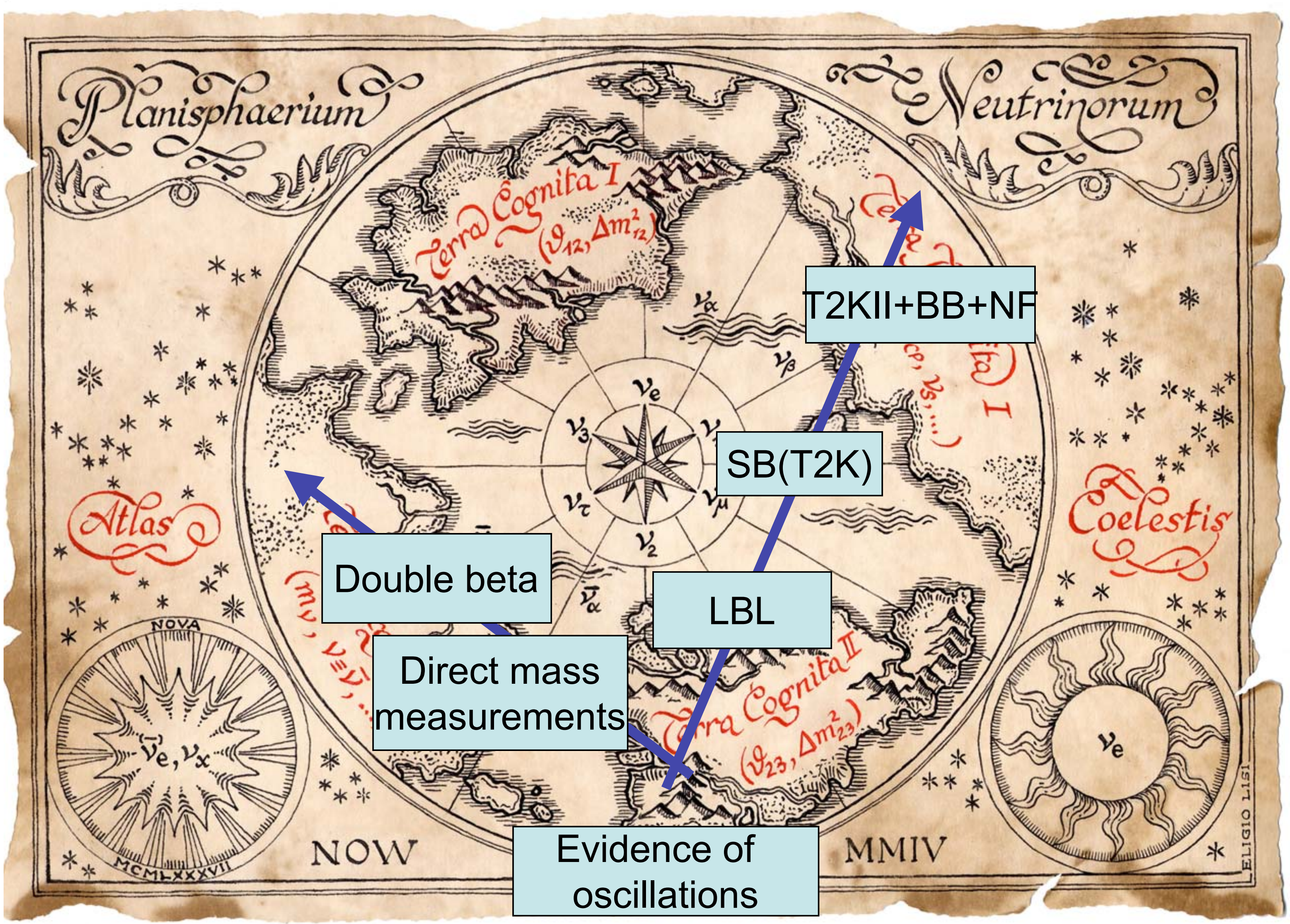
Extracting the parameters of the PMNS matrix from future neutrino oscillation experiments I

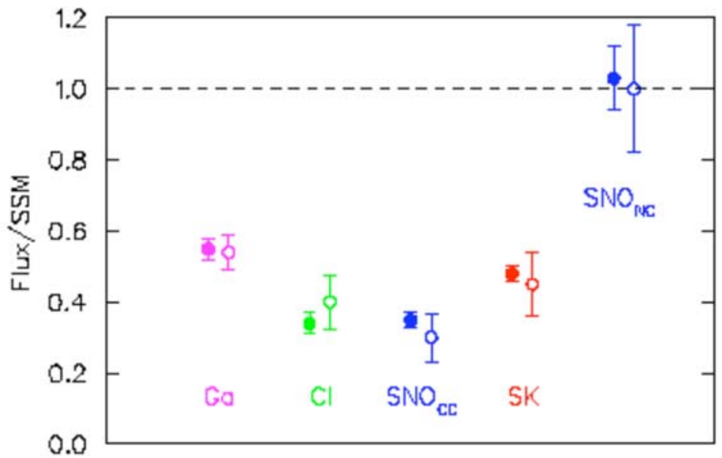
J.J. Gómez-Cadenas
U. Valencia/KEK

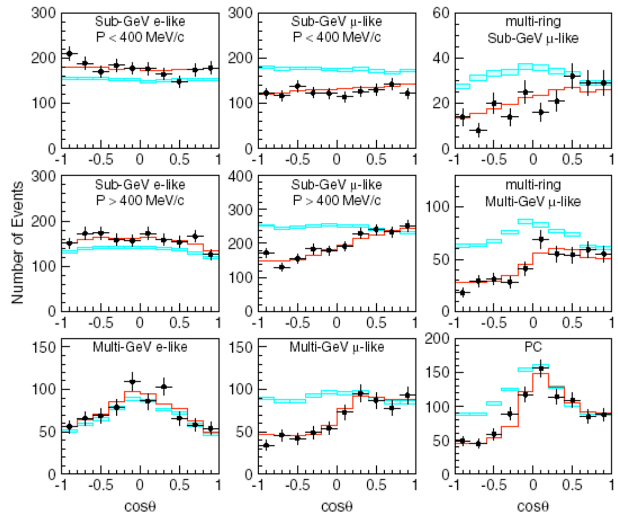
Original results presented in this talk based on work done in collaboration with P. Hernández, J. Burguet-Castell, D. Casper & P.Novella

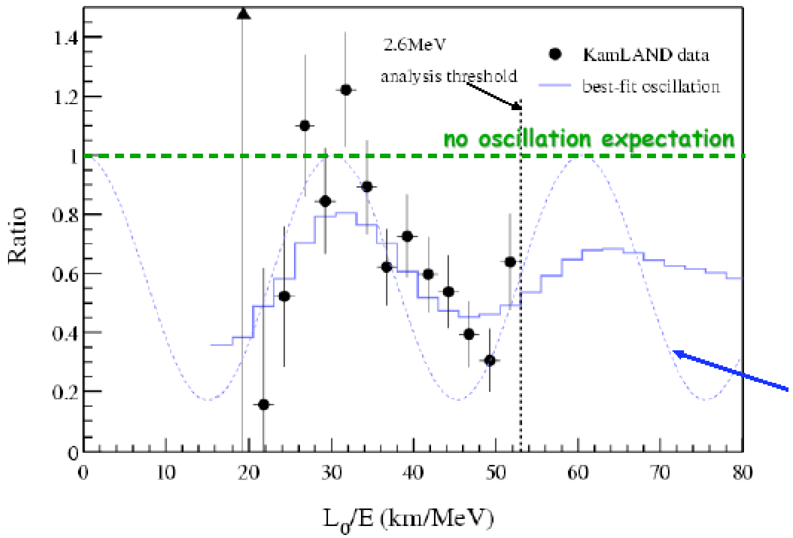
A trip to terra incognita

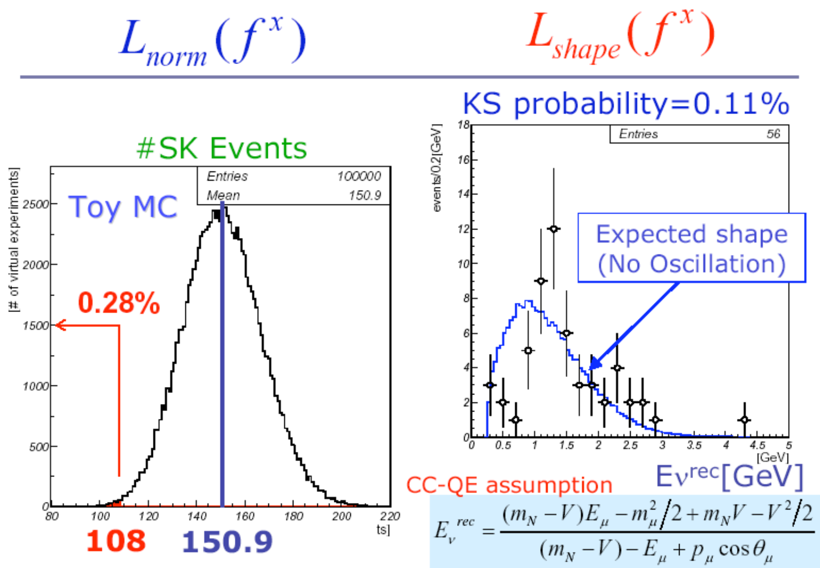


Evidence of neutrino oscillations



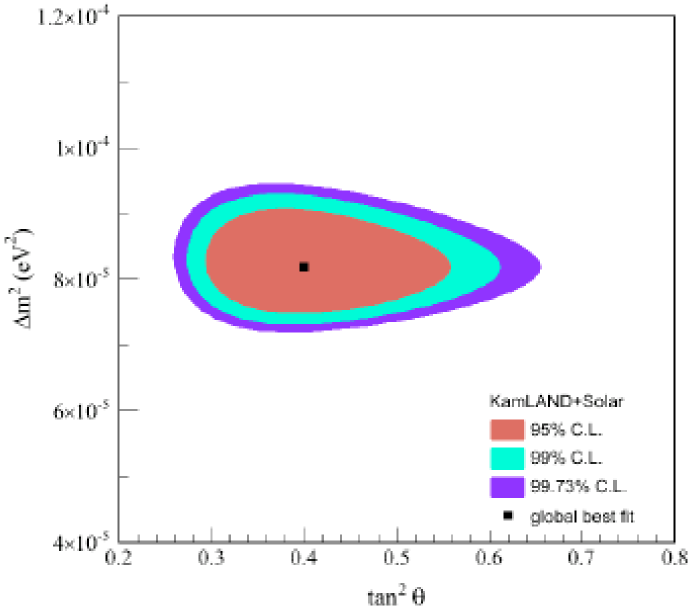






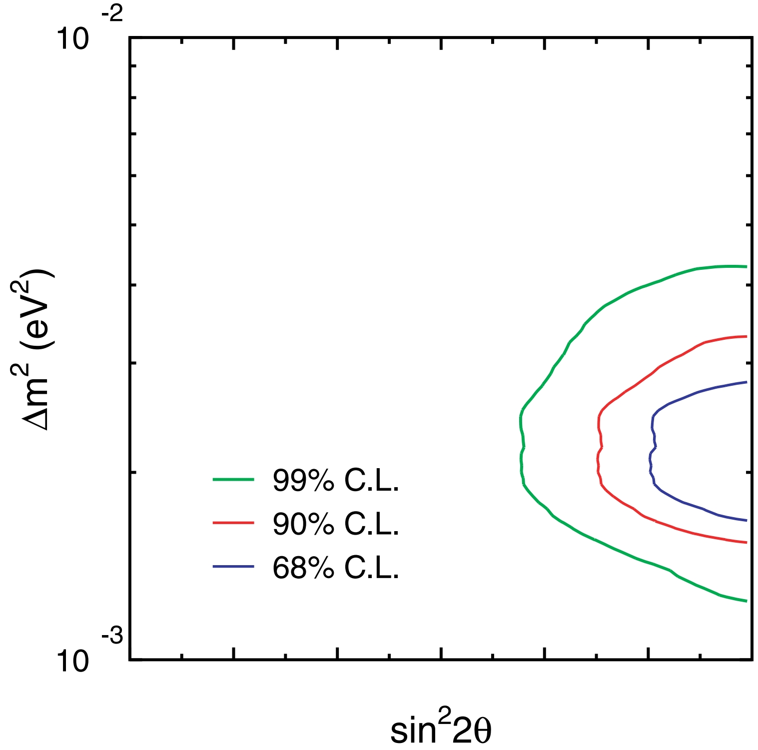
$$\Delta m_{12}^2 = 8.2^{+0.6}_{-0.5} \times 10^{-5}\, eV^2$$

$$\tan^2\theta_{12} = 0.40^{+0.09}_{-0.07}$$

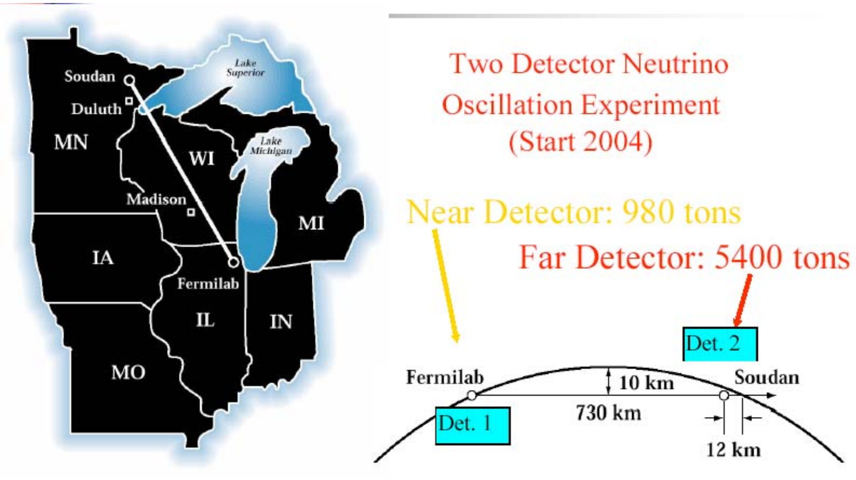


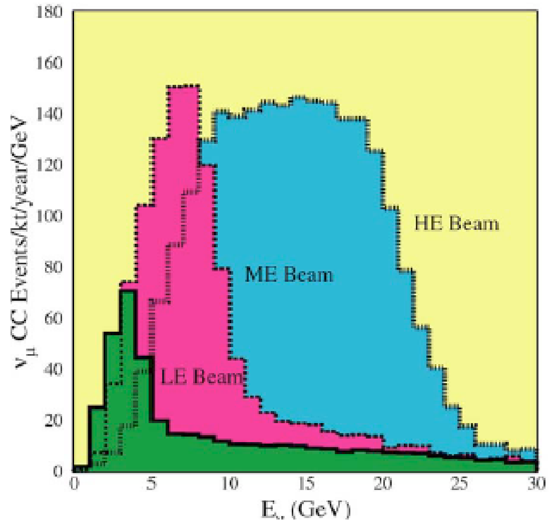
$$\Delta m^2 = 2.1 \cdot 10^{-3}\, eV^2$$

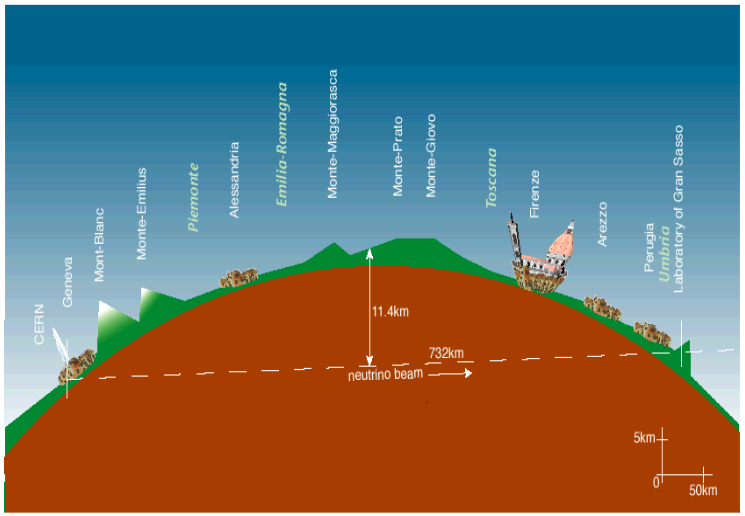
$$\sin^2 2\theta \approx 1$$

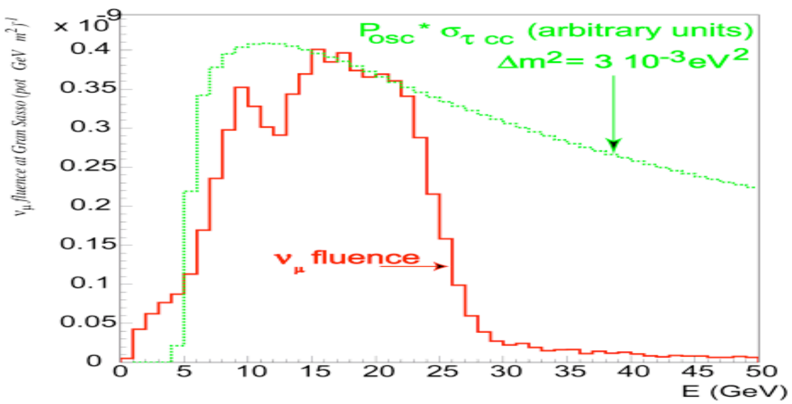


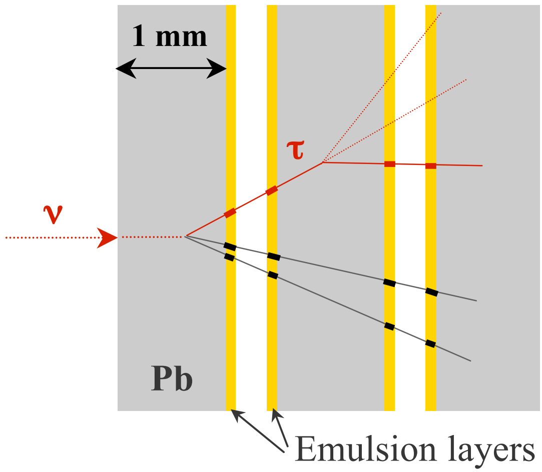
Long Base Line Experiments

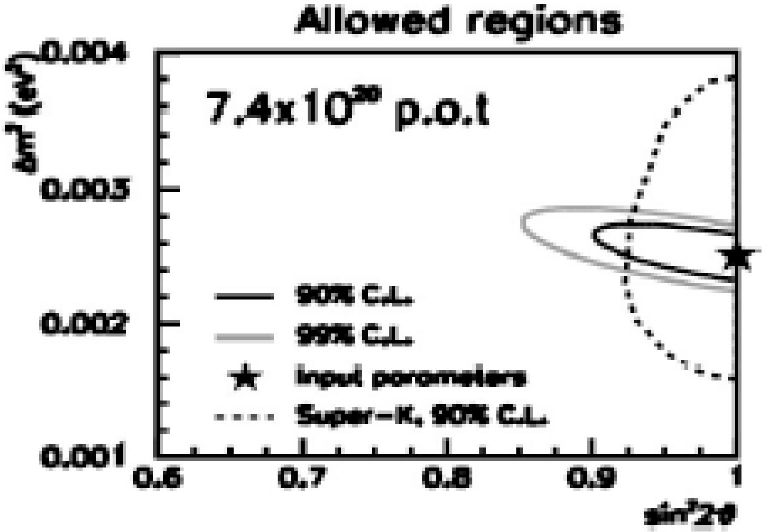


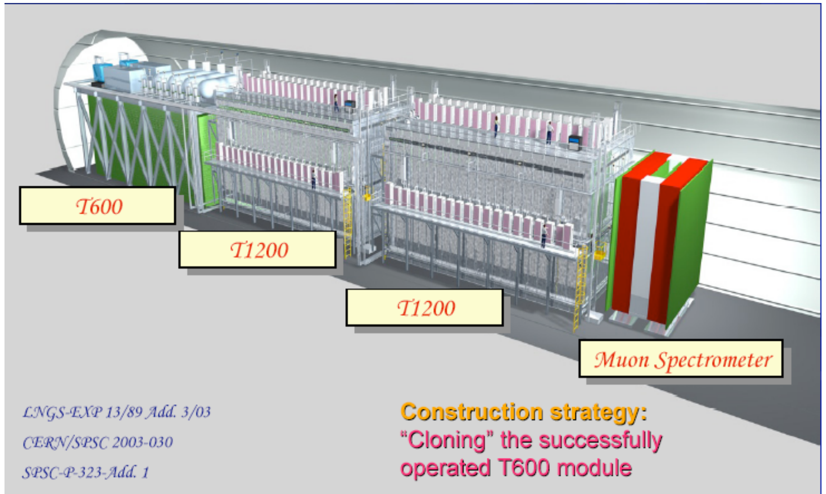


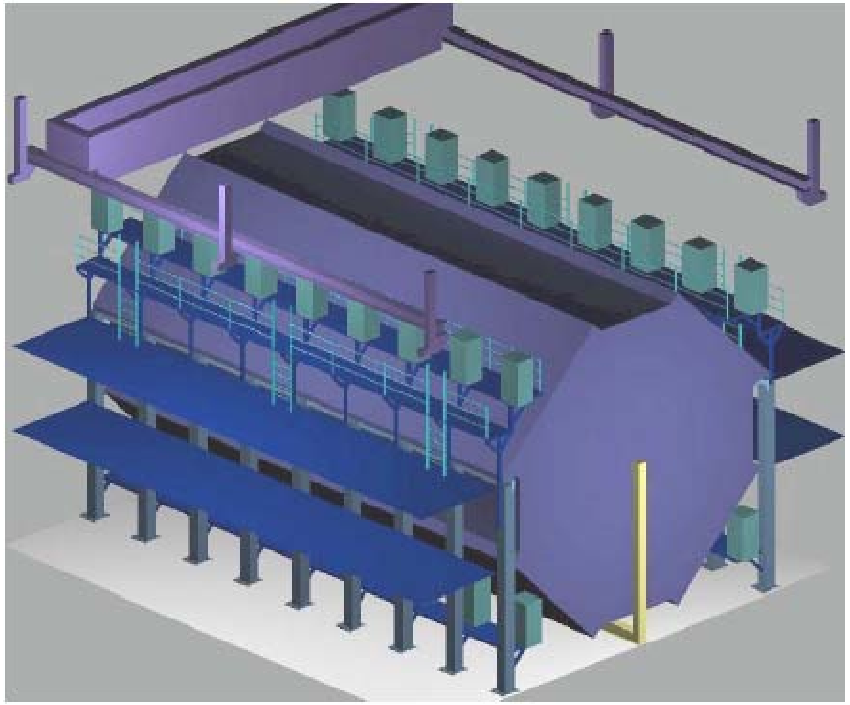





1 mm
ν
τ
Pb
Emulsion layers


Allowed regions
δm^2 (eV^2)
0.004
0.003
0.002
0.001
7.4×10^{20} p.o.t
90% C.L.
99% C.L.
input parameters
Super-K, 90% C.L.
0.6
0.7
0.8
0.9
1
$\sin^2 2\theta$

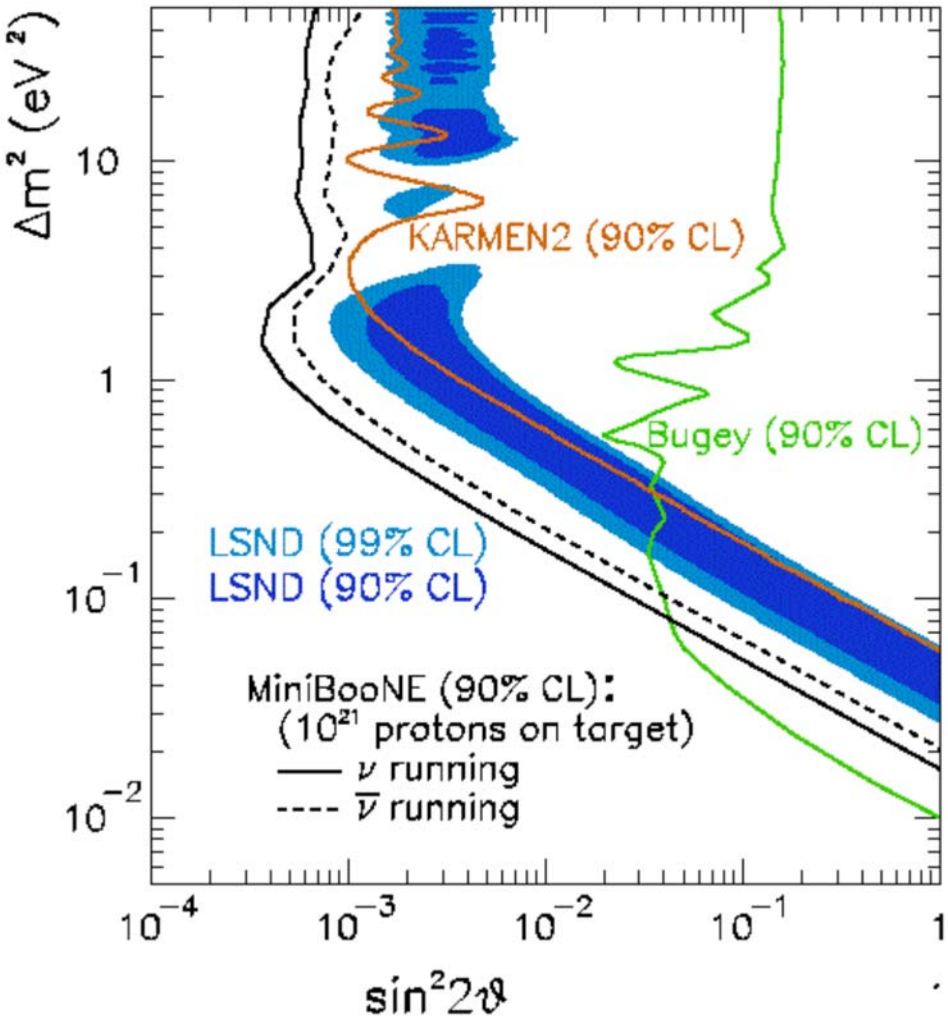

T600
T1200
T1200
Muon Spectrometer
LNGS-EXP 13/89 Add. 3/03
CERN/SPSC 2003-030
SPSC-P-323-Add. 1
Construction strategy:
"Cloning" the successfully
operated T600 module



The last anomaly

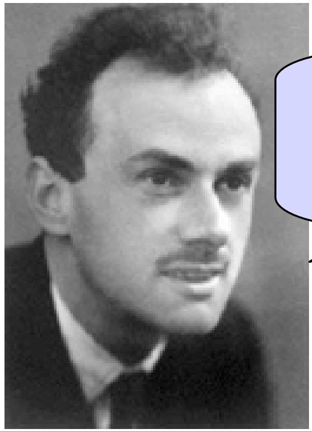
- LSND
  - Does not fit in a 3 family scenario
    - 2 Δm^2
- MiniBooNE (Fnal)
  - Testing it …
- If it is confirmed (2005)!?
  - Change our vision of ν
  - (has happened before…)

$$\overline{\nu}_\mu \rightarrow \overline{\nu}_e \, ?$$

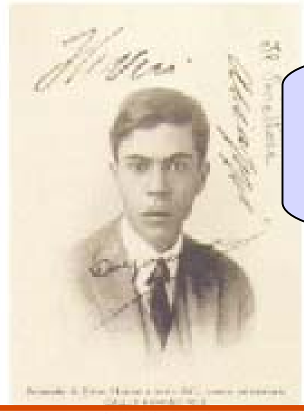


Neutrino masses mixing and oscillations

Neutrino masses. Dirac versus Majorana



ν_R is a distinct state



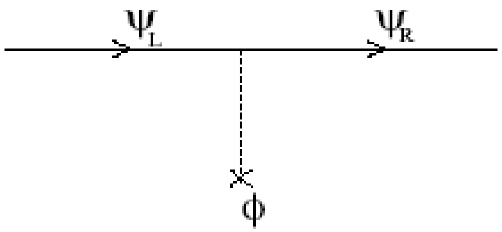
ν_R is the ν_L antineutrino

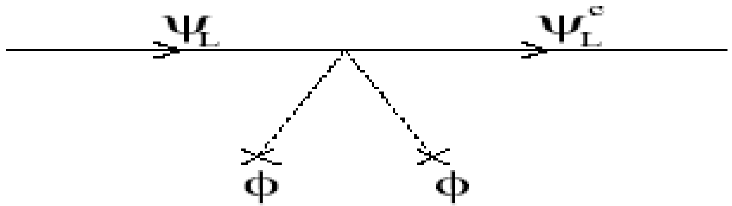
$$\lambda\, \overline{\psi}_R \phi \psi_L \xrightarrow{SSB} \lambda v\, \overline{\psi}_R \psi_L$$

$$m_\nu = \lambda v$$

$$\frac{1}{M}\overline{\psi^C_L}\, \lambda \phi^T \phi \psi_L \xrightarrow{SSB} \frac{\lambda v^2}{M}\overline{\psi^C_L}\psi_L$$

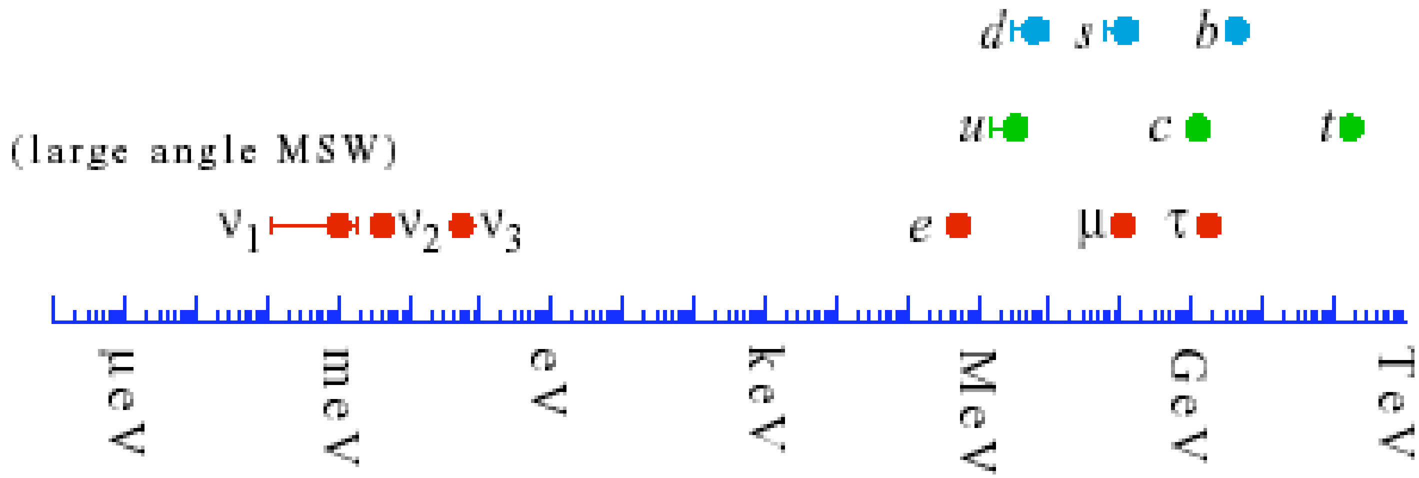
$$m_\nu = \lambda \frac{v^2}{\Lambda}, \quad \Lambda \to \textit{a new physics scale!?}$$



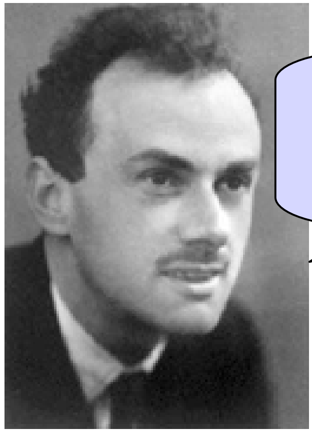


Smallness of neutrino masses

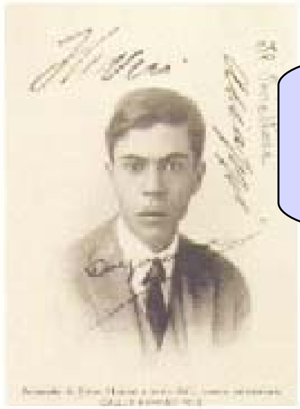
Why neutrino masses are so much smaller than the other fermion masses???



Smallness of Neutrino masses. Dirac versus Majorana



Give a small value to λ



M>>v

Why λ is so much smaller for neutrinos than for the other leptons?? (hierarchy problem!!!)

$$m_\nu = \lambda \frac{v^2}{\Lambda} \quad \textit{only for } \nu!$$

Same order than other fermions

New physics scale. Λ very large, so m_ν very small

Flavour basis

Mass couplings

$$\begin{pmatrix} \bar{\nu}_R^e & \bar{\nu}_R^\mu & \bar{\nu}_R^\tau \end{pmatrix} M_{3x3} \begin{pmatrix} \nu_L^e \\ \nu_L^\mu \\ \nu_L^\tau \end{pmatrix}$$

$$M_{3x3}^{Dirac} = \lambda_{3x3} v$$

$$M_{3x3}^{Majorana} = \lambda_{3x3} \frac{v^2}{\Lambda}$$

Weak couplings

$$\begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} W_\mu^+ \gamma^\mu \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_L^e \\ \nu_L^\mu \\ \nu_L^\tau \end{pmatrix}$$

Mass basis and mixing matrix

Diagonalize the mass matrix

$$\begin{pmatrix} \bar{\nu}_R^1 & \bar{\nu}_R^2 & \bar{\nu}_R^3 \end{pmatrix} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m3 \end{pmatrix} \begin{pmatrix} \nu_L^1 \\ \nu_L^2 \\ \nu_L^3 \end{pmatrix}$$

Weak couplings

$$\begin{pmatrix} \bar{e}_L & \bar{\mu}_L & \bar{\tau}_L \end{pmatrix} W_\mu^+ \gamma^\mu V_{PNMS} \begin{pmatrix} \nu_L^1 \\ \nu_L^2 \\ \nu_L^3 \end{pmatrix}$$

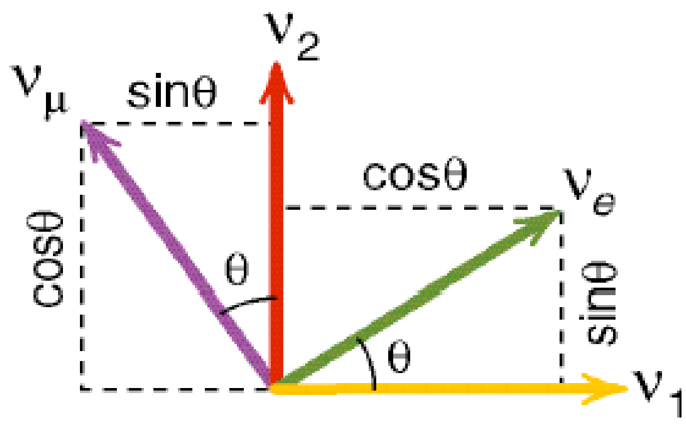
Mixing matrix connecting mass and weak eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

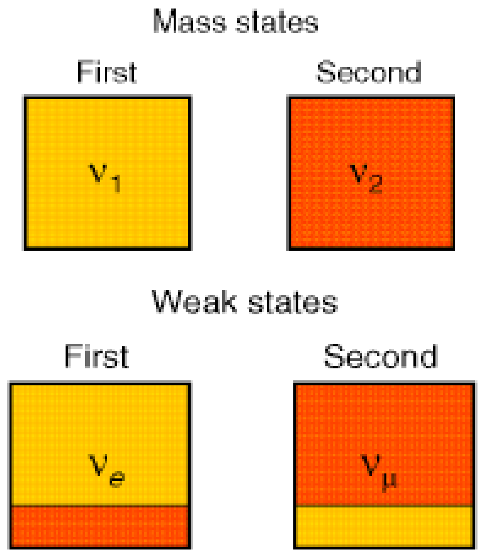
Pontecorvo
Maki
Nakagawa
Sakata

Mixing in two families

Consider for simplicity two families. Then the mixing matrix depends of a single parameter, the mixing angle θ



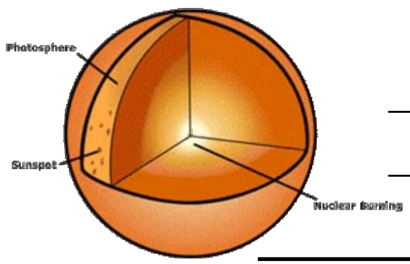
That is, the weak and mass eigenstates are connected by a simple two-dimensional rotation



$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Neutrino oscillations

source

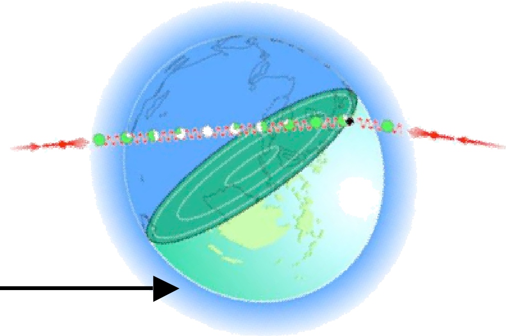


ν_e

propagation

L

detection



The weak interaction produces neutrinos of a given flavor

$$|\nu(x_0)\rangle = |\nu_e\rangle$$
$$= c|\nu_1\rangle + s|\nu_2\rangle$$

The mass eigenstates Propagate at different velocities

$$|\nu(x)\rangle = c|\nu_1\rangle e^{i(Et - \vec{k_1}\vec{x})}$$
$$+ \ s|\nu_2\rangle e^{i(Et - \vec{k_2}\vec{x})}$$

Detection again via weak interaction

$$\nu_\mu N \rightarrow \mu^- X$$
$$\nu_e N \rightarrow e^- X$$

$$P(\nu_e \rightarrow \nu_\mu) = |<\nu_\mu \,|\, \nu(t)>|^2$$

Oscillation probability

$$\left|\vec{k}\right| \approx E - \frac{m^2}{2E}$$

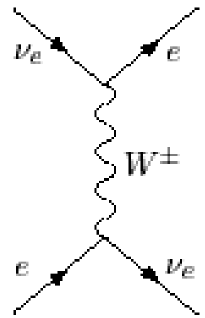
$$\left|\nu(L)\right\rangle \approx e^{-iEt}\left(c \cdot e^{-i\frac{m_1^2}{2E}L}\left|\nu_1\right\rangle + s \cdot e^{-i\frac{m_2^2}{2E}L}\left|\nu_2\right\rangle\right)$$

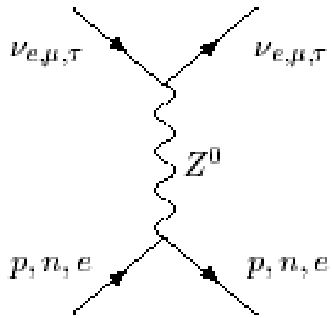
$$P(\nu_e \to \nu_\mu) = |<\nu_\mu | \nu(L)>|^2 = \left| -sce^{-i\frac{m_1^2}{2E}L} + cse^{-i\frac{m_2^2}{2E}L} \right|^2$$

$$= 4s^2c^2\left(1 - \cos\frac{m_1^2 - m_2^2}{2E}L\right) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{12}^2}{4E}L\right)$$

Neutrino oscillations in Matter

ν_e,ν_μ, ν_τ interact with e, p and n of matter via NC.interactions (Z). Only ν_e interact via (CC) with the electrons of the medium



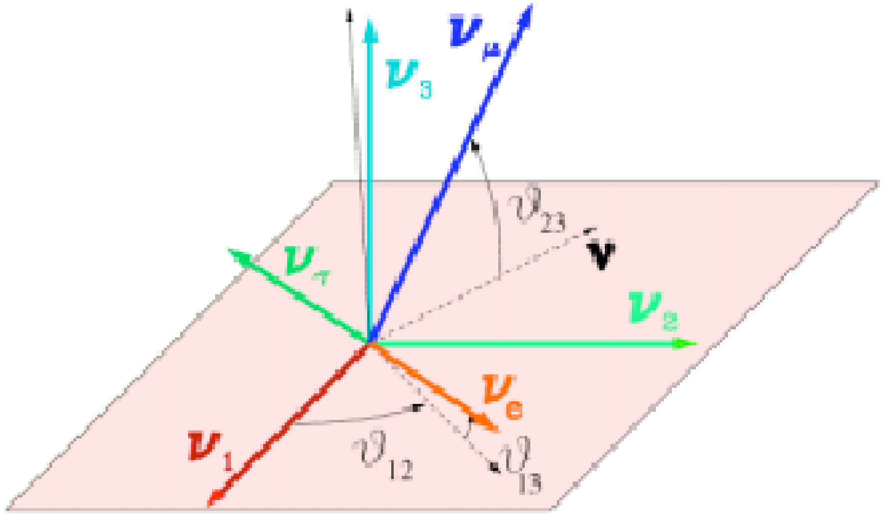


Oscillation probability change in matter. There can be a resonant enhancement of the oscillation probability. The Mikheyev-Smirnov-Wolfenstein (MSW) effect.

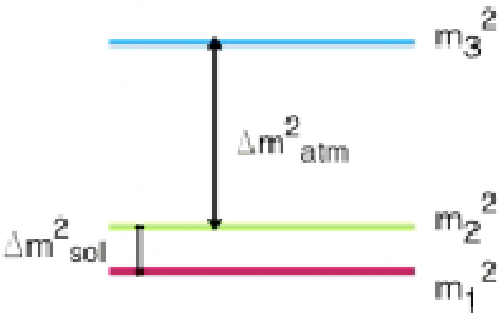
P_{osc}^{matter} can be large (≈1) even if mixing angle in vacuum is small

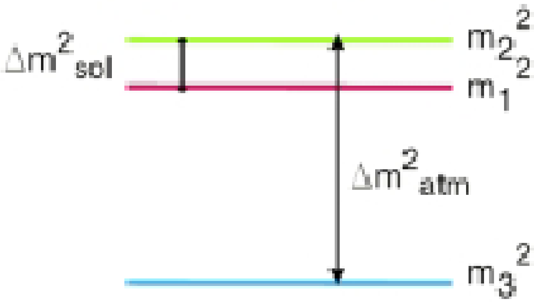
Oscillations in 3D

Solar data. The ν_e is oscillating (via enhanced matter resonance, MSW) to the other two flavours with $\Delta m_{12} \approx 10^{-4}$ eV^2, $\theta_{12} \approx 30^0$



Atmospheric data. Largely $\nu_\mu \rightarrow \nu_\tau$ (vacuum) oscillations with $\Delta m_{23} \approx 10^{-3}$ eV^2 $\theta_{23} \approx 45^0$





Two mass differences → need 3 neutrinos

The PMNS matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Unless the other two angles θ_{13} is small (experimental upper limit $\theta_{13} < 10^0$)

If $\delta \neq 0, \pi, 2\pi \ldots$ then weak interactions violate CP symmetry in the lepton sector (as in the quark sector)

atmospheric

CP violation phase

solar

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Links atmospheric & solar sectors

Neutrino oscillation physics: you are here

atmospheric

CP violation phase

solar

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Links atmospheric & solar sectors

Observed by Super-Kamiokande, confirmed by K2K. CNGS+NUMI will measure parameters to about 20-30 %

θ_{13} limited by CHOOZ to be smaller than about 10^0. Nothing known about δ.

Observed by 30 years of solar neutrinos experiments, nailed down by SNO and Kamland. Parameters known at 10%

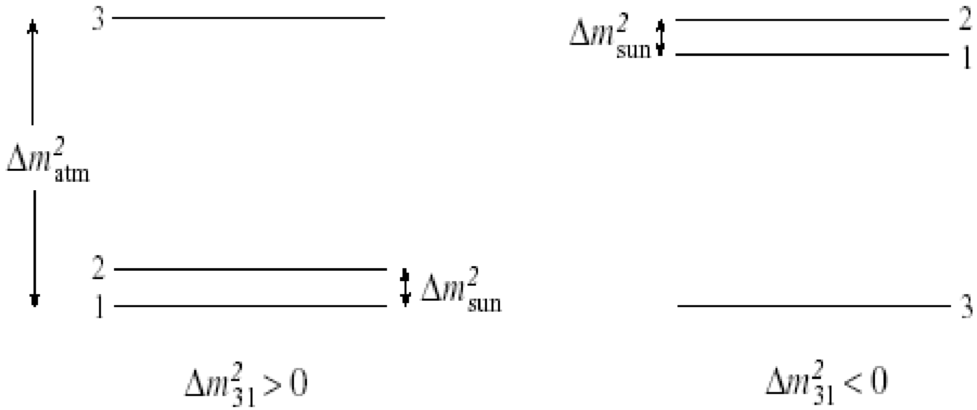
The quest

$$\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

What is the value of θ_{13}?

What is the value of δ?

Is there CP violation?



Which mass spectrum?

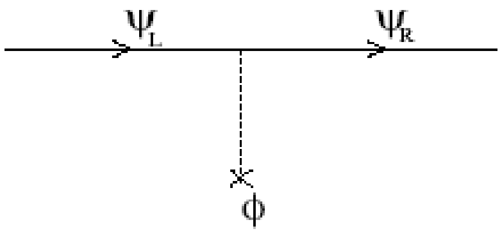
The quest (II)

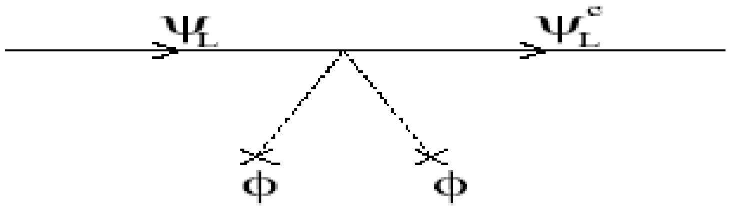
Dirac or Majorana?

If Majorana, what is the value of Λ?

$$m_\nu = \lambda v$$

$$m_\nu = \lambda \frac{v^2}{\Lambda}$$





θ_{13}: link between atmospheric and solar oscillations

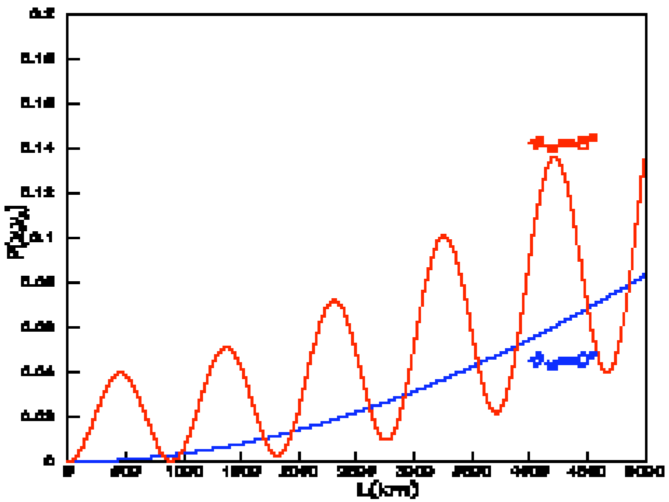
$\theta_{13} = 0$	$P(\nu_e \to \nu_\mu) = c_{23}^2 \sin^2 2\theta_{12} \sin^2(\frac{\Delta m_{12}^2 L}{4E})$	solar
$\theta_{13} \neq 0$	$P(\nu_e \to \nu_\mu) = c_{23}^2 \sin^2 2\theta_{12} \sin^2(\frac{\Delta m_{12}^2 L}{4E})$	solar
	$+ s_{23}^2 \sin^2 2\theta_{13} \sin^2(\frac{\Delta m_{23}^2 L}{4E})$	atmospheric
	$+ J \cos(\pm\delta - \frac{\Delta m_{23}^2 L}{4E}) \frac{\Delta m_{12}^2 L}{4E} \sin(\frac{\Delta m_{23}^2 L}{4E})$	interference

$$J = c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$$

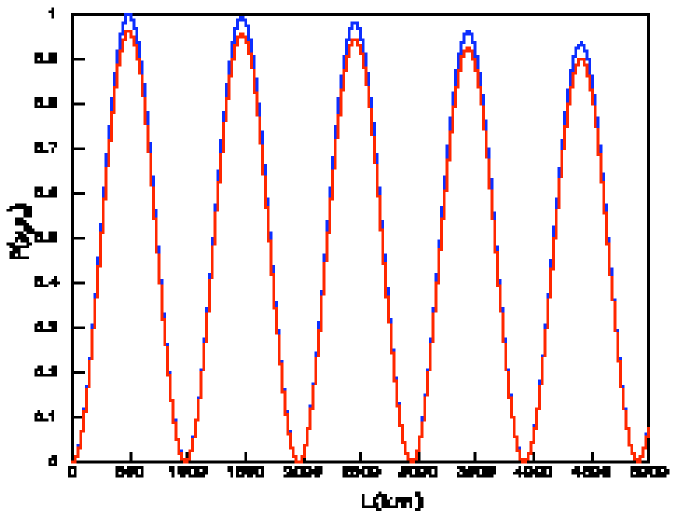
Sensitivity to θ_{13}: subleading transitions

Subleading: $\nu_e \rightarrow \nu_\mu$, $\nu_e \rightarrow \nu_\tau$: sensitive to θ_{13} and δ

Leading: $\nu_\mu \rightarrow \nu_\tau$: rather insensitive to θ_{13} and δ



$P(\nu_e \rightarrow \nu_\mu)$



$P(\nu_\mu \rightarrow \nu_\tau)$

CP violation in ν oscillations

CP violation in ν oscillations →Oscillation probability is different for neutrinos and antineutrinos.

Thus, one can measure non-vanishing asymmetries A_{CP}

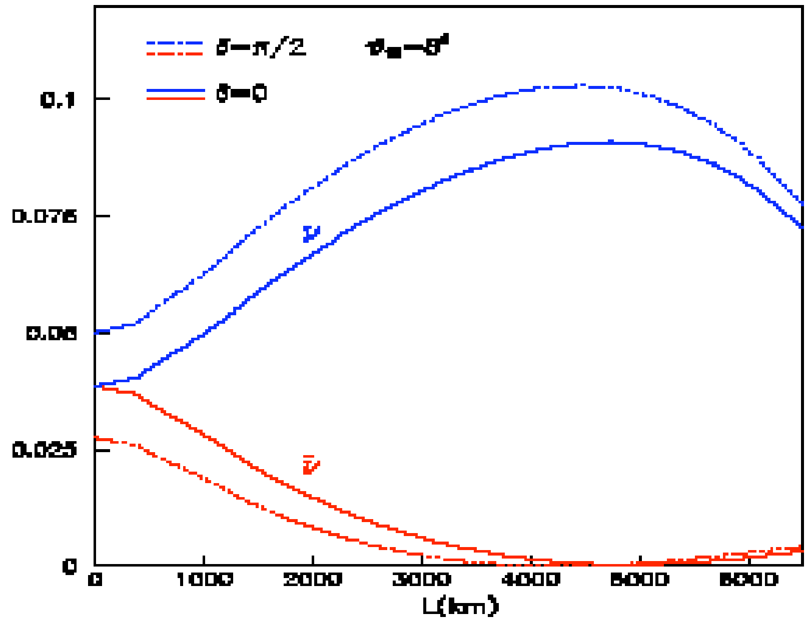
$$A^{CP}_{\nu_e\nu_\mu} = \frac{P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu)}{P(\nu_e \to \nu_\mu) + P(\bar{\nu}_e \to \bar{\nu}_\mu)}$$

$$= \frac{2 \overbrace{\sin\delta}^{CP\ violating\ phase} \overbrace{c_{13}\sin 2\theta_{13}}^{Interference} \overbrace{\sin 2\theta_{12}\frac{\Delta m^2_{12}L}{4E}}^{solar} \overbrace{\sin 2\theta_{23}\sin^2\frac{\Delta m^2_{13}L}{4E}}^{atmospheric}}{P^{CP-even}_{\nu_e\nu_\mu}}$$

Determine mass spectrum

The same experiments that will measure δ and θ_{13} can establish the ν mass hierarchy by studying the matter effects on Earth

- One gets a large amplification/supression of $P(\nu_e \rightarrow \nu_\mu)$ depending on whether the hierarchy is "natural" or "inverted"



Extracting the parameters of the PMNS matrix from future neutrino oscillation experiments I

J.J. Gómez-Cadenas
U. Valencia/KEK

Original results presented in this talk based on work done in collaboration with P. Hernández, J. Burguet-Castell, D. Casper & P.Novella

Measurement of θ_{13}. Correlations

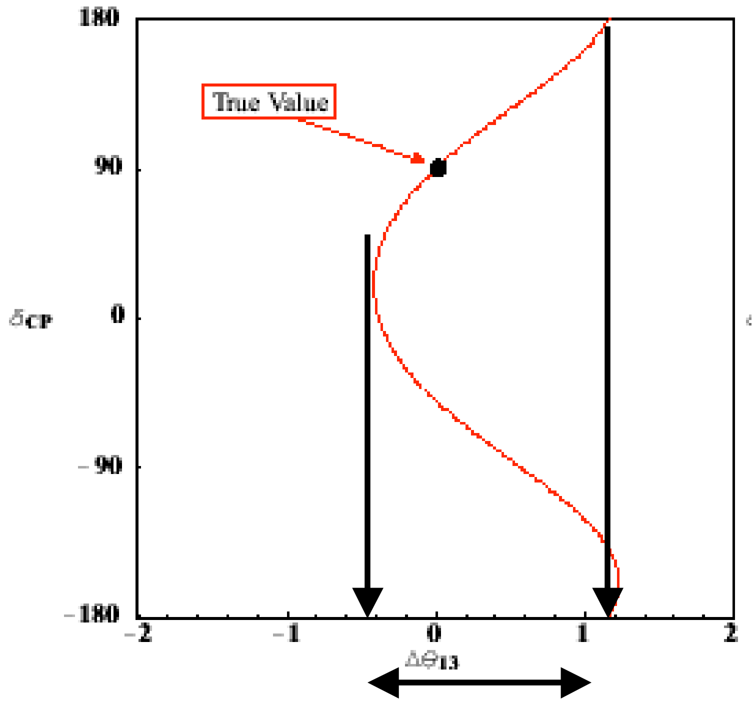
$$P^{\pm}_{\nu_e \nu_\mu}(\theta_{13}, \delta) \approx X_\pm \sin^2 2\theta_{13} + \left(Y^c_\pm \cos\delta \mp Y^s_\pm \sin\delta\right) \sin 2\theta_{13} + Z$$

(DeRujula99, Cervera00)

The appearance probability $P(\bar{\theta}_{13},\bar{\delta})$ obtained for neutrinos at fixed (E,L) with input parameters $(\bar{\theta}_{13},\bar{\delta})$ has no unique solution. Indeed the equation:

$$P_{\alpha\beta}(\bar{\theta}_{13},\bar{\delta}) = P_{\alpha\beta}(\theta_{13},\delta)$$

has a continuous number of solutions



Correlation error

Measurement of θ_{13}: Intrinsic degeneracy

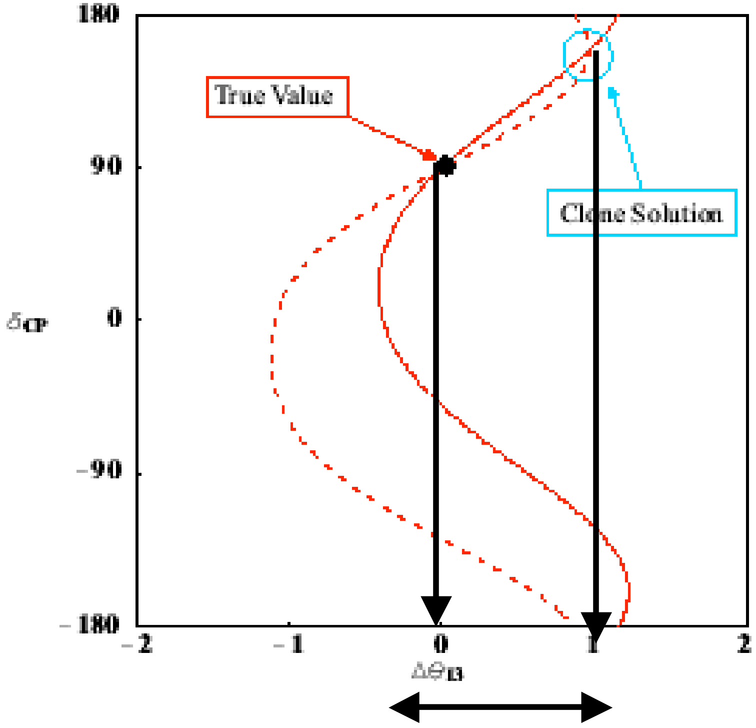
$$P^{\pm}_{\nu_e \nu_\mu}(\theta_{13}, \delta) \approx X_{\pm} \sin^2 2\theta_{13} + \left(Y^c_{\pm} \cos\delta \mp Y^s_{\pm} \sin\delta\right) \sin 2\theta_{13} + Z$$

J. Burguet-Castell *et al.* Nucl. Phys. B 608 (2001) 301;

For neutrinos and antineutrinos of the same energy and baseline the system of equations

$$P_{\alpha\beta}^{\pm}(\bar{\theta}_{13}, \bar{\delta}) = P_{\alpha\beta}^{\pm}(\theta_{13}, \delta)$$

has two intersections. The true one $(\bar{\theta}_{13}, \bar{\delta})$ and a second, energy dependent point (clone) that introduces and ambiguity in the determination of the parameters



Degeneracy error

Discrete degeneracies

3. H. Minakata and H. Nunokawa, JHEP 0110 (2001) 001.
4. V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D 65 (2002) 073023.

Two other sources of degeneracy.

1. Ignorance of the sign of Δm_{23}^2

$$s_{atm} = \text{sgn}(\Delta m_{23}^2)$$

2. Ignorance of the octant of θ_{23}

$$s_{oct} = \text{sgn}(\tan(2\theta_{23}))$$

These two discrete values assume the value ±1

Eightfold degeneracy

4. V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D 65 (2002) 073023.

Experimental measurement. Number of observed chaged leptons Nβ

Integrate P over Φ_ν, σ, and detector efficiencies.

$$N_\beta^\pm(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N_\beta^\pm(\theta_{13},\delta;s_{atm}=\bar{s}_{atm},s_{oct}=\bar{s}_{oct}) \quad \beta = e,\mu,\tau$$

Since s_{atm} & s_{oct} not known, one should consider also 2 other equations which result in an 8-fold degeneracy

$$N_\beta^\pm(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N_\beta^\pm(\theta_{13},\delta;s_{atm}=-\bar{s}_{atm},s_{oct}=\bar{s}_{oct})$$

$$N_\beta^\pm(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N_\beta^\pm(\theta_{13},\delta;s_{atm}=\bar{s}_{atm},s_{oct}=-\bar{s}_{oct})$$

$$N_\beta^\pm(\bar{\theta}_{13},\bar{\delta};\bar{s}_{atm},\bar{s}_{oct}) = N_\beta^\pm(\theta_{13},\delta;s_{atm}=-\bar{s}_{atm},s_{oct}=-\bar{s}_{oct})$$

How to solve degeneracies

1. Use spectral information on oscillation signals→ experiment with energy resolution
2. Combine experiments differing in E/L (and/or matter effects) → need two experiments
3. Include other flavor channels: silver channel $\nu_e \rightarrow \nu_\tau$. Need a tau-capable detector

Burguet et al, **Nucl.Phys.B608:301-318,2001**

Donini, Meloni, Miggliozzi, hep-ph/0206034

Donini, Meloni, Rigolin, hep-ph/hep-ph/0312072