Einstein centennial 2005

Tomonaga-Yukawa centennial 2006-7

Majorana centennial 2006

50 years since BCS

40 years since W-S

39+ years since Veneziano

23+ years since Green-Schwarz

Highways and byways of particle physics

Main concepts leading to SM

- Symmetries
- Renormalization
- Gauge theory
- Symmetry breakng

Heisenberg's S-Matrix
Dispersion theory and bootrstrap
Regge theory
Infinite multiplets
Classification of hadrons
Finite energy sum rules (FESR) and Duality
Veneziano (-Suzuki) formula
Emergence of string picture

1. Heisenberg's S-Matrix

Nuclear reactions a + b ->c + d

$\psi(k) = (e^{-ikr} + e^{i(kr+\delta)})$,

$S = e^{i\,\delta(k)}$, $\Psi_{out} = S\,\Psi_{in}$

(bound state: pole of $\delta(k)$ at $k = ik_B$)

Unitarity

$S^{+}S = SS^{+}$

$S = 1 + iT$, $-i(T - T^{\dagger}) = T^{\dagger}T$

T-invarance

$S = S^{-1*} = S^{T}$, $T = T^{T}$

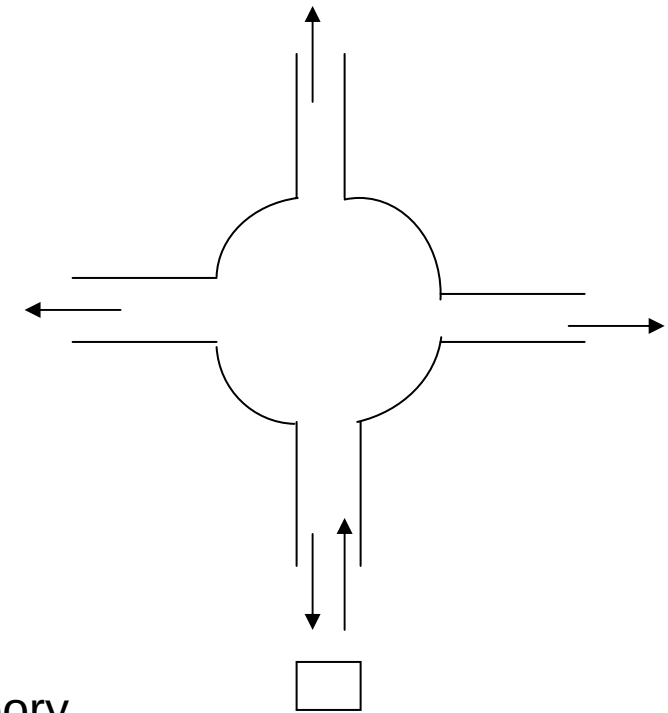
Applications

Nuclear reactions (Breit)

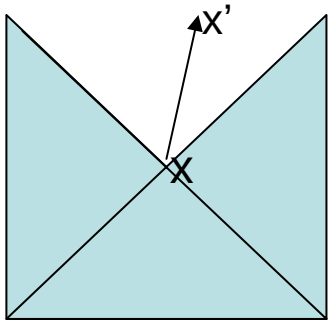
Wave guides (Kotani-Tomonaga)

Magnetron and klystron

Causality (analyticity) -> dispersion theory



2. Dispersion relations



$A \sim \psi(t')^* \psi(t) \sim e^{i\omega(t'-t)}$, ($\omega>0$, $t'-t>0$)

$d\sigma = \pi |A|^2 p^2 / v^2 d\Omega$, ($[A] = M^{-2}$)

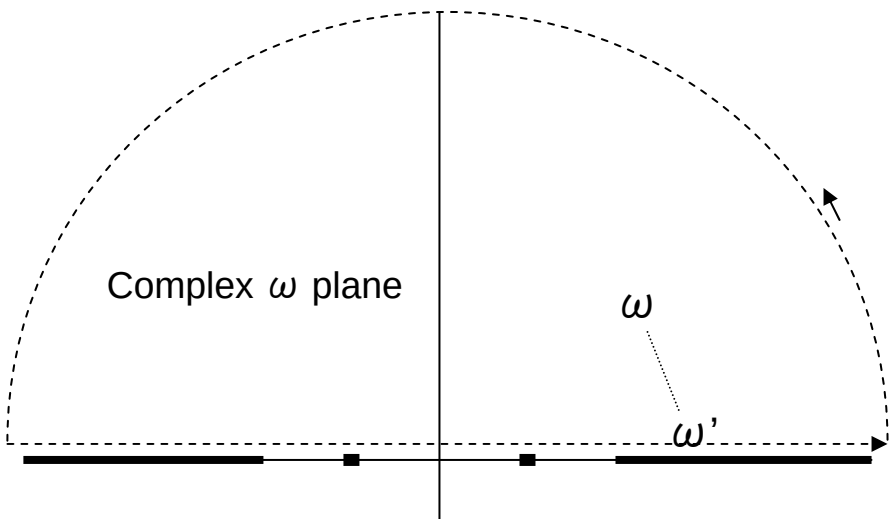
$- A \sim \sum_n < f|V|n><n|V|i> / (E_i - E_n)$

$$Re\, A(\omega) = (1/\pi) \int_{-\infty}^{\infty} Im\, A(\omega')/(\omega' - \omega))\, d\omega' + C$$

Lehmann representation of 2 point function

$G(x,x') = <0|T(\phi(x), \phi(x'))|0>$,

$G(p) = \int_{-\infty}^{\infty} \rho(m^2) dm^2 / (p^2 - m^2 + i\varepsilon)$, $\rho \geq 0$



3. Dispersion theory and bootrstrap

Crossing symmetry

$p + k \rightarrow p' + k'$, $p+(-k') \rightarrow p'+(-k)$

s,t,u channels

Mandelstam representation $A(\omega, \cos\theta) = A(s - u, t)$

$M(s,t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(s',t')/((s'-s)(t'-t))ds'dt'$

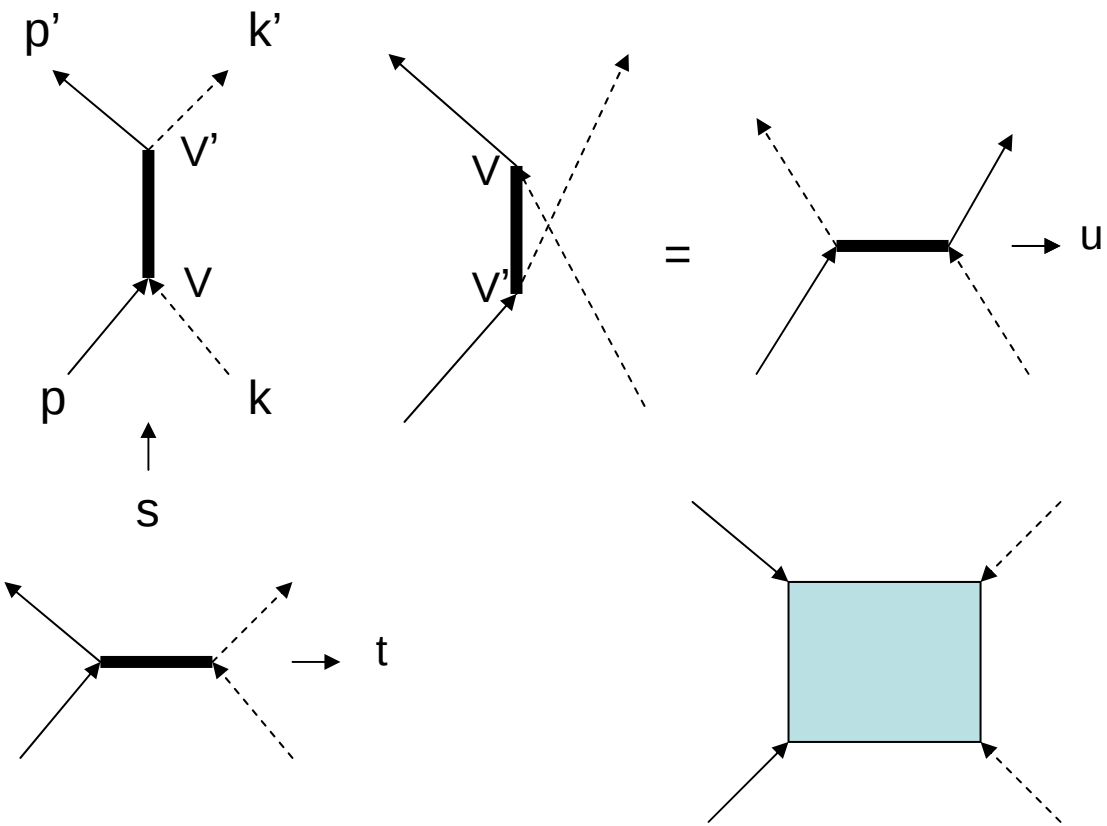
(double dispersion relation M(s,t) + M(u,t) + M(s,u))

Chew's bootstrap

hadron = hadron + hadron (s channel, $s > 0$, $\omega' > 0$ in DR)

force (potential) = hadron exchange (t & $u < 0$, $\omega' < 0$ in DR)

bootstrap: → s-channel = t-channel. etc



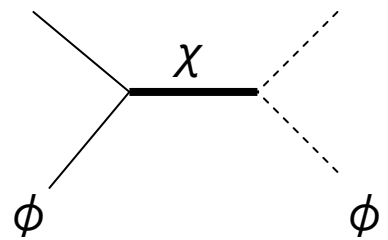
$s = (p + k)^2 = M^2 + m^2 + 2ME$

$u = (p - k')^2 = M^2 + m^2 - 2ME$

$t = (k - k')^2 = (\underline{k} - \underline{k}')^2$

$s + t + u = \Sigma\, m_i^2$

4.Regge theory



$\sigma \sim |A|^2 p^2$

	χ	[g]
$A \sim g(0)^2/(t - m(0)^2)$	scalar	M
$+ g(1)^2 p_i^2/(t - m(1)^2)$	vector	1
$+ g(2)^2 p_i^2 p_k^2/(t - m(2)^2)$	tensor	M^{-1}
$+ \ldots$		
$\sim \Sigma\, g(\ell)^2 p^{2\ell}/(t - m(\ell)^2)$		

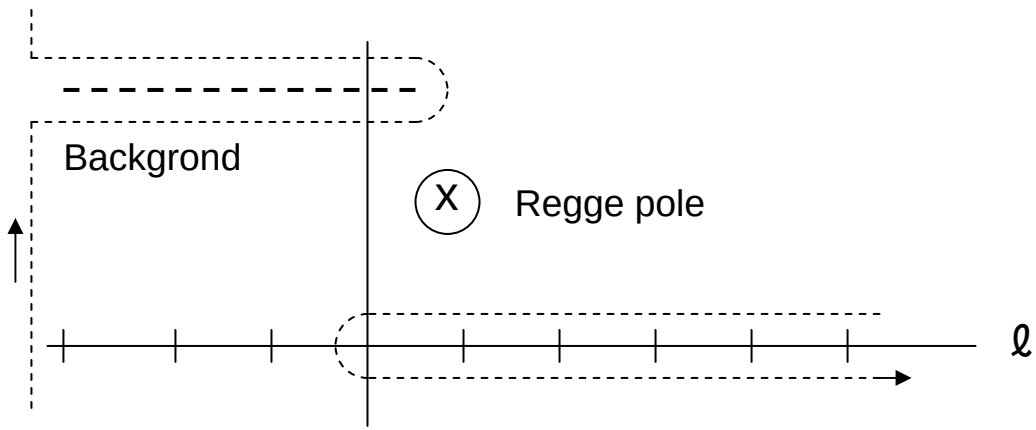
t-dispersion relation for A

$\ell = \alpha(t)$

$A(s,t) \sim s^{\alpha(t) - h}$ (h = helicity change)

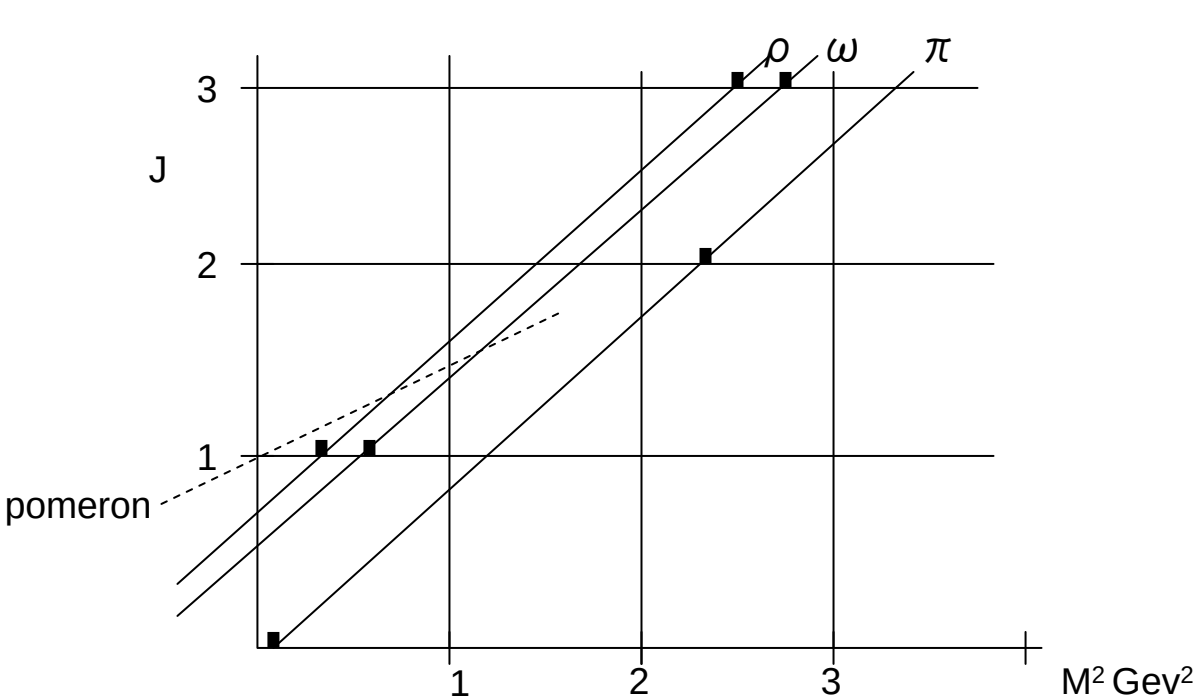
Sommerfeld-Watson transformation

$A(s,z) = \Sigma\,(2\ell+1) a_\ell(s) P_\ell(z) = (-1/2i) \int d\ell (2\ell+1) a(\ell) P_\ell(-z)/\sin(\pi\ell)$



5.Classification of hadrons

Horizontal: symmetries SU(3), SU(6), etc.
Vertical: Regge trajectories



6. Infinite multiplets

Wave equations with many masses
Unitary representation of Lorentz group
E. Majorana 1932
Gel'fand and Yaglom 1948
Barut, Fronsdal, Nambu,Takabayashi 1966..

Majorana equation (SO(3,2), (123,05))

$(p_\mu \Gamma^\mu - m_0)\,\psi = 0$

$\Gamma^\mu \sim L_{5\mu}$, $L_{54} = n+½ = 1/2,3/2,..$

$J = n$, $m_n = m_0/(n+½)$, $E = \sqrt{(m_n^2+p^2)} > 0$

$E_n = m_n + \langle n|\Gamma\cdot p|n'\rangle\langle n'|\Gamma\cdot p|n\rangle/(m_n - m_{n'})$
$= m_n + p^2/2m_n \rightarrow m_n > m_{n'}$,
$\rightarrow m_1 > m_2 > \ldots > 0$

Hydrogen atom (SO(4,2), (1234,06))

$(r\Delta + 2e^2m + 2Bmr)\,\psi = 0$
$r = L_{45} + L_{50}$, $r\Delta = L_{45} - L_{50}$,
$L_{50} = n = 1,2,3,..$
$B = me^4/2n^2$

Diseases: CPT, tachyon, etc.

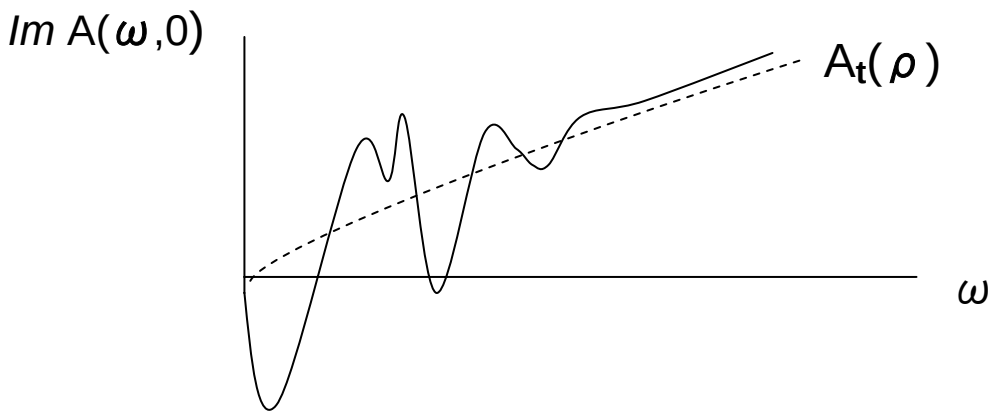
7. Finite energy sum rules (FESR) and Duality

Superconvergence

$A(\omega) = \int \rho(\omega')d\omega'/(\omega' - \omega)$

If $\rho(\omega) \sim \omega^{-\alpha}$, $\alpha > 1$ -> $\int_{-\infty}^{\infty} \rho(\omega')d\omega' = 0$

Duality $A = A_s(\text{Res}) - \langle A_s(\text{Res})\rangle + A_t(\text{Regge}) + A_t(\text{Pomeron})$

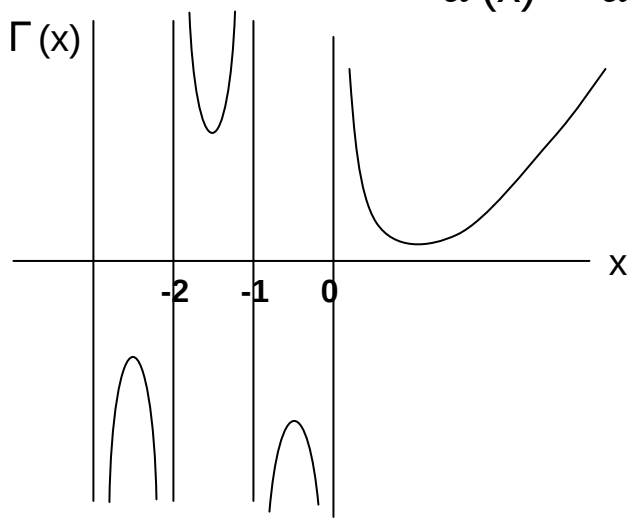


$\pi^- p \rightarrow \pi^0 n$, $\pi^+ n \rightarrow \pi^0 p$

: $Im\ A(\omega,0) \sim \sigma_{tot}(\pi^- p) - \sigma_{tot}(\pi^+ p)$

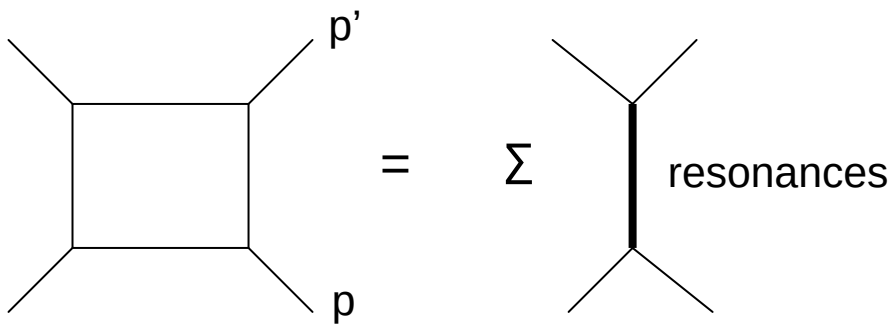
9. Veneziano (-Suzuki) formula

$$A(s,t) = \Gamma(-\alpha(s))\,\Gamma(-\alpha(t))/\Gamma(-\alpha(s) - \alpha(t))$$
$$\equiv B(-\alpha(s), -\alpha(t)), -\alpha(t)),$$
$$\alpha(x) = \alpha' x - \alpha(0)$$



$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt \text{ (Koba-Nielsen formula)}$$

10. String model
Factorization



$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ (Koba and Nielsen)
$= \int_0^1 t^{x-1} e^{(y-1)\ln(1-t)} dt$

$\ln(1 - t) = - (t + t^2/2 + t^3/3 + \ldots + t^n/n + ..)$
$x \sim s \sim (p+k)^2$
$y \sim -\alpha' t + c \sim \alpha'(k - k')^2 = -2k\cdot k' + 2m^2 + c$

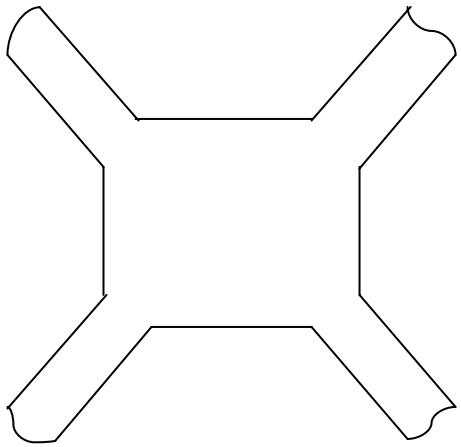
Field $\phi_\mu(x) = \sum_k (e^{ik\cdot x} a_k/(2E_k)^{1/2} + e^{-ik\cdot x} a^+_k/(2E_k)^{1/2}$
$\langle \phi(x), \phi(x') \rangle = \sum_k e^{k\cdot(x-x')}/2E_k$

$\langle e^{\lambda a} e^{\mu a^+} \rangle = e^{\lambda\mu}$

$\phi(x) \rightarrow \phi_\mu(x)$, $\mu = 1,2,3,4,0$

$\rightarrow e^{(y-1)\ln(1-t)} < e^{2ip'\cdot\phi(0)} e^{2ip\cdot\phi(0)} >$

11.String picture



$$\Sigma\ \alpha_0 + \Sigma\ m_{in}^2 = -1$$

Closed strings

Open strings with quarks at the ends

12. Virasoro algebra

unitarization (Kikkawa, Sakita,Virasoro)
splitting and joining of strings

world sheet action (Nambu, Goto)

fermionic string (Ramond)

string supersymmetry (Gervais and Sakita)

13. Demise of hadron string theory (ca 1974)

Relation to QCD

string confinement vs. integer color

large N behavior (t'Hooft)