Diffusion in Relativistic Systems

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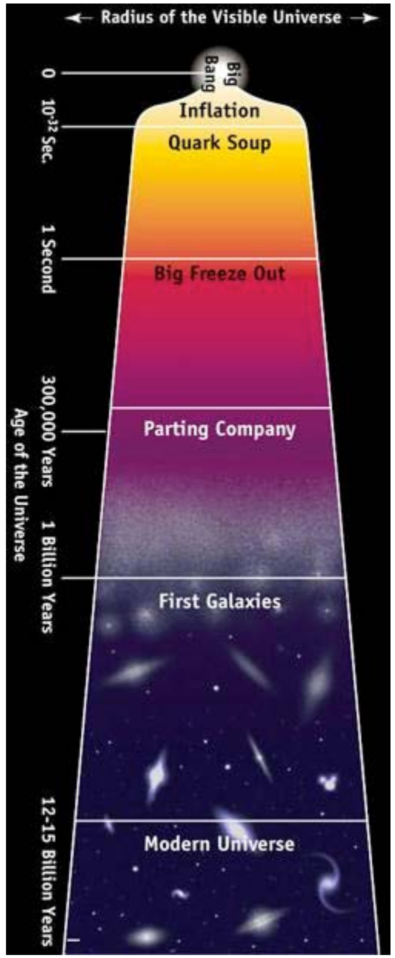
Theoretical Physics

Topics

- Introduction
- Relativistic Diffusion Model for $R(p_T,y;t)$ with three sources for symmetric / asymmetric systems
- Net protons dN/dy and produced charged particles dN/dη at 200 A GeV
- Importance of the equilibrated midrapidity source in d+Au compared to Au+Au
- Longitudinal collective expansion
- Conclusion

Introduction

- QGP in the early universe: quarks, gluons in thermal equilibrium
- Quark-hadron phase transition at ≈ 10μs
- Chemical freezeout of the produced hadrons at T≈170 MeV: hadron abundances remain fixed
- Primordial synthesis of H-2,He-3, He-4, Li-7 at t≈1s, T≈ 1 MeV



Recreate the QGP in collisions

- CERN SPS: fixed-target experiments.
- Pb-Pb @ $\sqrt{s_{NN}}$=17.3 GeV
- Relativistic heavy-ion collider RHIC: two counter-circulating rings, 3.8 km circumference; energies
- 62.4, 130, 200 GeV
- PHENIX, STAR, PHOBOS, BRAHMS experiments

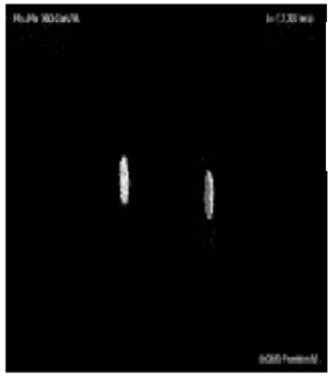
Relativistic heavy-ion collisions

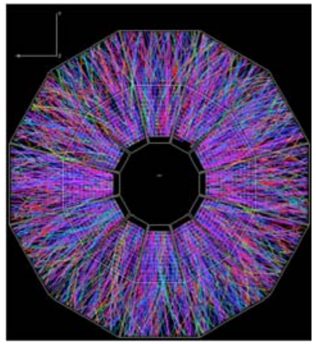
- Search for the primordial state of matter (equilibrated Quark-Gluon Plasma) in Relativistic Heavy-Ion collisions:

  Possible QGP signatures? (J/ψ suppression at SPS; jet quenching, properties of collective flow,..at RHIC) ?

  Signatures for intermediary deconfinement of the constituent quarks in the incoming baryons

- Distributions of transverse momentum p_T (or transverse energy E_T), and rapidity $y=1/2\cdot\ln[(E+p)/(E-p)]$





$\tau_{int} \approx 10^{-23}$ s
$\varepsilon \gtrsim 1.5$ GeV/fm^3

≈4600 charged hadrons produced at RHIC in central Au-Au collisions

Indications for local deconfinement/qgp?

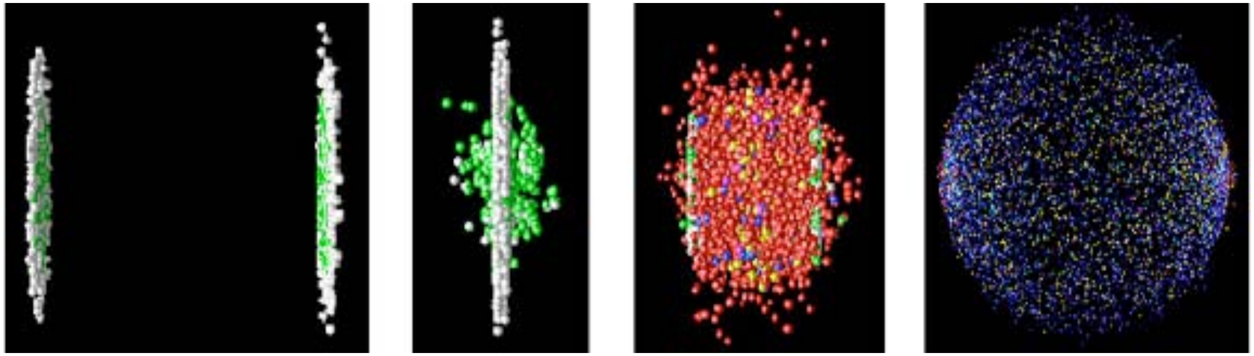
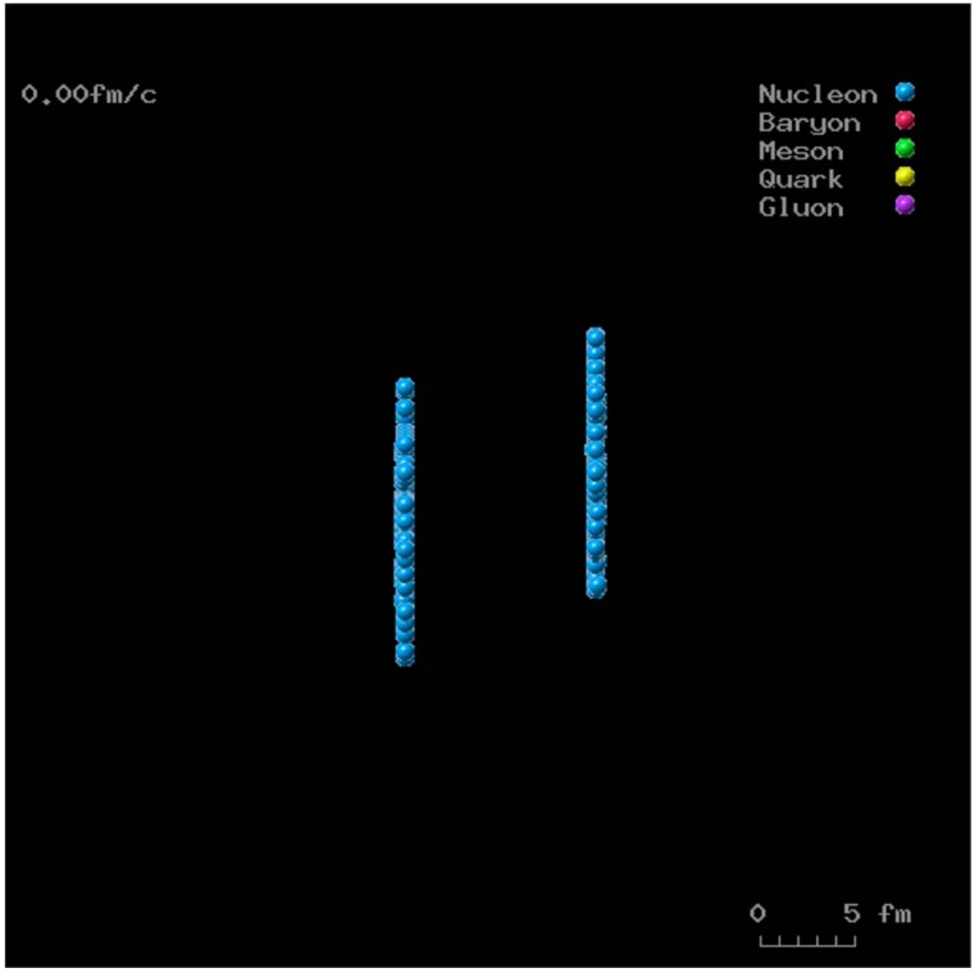


Fig. Courtesy U Frankfurt

1. Yes, in central collisions of Au+Au at $\sqrt{s}=200$ GeV/particle pair, the partons in 14% of the incoming baryons are likely to be deconfined. [cf. GW, Phys. Rev. C 69, 024906(2004)]

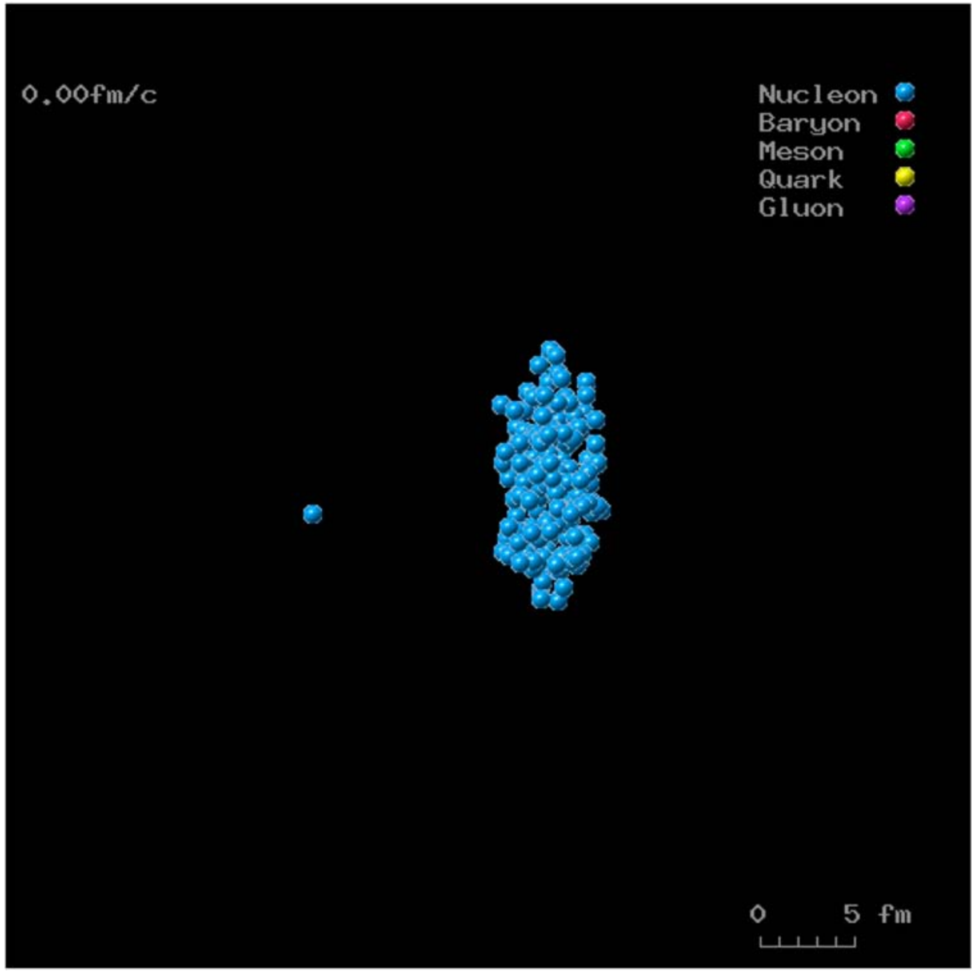
2. Yes, most of the produced particles are in local thermal equilibrium [cf. M. Biyajima et al., Prog. Theor. Phys. Suppl. 153, 344 (2004))]

Au-Au
200 GeV



JAM-code/courtesy Hokkaido Univ. Y.Nara, M.Ohtsuka et al.

p-Au
10GeV



JAM-code/courtesy Hokkaido Univ.

Relativistic Diffusion Model for R(y,t)

Nonequilibrium-statistical description of the distribution function R(y,t) in rapidity space,

y = 0.5 · ln((E + p)/(E − p)).

$$\frac{\partial}{\partial t} R_k(y,t) = \frac{1}{\tau_y}\frac{\partial}{\partial y}\Big[(y - y_{eq}) \cdot R_k(y,t)\Big] + \frac{\partial^2}{\partial^2 y}\Big[D_y^k \cdot R_k(y,t)\Big].$$

k=1: Au-like
k=2: d-like
k=3: central distr.

GW, EPJ A5, 85(1999), 2 sources;
Phys. Lett. B 569, 67 (2003), 3 sources

Relativistic Diffusion Model for R(y,t)

-The drift function $(y-y_{eq})/\tau_y$ determines the shift of the mean rapidity towards the equilibrium value

- The diffusion coefficient D accounts for the broadening of the distributions due to interactions and particle creations. It is related to τ_y via a dissipation-fluct. Theorem.

- Nonequilibrium-statistical approach to relativistic many-body collisions
- Macroscopic distribution function R(y,t) for the rapidity y

GW EPJ A5, 85 (1999)

Linear RDM in y- space

In a moments expansion and for δ–function initial conditions, the mean values become

$$< y_{1,2}(t) >= y_{eq}[1 - exp(-t/\tau_y)] \mp y_{max} \exp(-t/\tau_y)$$

$$\langle y_3(t) \rangle = y_{eq}[1-\exp(-t/\tau_y)]$$

and the variances are

$$\sigma^2_{1,2,eq}(t) = D_y^{1,2,eq} \tau_y [1 - \exp(-2t/\tau_y)]$$

The rapidity relaxation time τ_y determines the peak positions
The rapidity diffusion coefficient D_y determines the variances.

Dissipation-fluctuation relation in y- space

The rapidity diffusion coefficient D_y is calculated from τ_y and the equilibrium temperature T in the weak-coupling limit as

$$D_y(\tau_y, T) = \frac{1}{2\pi\tau_y}\left[c(\sqrt{s}, T)m^2 T \times \left(1 + 2\frac{T}{m} + 2\left(\frac{T}{m}\right)^2\right)\right]^{-2} \exp\left(\frac{2m}{T}\right)$$

GW, Eur. Phys. Lett. 47, 30 (1999)

Note that $D \sim T|\tau_y$ as in the Einstein relation of Brownian motion.
The diffusion coefficient as obtained from this statistical consideration is further enhanced (D_y^{eff}) due to collective expansion.

Linear RDM in y-space

The equilibrium value y_{eq} that appears in the drift function $J(y) = -(y_{eq}-y)/\tau_y$ (and in the mean value of the rapidity) is obtained from energy- and momentum conservation in the subsystem of participants as

$$y_{eq} := \frac{1}{2}\cdot\ln\left(\frac{m_1{}^t\cdot e^{y_b} + m_2{}^t\cdot e^{-y_b}}{m_2{}^t\cdot e^{y_b} + m_1{}^t\cdot e^{-y_b}}\right)$$

with the beam rapidity y_b, and transverse masses

$$m_k{}^t := \sqrt{m_k{}^2 + \left(p_k{}^t\right)^2}$$

m_k = participant masses (k=1,2)

y_{eq} = 0 for symmetric systems but ≠ 0 for asymmetric systems

Linear RDM in y-space

Analytical RDM-solutions with δ-function initial conditions at the beam rapidities $\pm y_b$ in the two-sources model

$$R(y,0) = \frac{1}{2}\cdot\left(\delta\left(y + y_b\right) + \delta\left(y - y_b\right)\right)$$

are obtained as

$$R(y,t) = \frac{1}{2\sqrt{(2\pi\sigma_1^2(t))}} exp\left[-\frac{(y- < y_1(t) >)^2}{2\sigma_1^2}\right] + \frac{1}{2\sqrt{(2\pi\sigma_2^2(t))}} exp\left[-\frac{(y- < y_2(t) >)^2}{2\sigma_2^2}\right]$$

In the 3-sources model, a third (equilibrium) term appears.
Is there evidence for a 3-sources-approach? Yes

Equilibrium temperature in the RDM

- The temperature of the stationary distribution in the relativistic diffusion model is identified with the kinetic freezeout temperature (which is somewhat below, but close to the chemical freezeout temperature)

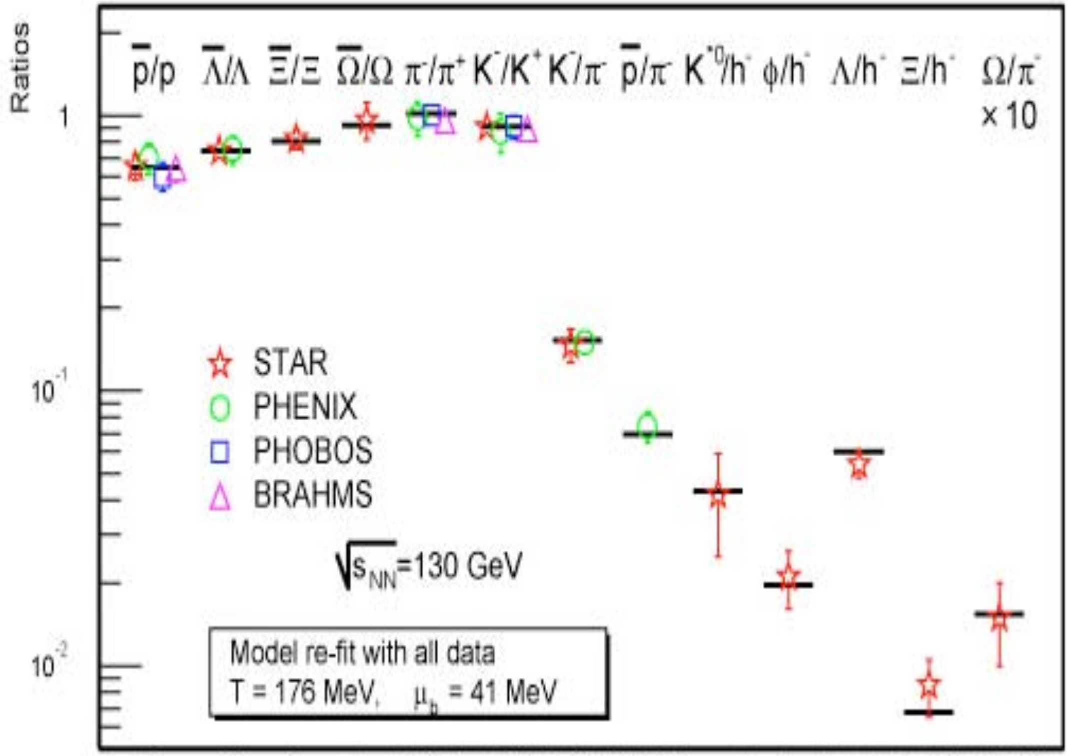
- Its numerical value ($T_{eq} \approx 170$ MeV at RHIC energies) is determined in accordance with fits of the abundances of produced hadrons such as in:

  F. Becattini et al., Phys. Rev. C 64, 024901 (2001);

  P. Braun-Munzinger et al., Phys. Lett. B 465, 15 (1999)

Chemical freezeout temperature

The equilibrium temperature T_{eq} is approximated by the chemical freezeout temperature - which is taken from fits of abundance ratios of produced charged particles in the Statistical Model.



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Hagedorn temperature

- The Hagedorn limiting temperature from statistical thermodynamics of strong interactions at high energies does not necessarily imply that the overall many-body system has reached, or gone through, thermal equilibrium.
- Hagedorn had therefore proposed to replace the term »equilibrium« by »constant temperature« in this context because "T_0..will rapidly be approached ..long before thermodynamical equilibrium..is reached".

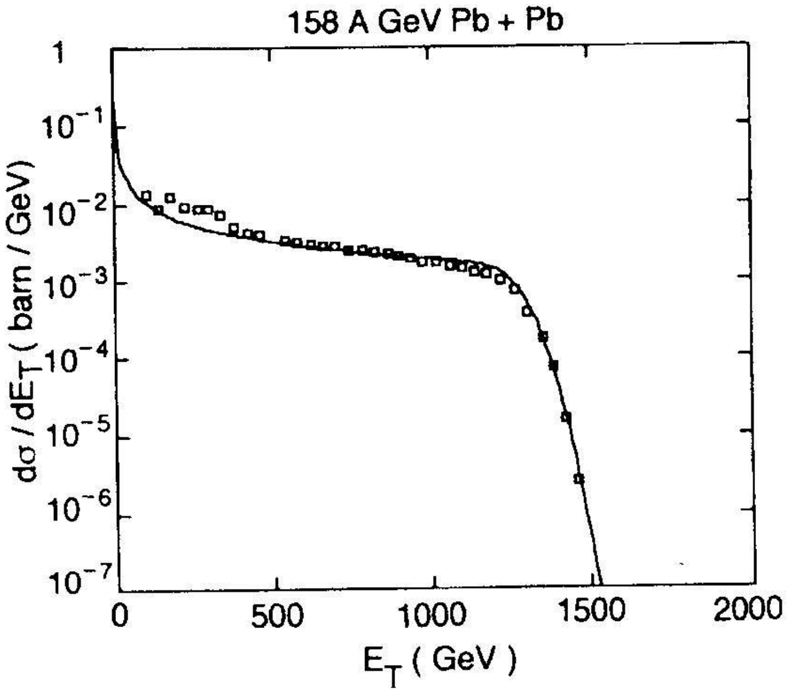
  [R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965); pp. 149,163].

Equilibrium vs. constant temperature

- **R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965); p. 163:**

(*) As actually it will turn out that $T_0 \approx m_\pi$ is rather « low », T_0 will rapidly be approached in collisions long before thermodynamical equilibrium in the usual sense is reached. We thus may have an almost constant temperature T_0 all over the volume V_0 without necessarily having a constant energy density and without having transformed all kinetic energy in V_0 into heat motion. Remaining collective motions of whole parts of V_0 will then be strongly correlated to the former motion of the incoming colliding particles. In the longitudinal directions this collective motion will in general suppress the isotropic but small heat motion (except in very central collisions).

Transverse energy spectra: SPS

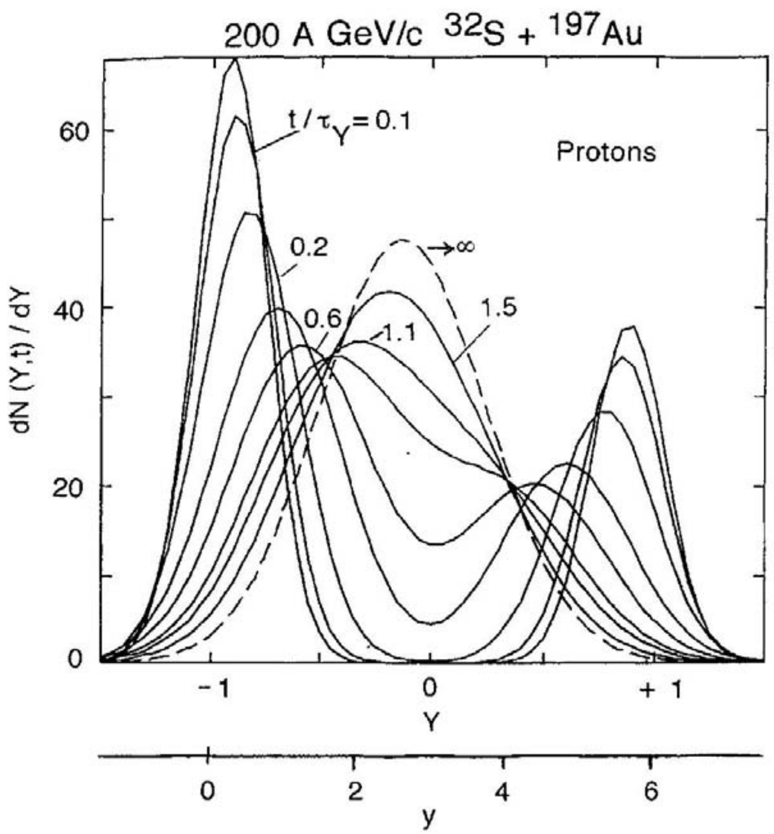


- RDM-prediction @SPS energies, p_L=157.7 A GeV
- $\sqrt{s_{NN}}$ = 17.3 GeV
- NA 49 data scaled to 4π acceptance
- Calorimeter data, integrated over all particle species

Parameters: energy relaxation time, interaction time

GW, Z. Phys. A 355, 301 (1996)

RDM: time evolution of the rapidity density for net protons



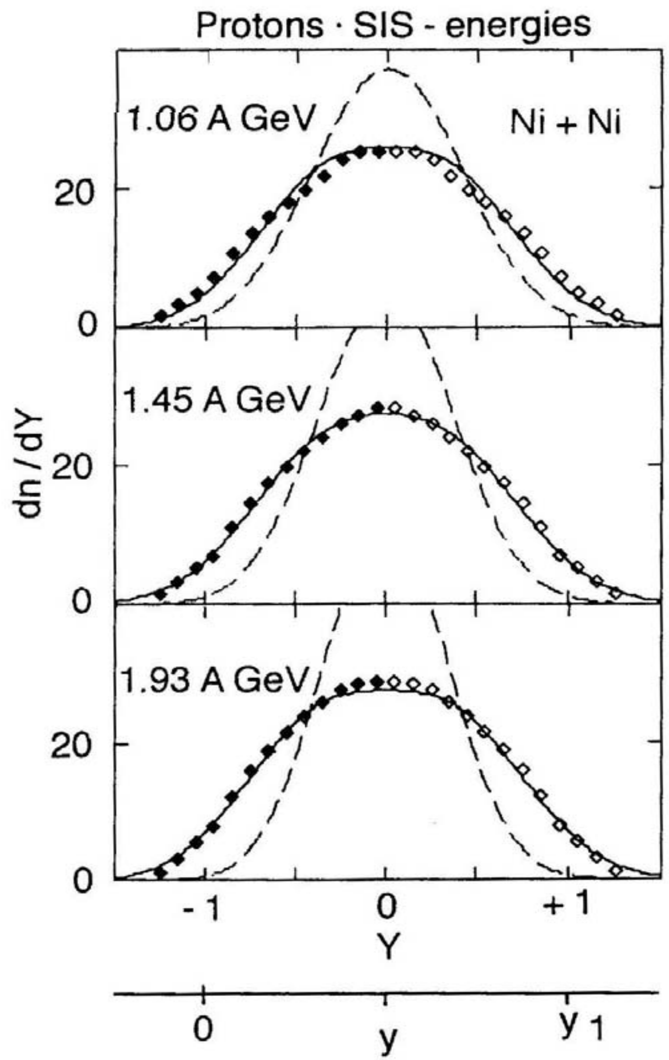
GW, Eur. Phys. J. A5, 85 (1999)

- Time evolution for 200 A GeV/c S+Au, linear model; δ–function initial conditions
- Selected weighted solutions of the transport eq. at various values of τ/τ_y
- The stationary solution (Gaussian) is approached for $t/\tau_y \gg 1$

Rapidity density distributions: Net protons, SIS

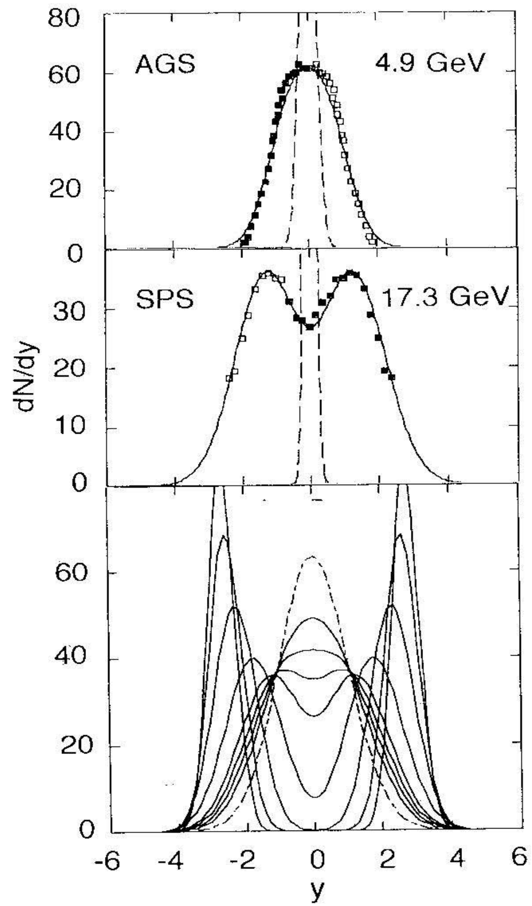
- Linear Relativistic Diffusion Model-calculations @SIS energies
- Ni-Ni, E_{cm} = 1.06-1.93 A GeV; FOPI data: bell-shaped distributions (dashed: thermal equil.)

GW, Eur. Phys. Lett. 47, 30 (1999)



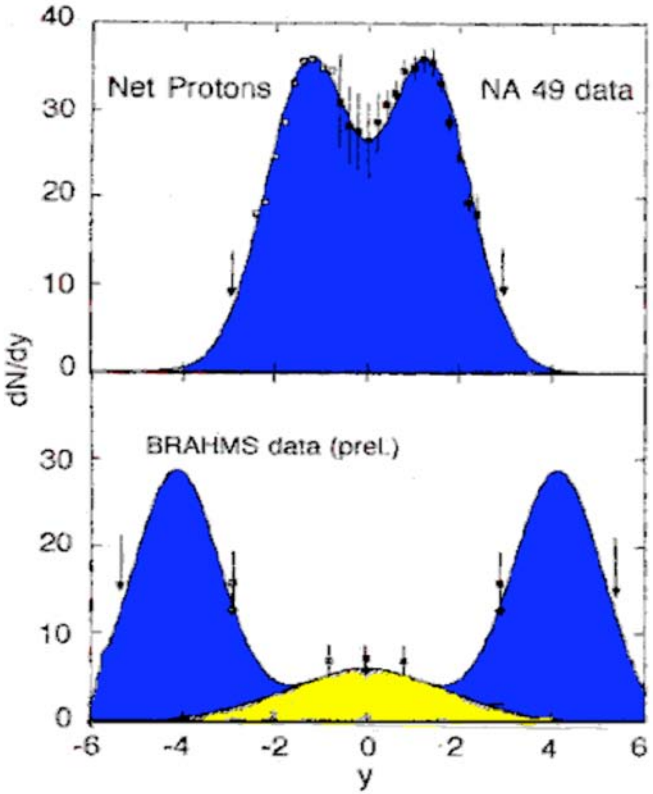
Central Collisions at AGS, SPS

- Rapidity density distributions evolve from bell-shaped to double-humped as the energy increases from AGS (4.9 GeV) to SPS (17.3 GeV)
- Diffusion-model solutions are shown for SPS energies



GW, Proc. INPC04 Göteborg

Central Au+Au @ RHIC vs. SPS



- BRAHMS data at $\sqrt{s_{NN}}$=**200** GeV for net protons
- Central 10% of the cross section
- Relativistic Diffusion Model for the nonequilibrium contributions
- Discontinuous transition to local statistical equilibrium at midrapidity indicates deconfinement.

GW, PLB 569, 67 (2003) and Phys. Rev. C 69 (2004)

3-sources RDM for Au+Au

- Incoherent superposition of nonequilibrium and equilibrium solutions of the transport equation yields a very satisfactory representation of the data
- Disontinuous evolution of the distribution functions with time towards the local thermal equilibrium distribution

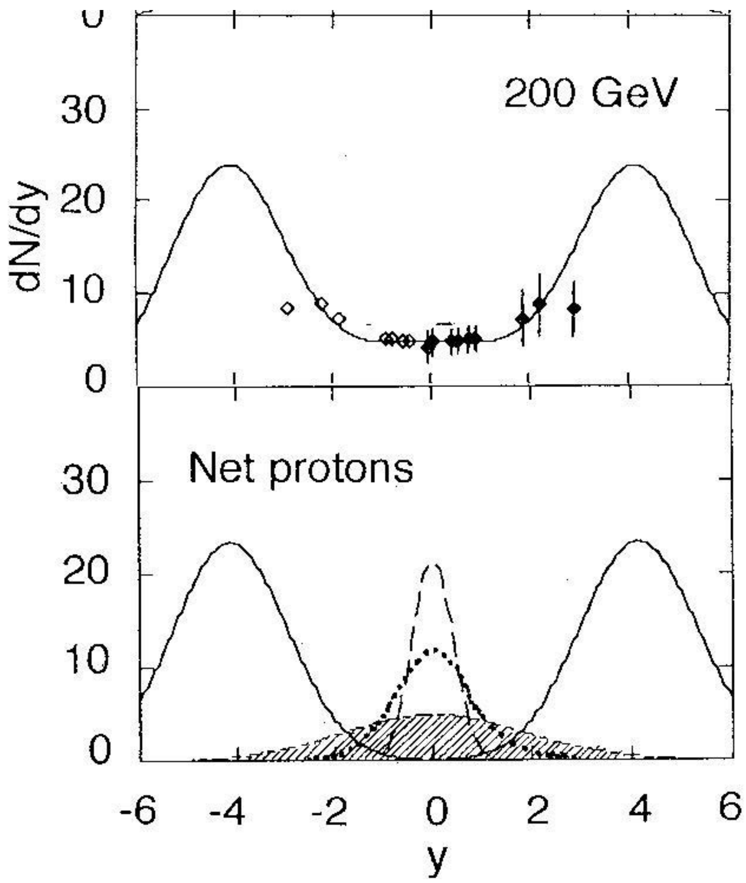
$$\frac{dN(y, t_{int})}{dy} = N_1 \cdot R_1(y, t_{int}) + N_2 \cdot R_2(y, t_{int}) + N_{eq} \cdot R_{eq}(y)$$

$N_{eq} \approx$ 56 baryons (22 protons)

$N_{1,2} \approx$ 169 baryons (68 protons)

GW, Phys. Lett. B 569, 67 (2003); Phys. Rev. C 69, 024906 (2004)

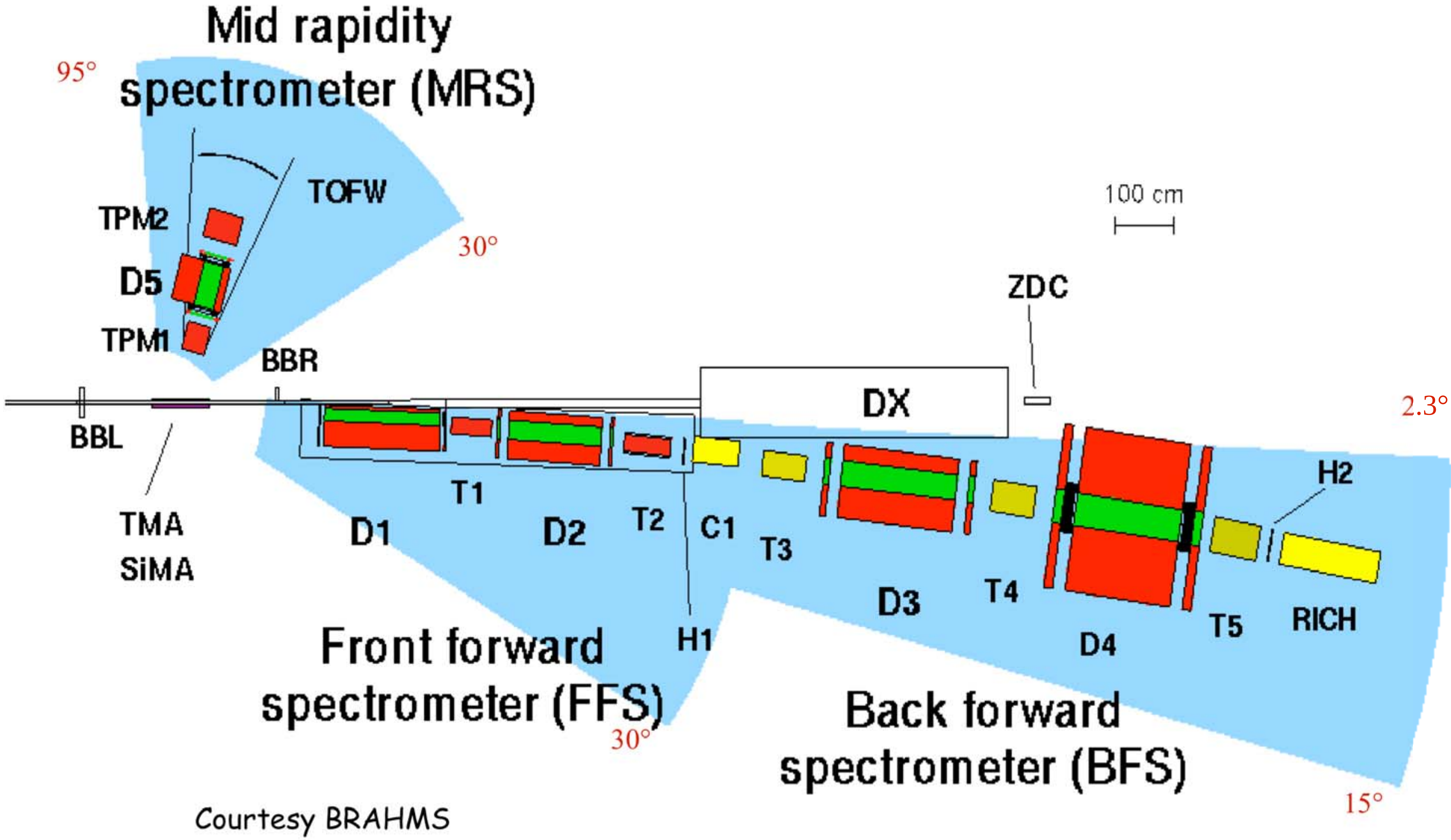
Central Au+Au @ RHIC



Calc. GW (2004); data P. Christiansen (BRAHMS), Priv. comm.

- BRAHMS data at $\sqrt{s_{NN}}$=**200** GeV for net protons
- Central 5% of the cross section
- Relativistic Diffusion Model for the nonequilibrium contributions
- Local statistical equilibrium at midrapidity.

The BRAHMS experiment

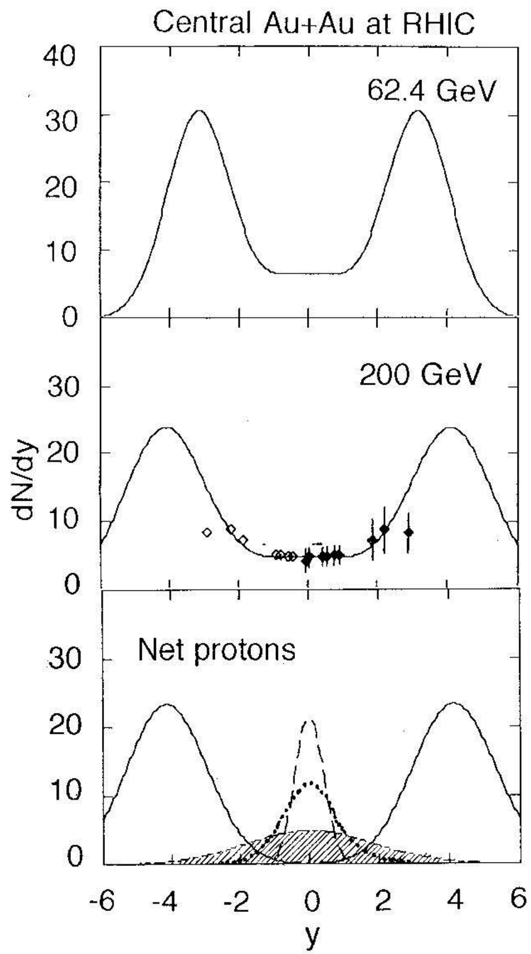


Courtesy BRAHMS

Au+Au at 62.4 GeV

RDM-prediction for 62.4 GeV

(the lower RHIC energy
measured by BRAHMS; data
analysis is underway)



Collective longitudinal expansion

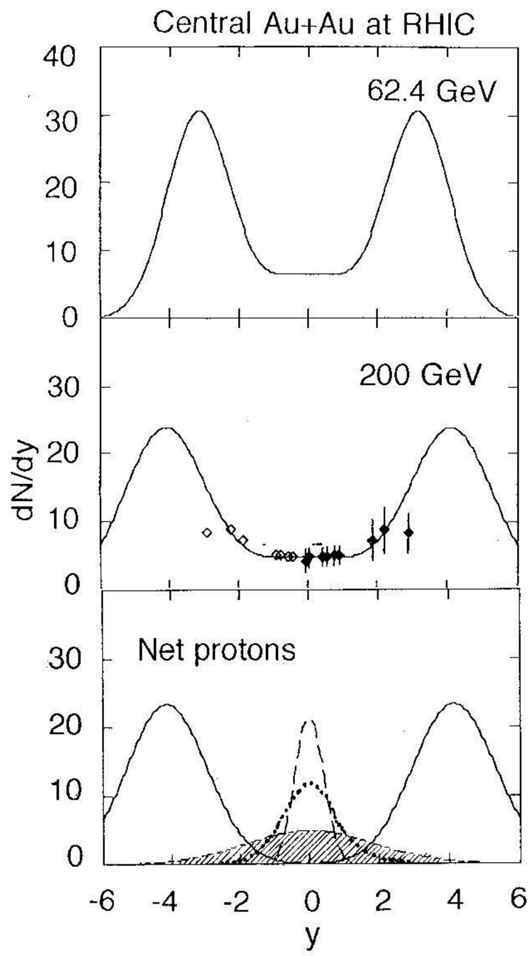
The longitudinal expansion velocity is obtained from the relativistic expression (c=1)

$$v := \sqrt{1 - \left(\frac{m_0}{m}\right)^2}$$

with the effective mass

$$m := m_0 + \frac{f}{2} \cdot (T_{eff} - T)$$

for f degrees of freedom



Collective longitudinal expansion

The longitudinal expansion velocity becomes

$$v_{coll} := \left[1 - \left[\frac{m_0}{m_0 + \frac{f}{2}\cdot\left(T_{eff} - T\right)}\right]^2\right]^{\frac{1}{2}}$$

with the effective temperature T_{eff} that corresponds to the value of the diffusion coefficient that is required from the data, and includes the effect of expansion.

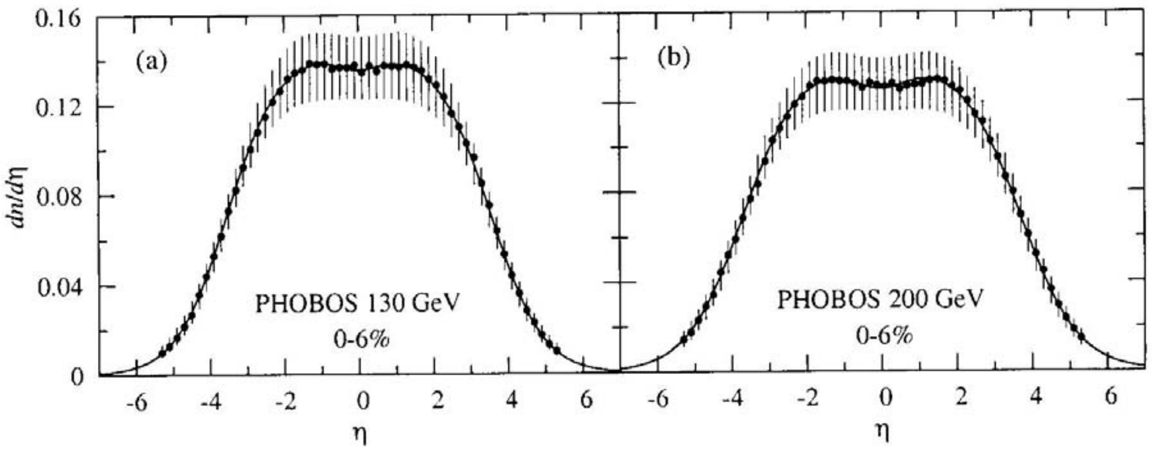
Heavy Relativistic Systems

Parameters for heavy relativistic systems at AGS, SPS and RHIC energies. The beam rapidity is expressed in the c.m. system. The ratio τ_{int}/τ_y determines how fast the net-baryon system equilibrates in rapidity space. The effective rapidity diffusion coefficient is D_y^{eff}, the longitudinal expansion velocity v_{coll}.

*At 62.4 GeV, D_y^{eff} will need adjustment to forthcoming data.

Lab	System	$\sqrt{s_{NN}}$ (GeV)	$y_b^{c.m.}$	τ_{int}/τ_y	D_y^{eff} $(10^{-4}\mathrm{fm}^{-1})$	$v_{coll}^{\parallel}$
AGS	Au - Au	4.9	1.61	1.08	5.1	0.34
SPS	Pb - Pb	17.3	2.91	0.81	9.4	0.48
RHIC	Au - Au	62.4	4.20	0.34	30*	0.64
RHIC	Au - Au	200	5.36	0.26	30	0.65

Produced particles in the 3 sources RDM:
Charged-hadron (pseudo-) rapidity distributions



PHOBOS data at $\sqrt{s_{NN}}$=130, 200 GeV for charged hadrons   Central collisions (0-6%)

Total number of particles in the 3 "sources": 448:3134:448 @ 130 GeV
551:3858:551 @ 200 GeV

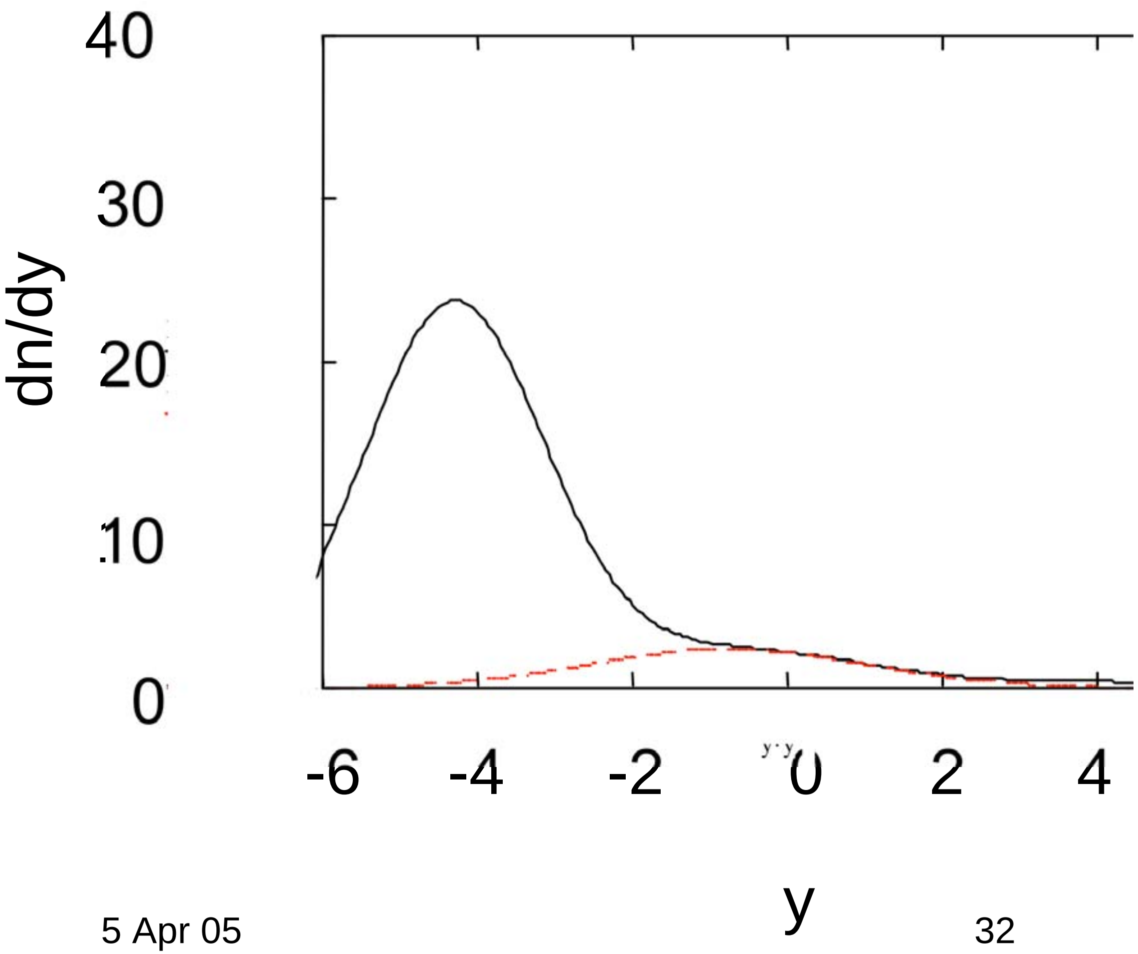
Most of the produced charged hadrons at RHIC are in the equilibrated midrapidity region

M. Biyajima et al., Prog. Theor. Phys.Suppl. 153, 344 (2004)

d+Au 200 GeV net protons (RHIC)

RDM: schematic calculation for Central d+Au:

- 3 sources model
- Net protons
- D, $v_{||}$ from Au+Au (overestimated)
- **y_{eq} from energy/momentum conservation**



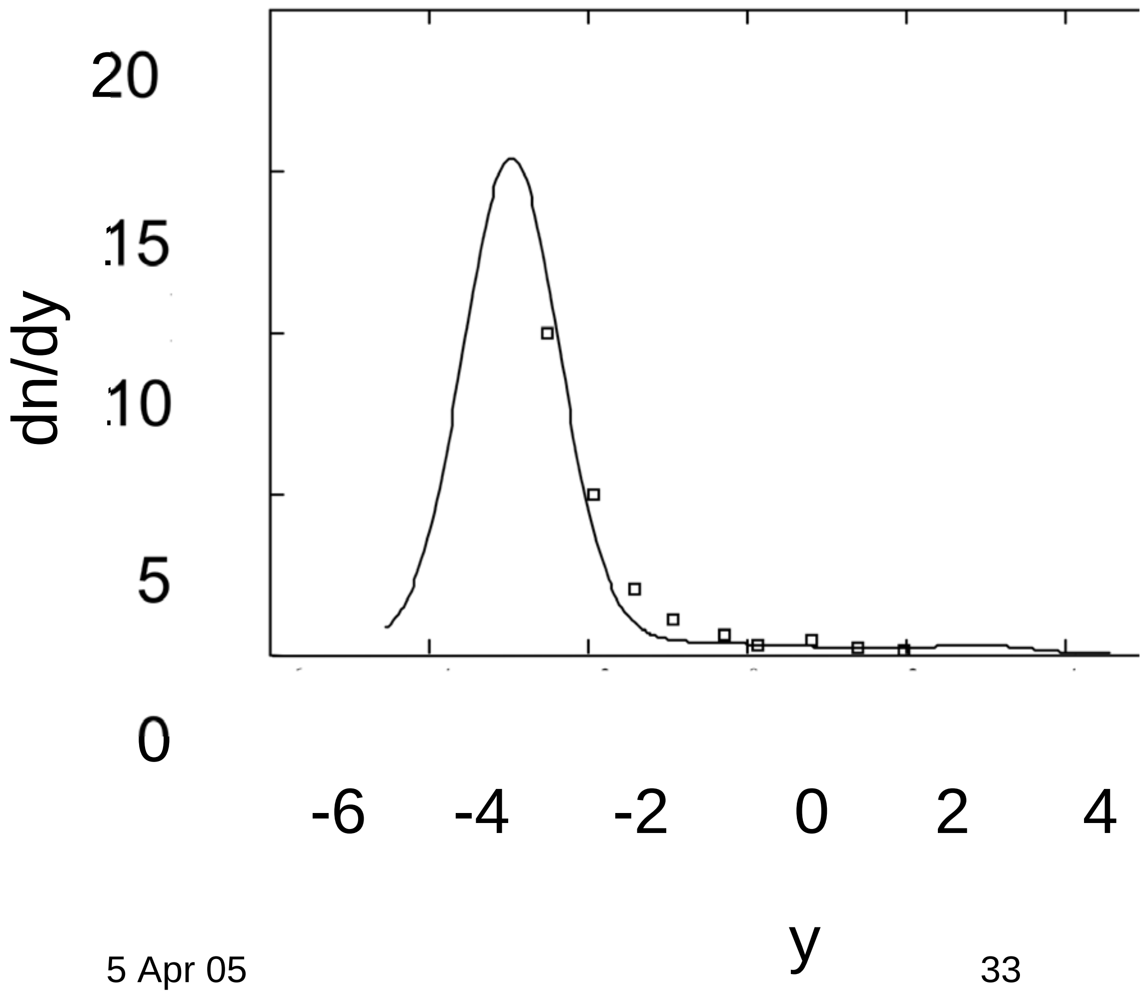
d+Au 19.4 GeV net protons (SPS)

RDM-calculation
for central d+Au:

- 3 sources model
- Net protons

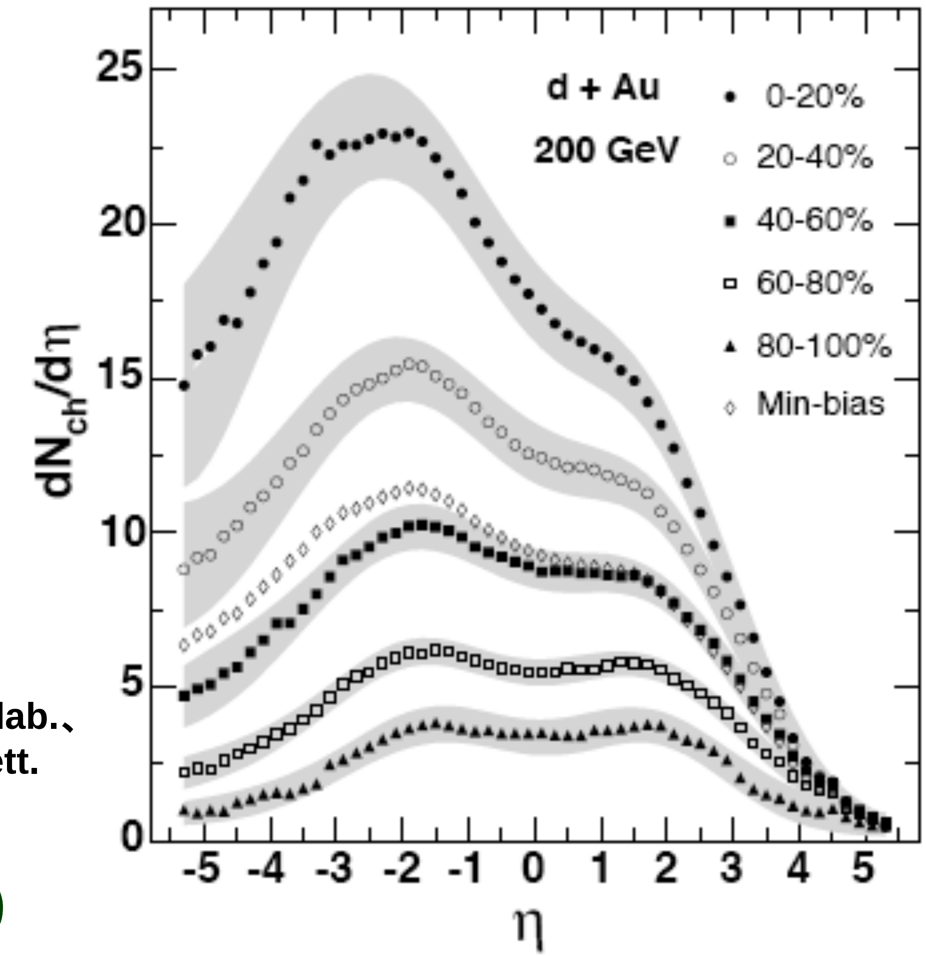
- NA35 data from

T.Alber et al.,
EPJ C2,643 (1998)



d+Au 200 GeV charged hadrons: PHOBOS data

- PHOBOS data in centrality bins
- Use 3-sources-RDM wit initial particle creatior at beam- and equilibrium (y_{eq})value of the rapidity in the analysis.



B.B. Back et al., PHOBOS collab.、 Submitted to Phys. Rev. Lett. (nucl-ex/0409021)

$\eta = -\ln(\tan(\theta/2))$

d+Au 200 GeV charged hadrons: RDM-analysis

Calculate the impact-parameter dependent equilibrium Value of the rapidity:

$$y_{eq} := \frac{1}{2} \cdot \ln\left(\frac{m_1{}^t \cdot e^{y_b} + m_2{}^t \cdot e^{-y_b}}{m_2{}^t \cdot e^{y_b} + m_1{}^t \cdot e^{-y_b}}\right)$$

H.J. Bhabha, Proc. Roy. Soc. (London) A 219, 293 (1953);

S. Nagamiya and M. Gyulassy, Adv. Nucl. Phys. 13, 201 (1984).

with the transverse masses m^t, and the masses determined by the respective numbers of participants

d+Au 200 GeV charged hadrons: RDM-analysis

	b_k	N_k	NGl_k	y_{eq_j}
1: 0-20 %	2.533	12.74	13.5	−0.944
2: 20-40%	4.848	10.237	8.9	−0.76
3: 40-60 %	6.177	6.744	5.4	−0.564
4: 60-80 %	7.258	4.327	2.9	−0.347
5: 80-100 %	8.195	2.008	1.6	−0.169
	8.494	7.372	6.6	−0.664

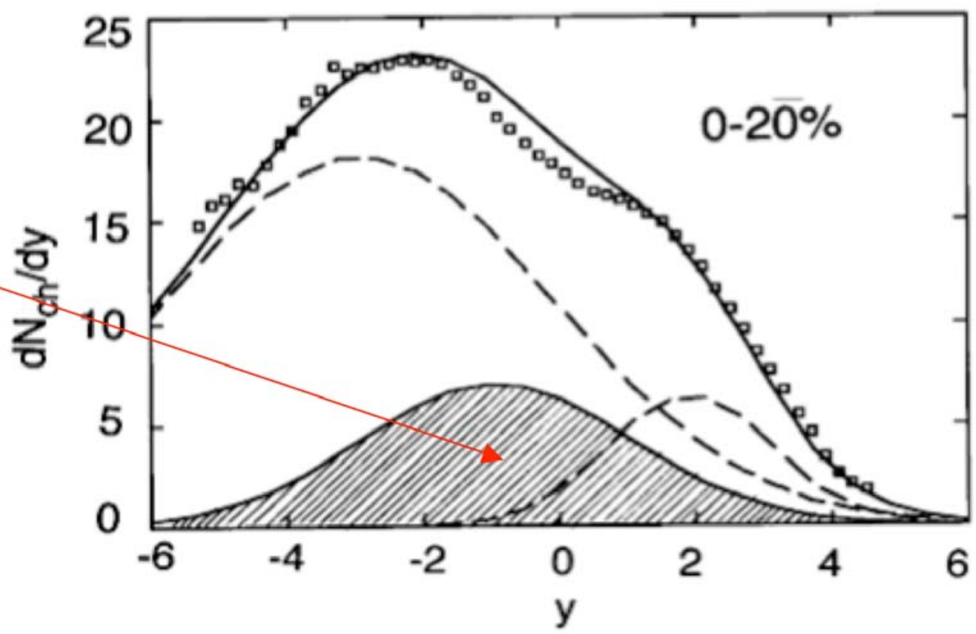
bmax=8.49 fm

Min.bias

B.B.Back et al.

d+Au 200 GeV charged hadrons: RDM-analysis

In the RDM-analysis, determine in particular the importance of the Moving equilibrium (gluonic) source



Data: B.B. Back et al., PHOBOS col
Submitted to Phys. Rev. Lett.

Data are for
$\eta = -\ln(\tan(\theta/2)) \approx y$

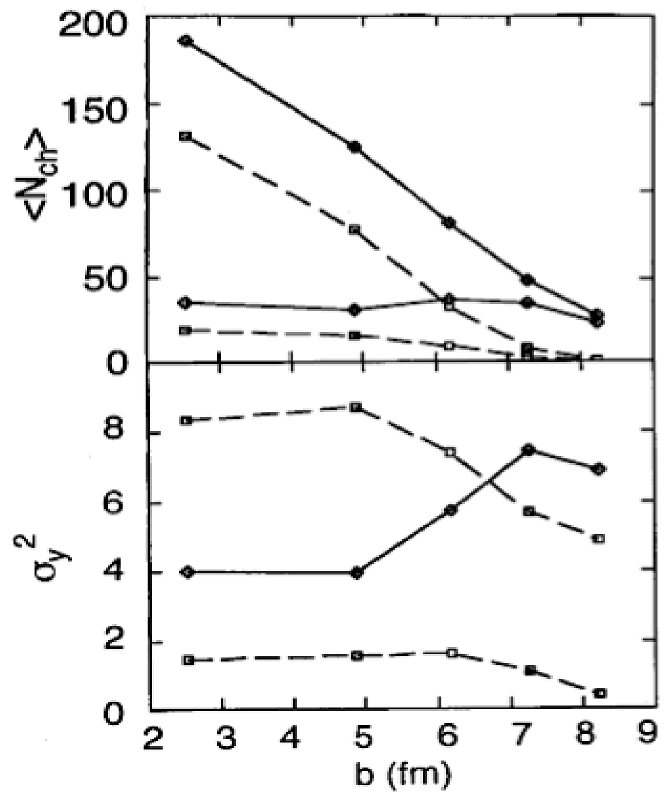
G. Wolschin, M. Biyajima, T. Mizoguchi, N. Suzuki,
hep-ph/0503212

d+Au 200 GeV charged hadrons: RDM-analysis

Mean value and variance of the 3 sources for particle production:

Dashed: d- and Au-like
Solid: equilibrated midrapidity source
Top: total no. of particles

GW, MB,TM, NS, hep-ph/0503212



Convert to pseudorapidity space

The conversion from rapidity (y-) to pseudorapidity space requires the knowledge of the Jacobian

$$J(\eta, \langle m \rangle / \langle p_T \rangle) = \cosh(\eta) \cdot [1 + (\langle m \rangle / \langle p_T \rangle)^2 + \sinh^2(\eta)]^{-1/2}$$

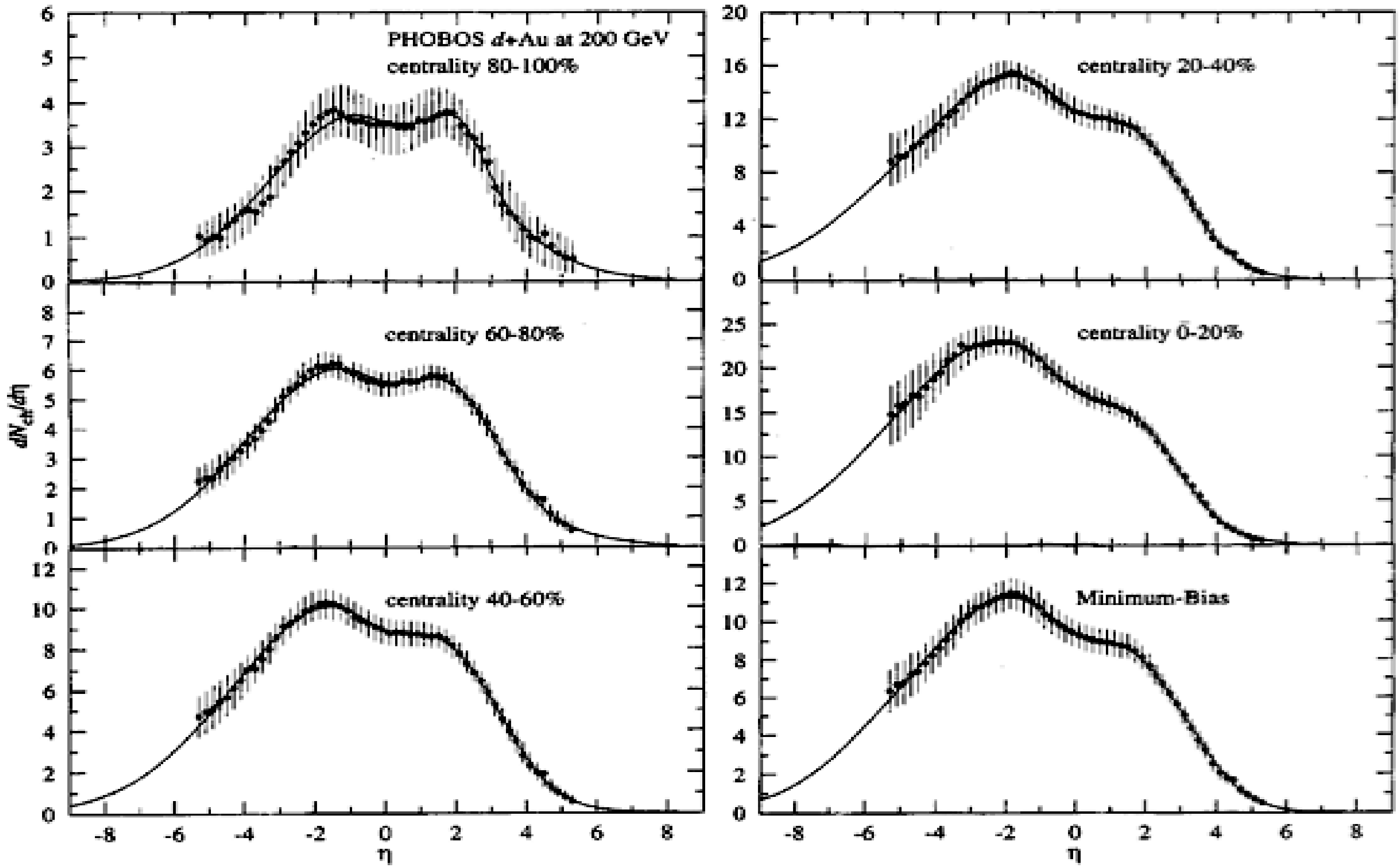
The mean transverse momentum is taken to be 0.4 GeV/c.

The average mass is approximated by the pion mass (0.14 GeV) in the central region. In the Au-like region, it is ≈ 0.17 GeV.

[$< m > \approx m_p \cdot Z_1/N_1^{ch} + m_\pi \cdot (N_1^{ch} - Z_1)/N_1^{ch} \approx 0.17 \mathrm{GeV}$]

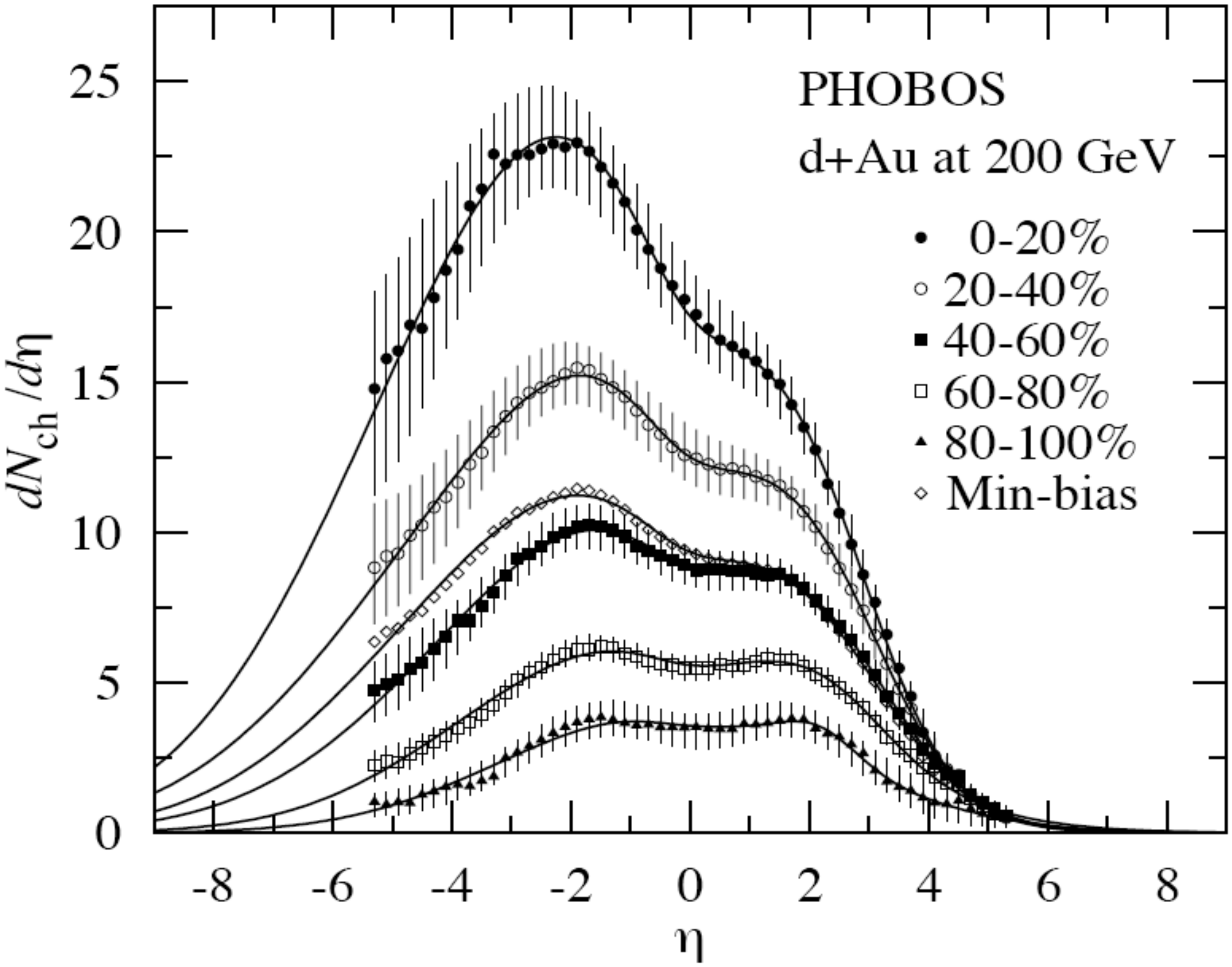
d+Au 200 GeV RDM-analysis

Data: PHOBOS coll.
Submitted to PRL.



GW, MB,TM, NS,
hep-ph/0503212

d+Au 200 GeV RDM-analysis



Data: PHOBOS coll. Submitted to PRL.

Full lines: RDM-results with χ2-minimization

GW, MB,TM, NS

Compare d+Au/Au+Au 200 GeV central source

TABLE I. Comparison of central d + Au (0-20%) and Au + Au (0-6%) [10] collisions at $\sqrt{s_{NN}}$ = 200 GeV, $y_b = \pm$ 5.36 in the Relativistic Diffusion Model. The ratio τ_{int}/τ_y determines how fast the system of produced charged particles equilibrates in rapidity space. The variance of the central source in y−space is σ^2_{eq}. The number of produced charged particles in central collisions is $N^{1,2}_{ch}$ for the sources 1 and 2 and N^{eq}_{ch} for the equilibrium source, the percentage of charged particles produced in the thermalized source is n^{eq}_{ch}.

System	τ_{int}/τ_y	σ^2_{eq}	N^1_{ch}	N^2_{ch}	N^{eq}_{ch}	$n^{eq}_{ch}(\%)$
d + Au	0.79	5.19	131	19	35	19
Au + Au	1.35	4.0	561	561	3928	78

Conclusion

- The Relativistic Diffusion Model accounts for rapidity distributions of net protons at SIS, AGS, SPS, RHIC energies.
- The locally equilibrated midrapidity region contains 13-14% of the net protons at RHIC energies: indirect evidence for deconfinement and longitudinal collective expansion.
- The Model with 3 sources describes the centrality dependence of pseudorapidity distributions for produced charged hadrons in d+Au and Au+Au collisions at RHIC energies (200A GeV) accurately.
- The equilibrated central source carries 78% of the charged-particle content in Au+Au central collisions, but only 19% in d+Au collisions.
- This locally thermalized central source may be interpreted as indirect evidence for a locally equilibrated quark-gluon plasma.

Relativistic Diffusion Model

The model is based on a generalized Fokker-Planck equation (FPE) for the distribution function W($p_\perp$,y;t) in transverse momentum ($p_\perp$) and rapidity (y) space ($p_{1,2} = p_\perp, y$),

$$y = 0.5 \cdot ln((E + p_{||})/(E - p_{||})) \quad (1)$$

$$\frac{\partial}{\partial t}\Big[W(p_\perp, y; t)\Big]^{\mu} = -\sum_{k=1}^{2} \frac{\partial}{\partial p_k} J(p_k)\Big[W(p_\perp, y; t)\Big]^{\mu} + \sum_{k,j=1} \frac{\partial^2}{\partial p_k \partial p_j} D(p_k p_j; t)\Big[W(p_\perp, y; t)\Big]^{\nu}. \quad (2)$$

with the drift function $J(p_\perp, y)$, and the diffusion tensor

$$\mathbf{D} = \begin{pmatrix} D_{p_\perp p_\perp} & 0 \\ 0 & D_{yy} \end{pmatrix}.$$

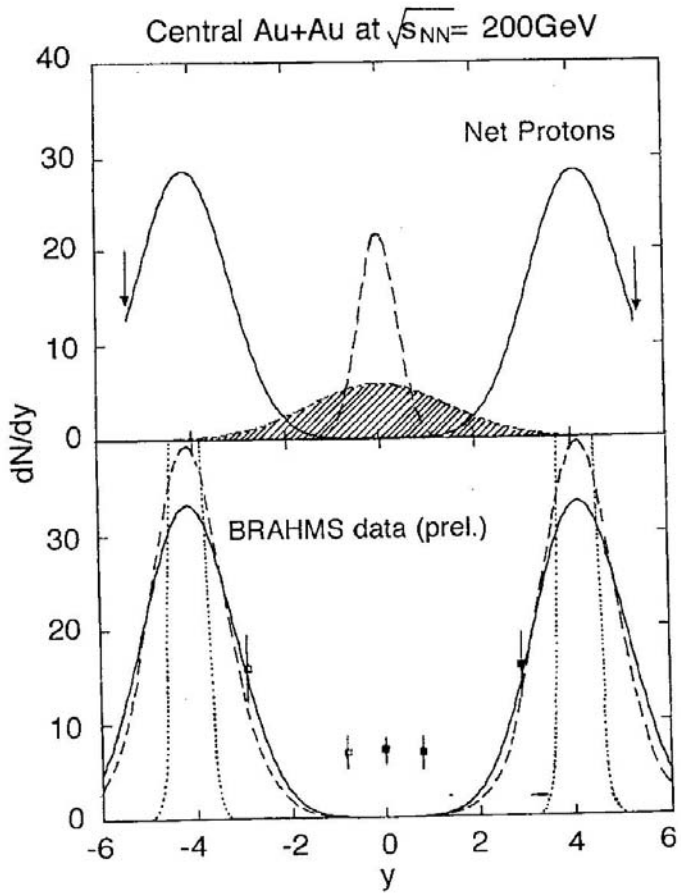
To conserve the norm, $\mu = 1$. Normally $\nu = 1; \nu < 1$ accounts for possible deviations from Maxwell-Boltzmann statistics.

ν=1: GW, Z. Phys. A 355, 301 (1996); EPJ A5, 85(1999)

RDM for Au-Au @ RHIC

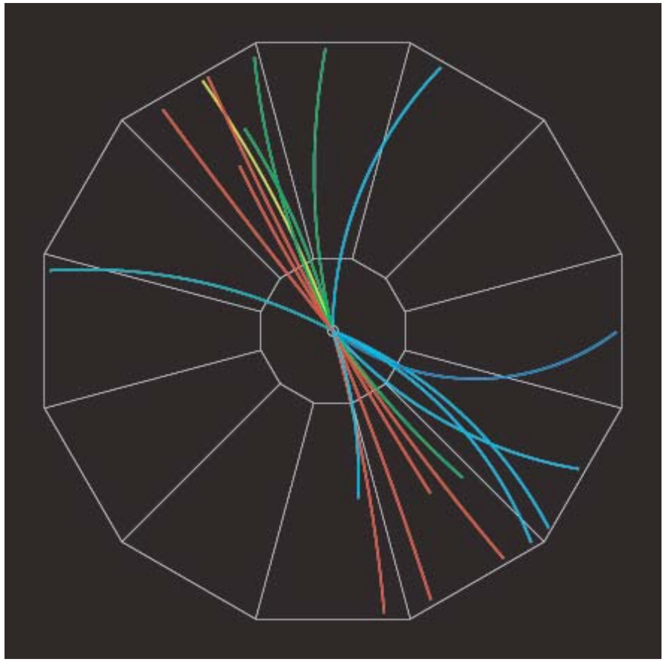
- Net protons in central collisions
- Linear (solid curves) and nonlinear RDM-results; weak-coupling solution is dotted
- Midrapidity data require transition to thermal equilibrium (dashed area)

GW, Phys. Lett. B 569, 67 (2003)

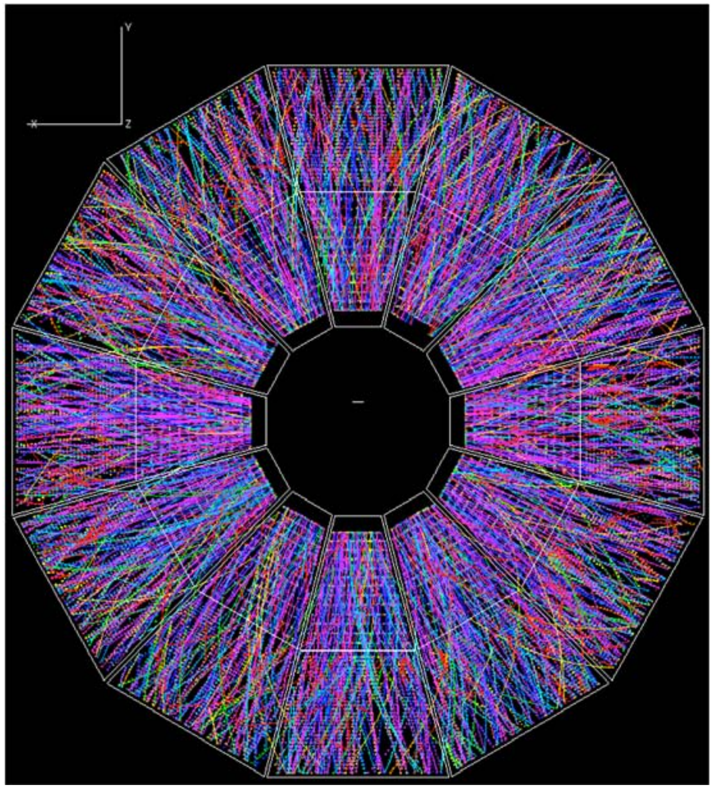


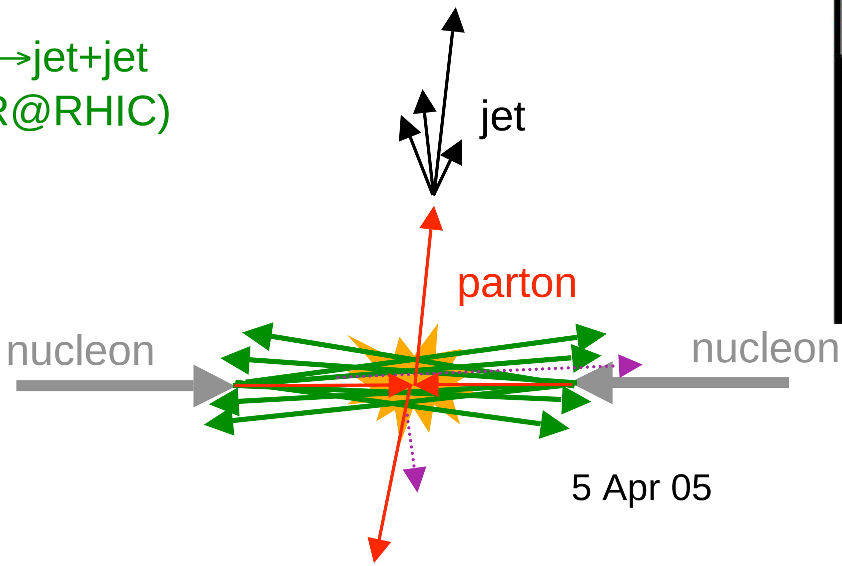
Jets @ RHIC

Find this……….in this



p+p →jet+jet
(STAR@RHIC)

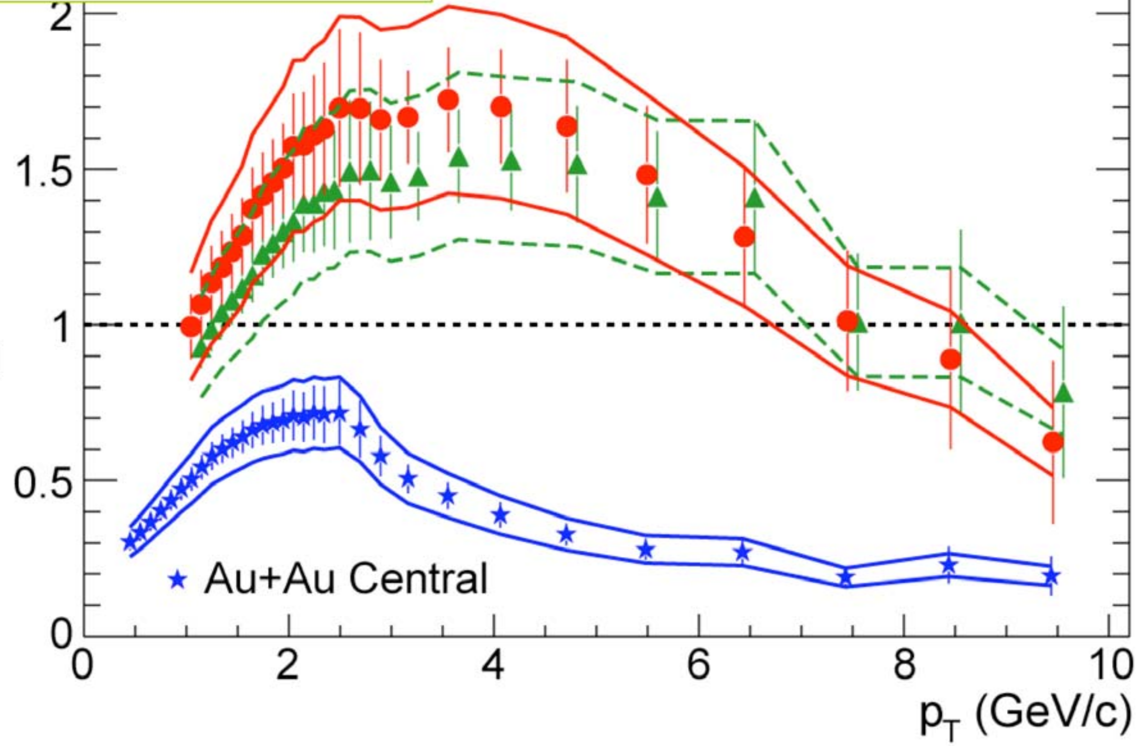




Au+Au →???
courtesy
STAR@RHIC

Inclusive yield relative to binary scaled pp: STAR

$$R_{AB} = \frac{dN^{AB} / dp_T d\eta}{T_{AB} d\sigma^{pp} / dp_T d\eta}$$



- d+Au : enhancement
  Au+Au: strong suppression

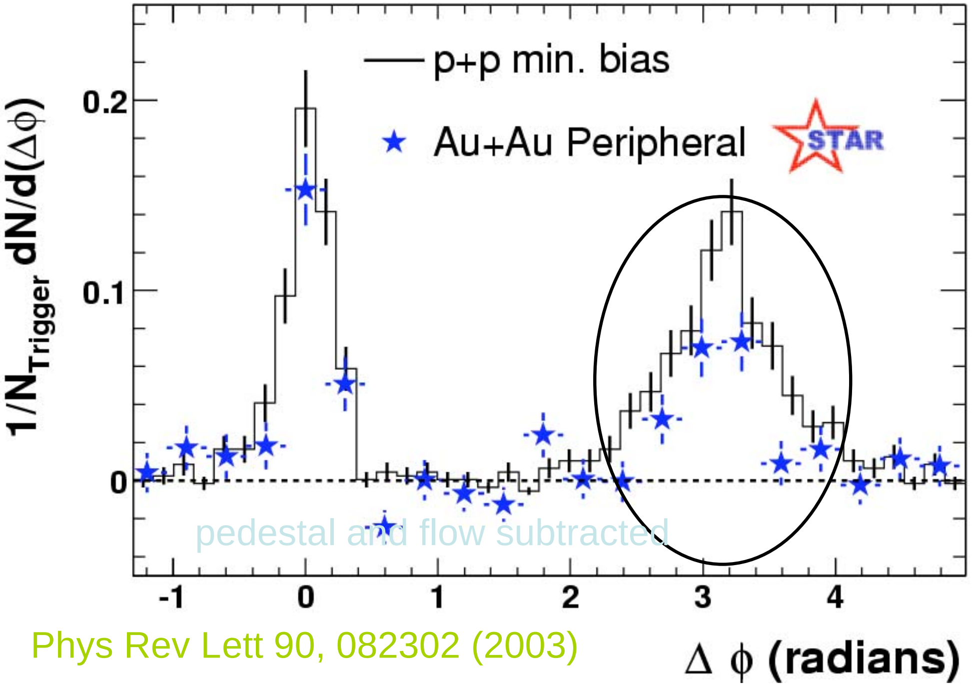
- p_T=4 GeV/c:
  cent/minbias = 1.11±0.03
  ⇒ central collisions enhanced wrt minbias

courtesy STAR

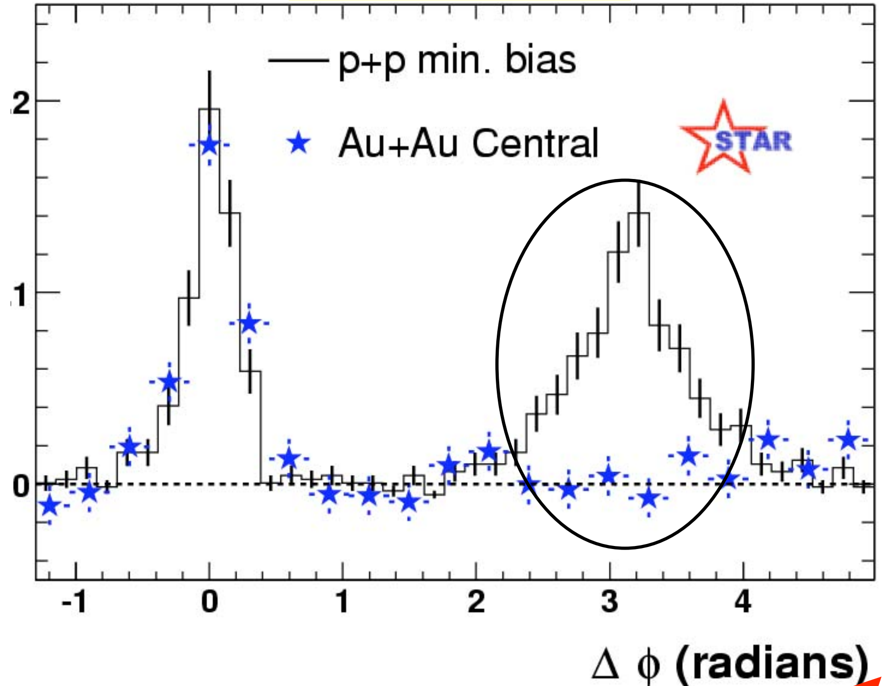
Suppression of the inclusive yield
in central Au+Au is a final-state effect

Azimuthal distributions in Au-Au: STAR

Au+Au peripheral



Au+Au central



Phys Rev Lett 90, 082302 (2003)

Near-side: peripheral and central Au+Au similar to p+p

Strong suppression of back-to-back correlations in central Au+Au