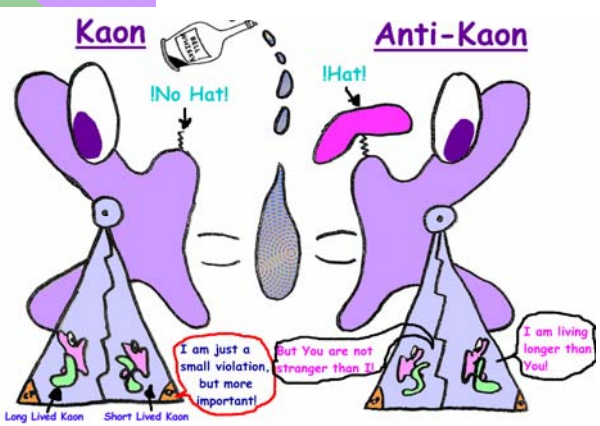
Massive entangled states put to test in theory and experiment



by
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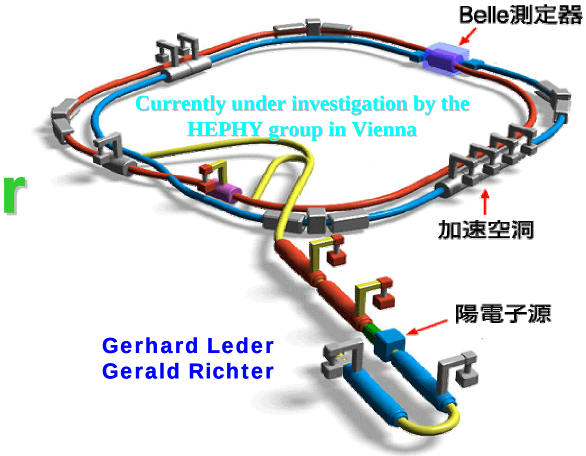


$$|Physics\rangle = \alpha\, |Particle\ Physics\rangle + \beta\, |Quantum\ Theory\rangle$$

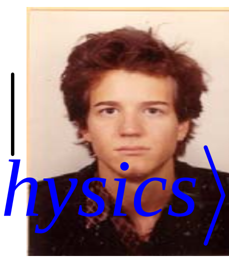
- **Part I:** **Entanglement related to a violation of a symmetry in high energy physics !?** In the morning!
- **Part II:** **How to test entanglement in high energy physics?**

UNIVERSITÄT WIEN

The Viennese Quantum Engineers

Philipp Krammer

Katharina Durstberger

$$|Physics\rangle = 1/N \{ |$$  $$\rangle \otimes |$$  $$\rangle - |$$  $$\rangle \otimes |$$  $$\rangle$$

$|Physics\rangle = \alpha\,|$ I … ePhysics$\rangle + \beta$ … ntur … ory$\rangle$

Reinhold Bertlmann

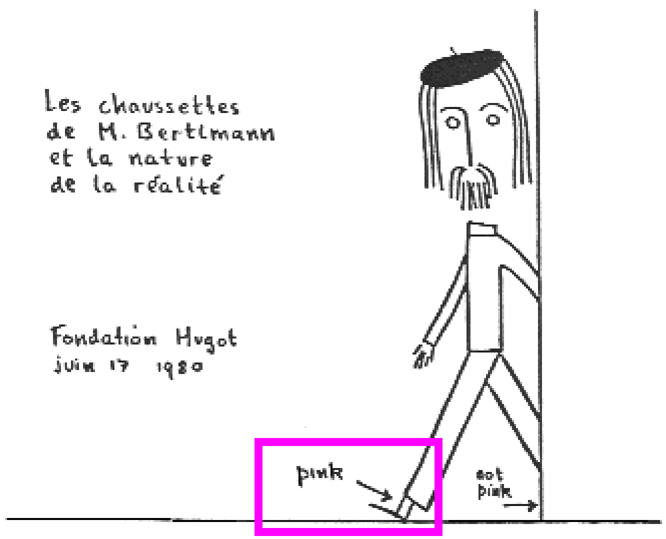
Beatrix Hiesmayr

$$+e^{i\theta} |$$  $$\rangle \otimes |$$  $$\rangle \}$$

Yuji Hasegawa

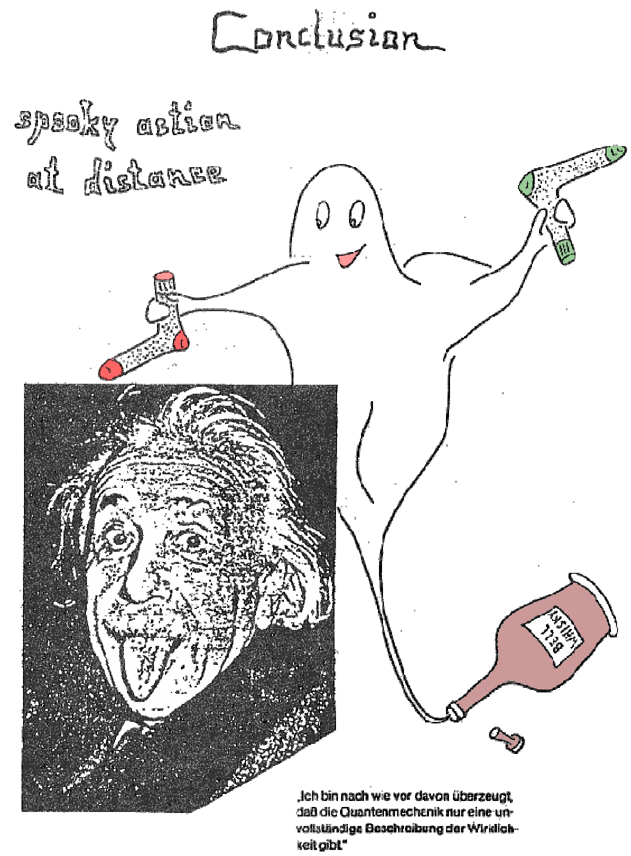
Stefan Filipp

A little history

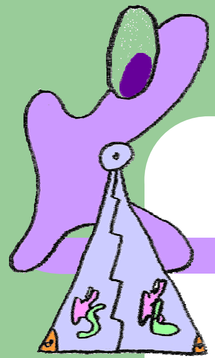


from: J.S. Bell, "Bertlmann's socks and the nature of reality", Journal de Physique, Tome 42, Colloque C-2, supplément au No. 3, mars 1981.

reprinted in: J.S. Bell, "Speakable and unspeakable in quantum mechanics", Cambridge University Press, p. 139, 1987.



drawn by R.A. Bertlmann to the 60th birthday of J.S. Bell



How to measure entanglement?

Einstein / Podolsky / Rosen: An entangled wavefunction does not describe the physical reality in a complete way.
E. Schrödinger: For an entangled state “the best possible knowledge of the whole does not include the best possible knowledge of its parts.”
Entanglement is…
J. Bell: …a correlation that is stronger than any classical correlation.
D. Mermin: …a correlation that contradicts the theory of elements of reality.
A. Peres: “…a trick that quantum magicians use to produce phenomena that cannot be imitated by classical magicians.”
C. Bennett: …a resource that enables quantum teleportation.
P. Shor: …a global structure of the wavefunction that allows for faster algorithms.
A. Ekert: …a tool for secure communication.
Horodecki family: …the need for first applications of positive maps in physics.

decoherence model

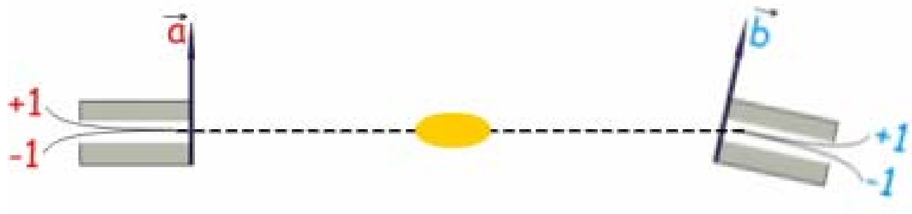
experimental ↔ phenomenological ↔ mathematical

aspects

K-mesons
($\bar{s}$ d)

B-mesons
($\bar{b}$ d)

The EPR scenario



Antisymmetric Bell state:

$$\left|\psi^-\right\rangle = \tfrac{1}{\sqrt{2}}\left\{\left|\Uparrow\right\rangle_l \otimes \left|\Downarrow\right\rangle_r - \left|\Downarrow\right\rangle_l \otimes \left|\Uparrow\right\rangle_r\right\}$$ **... spin 1/2**

$$= \tfrac{1}{\sqrt{2}}\left\{\left|\mathbf{0}\right\rangle_l \otimes \left|\mathbf{1}\right\rangle_r - \left|\mathbf{1}\right\rangle_l \otimes \left|\mathbf{0}\right\rangle_r\right\}$$ **... qubit**

$$= \tfrac{1}{\sqrt{2}}\left\{\left|H\right\rangle_l \otimes \left|V\right\rangle_r - \left|V\right\rangle_l \otimes \left|H\right\rangle_r\right\}$$ **... photon**

$$= \tfrac{1}{\sqrt{2}}\left\{\left|K^0\right\rangle_l \otimes \left|\overline{K}^0\right\rangle_r - \left|\overline{K}^0\right\rangle_l \otimes \left|K^0\right\rangle_r\right\}$$ **... kaon**

$$= \tfrac{1}{\sqrt{2}}\left\{\left|B^0\right\rangle_l \otimes \left|\overline{B}^0\right\rangle_r - \left|\overline{B}^0\right\rangle_l \otimes \left|B^0\right\rangle_r\right\}$$ **... B-meson**

$$= \tfrac{1}{\sqrt{2}}\left\{\left|I\right\rangle_l \otimes \left|\Uparrow\right\rangle_r - \left|II\right\rangle_l \otimes \left|\Downarrow\right\rangle_r\right\}$$ **... single neutron in interferometer**

$$= \cdots$$

Neutral kaons are kind of double slits

Kaon in time:

short-lived state ↓ **long-lived state** ↓

$$\left|K^0(t)\right\rangle \cong \frac{1}{\sqrt{2}}\left\{e^{-\frac{\Gamma_S}{2}t - i m_S t}\left|K_S\right\rangle + e^{-\frac{\Gamma_L}{2}t - i m_L t}\left|K_L\right\rangle\right\}$$

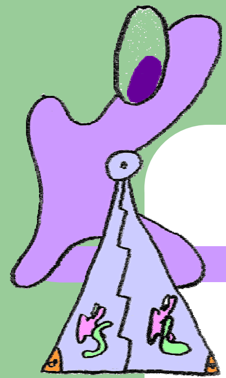
Antikaon in time:

$$\left|\overline{K}^0(t)\right\rangle \cong \frac{1}{\sqrt{2}}\left\{-e^{-\frac{\Gamma_S}{2}t - i m_S t}\left|K_S\right\rangle + e^{-\frac{\Gamma_L}{2}t - i m_L t}\left|K_L\right\rangle\right\}$$

$\Gamma_S \approx 10^{10}\,\frac{1}{s}$ ...decay width of K_S

$\Gamma_L \approx 1/600\,\Gamma_S$ ...decay width of K_L

$\Delta m = m_L - m_S \approx 0.5\,\Gamma_S$ ...mass difference



What are B-mesons?

Beauty: $\left|B^0\right\rangle \left|\overline{B}^0\right\rangle$

„B-mesons are a kind of double slit with equal slit widths"

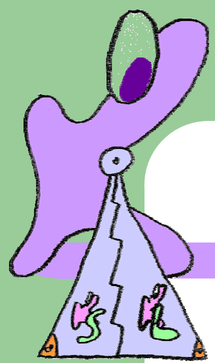
Mass-eigenstates: $\left|B_H\right\rangle, \left|B_L\right\rangle$

$$\left|B^0\right\rangle = \tfrac{1}{\sqrt{2}}\left\{\left|B_H\right\rangle + \left|B_L\right\rangle\right\}$$

$$\Gamma \approx \Gamma_H \approx \Gamma_L \approx 0.65 \cdot 10^{12}\ \tfrac{1}{s}$$

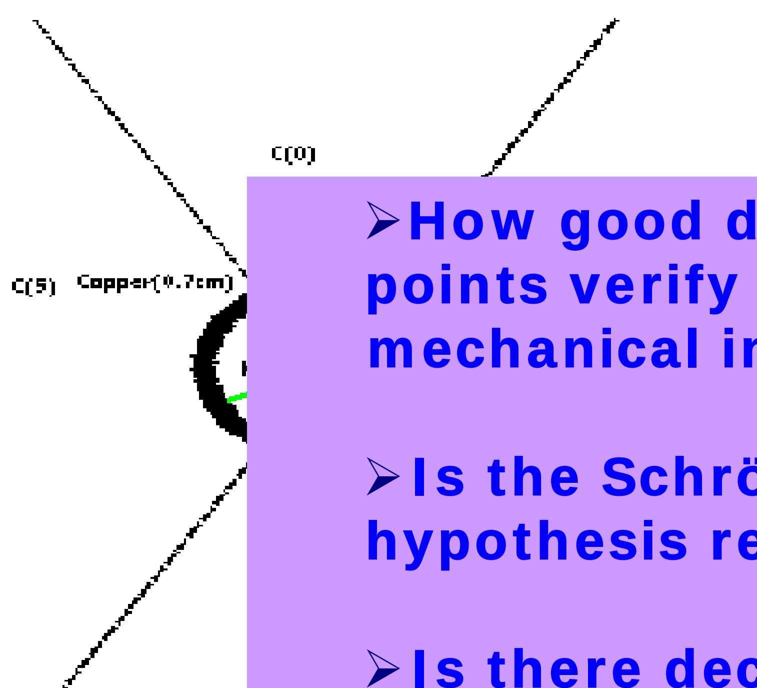
$$\Delta m \approx 0.49 \cdot 10^{12}\ \tfrac{1}{s}$$

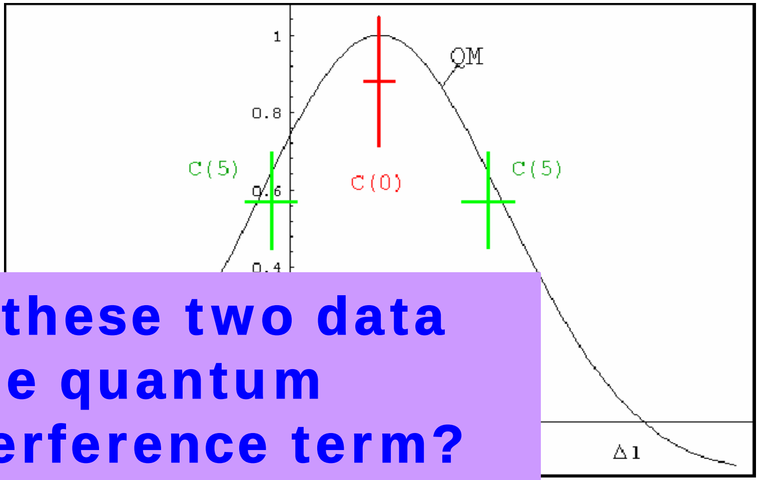
$$\left|B^0(t)\right\rangle = \frac{e^{-\Gamma t}}{\sqrt{2}}\left\{e^{-i m_H t}\left|B_H\right\rangle + e^{-i m_L t}\left|B_L\right\rangle\right\}$$



An experiment for kaons

CPLEAR-Experiment (1998)





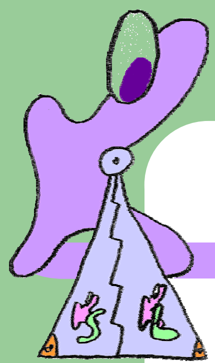
- How good do these two data points verify the quantum mechanical interference term?
- Is the Schrödinger-Furry hypothesis really ruled out?
- Is there decoherence in the system? Loss of entanglement?

$(K^0, t_l; K^0, t_r)$

$K^0 + p \longrightarrow K^+ + n$

$K^0 + p \longrightarrow \Lambda^0 + \pi^+$

$\bar{K}^0 + n \longrightarrow K^- + p$



Spontaneous factorization of the wave function

Schrödinger-Furry Hypothesis: $(\zeta = 1)$

$$|\psi^-\rangle \cong |K_S\rangle_l \otimes |K_L\rangle_r - |K_L\rangle_l \otimes |K_S\rangle_r$$

50% → $|K_S\rangle_l \otimes |K_L\rangle_r$

50% → $|K_L\rangle_l \otimes |K_S\rangle_r$

$$P^{\zeta}(f_1, t_l; f_2, t_r) = |\mathrm{A}_1|^2 + |\mathrm{A}_2|^2 - 2(1-\zeta)\,\mathrm{Re}\{\mathrm{A}_1^* \mathrm{A}_2\}$$

Observable:

$$A^{QM}(t_l, t_r) = \frac{P(K^0, t_l; \overline{K}^0, t_r) - P(K^0, t_l; K^0, t_r)}{\Sigma} = \frac{-\cos(\Delta m \Delta t)}{\cosh(\frac{\Delta\Gamma}{2}\Delta t)}$$

⇒ $$\boldsymbol{A^{\zeta}(\Delta t) = A^{QM}(\Delta t)\cdot(1-\zeta)}$$

CPLEAR-experiment (1998): $\zeta = 0.13^{+0.16}_{-0.15}$

Bertlmann, Grimus and Hiesmayr, Phys. Rev. D, 60, 114032 (1999)

Spontaneous factorization of the wave function

Schrödinger-Furry Hypothesis $(\zeta = 1)$:

$$|\psi^-\rangle \cong |K^0\rangle_l \otimes |\overline{K}^0\rangle_r - |\overline{K}^0\rangle_l \otimes |K^0\rangle_r$$

50% → $|K^0\rangle_l \otimes |\overline{K}^0\rangle_r$

50% → $|\overline{K}^0\rangle_l \otimes |K^0\rangle_r$

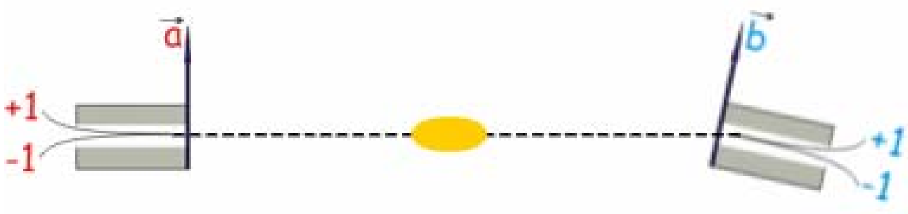
$$P^\zeta(f_1,t_l;f_2,t_r) = |A_1|^2 + |A_2|^2 - 2(1-\zeta)\,\mathrm{Re}\{A_1^* A_2\}$$

$$\Rightarrow\quad A^\zeta_{K^0,\overline{K}^0}(t_l,t_r) = \frac{\cos(\Delta m \Delta t) - \frac{1}{2}\zeta(\cos(\Delta m \Delta t) - \cos(\Delta m(t_l+t_r)))}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t) - \frac{1}{2}\zeta(\cosh(\frac{1}{2}\Delta\Gamma\Delta t) - \cosh(\frac{1}{2}\Delta\Gamma(t_l+t_r)))}$$

CPLEAR-experiment (1998): $\zeta_{K^0,\overline{K}^0} = 0.41^{+0.67}_{-0.57}$

Bertlmann, Grimus and Hiesmayr, Phys. Rev. D, 60, 114032 (1999)

Decoherence



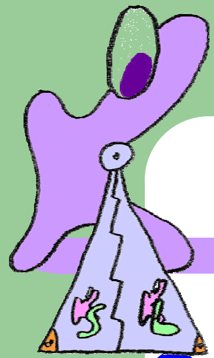
Liouville-von Neumann Eq.:

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] - D[\rho]$$

Dissipation / Decoherence

$$D[\rho] = \tfrac{1}{2}\sum_i \lambda_i \left(A_i^t A_i \rho + \rho A_i^t A_i - 2 A_i \rho A_i^t \right)$$

1976: Lindblad;
Gorini-Kossakowski-Sudarshan



The model

Generators:

$$A_i = \sqrt{\lambda}\, P_i = \sqrt{\lambda}\, |e_i\rangle\langle e_i| \qquad i = 1,2$$

$$|e_{1/2}\rangle = |K_{S/L}\rangle\langle K_{L/S}| \otimes |K_{L/S}\rangle\langle K_{S/L}|$$

$$D[\rho] = \lambda\left(P_1 \rho P_2 - P_2 \rho P_1\right)$$

$\lambda \geq 0$ **decoherence parameter**

- **Generates a completely positive map**
- **Conserves energy in case of Hermitian $H = H^t$ since $[P_i, H] = 0$**
- **Von Neumann entropy is not decreasing, because generators are Hermitian: $P_i = P^t_i$**

The solution

Initial state:

$$|\psi^-\rangle = \tfrac{1}{\sqrt{2}}\Big\{ \underbrace{|K_S\rangle_l \otimes |K_L\rangle_r}_{} - \underbrace{|K_L\rangle_l \otimes |K_S\rangle_r}_{} \Big\}$$

$$= \tfrac{1}{\sqrt{2}}\Big\{ \quad |e_1\rangle \quad - \quad |e_2\rangle \quad \Big\}$$

Solution:

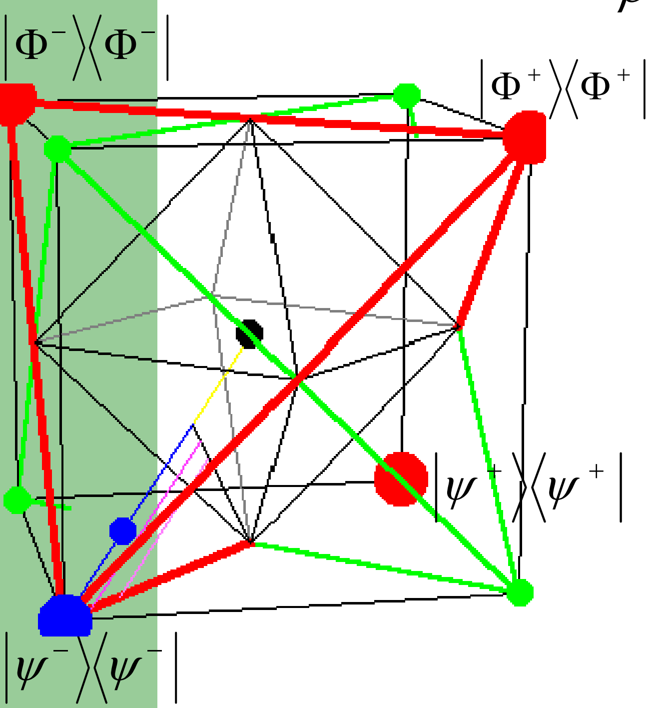
$$\rho(t) = \tfrac{1}{2} e^{-2\Gamma t} \Big\{ |e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - e^{-\lambda t} \big\{ |e_1\rangle\langle e_2| + |e_2\rangle\langle e_1| \big\} \Big\}$$

$$= \tfrac{1}{2} e^{-2\Gamma t} \Big\{ \sigma_{\Uparrow} \otimes \sigma_{\Downarrow} + \sigma_{\Downarrow} \otimes \sigma_{\Uparrow} - e^{-\lambda t} \big\{ \sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+ \big\} \Big\}$$

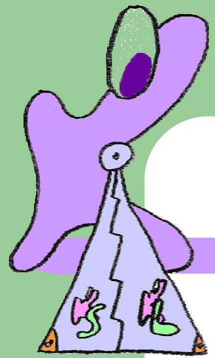
$$= \tfrac{1}{4} e^{-2\Gamma t} \Big\{ 1 \otimes 1 - \sigma_z \otimes \sigma_z - e^{-\lambda t} \big\{ \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y \big\} \Big\}$$

$$= e^{-2\Gamma t} \Big\{ \tfrac{1+e^{-\lambda t}}{2} |\psi^-\rangle\langle\psi^-| + \tfrac{1-e^{-\lambda t}}{2} |\psi^+\rangle\langle\psi^+| \Big\}$$

$$= \tfrac{1}{2} e^{-2\Gamma t} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -e^{-\lambda t} & 0 \\ 0 & -e^{-\lambda t} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



- ➢**Tetrahedron: positivity**
- ➢**Double pyramid: separability**
- ➢**Origin: totally mixed state**



Bounds from exp. Data

Observable:

$$A^{QM}(t_l,t_r)=\frac{P(M^0,t_l;\overline{M}^0,t_r)-P(M^0,t_l;M^0,t_r)}{\Sigma}=\frac{-\cos(\Delta m\Delta t)}{\cosh(\frac{\Delta\Gamma}{2}\Delta t)}$$

Model:

$$A^{\lambda}(t_l,t_r)=A^{QM}(\Delta t)\cdot e^{-\lambda\min\{t_l,t_r\}}$$

$$\zeta(t)=1-e^{-\lambda t}$$

K-mesons

$$\overline{\lambda}=(1.84^{+2.50}_{-2.17})\cdot 10^{-12}\ MeV$$

Bertlmann, Durstberger and H., Phys.Rev. A 68 (2003) 012111.

$$\overline{\Lambda}=\frac{\overline{\lambda}}{\Gamma_S}=0.25^{+0.34}_{-0.32}$$

B-mesons

$$\lambda_B=(-56\pm 61)\cdot 10^{-12}\ MeV$$

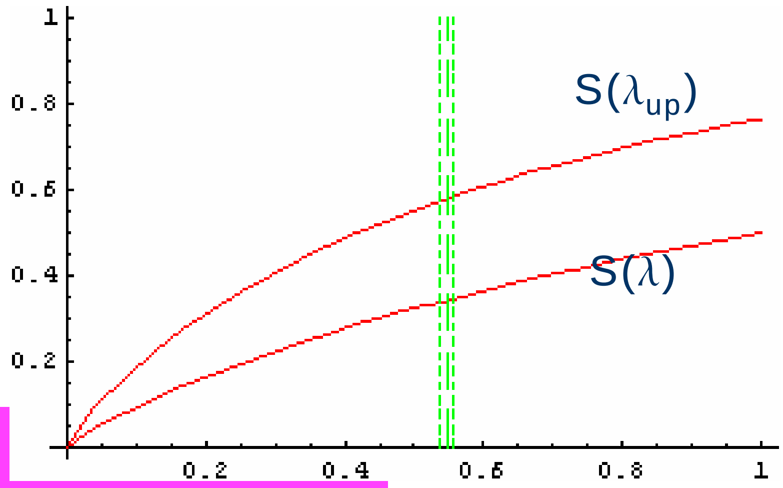
Bertlmann and Grimus., Phys.Rev. D 64 (2001) 056004.

$$\overline{\Lambda}=\frac{\overline{\lambda}}{\Gamma}=-0.13^{+0.14}_{-0.14}$$

Von Neumann entropy S

How much uncertainty is in the state?

$$\rho(t)=\frac{e^{-2\Gamma t}}{2}\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 1 & -e^{-\lambda t} & 0\\ 0 & -e^{-\lambda t} & 1 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$



System (2 mesons):

$$S\left(\rho(t)\right)=-Tr\left\{\rho(t)\log_2\rho(t)\right\}$$
$$=-\frac{1-e^{-\lambda t}}{2}\log_2\frac{1-e^{-\lambda t}}{2}-\frac{1+e^{-\lambda t}}{2}\log_2\frac{1+e^{-\lambda t}}{2}$$

Partial system (1 meson):

$$S\left(Tr_r\,\rho(t)\right)=1 \qquad \forall\ t\geq 0$$

Entanglement of Formation /Concurrence

How much resources are needed to create a given state?

$|\psi^-\rangle$ $|\psi^-\rangle$ $|\psi^-\rangle$

n

LOCC → **Local Operations and Classical Communication**

ρ

m

$$\rho = \sum p_i \rho_i = \sum p_i |\psi_i\rangle\langle\psi_i|$$

Entanglement of formation

$$E(\rho) = \min \sum p_i S(Tr_l(\rho_i)) \quad \sim n/m$$

Bennett et al. (1996)

$$= \mathcal{E}\left(C(\rho)\right)$$

Concurrence

Wootters,Hill (1997)

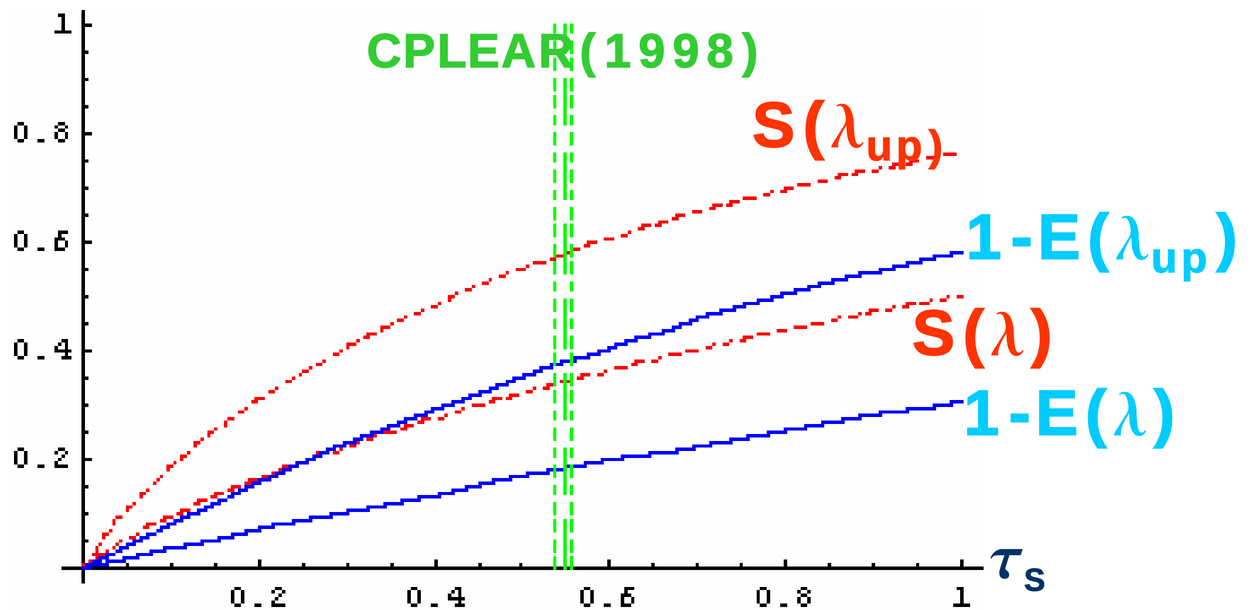
Measures of entanglement

S: How much uncertainty?

E: How much resources are needed to create a given state?

Loss of Concurrence: $1-C\big(\rho(t)\big)=1-e^{-\lambda t}=\zeta(t)$

Loss of entanglement: $1-E\big(\rho(t)\big)\doteq\frac{1}{\ln 2}\zeta(t)\doteq\frac{\lambda}{\ln 2}t$



Decoherence model put to test by BELLE

1. case: upper bound found -->
   several other models ruled out
2. case: model ruled out

model can be tested with other particles (kaons, photons, neutrons) → λ scales with kind of particle ?!

Summary

- **BI sensitive to S/B:** no violation possible because the ratio of oscillation to decay is too small

Phys.Rev. A 63, 062112 (2001)

- **BI sensitive to CP violation in K^0-$\bar{K}^0$ mixing (as loopholefree as possible):** the premises of LRT are only compatible with strict CP conservation, in contradiction to experiment

Phys.Lett. A 289, 21 (2001)

- In this way the nonzero result of δ is a manifestation of entanglement!!

- **decoherence model:** can be tested in experiments (KEK Japan); remarkable simple connected to measures of entanglement (loss of concurrence = ζ...spontaneous factorisation of the wave function)

Phys.Rev. D 60, 114032 (1999);

Phys. Rev. A 68, 012111 (2003)

Entanglement

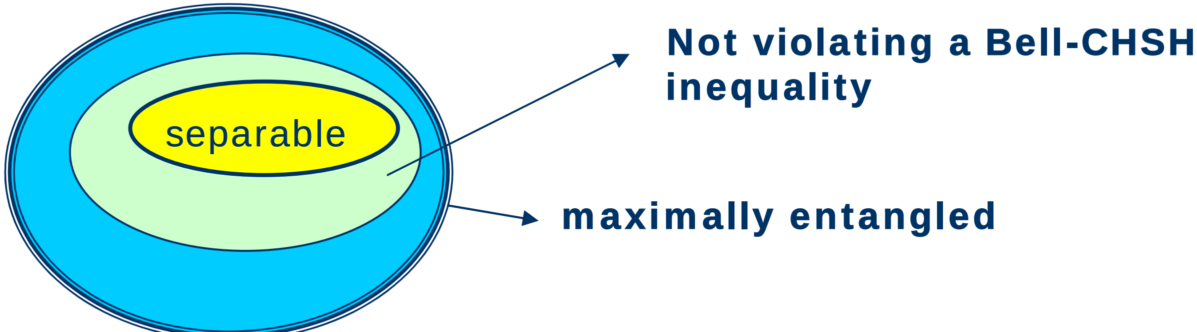
Pure states: $|\psi\rangle$

Mixed states (density matrix): $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \qquad \rho \geq 0$

Separable states: $\rho = \sum_i p_i \, \rho_i^A \otimes \tilde{\rho}_i^B$

$\forall\ 0 \leq p_i \leq 1 \quad \sum_i p_i = 1$

If the state ρ cannot be written in the form above, then the state is called entangled (=not separable).



Quantifying entanglement is an open question!