+0.10}_{-0.05}$	± 0.55
	RS 3	3.52	$^{+0.03}_{-0.00}$	$^{+0.31}_{-0.41}$	$^{+0.17}_{-0.11}$	$^{+0.35}_{-0.42}$	$^{+0.10}_{-0.05}$	± 0.55

- Scale uncertainty much larger than ±3% (usually assigned)

Branching Ratio

- Parameter uncertainties:

Table 3: $B \to X_s\gamma$ branching ratio (in units of 10^{-4}) with estimates of theoretical uncertainties due to input parameter variations as listed in Table 1. The upper (lower) sign refers to increasing (decreasing) a given input parameter.

Default	$m_b(\mu_*, \mu_*)$	$\overline{m}_b(\overline{m}_b)$	m_c/m_b	m_t	$\lvert V_{ts}^* V_{tb}\rvert$	τ_B	$\alpha_s(M_Z)$	Combined
3.44	± 0.15	± 0.18	$^{-0.19}_{+0.10}$	± 0.04	$^{+0.24}_{-0.10}$	± 0.03	± 0.08	$^{+0.36}_{-0.33}$
3.52	± 0.13	± 0.18	$^{-0.20}_{+0.10}$	± 0.04	$^{+0.25}_{-0.10}$	± 0.04	± 0.10	$^{+0.37}_{-0.33}$

(correlation between m_b and $|V_{ts}{}^*V_{tb}|$ not yet included)

Branching Ratio

- Combined theory result:

$$\mathrm{Br}(B \to X_s\gamma)\Big|_{E_0=1.8\,\mathrm{GeV}} = (3.44 \pm 0.53\,[\mathrm{pert.}] \pm 0.35\,[\mathrm{pars.}]) \times 10^{-4}$$

  - Significant perturbative uncertainty from sensitivity to low scales!

- Experiment (Belle 2004):

$$\mathrm{Br}(B \to X_s\gamma)\Big|_{E_\gamma>1.8\,\mathrm{GeV}} = (3.38 \pm 0.30 \pm 0.29) \cdot 10^{-4}$$

Implications for New Physics

- Larger theory error, and better agreement between theory and experiment, weaken constraints on parameter space of New Physics models!
- E.g., type-II two-Higgs doublet model:
  - $m(H+) > 200$ GeV (95% CL)
    compared with previous bound of 500 GeV *[Gambino, Misiak]*

Shopping List

- Needed for a 5-10% calculation of the rate:
  - 2-loop corrections (at least $\beta_0\,\alpha_s^2$ terms) for matching coefficients at the scales μ_h, μ_i, and μ_0
  - "Contour-improved perturbation theory" at low scale μ_0 ?
  - Leading-order RG analysis (operator mixing) for $\Lambda_{\mathrm{QCD}}/m_b$ power corrections (NNLO in SCET expansion)
- "Straightforward, but tedious …"
- A lot of work is required to get a truly precise prediction for the $B \to X_s \gamma$ Branching Ratio

Summary

- For past few years, focus of heavy-flavor theory has been on understanding interplay of soft and energetic light partons
- Soft-collinear effective theory and QCD factorization theorems provide field-theoretical language for systematic studies of Sudakov logarithms and power-suppressed corrections
- Long list of potential applications