Tetraquarks

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Outline:

- **Experiment: new charmed mesons**
- **Three theoretical scenarios**
- **Quark model with tHooft force**
- **Mesons and baryons**
- **Colour dependent confining forces**
- **Tetraquarks**
- **Pentaquarks**
- **Summary**

Experiment

- New charmed (nonexotic) mesons:
- $D^+(2308)$, $D_s^+(2317)$ and $D_s^+(2632)$
- Too many states: Tetraquarks?
- Other candidates: X(3872) ?
- Exotic hyperons: $\Theta^+(1540)$ pentaquark!
- Other candidates: $\Xi^{--}(1862)$, $\Theta^c(3099)$?

Three theoretical scenarios

- The $D^+(2308)$, $D_s^+(2317)$, $D_s^+(2632)$, and $X(3872) = (q^2\bar{q}^2)$ "true tetraquarks", bound by QCD forces. Colour quark force important:

  Multiquarks related to additional colour singlets (expected since 1976).

  Consistent with "true pentaquark" interpretation of $\Theta^+(1540)$: $\Theta^+ = (q^4\bar{q})$.

- The hadronic molecule model of $D^+(2308)$, $D_s^+(2317)$, $D_s^+(2632)$, and $\sigma^o(600)$, $a_0(980)$, $f_0(980)$ resonances (I. Nakamura +V.D., many others). Consistent with $\Theta^+(1540)$ as a molecular bound state of three hadrons (Kishimoto and Sato): $\Theta^+ = K^+ n \pi^o$ No colored quarks, just hadrons. Chiral symmetry the only guide.

- The $\Theta^+(1540)$ is a chiral soliton state; no quarks, just meson fields. What are the new mesons?

Constituent quark model with instanton-induced interaction

- Nonrelativistic constituent quarks (massive, 330 MeV): no chiral symmetry!
- Confinement via chromo-harmonic 2-body force: a) all colour singlets stable; b) mesons and baryons asymptotic states; c) predicts new confined "hidden colour" states
- HFI determines "fine structure" of the spectrum:

  a) colour-spin (Fermi-Breit one-gluon exchange)
  b) flavour-spin (tHooft instanton induced)

Hyperfine Interactions

- The spin-dependent "color-magnetic" CS (Fermi-Breit) interaction was believed to be the main source of SU(6) multiplet splittings in QCD.
- CS model cannot solve the "$U_A(1)$ problem" in mesons and several problems in baryons.
- The 't Hooft interaction solves the $U_A(1)$ problem, but it changes other mass splittings. How?
- "Calibrate" HFI in mesons and baryons, then use it in multiquarks

What is the "$U_A(1)$ problem"?

1. **The sum of η and $\eta^{'}$ masses does not satisfy the Gell-Mann–Okubo relation:**

$$m_\eta^2 + m_{\eta'}^2 = (1111 \text{ MeV})^2 \neq 2m_K^2 = (700 \text{ MeV})^2$$

2. **The η - $\eta^{'}$ mixing angle $\theta_{p.s.}$ is not the ideal one:**

$$\theta_{p.s.} \simeq -20^o \neq 35.3^o .$$

Two solutions

QCD Instantons ⇒ 't Hooft-Kobayashi-Kondo-Maskawa quark interaction.

$$\mathcal{L}_{\rm tH} = G_{\rm tH}\left[\det\left(\bar{\psi}(1+\gamma_5)\psi\right) + \det\left(\bar{\psi}(1-\gamma_5)\psi\right)\right]$$
$$= 2G_{\rm tH}{\rm Re}\ \det\left(\bar{\psi}(1+\gamma_5)\psi\right)$$

where

$$G_{\rm tH} = \frac{f_\eta^2}{12}\left[m_\eta^2 + m_{\eta'}^2 - 2m_K^2\right]\langle\bar{q}q\rangle_0^{-2}$$

QCD $1/N_c$ limit ⇒ Veneziano-Witten quark interaction.

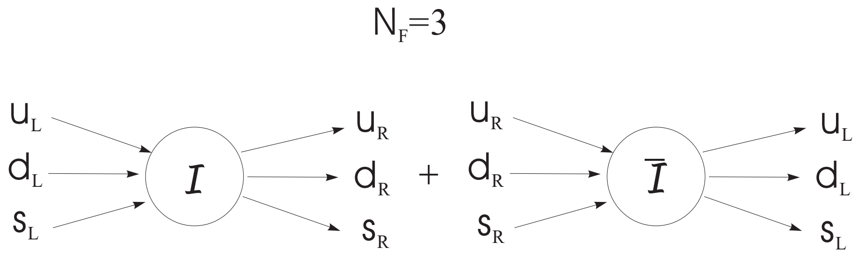
$$\mathcal{L}_{\rm VW} = G_{\rm VW}\left[\det\left(\bar{\psi}(1+\gamma_5)\psi\right) - \det\left(\bar{\psi}(1-\gamma_5)\psi\right)\right]^2$$
$$= -4G_{\rm VW}\left[{\rm Im}\ \det\left(\bar{\psi}(1+\gamma_5)\psi\right)\right]^2$$

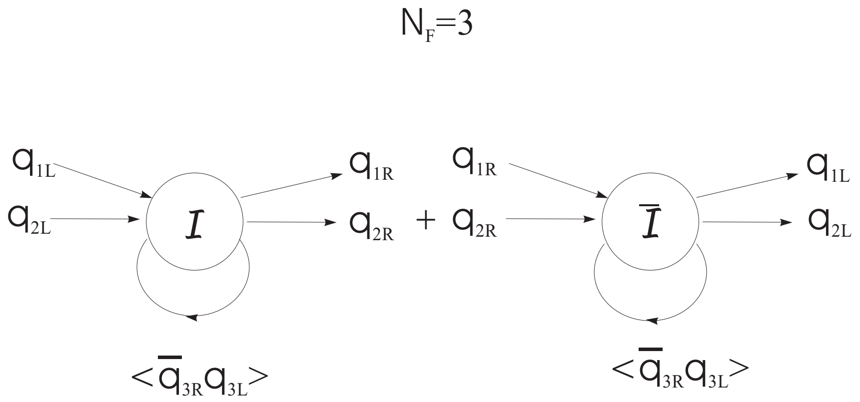
where

$$G_{\rm VW} = \frac{f_\eta^2}{48}\left[m_\eta^2 + m_{\eta'}^2 - 2m_K^2\right]\langle\bar{q}q\rangle_0^{-6}$$

Instanton induced interaction in QCD

- Instantons induce a new three-body, flavour-dependent, contact interaction
- Closing one pair of “legs” leads to a two-body interaction that depends on flavour and spin.





Two new $U_A(1)$ symmetry breaking interactions

There are also two “antisymmetric tensor” interactions: [Phys. Rev. D 56, 247 (1997)]

‘t Hooft

$$\mathcal{L}_{\mathrm{T1}} = G_{\mathrm{T1}} \left[\det{}_f \left(\bar\psi \sigma_{\mu\nu}(1+\gamma_5)\psi\right) + \det{}_f \left(\bar\psi \sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right]$$

Veneziano-Witten

$$\mathcal{L}_{\mathrm{T2}} = G_{\mathrm{T2}} \left[\det{}_f \left(\bar\psi \sigma_{\mu\nu}(1+\gamma_5)\psi\right) - \det{}_f \left(\bar\psi \sigma_{\mu\nu}(1-\gamma_5)\psi\right)\right]^2$$

where $\sigma_{\mu\nu}$ is the Pauli tensor

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]$$

for two flavours this leads to an extended NJL Lagrangian [Phys. Rev. D 62, 096010(8) (2000)]

$$\begin{aligned}\mathcal{L}_{\mathrm{ENJL}} = \bar\psi[\mathrm{i}\partial\!\!\!/ - m_q^0]\psi + G_S[(\bar\psi\psi)^2 + (\bar\psi \mathrm{i}\gamma_5 \boldsymbol{\tau}\psi)^2] \\ -G_T[(\bar\psi\sigma_{\mu\nu}\psi)^2 + (\bar\psi i\gamma_5\sigma_{\mu\nu}\boldsymbol{\tau}\psi)^2]\end{aligned}$$

What is the physics of the Pauli tensor interaction?

The two-body **'t Hooft** interaction leads to the following two-quark potential

$$\begin{aligned} V_{12} &= 4K\langle\bar{q}q\rangle_0 P^{\bar{3}}_{12}\,(1-\vec{\sigma}_1\cdot\vec{\sigma}_2)\,\delta(\mathbf{r}_1-\mathbf{r}_2) \\ P^{\bar{3}}_{12} &= \left[\frac{1}{3}-\frac{1}{4}\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2\right]. \end{aligned} \tag{2}$$

affects only spin singlets (PS mesons) and cures the $U_A(1)$ problem.

The 't Hooft interaction also leads to the three-quark potential

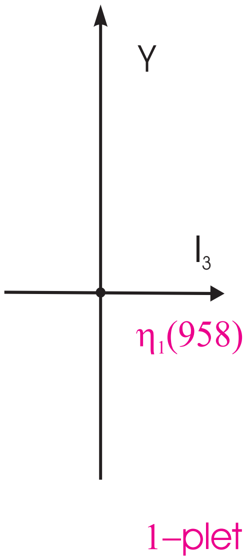
$$\begin{aligned} V_{123} &= 12KP^{1}_{123}\left(1-\sum_{i<j}^{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_3-\mathbf{r}_2) \\ P^{1}_{123} &= \frac{1}{12}\left[\frac{4}{9}-\frac{1}{3}\sum_{i<j}^{3}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j+d^{abc}\boldsymbol{\lambda}^a_1\boldsymbol{\lambda}^b_2\boldsymbol{\lambda}^c_3\right]. \end{aligned} \tag{3}$$

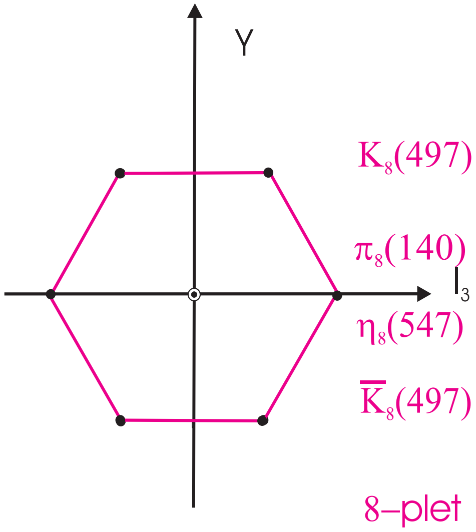
Basic check: mesons and baryons

- Must check our “new” model against known phenomena (see also Bonn and Tokyo groups)
- Mesons: tHooft interaction yields OK spectra in chiral models (e.g. NJL, OGE BSE), but in nonrelativistic ones?
- Baryons: Little understanding, in spite (because?) of Bonn U. papers.

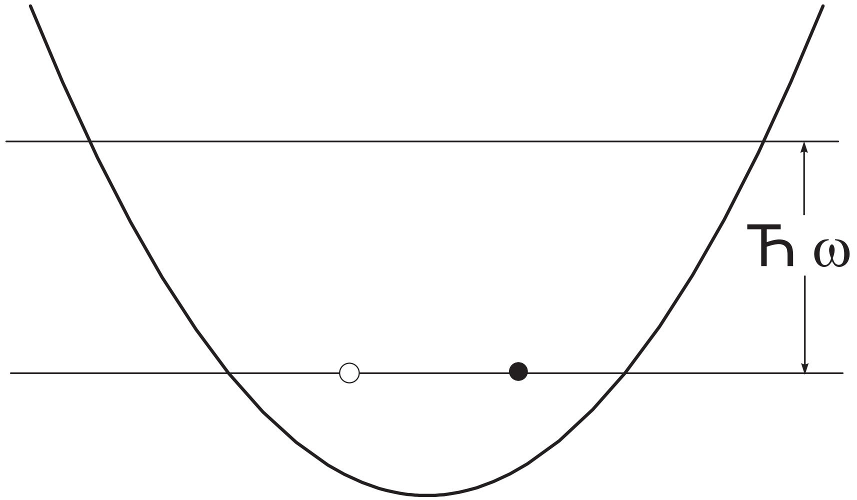
Mesons

- Mesons form a nonet = singlet + octet
- For every spin and parity allowed by angular momentum conservation, P- and C-conjugation



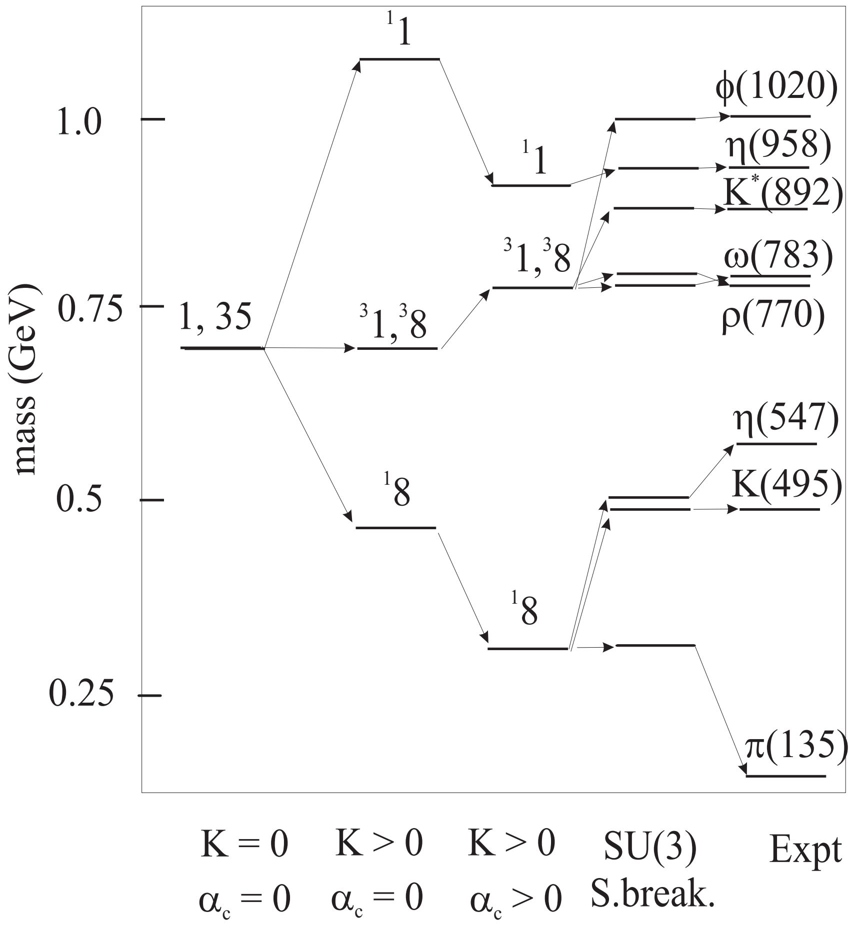


- Confining two-body potential (simple= HO realistic= linear + Coulomb) determines overall masses (shell separation)
- Mass splitting determined by HFI



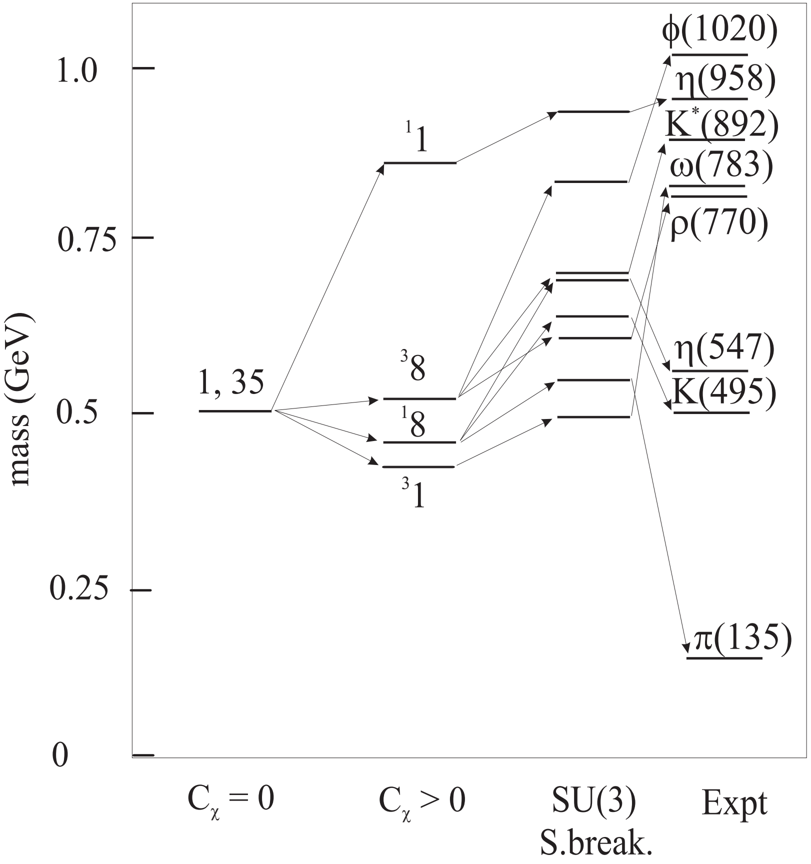
Light meson spectrum with nonrelativistic KKMT interaction

- Vector and pseudoscalars
- Only p.s. change.
- Vector mesons still ideally mixed.
- Only the pion still too heavy.



Comparison with Glozman-Riska interaction

- Simple flavour-spin (Gamow-Teller) HF interaction normalized on baryons leads only to ή(960) meson OK.
- The rest is bad.
- Accident? No, GR guessed one piece of tHooft HFI correctly.



Modelling Confinement

- Two-body interaction with the “saturating” colour factor has been used.
- “Saturation” of color forces = no confining forces between colour singlets
- Lorentz *vector* confining interaction
- Nogami and Bhaduri’s or Grenoble parametrization.

$$V(r_{ij}) = \frac{1}{4} \sum_{a=1,..,8} \lambda_i^a \lambda_j^a v(r_{ij}) \equiv (F_i \cdot F_j) v(r_{ij})$$

$$(F_i \cdot F_j)$$

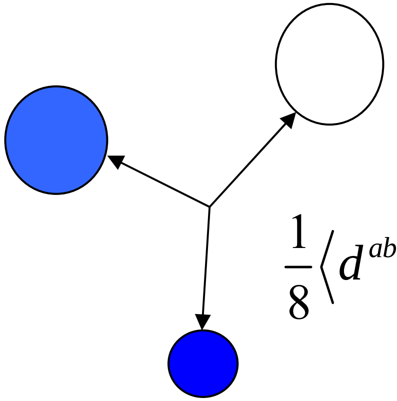
Lorentz vector vs. scalar

- Lorentz *scalar* confining two-body potentials are *not* positive definite, i.e., *some colour singlets are unconfined*
- They lead to small spin-orbit splitting /potential
- They *demand* a strong three-body potential to confine the baryon!
- May require four-body forces to stabilize the tetraquark.

- Lorentz *vector* confining two-body potentials are stable and saturating.
- They lead to large spin-orbit splitting /potential.
- A fit to meson and baryon spectra leaves little room for a three-body force.
- May lead to bound tetraquarks for sufficiently asymmetric masses .

Three-body force: stability and saturation

- A “saturating” three-body force has been introduced (PLB499,136)
- Two scenarios: (a) Allow all color states, demand that color 8, 10 be heavy; (b) Forbid all non-singlets
- Scenario (a) demands 3-, 4-,5-body forces etc.; (b) allows only dominant 2-body and weak 3-body.



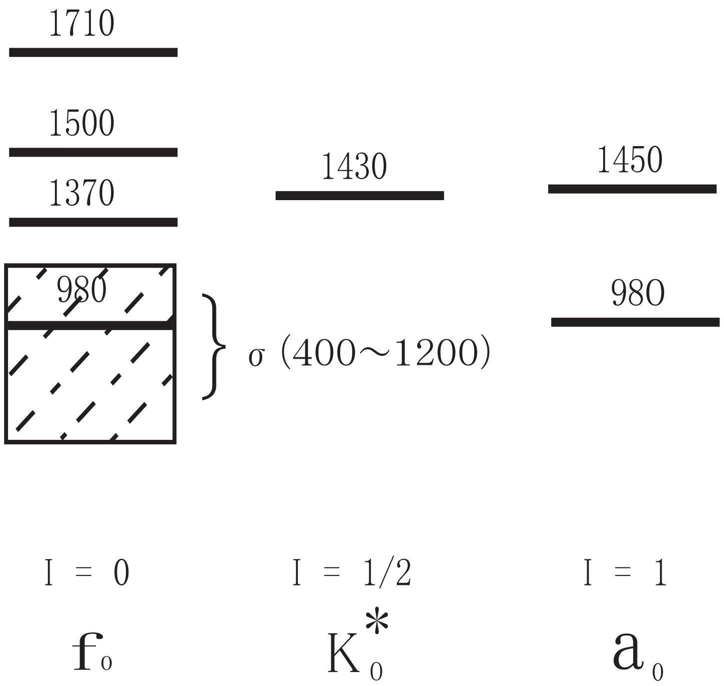
$$\frac{1}{8}\left\langle d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\right\rangle$$

$$V \rightarrow V + V_{3b}$$

$$V_{3b}\left(\vec{r}_i,\vec{r}_j,\vec{r}_k\right) = \frac{1}{8} d^{abc}\lambda_i^{\ a}\lambda_j^{\ b}\lambda_k^{\ c} U_0 \exp[-(r_i^2 + r_j^2 + r_k^2)/a_0]$$

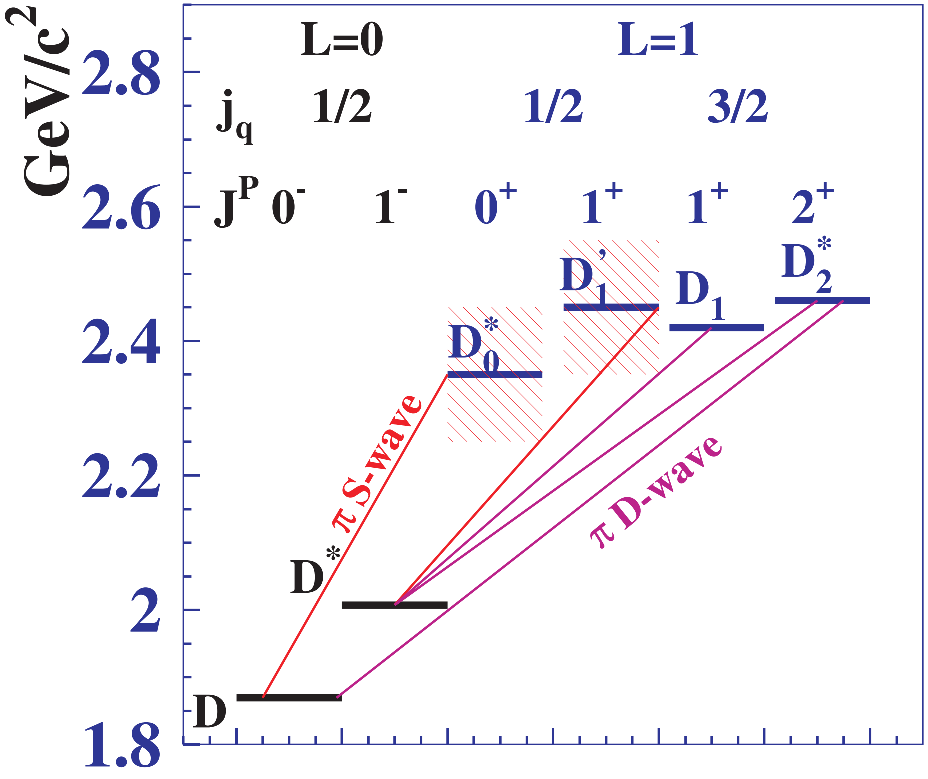
Why tetraquarks? Scalar mesons

- Similar problems already exist in the spectrum of light scalar mesons:
- More states than fill one nonet, but not enough for two nonets.
- Supernumerary flavour singlets: glueballs? The second isovector a(1450)?



Why tetraquarks? Charmed mesons

- Expected spectrum in potential models:
- Too many observed scalar (0^+) states: strange $D_s^+(2317)$ (Belle) and $D_s^+(2632)$ (SELEX); and nonstrange $D^+(2308)$ (Belle)
- Too light; strange and nonstrange degenerate!



Tetraquarks: colour states

- Simplest multiquark with two colour singlets is the tetraquark:

$$[\bar{q} \quad q \quad \bar{q} \quad q]$$

- In the “asymptotic” basis: are the “two-meson” and the “hidden-colour” state

$$|1_{12}1_{34}\rangle , |8_{12}8_{34}\rangle$$

- Mathematically equivalent (“Pauli”) basis

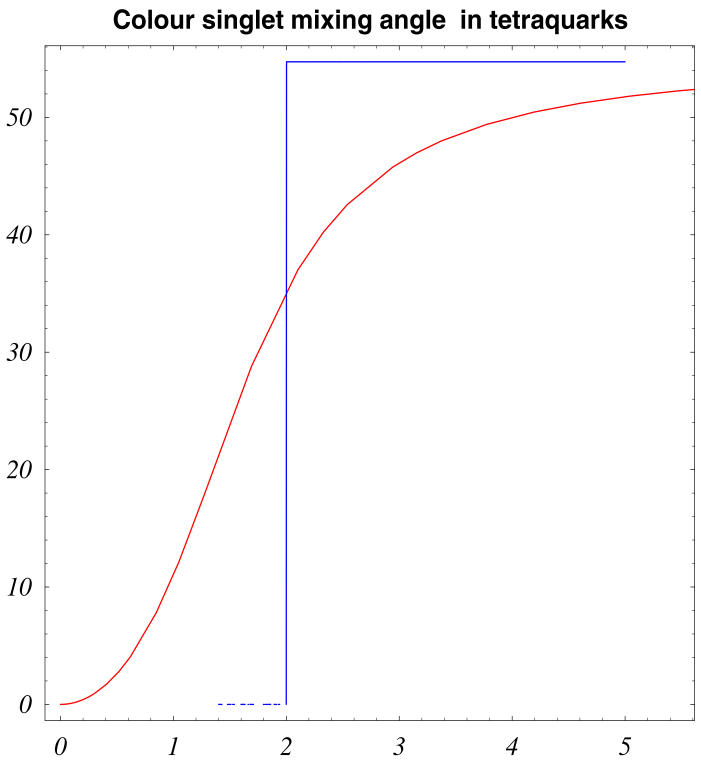
$$|\bar{3}_{13}3_{24}\rangle , |6_{13}\bar{6}_{24}\rangle$$

- “Asymptotic basis” is physically preferable;
- “Pauli” basis is more convenient (Pauli princ.)

More on tetraquark colour states

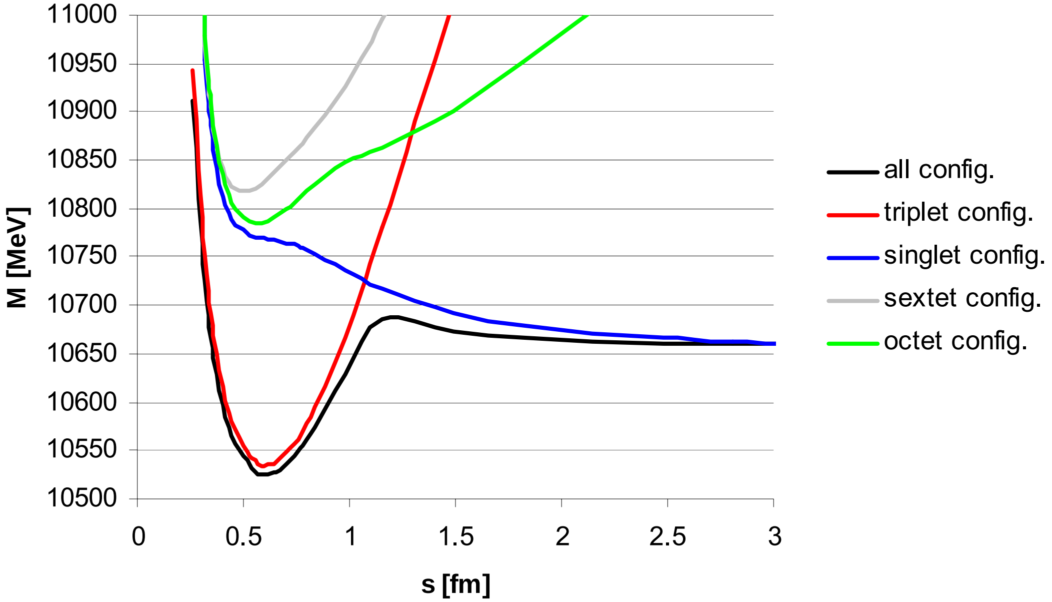
- The quark interactions mix the two colour singlets
- One two-meson state at asymptotic distances; another is a confined hidden-colour tetraquark state.
- Latter state cannot decay into two mesons.
- Two diagonal states are orthogonal: two separate “worlds”.
- No physical process (in NRCQM) can take one “world” into another. Need anihilation processes.

$$\sin\theta\left|1_{12}1_{34}\right\rangle+\cos\theta\left|8_{12}8_{34}\right\rangle$$



Example of a bound two-meson state due to two-body colour potential (Courtesy D. Janc)

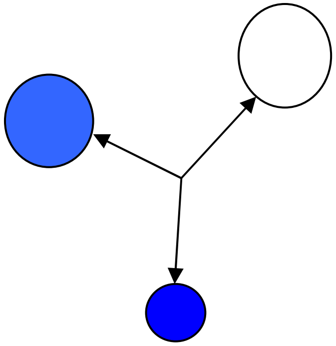
- At short distances new state is admixed in the two-meson continuum: a resonance or bound state.
- Tetraquark is bound for large enough mass ratio: double-t heavy-light tetraquark mass as a function of separation.
- Both Pauli- and hidden -colour states confined! Saturation of the two-meson state!



Three-quark colour force in tetraquarks

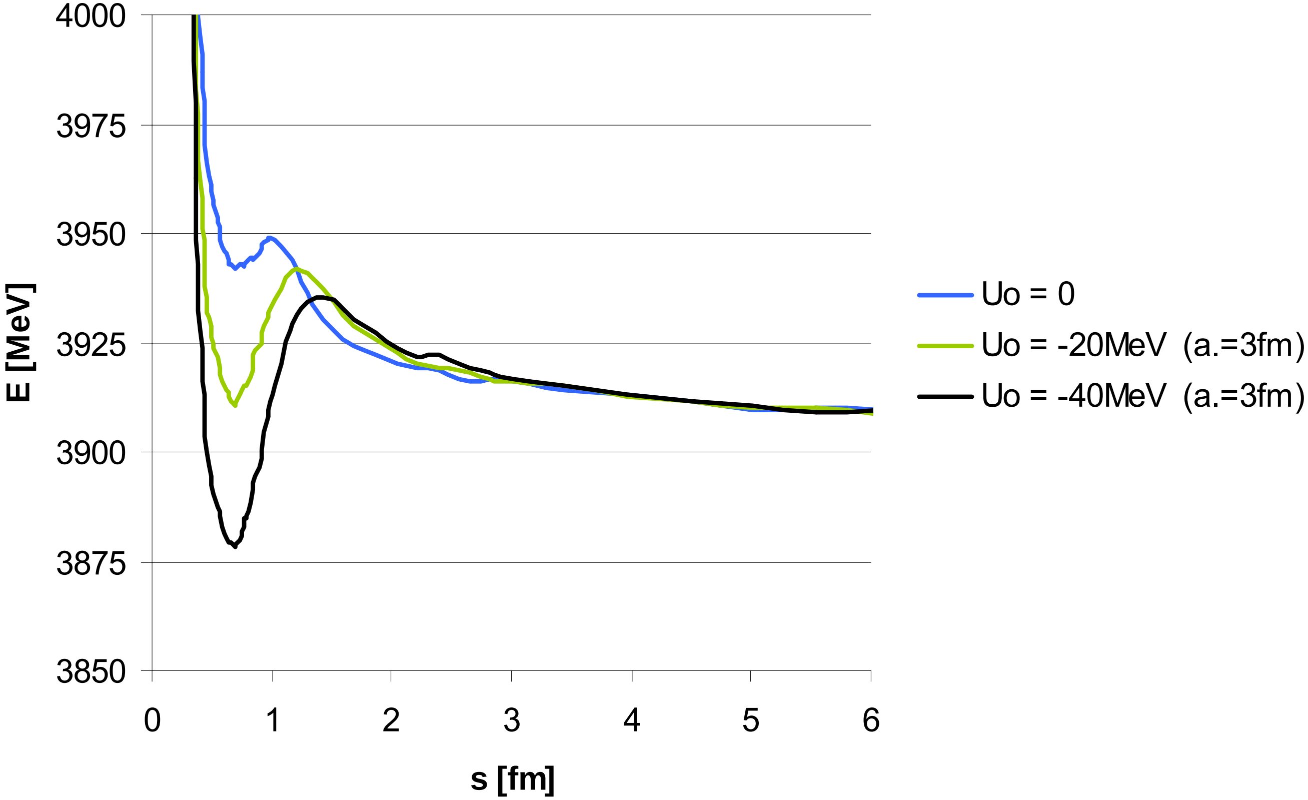
- The 3-body force:
- Saturation: mixes states in the asymptotic basis, but changes only the hidden-colour state energy
- Does not mix states in the Pauli basis, but changes their energies. Unstable!

$$\frac{1}{8}\left\langle d^{abc}\lambda_i^a\lambda_j^b\lambda_k^c\right\rangle$$



	$1_{12}1_{34}$	$8_{12}8_{34}$	$\bar{3}_{13}3_{24}$	$6_{13}6_{24}$
$1_{12}1_{34}$	0	$-5\sqrt{2}/18$	$5/(9\sqrt{3})$	$-5\sqrt{2}/(18\sqrt{3})$
$8_{12}8_{34}$	$-5\sqrt{2}/18$	$5/18$	$-5\sqrt{2}/(9\sqrt{3})$	$-5/(18\sqrt{3})$
$\bar{3}_{13}3_{24}$	$5/(9\sqrt{3})$	$-5\sqrt{2}/(9\sqrt{3})$	$5/9$	0
$6_{13}6_{24}$	$-5\sqrt{2}/(18\sqrt{3})$	$-5/(18\sqrt{3})$	0	$-5/18$

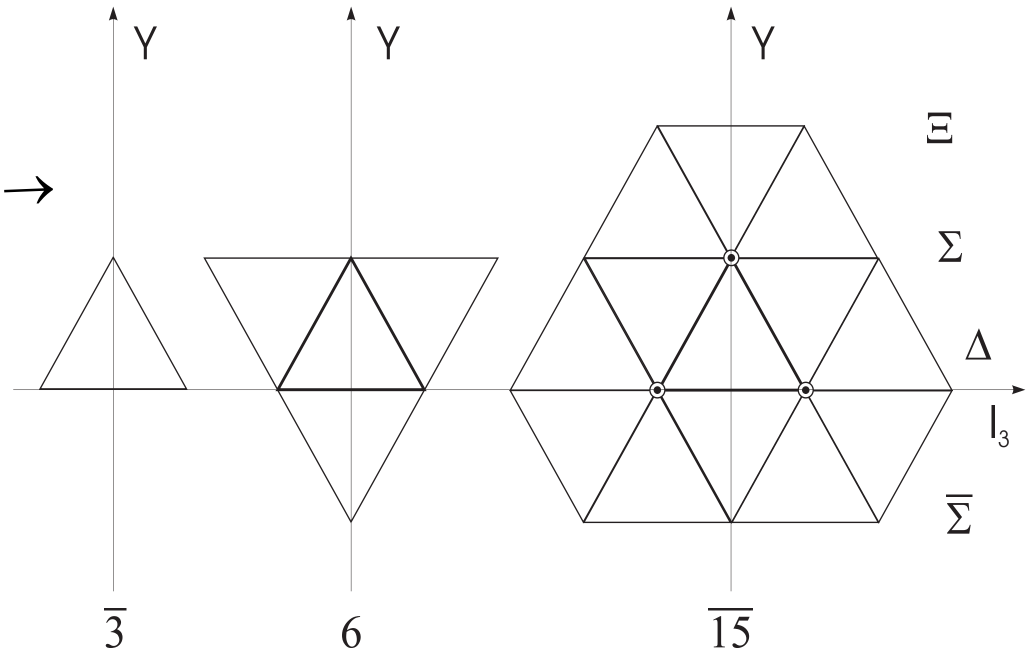
Effects of three-quark colour force on double-charm tetraquarks (D. Janc)



Single-charm tetraquark SU(3) contents

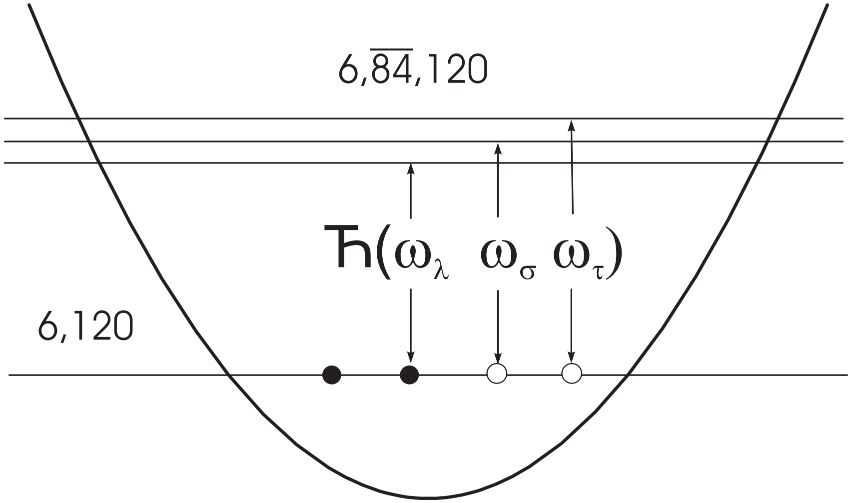
- Tetraquark SU(3) C.G. series: 3-,6-, 15-plets.
- Two 3-plets and “inner” triplet in 15-plet may mix!

→



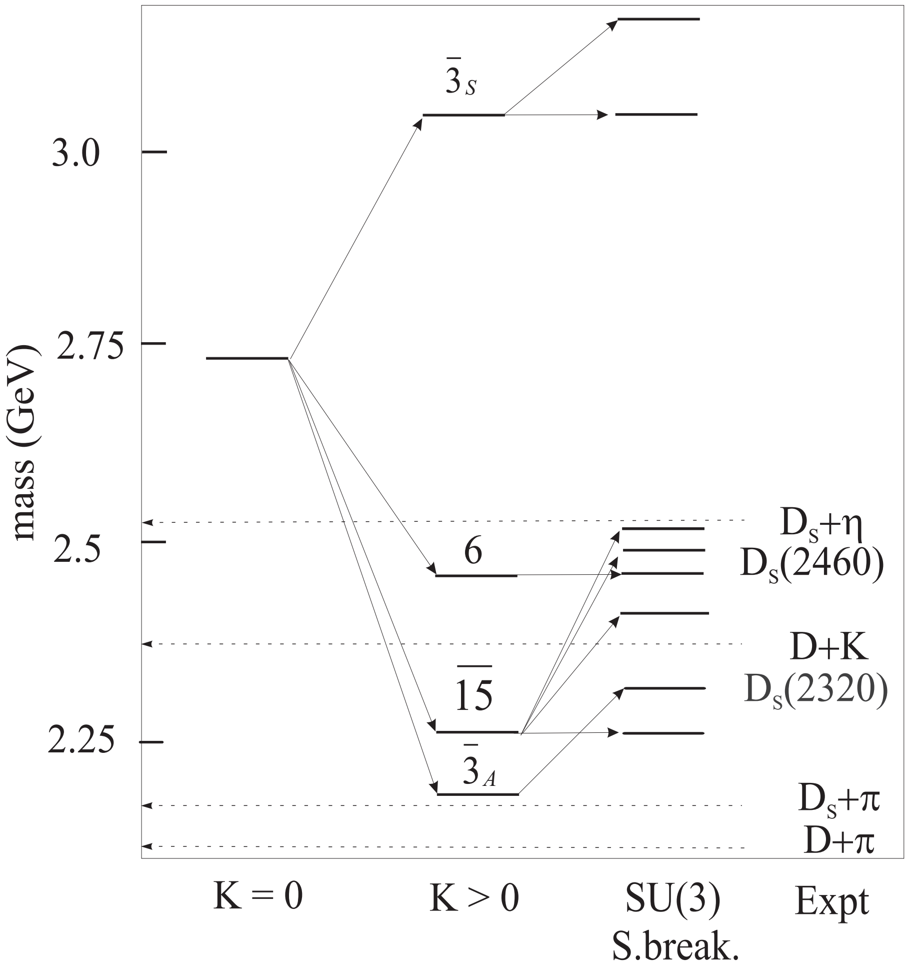
Single-charm tetraquark SU(6) contents

- SU(6) multiplets:
  6, 84, 120.
- Pauli principle allows only 6, and 120-plet in the ground state.



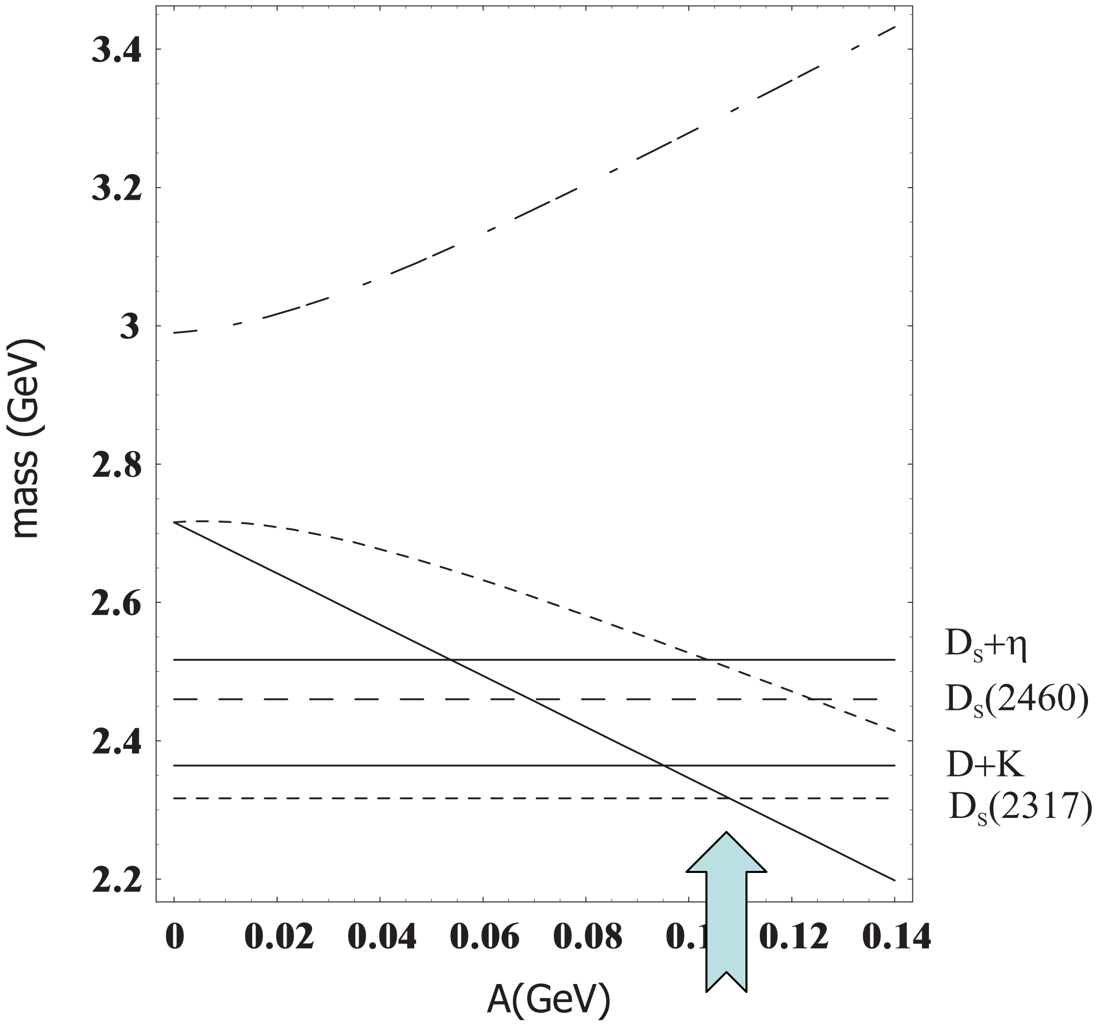
Charmed tetraquark mass splitting

- KKMT interaction splits the degeneracy.
- SU(3) symmetry breaking s-u/d quark mass difference adds to the splitting.



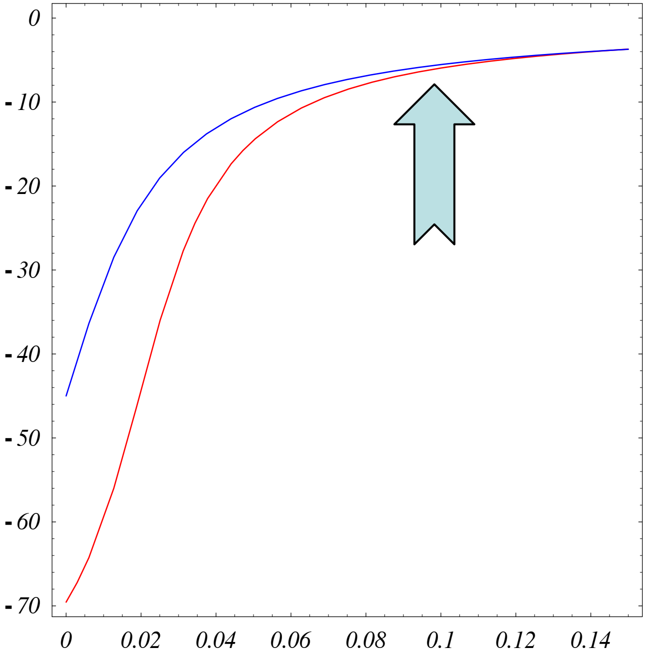
3-15 mixing

- Only symmetric 3-plet mixes with 15-plet
- Mixing separates two states
- Low mass asymm. 3-plet



Mixing angles

- The symmetric 3-plet mixes with the 15-plet
- Two mixing angles: strange D_s^+ and nonstrange D mesons.
- Both mixing angles small: - 5 to -10 degrees.
- $D_s^+(2632)$ partial decay widths indicate 15-plet.



$D^+(2308)$ - $D_s^+(2317)$ Mass Puzzle

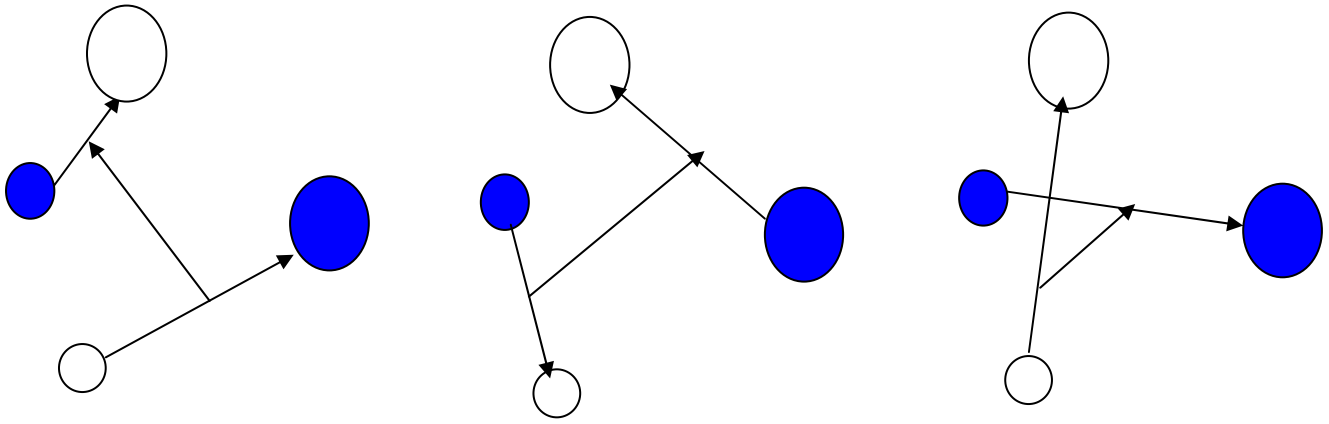
- Two SU(3) multiplets have unusual mass patterns due to their permutation symmetries:
- All members of the antisymmetric 3-plet and 6-plet are degenerate irrespective of their strangeness! Only in tetraquarks.
- Explains degeneracy of $D^+(2308)$ and $D_s^+(2317)$.

Summary of tetraquarks

- Strange resonances: $D_s^+(2317)$ (Belle) and $D_s^+(2632)$ (SELEX); and nonstrange $D^+(2308)$ may be tetraquarks lowered in mass due to KKMT interaction.
- Small strange-nonstrange meson mass difference $D_s^+(2317) - D^+(2308)$ is “smoking gun” evidence for tetraquarks?
- $D_s^+(2632)$ partial decay widths indicate 15-plet.
- Model predicts many exotic tetraquarks. Experiment?

Appendix: Variational calculation (D. Janc)

- Gaussian spatial configurations:



$$R_1 \propto exp\left(-a_1 x_1{}^2 - a_2 x_2{}^2 - a_3 x_3{}^2\right)$$

Courtesy of D. Janc

- The two coupled channel Schroedinger equations are solved variationally:
- The Ansatz consists of 3 parts:
- Orbital part:

$$R_1 \propto exp\left(-a_1 x_1{}^2 - a_2 x_2{}^2 - a_3 x_3{}^2\right)$$

- Colour part:

  or

$$C_i = \begin{matrix} \left|1_{12}1_{34}\right\rangle, \left|8_{12}8_{34}\right\rangle \\ \left|\overline{3}_{13}3_{24}\right\rangle, \left|6_{13}\overline{6}_{24}\right\rangle \end{matrix}$$

- Spin part :

$$\left|1_{12}0_{34}\right\rangle, \left|0_{12}1_{34}\right\rangle, \left|1_{12}1_{34}\right\rangle_1$$

- → spin 0 tetraq. ... 24 config. Ri(s1,s2,s3)CjSj
- spin 1 tetraq. ... 36 config.
- optimization of parameters

  si, s = a, b, c
- select best configuration
- build up basis (~50 config.)

Results of two-body potential variational calculation

- Only heavy-light systems may bind. Light flavour tetraquarks are resonances e.g..

$$a_0(980),\ f_0(980)$$

- Example: Double-heavy (open bottom) double-light tetraquark
- Tbb (S = 1, I = 0, P = +)
- (B.Silvestre-Brac, C.Semay (1993); D.M.Brink, Fl.Stancu (1998); D.Janc (2003))

$$bb\bar{u}\bar{d}\ldots\ldots b\bar{d} + b\bar{u}$$

- Threshold:
- M(Tbb) = 10523MeV
- M (B + B*) = 10651MeV →

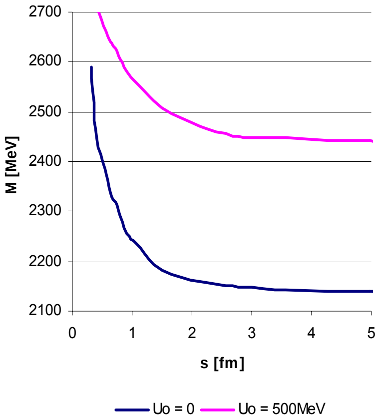
  binding energy: -128 MeV
- Expansion in basis:

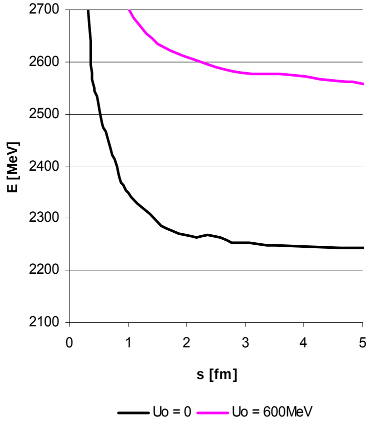
  Ri(s1,s2; s3 = fixed)*CjSj*

Role of tHooft in charm-strange channel

- S=0: thresholds:
  D + K = 1869 + 494

  = 1865 + 498 MeV
- = 2363 MeV (expt.)
- = 2406 MeV (theory)
- Ds + π = 1969 +140 =
  2109 MeV (exp.)
- = 2132 MeV (theory.)





Role of tHooft force in charm-strange channel

- Ds + η = 1969 + 547 = 2416 MeV
- = 2441 MeV (tHooft force Uo =600 MeV)
- S=1: thresholds:
- D* + K = 2540MeV
- D + K* = 2540MeV
- Ds + ρ = 2773MeV
- Ds* + π =2237MeV
- Ds* + η = 2535MeV
- (Uo =600MeV)
- Silvestre-Brac, Semay:
- S=0, M=2357MeV
- S=1, M=2456MeV