Exploring high energy frontier by high quality laser

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- 3rd approach to micro frontier, besides high energy and large scale detector
- High precision test of lepton number nonconservation possible
- If successful, baryon nonconservation, CP violation may also be explored

- **Type of processes considered**

$$Laser + Atom \to Laser' + Atom' + particles$$

Examples;

LENNON to abbreviate LEpton Number NONconservation

$$^{A}_{Z}X \to ^{A}_{Z-2} W' + e^{-} + e^{+} \,,$$

e^+ of a few MeV energy (monochromatic), and e^- almost at rest.

BARNNON (BARyon Number NONconservation)

$$^{A}_{Z}X \to ^{A-1}_{Z-1} W' + \pi + \pi \,, \qquad ^{A-2}_{Z-2}W' + e^{-} + N + \pi \; etc.$$

π of energy $O[m_N/2]$, back to back, and e^- almost at rest.

Crucial to treat **laser** γ **non-perturbatively**

From decay(passive) to reaction(active)

- High intensity flux

- Reaction time

- **Why laser ?**

  Large intensity and large reaction time

$$\text{flux} = 1.2 \times 10^{21} cm^{-2} s^{-1} \left(\frac{\omega}{eV}\right)^{-1} \left(\frac{P}{W\,mm^{-2}}\right)$$

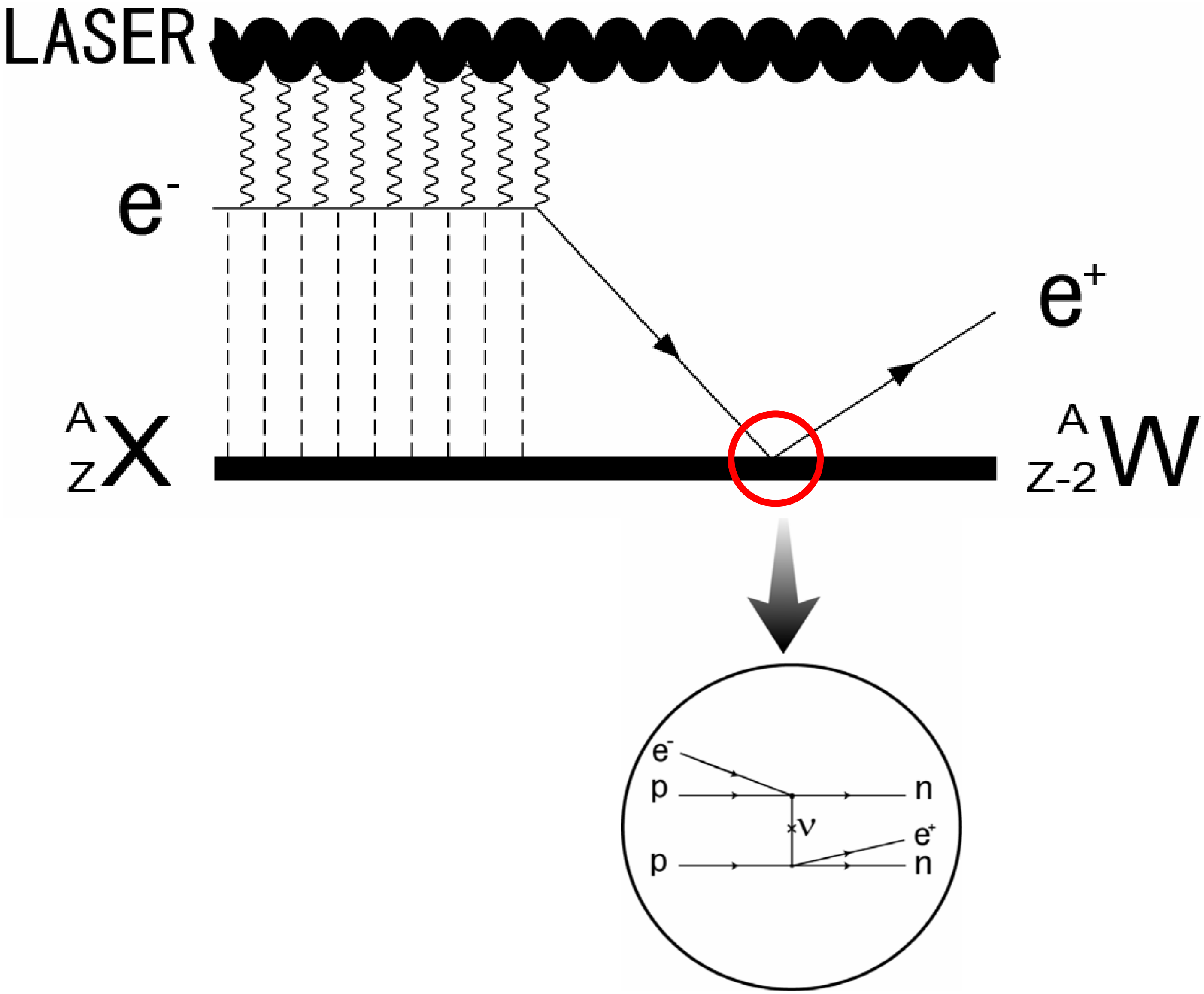
$$\Delta t = \frac{\hbar}{\Delta E} \approx \frac{1}{1\,MHz} \sim 10^{-6} sec$$

Main conclusion of this work

Rare atomic processes are enhanced by

$$Q \approx 1.6 \times 10^6 \, r \, (\frac{P}{W\, mm^2})(\frac{\omega_0}{eV})^{-4}(10^{-9}\frac{\omega_0}{\Delta E}) \tag{4}$$

with P the laser power, ω_0 the photon energy, ΔE the photon energy resolution, and r the wave function ratio squared like $r \approx 1/m^6$ ($m =$ principal quantum number).


LASER
e-
e+
A Z X
A Z-2 W
e-
p
n
ν
e+
p
n

Relevant Hamiltonian

QCD solved, Coulomb gauge QED, and lowest weak interaction, with Majorana neutrino masses

$$H = H_0 + H_{int} \,, \tag{4}$$

$$H_0 = \sum_{nucleus} E_n |n\rangle\langle n| + H'_{QCD} + \int d^3x [\, \psi_e(\vec{x})^\dagger (m_e - \frac{\vec{\nabla}^2}{2m_e} - \frac{Z\alpha}{r}) \psi_e(\vec{x}) + \overline{\psi_e^c}(x)(-i\partial\cdot\gamma + m_e)\psi_e^c(x) - \overline{\nu_j} i\partial\cdot\gamma\nu_j + m_{ij}\overline{\nu_i^c}\nu_j + \frac{1}{2}(\vec{E}^2 - \vec{B}^2)\,] \,, \tag{5}$$

$$H_{int} = \int d^3x [\, \psi_e(\vec{x})^\dagger (\frac{ie}{2m_e}(\vec{\nabla}\cdot\vec{A}(\vec{x}) + \vec{A}(\vec{x})\cdot\vec{\nabla}) + \frac{e^2}{2m_e}\vec{A}(\vec{x})^2)\psi_e(\vec{x}) + \frac{e}{m_e}\psi_e(\vec{x})^\dagger\vec{S}\psi_e(\vec{x})\cdot\vec{\nabla}\times\vec{A} + \sum_{nucleus}\frac{e(g_n - 2)}{2m_N}|n\rangle\langle n|\vec{S}|n\rangle\langle n|\cdot\vec{\nabla}\times\vec{A} + \mathcal{H}_{LENNON} + \mathcal{H}_{BARNNON}\,] \tag{6}$$

2 component notation used.

$$\mathcal{H}_{LENNON} = \frac{G_F}{\sqrt{2}} J_\mu^\dagger(\vec{x}) \cdot J^\mu(\vec{x}) \,, \tag{7}$$

$$\mathcal{H}_{BARNNON} = \frac{G_X}{\sqrt{2}} q(\vec{x})q(\vec{x})q(\vec{x})l(\vec{x}) \,. \tag{8}$$

$J_\mu(x)$ = weak charged current. Helicity structure not speified.

$B - L$ conservation assumed for BARNNON

Mixed Feynman rule

Wick theorem giving **propators**, **vertex**, and **initial and final wave functions**, as usual.

Mixture of **non-relativistic** and relativistic forms most convenient.

Non-trivial treatment of **laser-atom interaction** prior to photo-absorbed rare process.

Electron propagator in atom (non-trivial)

Nonrelativistic form with finite lifetime of atomic levels according to Breit-Wigner,

$$\sum_m \frac{-i}{E - E_m + i\gamma_m/2} |m\rangle\langle m| \,, \tag{9}$$

with γ_m decay rate of atomic level (hole + excited 1 electron states).

m_e is subtracted from E.

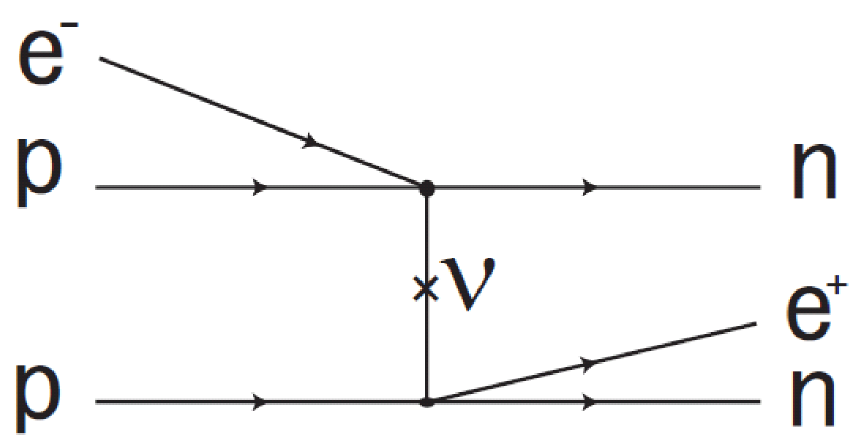
Neutrino propagator

For electron capture, 4 momentum transfer between nucleons having $|q_0| \ll |\vec{q}|$, hence $q^2 \approx -\vec{q}^2$

$$\langle 0|\overline{\psi^c}(x)\psi(y)|0\rangle = \int \frac{d^4(x-y)}{(2\pi)^4} e^{iq\cdot(x-y)} \frac{m_\nu}{q^2 - m_\nu^2 + i\epsilon} \approx -\frac{m_\nu \delta(x_0 - y_0)}{4\pi|\vec{x}-\vec{y}|} \tag{12}$$

giving the instantaneous Coulomb-like potential,

$$\frac{-\delta(x_0 - y_0)}{4\pi} \langle n_f| \frac{J(x)\cdot J(y)}{|\vec{x}-\vec{y}|} |n_i\rangle_N \tag{13}$$



Note that electron field operator e in the weak current $J(\vec{x})$ contains both **bound and free** state contributions, since H_0 includes the Coulomb potential from nucleus. Thus,

$$e(\vec{x})\,|n\rangle = \psi_n(\vec{x})\,|n\rangle \tag{17}$$

with $\psi_n(\vec{x})$ the atomic wave function

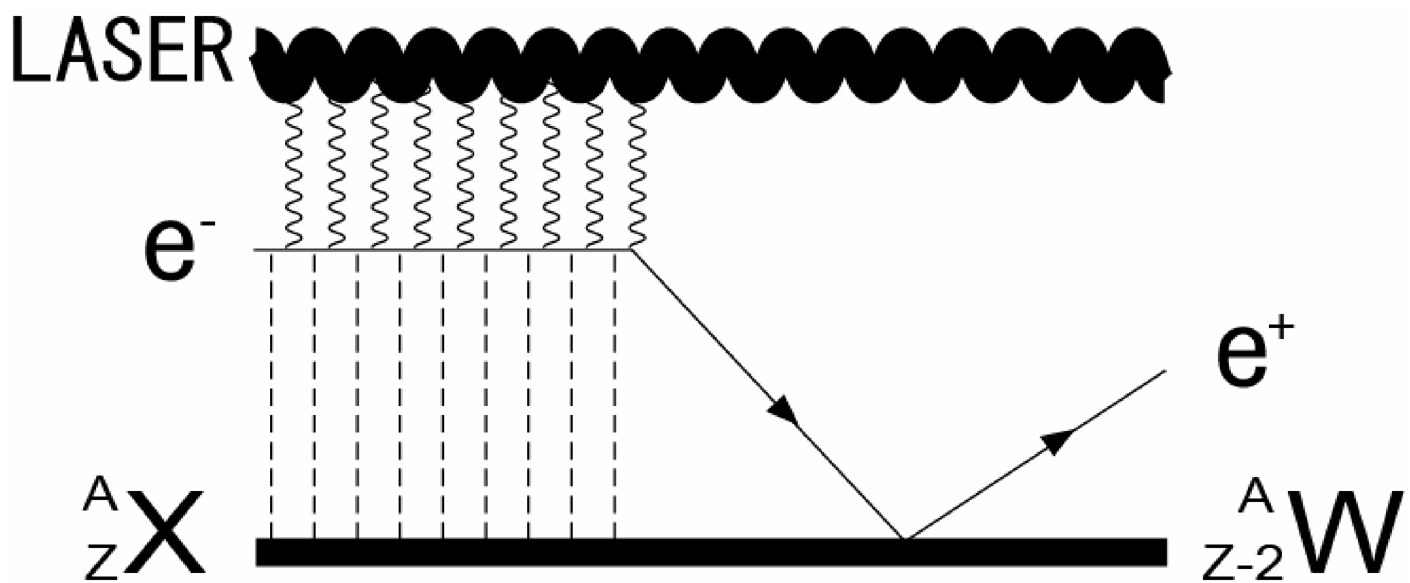
Initial and final states

For initial, with $\sum_n |c_n^{(i)}|^2 = 1$

$$|n_i\rangle_A \times |i\rangle_N \times |I(\vec{k}\,,\epsilon_k)\rangle_\gamma = |n_i\rangle_A \times |i\rangle_N \times \sum_n c_n^{(i)} |n(\vec{k}\,,\epsilon_k)\rangle_\gamma \quad (18)$$

For final, any photon state that may occur,

$$|n_f\rangle_A \times |f\rangle_N \times |X(\vec{k}\,,\epsilon_k)\rangle_\gamma \times |(\vec{q}\,,s_q)\rangle_+ \quad (19)$$



Laser and lasing atom

Photon state in laser beam characterized by **large N, good** $\Delta\omega$**, and coherence**

Atomic state within resonator characterized by **inverted population, Rabi oscillation between 2 levels** of frequency,

$$\Omega \sim \Omega_R = 3 \times 10^{10} sec^{-1}\,(P/W\,mm^2)^{1/2}(\omega/eV)^{-3/2}(\gamma/10^9 s^{-1})^{1/2} (23)$$

We wish to realize a **target state in the sort of atomic state as in the lasing medium** by **frequency tuning**.

Simplest realization is a **united beam-target system**. But with **frequency tunable lasers**, this is not necessary.

Truncation to 2 level laser

$$H_{atom+radiation} = \frac{\omega_{eg}}{2}\sigma_3 + \omega a^\dagger a + \tilde{s}(\sigma_+ a + \sigma_- a^\dagger)$$

σ_i acting on 2 levels of ground $|g\rangle$ and excited $|e\rangle$ state

Coupling strength given by

$$\langle e|\tilde{s}\sigma_+|g\rangle \equiv s = -\langle e|\vec{d}|g\rangle \cdot \vec{e}\,, \quad \vec{e} = i\frac{\omega}{\sqrt{2\omega V}}\vec{\epsilon}_k$$

Exactly solvable in each $2 \times \infty$ sector independently.

Assumption; **Target atom in stationary laser irradiation**. Essentially the same as in lasing medium.

Preliminary: electron displacement

Define electron displacement squared

$$\delta\mathcal{D}^2(t) \equiv d^2 \sum_n \langle \frac{1}{2}, n; t | \sigma_- \sigma_+ a^+ a | \frac{1}{2}, n; t \rangle$$
$$= \frac{d^2}{2} \sum_n \langle \frac{1}{2}, n; t | (1 - \sigma_3) a^+ a | \frac{1}{2}, n; t \rangle \quad (58)$$

Calculation similar to the following leads to

$$\delta\mathcal{D}^2(t) \approx \frac{d^2}{2} N [1 - \frac{1}{\sqrt{2\pi N}} \exp(-2N \sin^2 \frac{st}{2\sqrt{N}}) \sin(N \frac{st}{\sqrt{N}} - \sqrt{N}|s|t)] \quad (59)$$
$$\approx \frac{d^2}{2} N [1 - O[\frac{|s|^3}{\sqrt{2\pi N}}]] \quad (60)$$

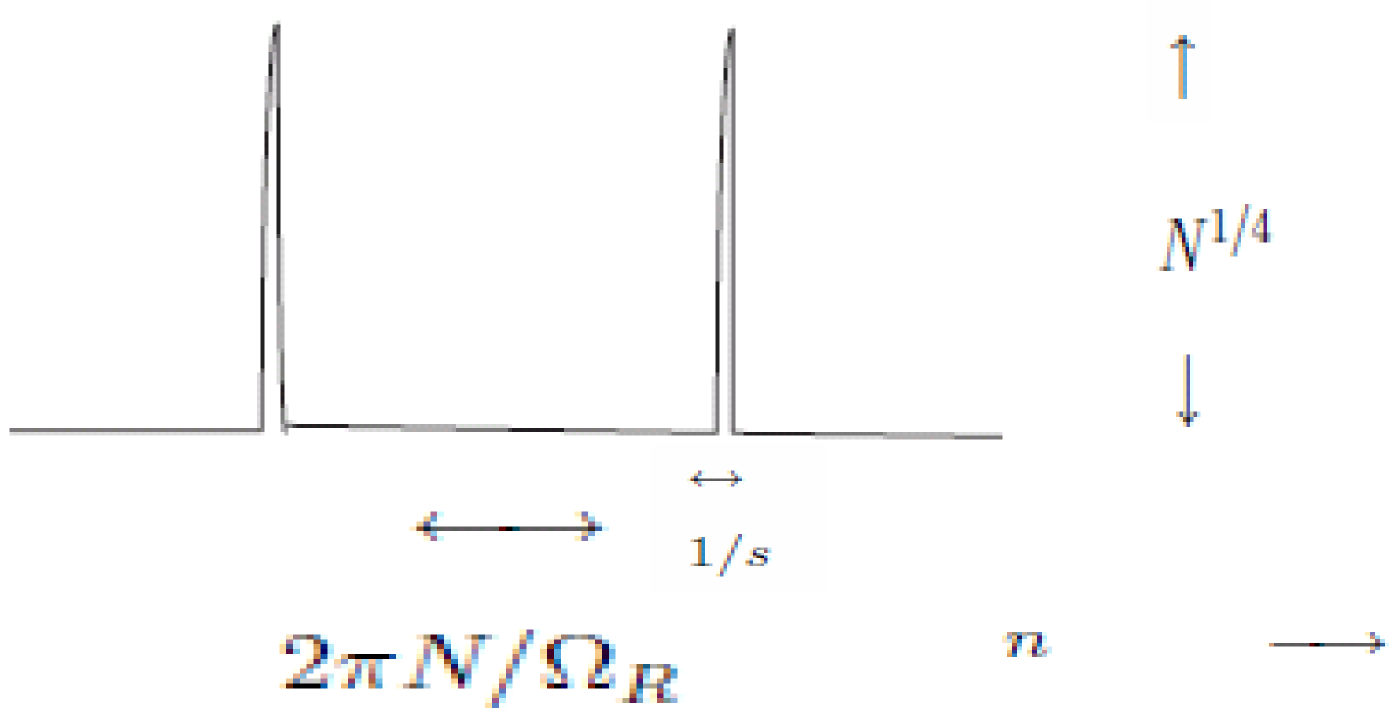
giving "enhancement" $\propto N$ in accord to the classical Lorentz model of oscillator collection of dipoles. Essentially due to $a^+ a \approx N$

More directly on the **displacement under laser irradiation**,

$$\mathcal{D}(t) = d\,\langle\frac{1}{2}, n\,; t|\sigma_+ a|\frac{1}{2}, n\,; t\rangle \tag{61}$$

$$\approx (\frac{N}{2\pi})^{1/4} \exp[-N \sin^2 \frac{st}{\sqrt{N}} + \sqrt{N} st \sin \frac{2st}{\sqrt{N}}] \cos[\frac{N}{2} \sin \frac{2st}{\sqrt{N}} + \sqrt{N} st (2 - \cos \frac{2st}{\sqrt{N}})] \tag{62}$$

Order $N^{1/4}$ **contributions, periodically at** $t = 2\pi N/\Omega_R$.



Back to 2 level problem

Seek exact solution using many-photon states, $|e\,,n\rangle = (a^\dagger)^n/\sqrt{n!}|e\rangle$ and $|g\,,n\rangle = (a^\dagger)^n/\sqrt{n!}|g\rangle$ in the Schrodinger picture,

$$|\psi(t)\rangle = \sum_n (c_{g\,n+1}(t)|g\,,n+1\rangle + c_{e\,n}(t)|e\,,n\rangle)\,, \qquad (28)$$

$$\sum_n (|c_{g\,n+1}(t|^2 + |c_{e\,n}(t)|^2) = 1 \qquad (29)$$

Schroedinger equation to be solved,

$$i\frac{d}{dt}\begin{pmatrix} c_{e\,n}(t) \\ c_{g\,n+1}(t) \end{pmatrix} = \mathcal{H}\begin{pmatrix} c_{e\,n}(t) \\ c_{g\,n+1}(t) \end{pmatrix}\,,$$

$$\mathcal{H} = \begin{pmatrix} n\omega + \frac{\omega_{eg}}{2} & s\sqrt{n+1} \\ s\sqrt{n+1} & (n+1)\omega - \frac{\omega_{eg}}{2} \end{pmatrix}\,,$$

Block-diagonal Hamiltonian

$$
\begin{pmatrix}
-\omega_{eg}/2 & & & & \\
 & \omega_{eg}/2 & S & & \\
 & S & -\omega_{eg}/2+\omega & & \\
 & & & \omega_{eg}/2+\omega & \sqrt{2}S \\
 & & & \sqrt{2}S & -\omega_{eg}/2+2\omega \\
 & & & & \\
 & & & &
\end{pmatrix}
$$

Dressed states

Assuming temporarily constant laser field (valid in the long wave approximation), Hamiltonian diagonalization is possible;

$$|+\,,n\rangle = \cos\frac{\varphi_n}{2}|e\,,n\rangle + \sin\frac{\varphi_n}{2}|g\,,n+1\rangle \tag{31}$$

$$|-\,,n\rangle = -\sin\frac{\varphi_n}{2}|e\,,n\rangle + \cos\frac{\varphi_n}{2}|g\,,n+1\rangle \tag{32}$$

with solutions,

$$\tan\varphi_n = \frac{2s\sqrt{n+1}}{\omega-\omega_{eg}}, \tag{33}$$

$$\omega_{\pm} = (n+\frac{1}{2})\omega \pm \frac{\sqrt{(\omega-\omega_{eg})^2+4s^2(n+1)}}{2}. \tag{34}$$

Time evolution

Taking as the **initial condition** atomic electron in the excited state of ms state (like the pumped laser state),

$$c_{e\,n}(0) = c_n^{(\gamma)}(0)\,, \qquad c_{g\,n+1}(0) = 0\,, \tag{37}$$

$$\sum_n |c_n^{(\gamma)}(0)|^2 = 1 \tag{38}$$

time evolution for the **appeararance** of ground level is given by

$$c_{g\,n+1}(t) = \langle g\,, n+1 | e\,, n\,; t\rangle = \frac{1}{2}\sin\varphi_n (e^{-i\omega_+ t} - e^{-i\omega_- t}) c_n^{(\gamma)}(0)$$

$$= -i \frac{2s\sqrt{n+1}c_n^{(\gamma)}(0)}{\sqrt{(\omega-\omega_{eg})^2 + 4s^2(n+1)}} e^{-i(n+\frac{1}{2})\omega t} \sin \frac{t\sqrt{(\omega-\omega_{eg})^2 + 4s^2(n+1)}}{2}$$

Rabi oscillation

$$\sin^2 \frac{t\sqrt{(\omega-\omega_{eg})^2+4s^2N}}{2}$$

Frequency at resonance, $\omega = \omega_{eg}$

$$\Omega_R = 2|s|\sqrt{N} = \sqrt{\frac{N\pi\gamma_d}{2\omega_0^2 V}} = \sqrt{\frac{\pi\gamma_d P}{\omega_0^3}}$$

in terms of the laser power $P = \omega N/(2V)$. Numerically,

$$\Omega_R \approx 2.8\times 10^{10} sec^{-1}\,(P/W\,mm^2)^{1/2}(\omega/eV)^{-3/2}(\gamma/10^9 s^{-1})^{1/2}\,,$$

Often $\Omega_R \gg \gamma$ a natural width.

Equilibrium state of two levels,

$$|\frac{1}{2}\rangle \equiv \frac{1}{\sqrt{2}}(e^{i\delta}|g\rangle + |e\rangle) \tag{45}$$

is perhaps most appropriate after constant laser irradiation. Take $\delta = \pi/2$ in the following for convenience.

Effect of natural width

Simply multiply

$$e^{-(\gamma_e+\gamma_g)t/2} \tag{46}$$

to the rate above. The enhancement thus lasts until the lifetime $2/(\gamma_e + \gamma_g)$.

2 cases of photon states

Case 1: n photon state applicable to randomly emitted photon source.

$$c_n^{(\gamma)}(0) = \delta_{n,N} \tag{47}$$

Case 2: coherent state

$$|\alpha\rangle = \exp(\alpha a^\dagger - \alpha^* a)|0\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{49}$$

$$\langle n \rangle = |\alpha|^2, \quad \langle (\Delta n)^2 \rangle = |\alpha|^2 \tag{50}$$

$$\rho_E \propto \omega |\alpha + \alpha^*|^2 \propto \langle n \rangle \tag{51}$$

Take α real and positive in the following such that

$$c_n^{(\gamma)}(0) = \frac{\langle n \rangle^{n/2} e^{-\langle n \rangle/2}}{\sqrt{n!}} \tag{52}$$

For large n,

$$c_n^{(\gamma)}(0) \approx \frac{e^{-\langle n \rangle/2}}{(2\pi)^{1/4}} \langle n \rangle^{n/2} e^{(n+1)/2} (n+1)^{-(n+\frac{1}{2})/2} \tag{53}$$

Convoluting photo-absorbed rare process

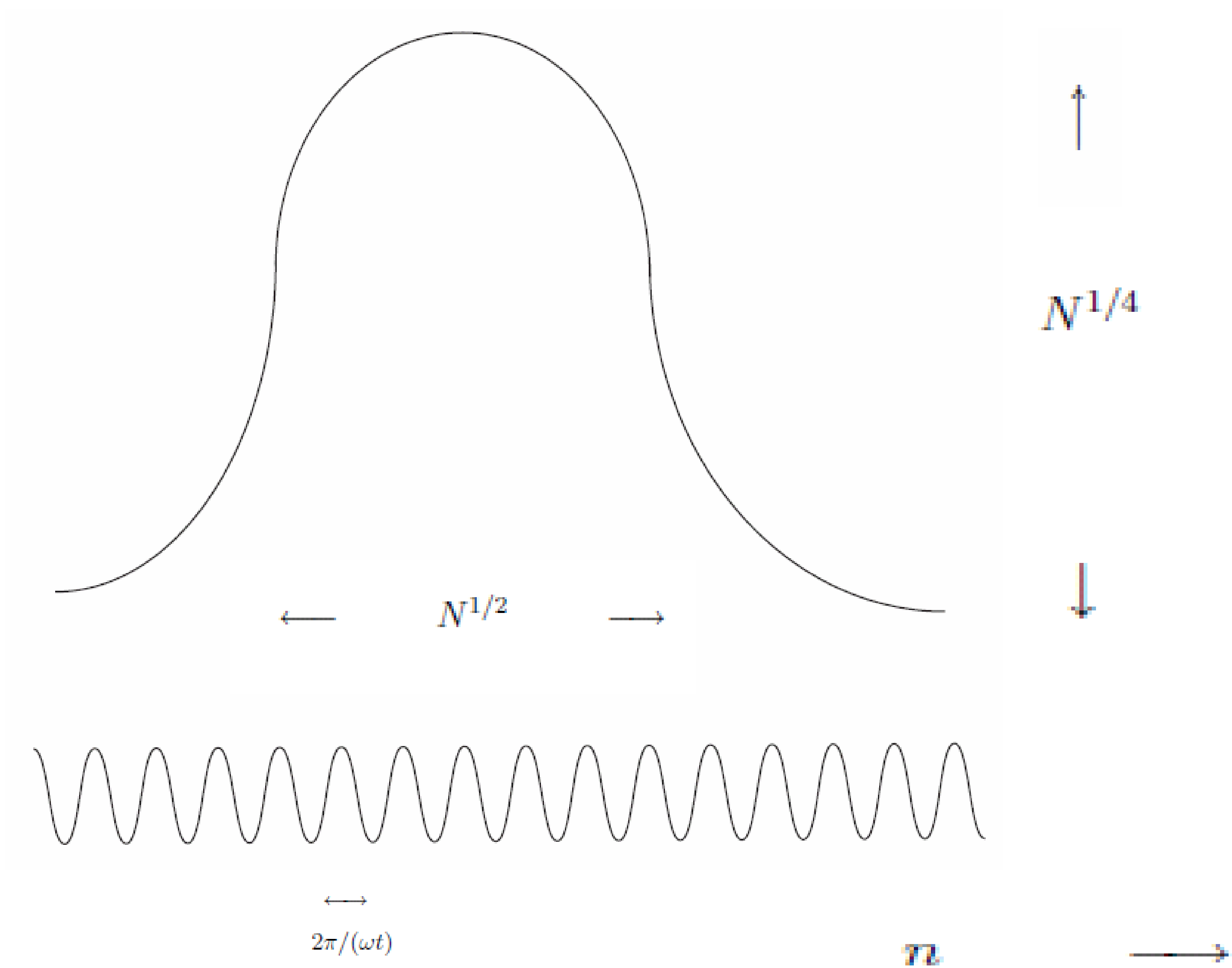
$$\tilde{\mathcal{M}}_{1/2}(t) \approx i \int_0^\infty dn\, c^{(\gamma)}_{n+1}(0)\, e^{-i(n+1/2)\omega t - (\gamma_e + \gamma_g)t/4}$$

$$\times \cos\left(\frac{\Omega_n t}{2} + \frac{\pi}{4}\right) \frac{s\sqrt{n+1}}{\omega - \omega_{eg} + i\gamma/2} \mathcal{M}_X(t)\,,$$

$$\Omega_n = \sqrt{(\omega - \omega_{eg})^2 + 4s^2(n+1)} \approx 2s\sqrt{n+1}$$

noting the mixing factor at resonance,

$$\sin\varphi_{n+1} = \frac{2s\sqrt{n+1}}{\sqrt{(\omega - \omega_{eg})^2 + 4s^2(n+1)}} \approx 1$$



$N^{1/4}$
$N^{1/2}$
$2\pi/(\omega t)$
n

Gaussian approximation around stationary point

For $t \leq 2/(\gamma_e + \gamma_g)$ and $\sin\varphi_n \approx 1$ near the resonance,

$$\frac{\partial}{\partial n}[-i(n+\frac{1}{2})\omega t \pm \frac{it}{2}\sqrt{(\omega-\omega_{eg})^2+4s^2(n+1)}$$
$$-\frac{1}{2}(n+\frac{3}{2})\ln(n+2)+\frac{1}{2}(n+2)+\frac{n+1}{2}\ln\langle n\rangle-\frac{1}{4}\ln(2\pi)+\frac{1}{2}\ln(n+1)]_{n_0}=0$$
$$n_0 \approx \langle n\rangle e^{-2i\omega t} \tag{66}$$

$$\frac{\partial^2}{\partial n^2}[-i(n+\frac{1}{2})\omega t \pm \frac{it}{2}\sqrt{(\omega-\omega_{eg})^2+4s^2(n+1)}$$
$$-\frac{1}{2}(n+\frac{3}{2})\ln(n+2)+\frac{1}{2}(n+2)+\frac{n+1}{2}\ln\langle n\rangle-\frac{1}{4}\ln(2\pi)+\frac{1}{2}\ln(n+1)]_{n_0}$$
$$\approx -\frac{1}{2\langle n\rangle}e^{2i\omega t} \tag{67}$$

With $N = \langle n \rangle$, neglecting small $O[st]$ terms,

$$\tilde{\mathcal{M}}_{1/2}(t) \approx i(\frac{\pi N}{2})^{1/4} \frac{s\sqrt{N}\,\mathcal{M}_X(t)}{\omega - \omega_{eg} + i\gamma/2} \exp[-(\gamma_e + \gamma_g)t/4 - N\sin^2(\omega t) - \frac{i}{2} N \sin(2\omega t)]$$
$$\times [\cos(\sqrt{N} st \cos\omega t)\cosh(\sqrt{N} st \sin\omega t) + i\sin(\sqrt{N} st \cos\omega t)\sinh(\sqrt{N} st \sin\omega t)]\,, \tag{68}$$

$$|\tilde{\mathcal{M}}_{1/2}(t)|^2 \approx (\frac{\pi}{2})^{1/2} N^{3/2} \frac{s^2\,|\mathcal{M}_X(t)|^2}{(\omega - \omega_{eg})^2 + \gamma^2/4} \exp[-N(1 - \cos 2\omega t)]$$
$$\times [\sinh^2(\sqrt{N} st \sin\omega t) + \cos^2(\sqrt{N} st \cos\omega t)]\,, \tag{69}$$

Small and intermediate time behavior

Small time behavior of $\omega t \ll 1$

Expanding in terms of ωt and using (57),

$$|\tilde{\mathcal{M}}_{1/2}(t)|^2 \approx \frac{s^2\sqrt{\frac{\pi}{2}}N^{3/2}}{(\omega-\omega_{eg})^2+\gamma^2/4}\,|\mathcal{M}_X(t)|^2 = |\tilde{\mathcal{M}}_{p\to s}(t)|^2\,, \quad (73)$$

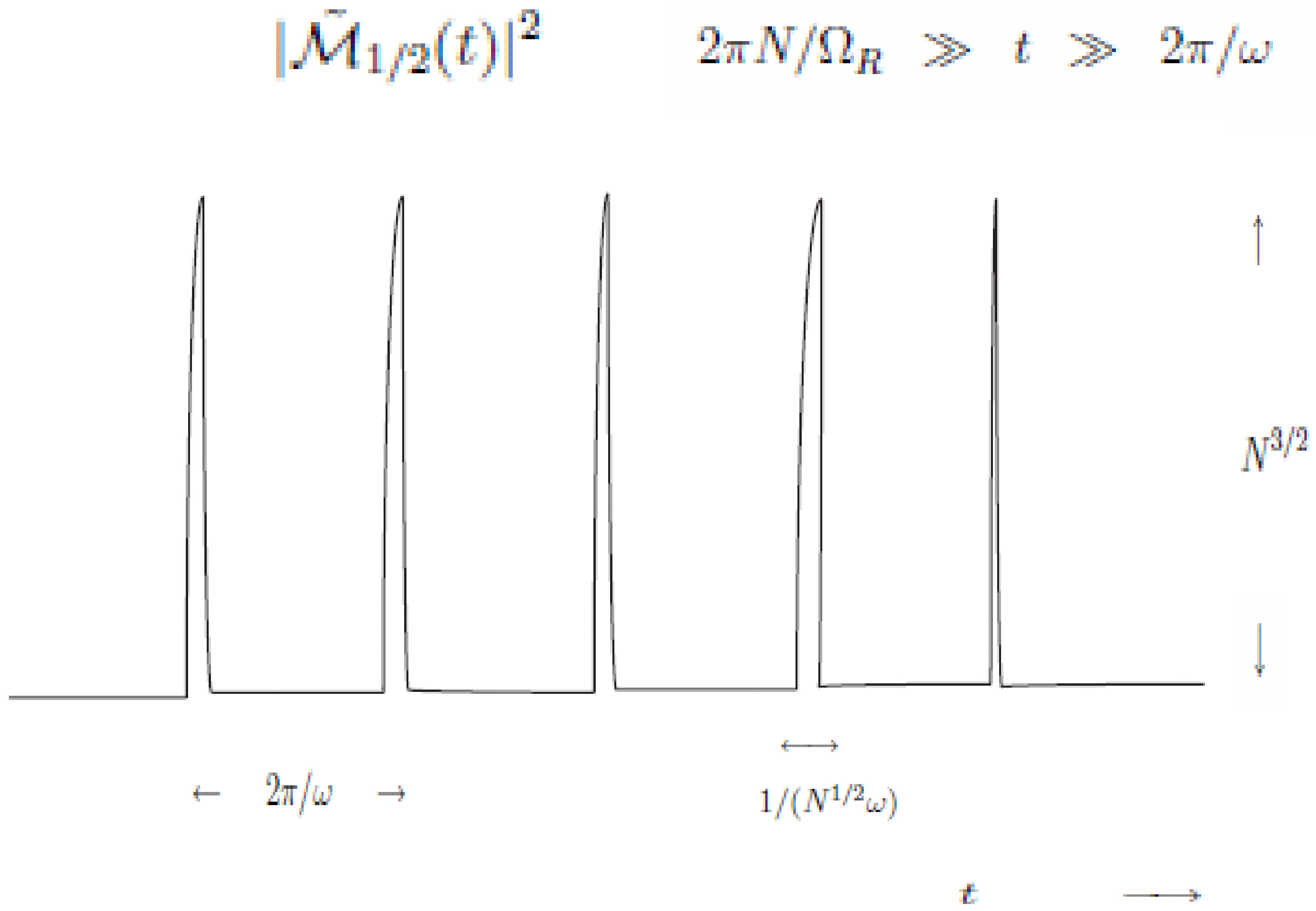
Iintermediate time behavior

$$1/\omega \ll t \ll N/\Omega_R \qquad (75)$$

Large contributions, periodically at $\cos 2\omega t = 1$. Using the following approximation,

$$\begin{aligned}
&\exp[-N(1-\cos 2\omega t)]\,[\sinh^2(\sqrt{N}st\sin\omega t)+\cos^2(\sqrt{N}st\cos\omega t)] \\
&\approx \sum_k \exp[-2\omega^2 N(t-\frac{k\pi}{\omega})^2 + 2\sqrt{N}st\omega|t-\frac{k\pi}{\omega}|] \\
&\qquad \approx \sum_k \sqrt{\frac{\pi}{2N}}\frac{1}{\omega}\,\delta(t-\frac{k\pi}{\omega}) \qquad (76)
\end{aligned}$$

with integer $k = 0\,,1\,,2\,,\cdots$.


$|\tilde{M}_{1/2}(t)|^2$
$2\pi N/\Omega_R \gg t \gg 2\pi/\omega$
$N^{3/2}$
$2\pi/\omega$
$1/(N^{1/2}\omega)$
t

Time averaged over $\Delta t \gg \pi/\omega$ for $\omega t \gg 1$,

$$|\tilde{\mathcal{M}}_{1/2}(t)|^2 \approx \frac{N}{2}\,\frac{s^2\,e^{-(\gamma_e+\gamma_g)t/2}}{(\omega-\omega_{eg})^2+\gamma^2/4}\,|\mathcal{M}_X(t)|^2$$

giving a time variant rate,

$$\tilde{\mathcal{R}}_{1/2}(t) = \frac{N}{2}\,\frac{s^2\,e^{-(\gamma_e+\gamma_g)t/2}}{(\omega-\omega_{eg})^2+\gamma^2/4}\,\mathcal{R}_X(t)$$

with

$$\mathcal{R}_X(t) = \frac{d|\mathcal{M}_X(t)|^2}{dt}$$

Time-averaged rate for the entire lifetime of oscillating levels,

$$\frac{\gamma_e+\gamma_g}{2}\int_0^\infty dt\,\tilde{\mathcal{R}}_{1/2}(t) \approx N\frac{s^2\mathcal{F}(\omega)}{2[(\omega-\omega_{eg})^2+\gamma^2/4]}\,\mathcal{R}_X(t)$$

$s = -\langle e|\vec{d}\sigma_+|g\rangle \cdot \vec{e}$ related to the dipole transition rate

$$s^2 = \frac{4\pi\alpha}{3}\frac{\omega_0|\langle|\vec{x}|\rangle|^2}{V} = \frac{\pi\gamma_d}{2\omega_0^2 V},$$

Large time behavior

Valid for $t \gg N/\Omega_R$

More precise formula of the saddle is necessary;

$$n_0 = \langle n \rangle e^{-2i\omega t} \exp[\pm \frac{iste^{i\omega t}}{\sqrt{\langle n \rangle}}] \tag{80}$$

Previous formula replaced like

$$e^{-2N \sin -2\omega t} \;\rightarrow\; \exp[-N\,(\sin \omega t \mp \frac{\Omega_R t}{4N} \cos^2 \omega t)^2] \tag{81}$$

$t_k = \frac{k\pi}{\omega}$ replaced by infinitely many solutions of

$$\sin \omega t_k \mp \frac{\Omega_R t}{4N} \cos^2 \omega t_k = 0 \tag{82}$$

Rate formula eq.(78) modified by a multiplication factor,

$$\times \frac{\pi}{\overline{\delta_k}\, \omega \overline{\Delta t_k}} \tag{83}$$

of order unity. Here $\overline{\Delta t_k}$ is the average interval of solutions to eq.(82) with $\overline{\delta_k}$ its curvature.

Quality factor of laser

Rate enhancement factor $Q(\omega)$

$$dQ(\omega) \equiv \frac{\pi}{2} \frac{\gamma_d \mathcal{P}(\omega) d\omega}{\omega_0^2 \omega[(\omega - \omega_0)^2 + \gamma^2/4]} \, r \,, \tag{86}$$

where

$$\int d\omega \mathcal{P}(\omega) = P \tag{87}$$

is the total power in the unit of energy /(time × area). r is the wave function ratio sqaured, for instance, for LENNON

$$r = \frac{|\psi_{ms}(0)|^2}{|\psi_{ns}(0)|^2} = (\frac{r_{ns}}{r_{ms}})^3 = O[(\frac{n}{m})^6] \,, \qquad n = 1 \,, \tag{88}$$

For laser beam of **energy resolution** $\Delta E \gg \gamma$

$$Q = \int dQ(\omega) \approx r \frac{\pi^2 P}{\omega_0^4} \frac{\gamma_d}{\gamma} \frac{\omega_0}{\Delta E}$$

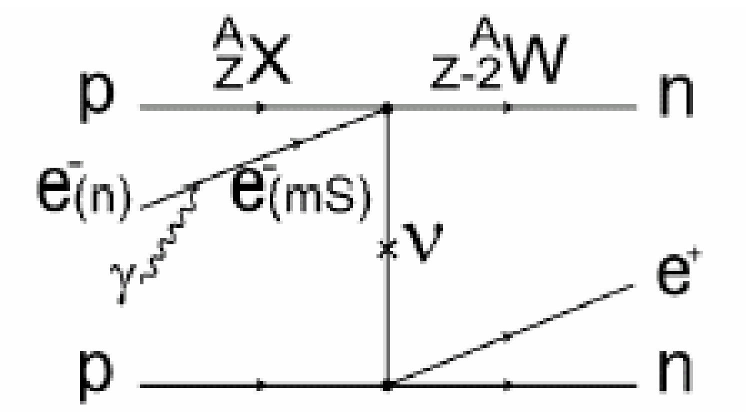
$$\approx 1.6 \times 10^6 r \frac{P}{W\, mm^2} (\frac{\omega_0}{eV})^{-4} (10^{-9} \frac{\omega_0}{\Delta E})$$

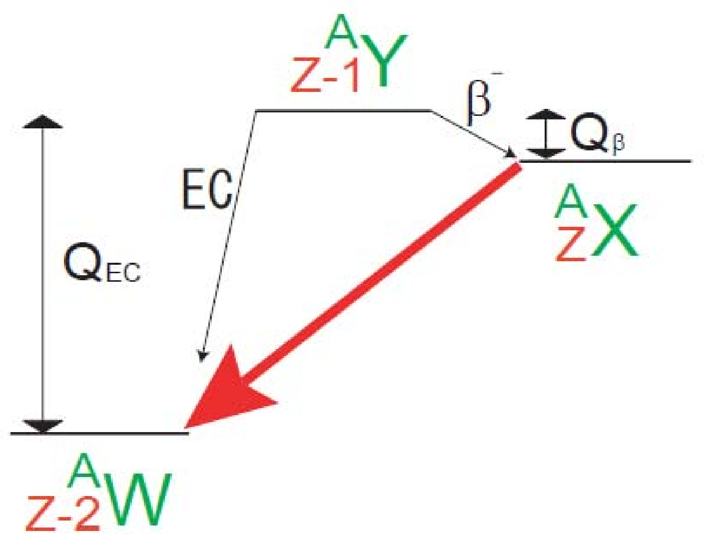
$\frac{\gamma_d}{\gamma} = O[1]$, thus $P/\Delta E$ is the crucial factor of laser quality.

LENNON

- Hep-ph /0506062 v2 "New method of enhancing lepton number nonconservation", Ikeda, Nakano, Sakuda,Tanaka, Yoshimura

$$\gamma + e^{-}(n) + {}^{Z}_{A}X \rightarrow e^{+} + {}^{Z-2}_{A}W$$





Transition amplitude $\mathcal{M}_{e^- \to e^+}$ without laser-atom interaction, calculable by perturbation theory,

$$= \frac{G_F^2}{2} \int d^4x d^4y \langle n_f | J_\alpha(x) \cdot J_\beta(y) \times \langle m\,; (\vec{q}, s_q) | \overline{e'^c}(x) \gamma_\alpha (1 + \gamma_5)$$

$$S_F(x-y) \gamma_\beta (1 - \gamma_5) e(y) | n \rangle \times | n_i \rangle_N \,, \tag{82}$$

$$S_F(x-y) \approx \frac{-m_{ee}}{4\pi} \frac{\delta(x_0 - y_0)}{|\vec{x} - \vec{y}|} \,, \qquad m_{ll'} = \sum_k U_{lk} U_{l'k} m_k \tag{83}$$

Introducing the average inter nucleon distance $\overline{R}$,

$$\approx \frac{G_F^2 \, m_{ee}}{4\pi \overline{R}} \delta(E_i + m_e - E_f - E_q) \, \delta(\vec{q} - \vec{P}_N) \sum_n \langle n_f | J_\alpha | n \rangle \langle n | J_\alpha | n_i \rangle_N$$

$$\langle m\,; (\vec{q}, s_q) | \overline{e'^c} (1 - \gamma_5) e | n \rangle \,, \tag{84}$$

or put it differently,

$$\mathcal{M}_{e^- \to e^+} = \frac{G_F^2 m_{ee} \bar{m}_\pi}{4\pi} \psi_{mS}(0) \,, \tag{85}$$

$$\bar{m}_\pi \equiv \int d^3x d^3y \frac{\langle f | \vec{J}(\vec{x}) \cdot \vec{J}(\vec{y}) | i \rangle}{|\vec{x} - \vec{y}|} \tag{86}$$

This gives a theoretical **rate** of

$$\mathcal{R}_X = \Gamma_{0\nu} = \frac{G_F^4 \tilde{m}_\pi^2 |m_{ee}|^2}{16\pi^3} |\psi_{ms}(0)|^2 p_+ E_+$$

$$= |\psi_{ms}(0)|^2 \times G_F^2 \tilde{m}_\pi^2 \times \frac{|m_{ee}|^2}{16\pi^2 \tilde{m}_\pi} \times \tilde{m}_\pi \times \frac{G_F^2 p_+ E_+}{\pi}$$

Indivisual factors are interpreted as

Initial effective flux of atomic electron $\times$ X-section for e-capture $\times$ Rate of $\nu \to \overline{\nu}$ conversion $\times$ $\frac{\text{flux}}{\text{phase space}}$ $\times$ X-section for $\overline{\nu}$ nucleus scattering

In this picture, $1/\tilde{m}_\pi$ is the target size and $\tilde{m}_\pi$ is also the virtual ν energy.

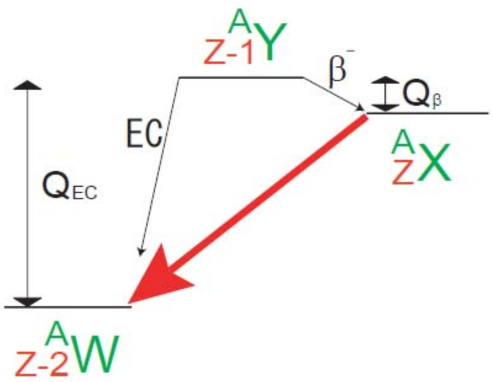
Nuclear radius,

$$\langle \frac{1}{|\vec{x} - \vec{y}|} \rangle = \frac{1}{R_n}\,, \qquad R_n \approx (0.82 A^{1/3} + 0.58) fm \tag{6}$$

following Bernabeu et al

Replacing nuclear matrix elements of intremediate nucleus ${}^{A}_{Z-1}Y$ by experimental data of β_- decay and EC,

$$\Gamma_{0\nu} = \frac{3\pi^3 |m_{ee}|^2}{4R_n^2} \frac{p_+ E_+}{p_\nu^2 \Delta_\beta^5 I} \Gamma_{EC} \Gamma_\beta\,, \tag{9}$$

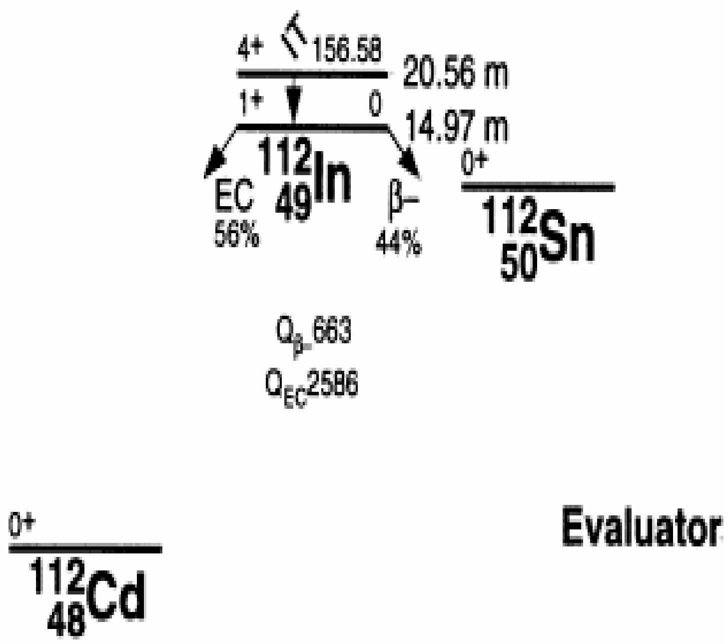


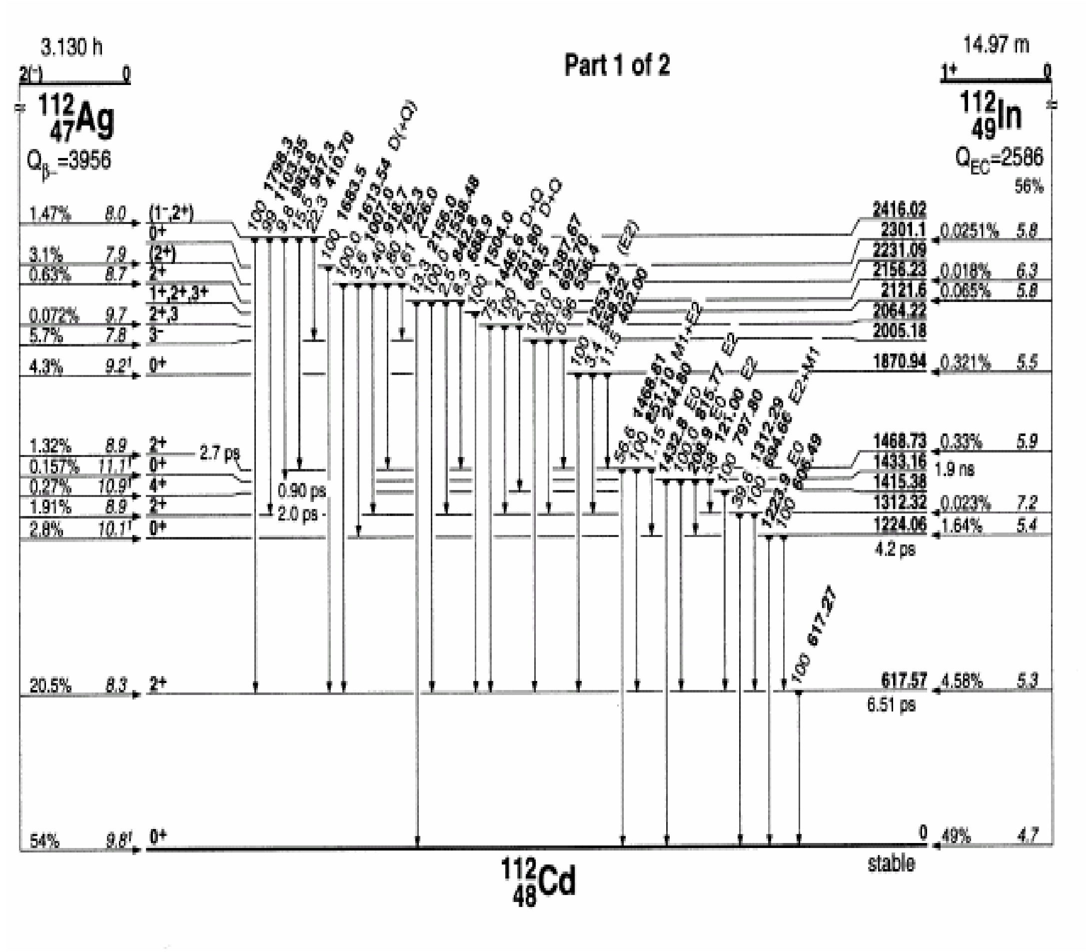
e.g. $\approx (2 \times 10^{29} y)^{-1} (|m_{ee}|/0.1eV)^{-2}$ for ${}^{112}_{50}Sn$

Atom	abnd.%	Q_{e^+} (keV)	$\tau_{0\nu\frac{1}{2}}$ y $\lvert\frac{m_{ee}}{0.1eV}\rvert^{-2}$
$^{78}_{36}Kr$	0.35	1846	$1.5 \times 10^{29*}$
$^{112}_{50}Sn$	0.97	901	5.5×10^{28}
$^{130}_{56}Ba$	0.11	1588	2.3×10^{29}

Table 1: Half-lifetime and useful characteristics for a few isotopes. $*$ For Kr half-life, we follow the calculation in Bernabeu et al. intead of eq.(9).

Example of large matrix element and nuclear gamma





Important physical background

Important physical background of **two accompanying neutrinos** (2ν) caused by the second order weak interaction, which itself is of interest.

Its rate

$$\frac{\tilde{\Gamma}_{0\nu}}{\tilde{\Gamma}_{2\nu}} \approx \frac{20160\pi^2 |m_{ee}|^2 m_\pi^2}{\Delta_\beta^6 \overline{R}^2} \tag{93}$$

$$\approx 0.66 \times 10^{-4} |m_{ee}/1meV|^2 (m_\pi \overline{R})^{-2} (\Delta_\beta / MeV)^{-6} \tag{94}$$

LENNON rate, when the positron energy is limited **near the end point** of e^+ energy width δE,

$$\frac{\tilde{\Gamma}_{0\nu}}{\tilde{\Gamma}_{2\nu}(\delta E)} \approx \frac{45\pi^2 m_\pi^2 |m_{ee}|^2}{(\delta E)^6 \overline{R}^2} \tag{97}$$

$$\approx O[7] |\frac{m_{ee}}{1meV}|^2 (\frac{\delta E}{100keV})^{-6} \tag{98}$$

Positron energy spectrum for two neutrino process

$$d\tilde{\Gamma}_{2\nu} \propto E\sqrt{E^2 - m_e^2}(E - \Delta)^5 dE\,. \tag{99}$$

Induced nucleon decay

$$e^- + N \to \pi + \pi$$

Enhancement factor $\propto N$ may compensate the **small nuclear overlap factor** of order $(a_B m_\pi)^3 \approx 10^{-15}$ of atomic electrons that otherwise disfavors this process.

Rough rate given by the free nucleon decay rate Γ_B times the enhancement factor of

$$Q\,(r_A A^{1/3} m_\pi)^{-3}\,, \tag{108}$$

where the pion mass m_π times $A^{1/3}$ gives a measure of the inverse nuclear size, and r_A is the atomic size.