Towards Realistic Models in String Theory with D-branes

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Plan of Lectures

1. Basic idea to construct realistic models
   - a brief review of string world-sheet theory
   - the basics of D-branes
   - the realization of chiral fermions

2. Construction of models without supersymmetry
   - "brane supersymmetry breaking"
   - NS-NS tadpole problem and back reactions
   - dynamics of gauge symmetry breaking

3. Stabilization of compact spaces
   - a sketch of KKLT scenario
   - other idea with magnetic fluxes on D-branes
   - dynamical stabilization by fractional branes

"Brane supersymmetry breaking"

Orientifold projection: projection by world-sheet parity

World-sheet parity transformation is the symmetry in type IIB theory.

$$\Omega \ : \quad \sigma \to 2\pi - \sigma, \quad \tau \to \tau \quad \text{(closed string)}$$

$$\Omega \ : \quad \sigma \to \pi - \sigma, \quad \tau \to \tau, \quad \lambda_{ij} \to (\gamma_\Omega)_{ii'} \lambda_{j'i'} (\gamma_\Omega^{-1})_{j'j} \quad \text{(open string)}$$

In complex coordinates:

$$\Omega \ : \quad z \to \bar{z}, \quad \bar{z} \to z, \quad \text{(closed string)}$$

$$\text{or} \quad \Omega\, X^\mu(z, \bar{z})\, \Omega^{-1} = X^\mu(\bar{z}, z) \quad \text{etc.}$$

$$\Omega \ : \quad z \to -\bar{z}, \quad \bar{z} \to -z, \quad \lambda_{ij} \to (\gamma_\Omega)_{ii'} \lambda_{j'i'} (\gamma_\Omega^{-1})_{j'j} \quad \text{(open string)}$$

$$\text{or} \quad \Omega\, X^\mu(z, \bar{z})\, \Omega^{-1} = X^\mu(-\bar{z}, -z) \quad \text{etc.}$$

Transformations of creation and annihilation operators

closed string

$$\Omega\,\alpha_m^\mu\,\Omega^{-1} = \tilde{\alpha}_m^\mu \qquad \Omega\,\tilde{\alpha}_m^\mu\,\Omega^{-1} = \alpha_m^\mu$$

$$\Omega\,\psi_r^\mu\,\Omega^{-1} = e^{-i2\pi r}\tilde{\psi}_r^\mu \qquad \Omega\,\tilde{\psi}_r^\mu\,\Omega^{-1} = e^{i2\pi r}\psi_r^\mu$$

$$\Omega\,b_m\,\Omega^{-1} = \tilde{b}_m \qquad \Omega\,\tilde{b}_m\,\Omega^{-1} = b_m$$

$$\Omega\,c_m\,\Omega^{-1} = \tilde{c}_m \qquad \Omega\,\tilde{c}_m\,\Omega^{-1} = c_m$$

$$\Omega\,\beta_r\,\Omega^{-1} = e^{-i2\pi r}\tilde{\beta}_r \qquad \Omega\,\tilde{\beta}_r\,\Omega^{-1} = e^{i2\pi r}\beta_r$$

$$\Omega\,\gamma_r\,\Omega^{-1} = e^{-i2\pi r}\tilde{\gamma}_r \qquad \Omega\,\tilde{\gamma}_r\,\Omega^{-1} = e^{i2\pi r}\gamma_r$$

Transformation of closed string (type IIB) ground states

$$\Omega\,|0;0;k\rangle_{\mathrm{NS-NS}} = -|0;0;k\rangle_{\mathrm{NS-NS}} \qquad \Omega\,|\mathbf{s}_+;\mathbf{s}'_+;k\rangle_{\mathrm{R-R}} = -|\mathbf{s}'_+;\mathbf{s}_+;k\rangle_{\mathrm{R-R}}$$

$$\Omega\,|0;\mathbf{s}_+;k\rangle_{\mathrm{NS-R}} = -|\mathbf{s}_+;0;k\rangle_{\mathrm{R-NS}} \qquad \Omega\,|\mathbf{s}_+;0;k\rangle_{\mathrm{R-NS}} = -|0;\mathbf{s}_+;k\rangle_{\mathrm{NS-R}}$$

Massless states in NS-NS sector

$$\Omega\,\psi^{\mu}_{-1/2}\tilde{\psi}^{\nu}_{-1/2}|0;0;k\rangle_{\mathrm{NS-NS}} = -\tilde{\psi}^{\mu}_{-1/2}\psi^{\nu}_{-1/2}|0;0;k\rangle_{\mathrm{NS-NS}} = \psi^{\nu}_{-1/2}\tilde{\psi}^{\mu}_{-1/2}|0;0;k\rangle_{\mathrm{NS-NS}}$$

Anti-symmetric $B^{\mu\nu}$ field is projected out

Massless states in NS-R and R-NS sector

$$\Omega\,\psi^{\mu}_{-1/2}|0;\mathbf{s}_+;k\rangle_{\mathrm{NS-R}} = \tilde{\psi}^{\mu}_{-1/2}|\mathbf{s}_+;0;k\rangle_{\mathrm{R-NS}} \qquad \Omega\,\tilde{\psi}^{\mu}_{-1/2}|\mathbf{s}_+;0;k\rangle_{\mathrm{R-NS}} = \psi^{\mu}_{-1/2}|0;\mathbf{s}_+;k\rangle_{\mathrm{NS-R}}$$

A combination of (NS-R)+(R-NS) survives
and one of two gravitino is projected out.

10D N=2 SUSY is reduced to 10D N=1 SUSY.

Transformations of creation and annihilation operators

open string (Neumann-Neumann b.c.)

$$\Omega\,\alpha_m^\mu\,\Omega^{-1} = e^{i\pi m}\alpha_m^\mu$$

$$\Omega\,\psi_r^\mu\,\Omega^{-1} = e^{-i\pi r}\psi_r^\mu$$

$$\Omega\,b_m\,\Omega^{-1} = e^{i\pi m}b_m \qquad \Omega\,c_m\,\Omega^{-1} = e^{i\pi m}c_m$$

$$\Omega\,\beta_r\,\Omega^{-1} = e^{-i\pi r}\beta_r \qquad \Omega\,\gamma_r\,\Omega^{-1} = e^{-i\pi r}\gamma_r$$

Transformation of ground states

$$\Omega\,|k;ij\rangle_{\rm NS} = i(\gamma_\Omega)_{ii'}|k;j'i'\rangle_{\rm NS}(\gamma_\Omega^{-1})_{j'j} \qquad \Omega\,|\mathbf{s}_+;k;ij\rangle_{\rm R} = -(\gamma_\Omega)_{ii'}|\mathbf{s}_+;k;j'i'\rangle_{\rm R}(\gamma_\Omega^{-1})_{j'j}$$

Transformation of massless states

$$\Omega\,\psi^\mu_{-1/2}|k;ij\rangle_{\rm NS} = -\psi^\mu_{-1/2}(\gamma_\Omega)_{ii'}|k;j'i'\rangle_{\rm NS}(\gamma_\Omega^{-1})_{j'j}$$

$$\Omega\,|\mathbf{s}_+;k;ij\rangle_{\rm R} = -(\gamma_\Omega)_{ii'}|\mathbf{s}_+;k;j'i'\rangle_{\rm R}(\gamma_\Omega^{-1})_{j'j}$$

Transformations of massless states (cont.)

$$\Omega\, \psi^{\mu}_{-1/2}|k;ij\rangle_{\rm NS} = -\psi^{\mu}_{-1/2}(\gamma_\Omega)_{ii'}|k;j'i'\rangle_{\rm NS}(\gamma_\Omega^{-1})_{j'j}$$

$$\Omega\,|\mathbf{s}_+;k;ij\rangle_{\rm R} = -(\gamma_\Omega)_{ii'}|\mathbf{s}_+;k;j'i'\rangle_{\rm R}(\gamma_\Omega^{-1})_{j'j}$$

Two choices of the action to Chan-Paton factor

$$(\gamma_\Omega)_{ii'}|j'i'\rangle(\gamma_\Omega^{-1})_{j'j} = |ji\rangle$$

$$\gamma_\Omega = \mathbf{1}$$

Chan-Paton factor should be anti-symmetric: $\mathrm{U}(N) \to \mathrm{SO}(N)$

$$(\gamma_\Omega)_{ii'}|j'i'\rangle(\gamma_\Omega^{-1})_{j'j} = -|ji\rangle$$

$$\gamma_\Omega = \begin{pmatrix} 0 & -\mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix} \quad \text{with} \quad |ji\rangle \sim \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

Chan-Paton factor should be symmetric: $\mathrm{U}(N) \to \mathrm{USp}(N)$

10D N=1 SUSY remains.

Construction of type I string theory

Unoriented closed string with D=10 N=1 SUSY
can couple with open string with D=10 N=1 SUSY

Introduction of open string with Chan-Paton factor
= Introduction of n D9-branes

Closed string massless states (unoriented type IIB theory)

(NS+ NS+) (R+ R+) (NS+ R+) (R+ NS+)

(NS+ NS+)	(R+ R+)	(NS+ R+) (R+ NS+)
$1+\cancel{28}+35$	$\cancel{1}+28+\cancel{35}$	8'+56 $\longleftrightarrow$ 8'+56
$\phi,\ \cancel{B_{\mu\nu}},\ g_{\mu\nu}$	$\cancel{C},\ C_{\mu\nu},\ \cancel{C_{\mu\nu\rho\sigma}}$	$\downarrow$ 8'+56

Open string massless states

NS	R
8	8
A_μ^a	λ^a

The gauge group is fixed
by tadpole cancelations.

Vacuum amplitudes (closed string)

$$A_{\text{closed}} = \int_F \frac{d^2\tau}{4\tau_2}\,\text{tr}\left(\frac{1+\Omega}{2} P_{\text{GSO}}\tilde{P}_{\text{GSO}}\, q^{L_0}\bar{q}^{\tilde{L}_0}\right)$$

torus

Klein bottle

$$\mathcal{T} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_F \frac{d^2\tau}{\tau_2}\frac{1}{\tau_2^5}\frac{1}{|\eta(\tau)|^{16}}|V_8(\tau) - S_8(\tau)|^2$$

$$\mathcal{K} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty \frac{dt}{t}\frac{1}{t^5}\frac{1}{\eta(2it)^8} \times \frac{1}{2}\left(V_8(2it) - S_8(2it)\right)$$

("cylinder with modulus 2t" due to projection (left=right, effectively))

$$V_{2n}(\tau) \equiv \frac{(\theta_3(\tau))^n - (\theta_4(\tau))^n}{2(\eta(\tau))^n} \qquad S_{2n}(\tau) \equiv \frac{(\theta_2(\tau))^n - (\theta_1(\tau))^n}{2(\eta(\tau))^n}$$

"NS sector" "R sector"

Vacuum amplitudes (open string)

$$A_{\text{open}} = \int_0^\infty \frac{dt}{2t}\,\mathrm{tr}\left(\frac{1+\Omega}{2} P_{\text{GSO}}\, q^{L_0}\right)$$

cylinder Möbius strip

$$\mathcal{A} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty \frac{dt}{t}\frac{1}{t^5}\frac{1}{\eta(it/2)^8} \times \frac{n^2}{2}\left(V_8(it/2) - S_8(it/2)\right)$$

$$\mathcal{M} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty \frac{dt}{t}\frac{1}{t^5}\frac{1}{\eta(\frac{1}{2}+i\frac{t}{2})^8} \times \frac{-\epsilon n}{2}\left(V_8(\frac{1}{2}+i\frac{t}{2}) - S_8(\frac{1}{2}+i\frac{t}{2})\right)$$

(projection makes $q \to q e^{i\pi}$ by $\Omega\,\psi_r^\mu\,\Omega^{-1} = e^{-i\pi r}\psi_r^\mu$)

n : number of D9-branes

$$\begin{cases} \epsilon = 1 & \text{: SO type projection} \\ \epsilon = -1 & \text{: USp type projection} \end{cases}$$

Change variables of integrations to closed string picture

$$\mathcal{K} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty dl \frac{1}{\eta(il)^8} \times \frac{2^5}{2}\left(V_8(il) - S_8(il)\right)$$

$$\mathcal{A} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty dl \frac{1}{\eta(il)^8} \times \frac{2^{-5}n^2}{2}\left(V_8(il) - S_8(il)\right)$$

$$\mathcal{M} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty dl \frac{1}{\eta(\frac{1}{2}+il)^8} \times \frac{-\epsilon 2n}{2}\left(V_8(\frac{1}{2}+il) - S_8(\frac{1}{2}+il)\right)$$

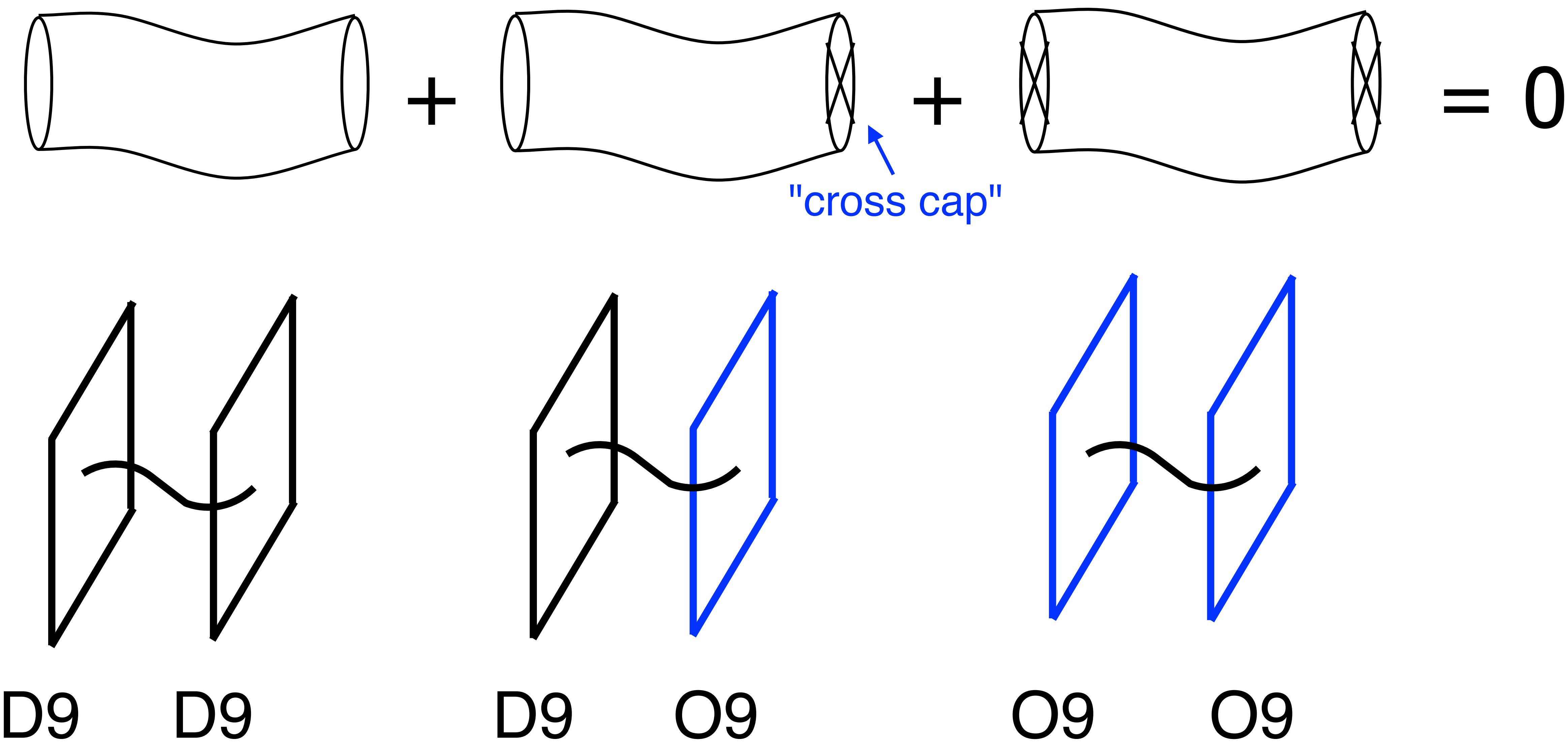
Anomaly (gravitational) cancelation requires
cancelation of massless Ramond-Ramond tadpoles.

$$2^5 + 2^{-5}n^2 - \epsilon 2n = 0 \quad \Rightarrow \quad (n - \epsilon\, 2^5)^2 = 0$$

Therefore, $\epsilon = 1$ and $n = 2^5$ and the gauge group is SO(32).

Massless NS-NS tadpoles are canceled simultaneously by SUSY.

A schematic picture of tadpole cancelation



We may introduce orientifold fixed planes which have R-R and NS-NS charges, though they are not dynamical objects.

A simple model of "brane supersymmetry breaking": USp(32) type I model or Sugimoto model.

Consider n anti-D9-branes.
The sign of RR sector in Möbius amplitude is flipped.

$$\mathcal{M} = \frac{iV_{10}}{2(4\pi^2\alpha')^5}\int_0^\infty dl\,\frac{1}{\eta(\frac{1}{2}+il)^8} \times \frac{-\epsilon 2n}{2}\left(V_8(\frac{1}{2}+il)+S_8(\frac{1}{2}+il)\right)$$

Take USp type orientifold: $\epsilon = -1$, then R-R tadpoles are canceled, though NS-NS tadpoles are not canceled.

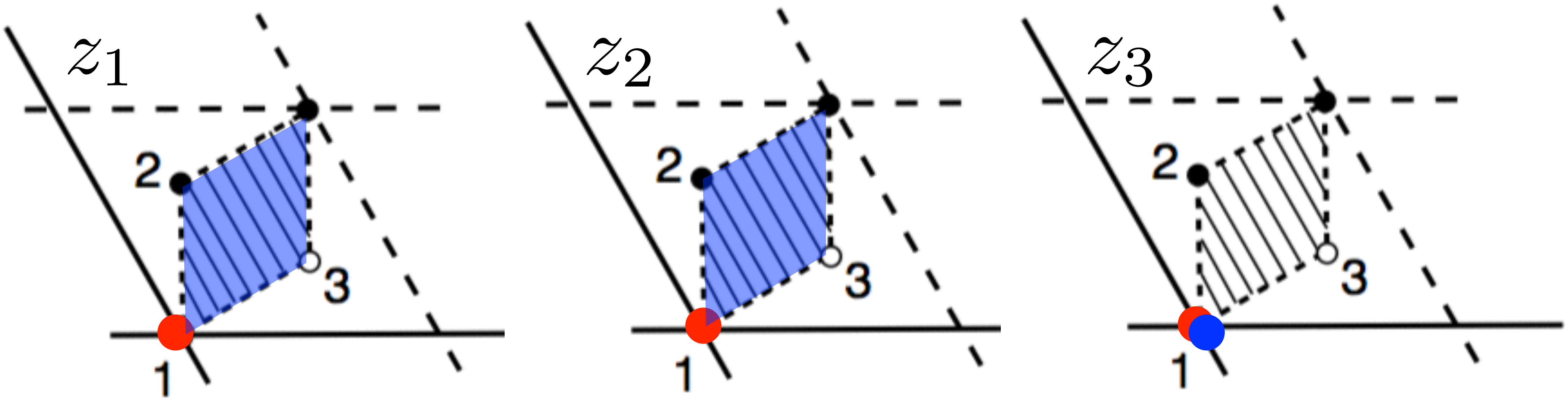
This is a consistent model with USp(32) gauge symmetry, but there is no supersymmetry in open string sector.

On anti-D9-branes there are USp(32) gauge bosons 528 and fermions in anti-symmetric tensor representation 496=495+1.

(An additional (-) sign in orientifold projection of R states is required for the open string on anti-D-branes)

Brane supersymmetry breaking without orientifold

Four D3-branes and Three anti-D7-branes
on a $\mathbf{Z}_3$ orbifold singularity



$\mathbf{Z}_3$ operation matrices on each D-brane

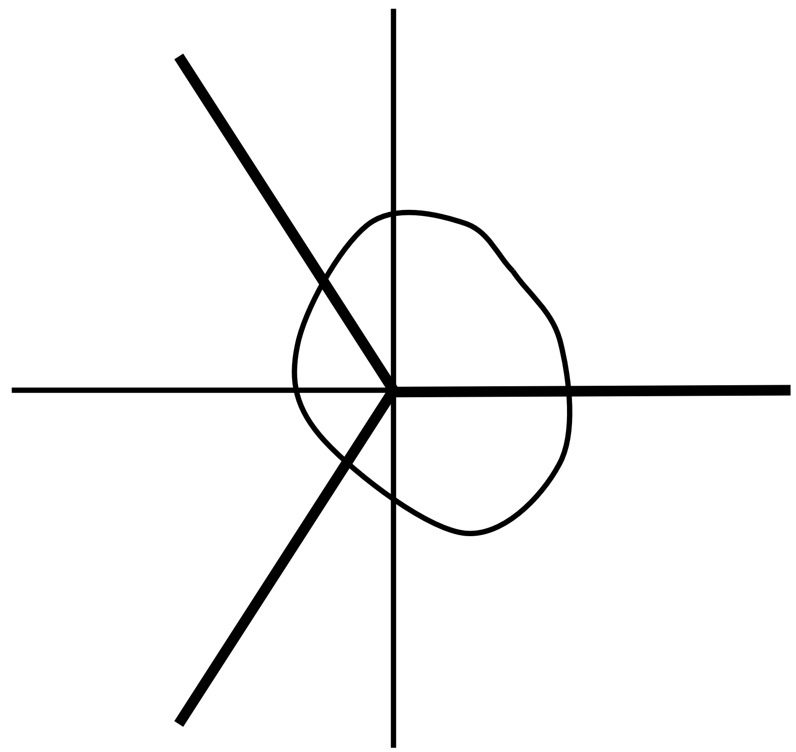
$\gamma_3 = \mathrm{diag}(\mathbf{1}_2, \alpha\mathbf{1}_1, \alpha^2\mathbf{1}_1)$ $\quad\quad$ $\mathrm{U}(2) \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_2$

$\gamma_{\bar{7}_3} = \mathbf{1}_3$ $\quad\quad$ $\mathrm{U}(3)$ $\quad\quad$ $(\alpha \equiv e^{i2\pi/3})$

The condition of twisted R-R tadpole cancelation is satisfied.

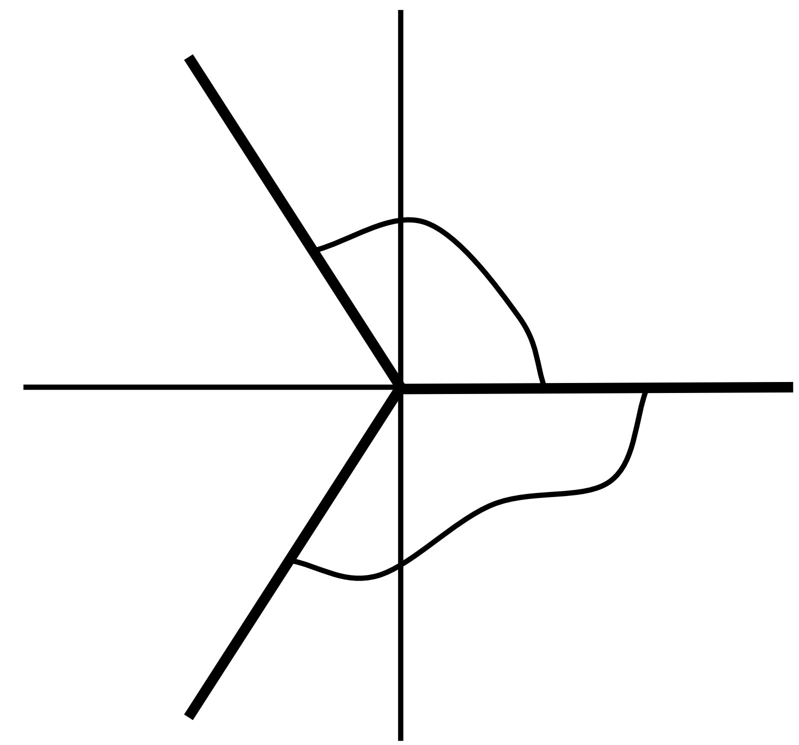
$$3\mathrm{Tr}(\gamma_3) - \mathrm{Tr}(\gamma_{\bar{7}_3}) = 0$$

"untwisted" and "twisted" closed strings



untwisted

actually closed



twisted

closed up to orbifold transformation

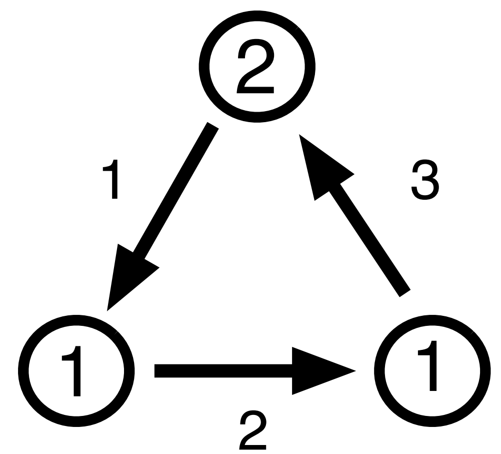
Cancelation of R-R tadpoles of twisted closed string is required for the cancelation of gauge anomaly. Cancelations of NS-NS tadpoles are not necessary for anomaly cancelations ("consistency") of the theory.

Massless modes under $\mathrm{U}(2)\times\mathrm{U}(3)\times\mathrm{U}(1)_{1+2}$

non-anomalous

D3-D3 open string (supersymmetric)

$$
\begin{aligned}
(\Phi_1^a, \Psi_1^a) &: \quad (2^*, 1)_{+1},\\
(\Phi_2^a, \Psi_2^a) &: \quad (1, 1)_{0},\\
(\Phi_3^a, \Psi_3^a) &: \quad (2, 1)_{-1}.
\end{aligned}
$$



$$a = 1, 2, 3$$

D3-anti-D7 open string (non-SUSY)

$$
\begin{aligned}
\phi_1 &: \quad (2, 3^*)_0,\\
\phi_2 &: \quad (2^*, 3)_0,\\
\psi_1 &: \quad (1, 3)_{-1},\\
\psi_2 &: \quad (1, 3^*)_{+1},
\end{aligned}
$$

NS-NS tadpole problem and back reactions

In general string models without supersymmetry have tadpoles of massless dilaton and graviton (NS-NS sector).

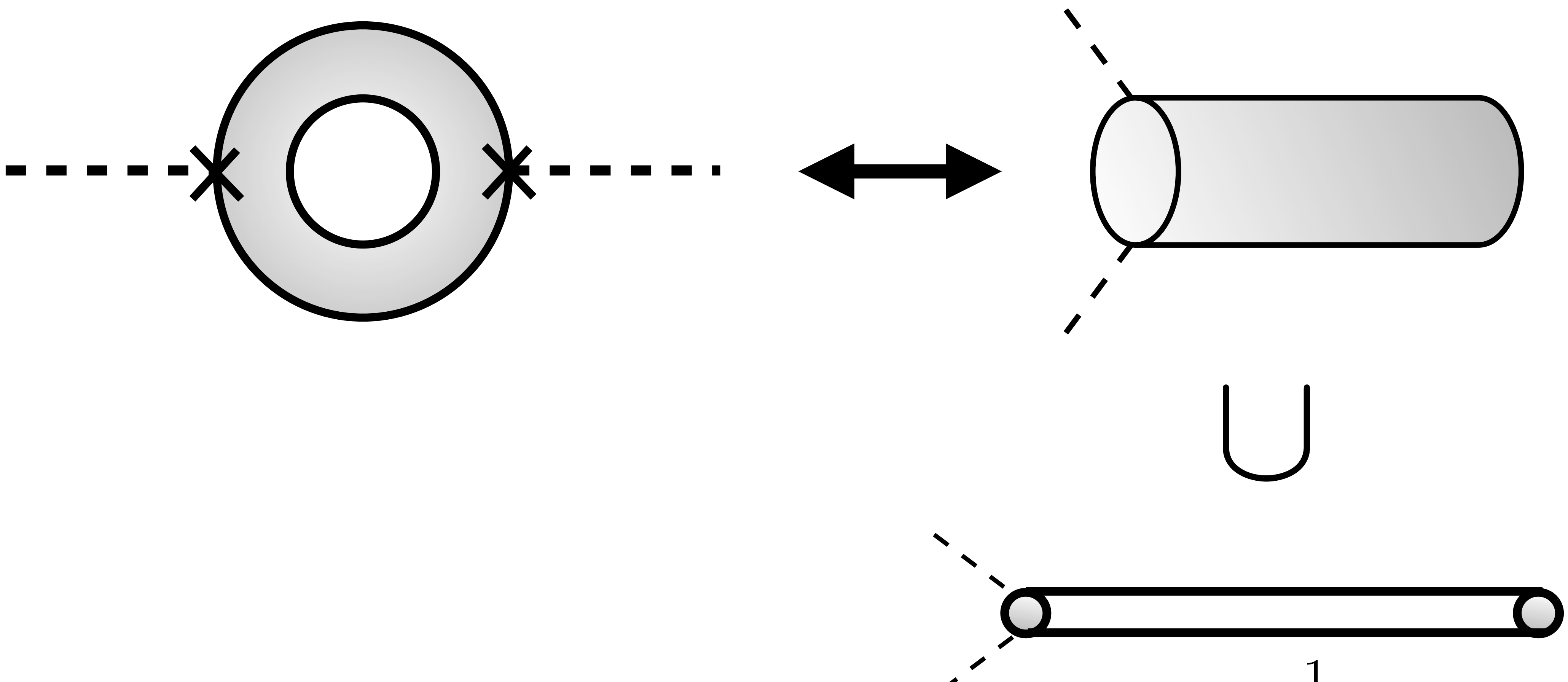
Background geometry and field configurations, which are assumed to be trivial at the beginning, are not the "solution" of String Theory, or "back reaction" is not included.

This could be cured by "Fishler-Susskind mechanism", but it would be very difficult to do it in general.

It could be even possible that the present understanding of "brane supersymmetry breaking" would be incomplete.

There is a more practical difficulty.

Open-string one-loop calculations get infrared divergences.



$$\lim_{p \to 0} \frac{1}{p^2}$$

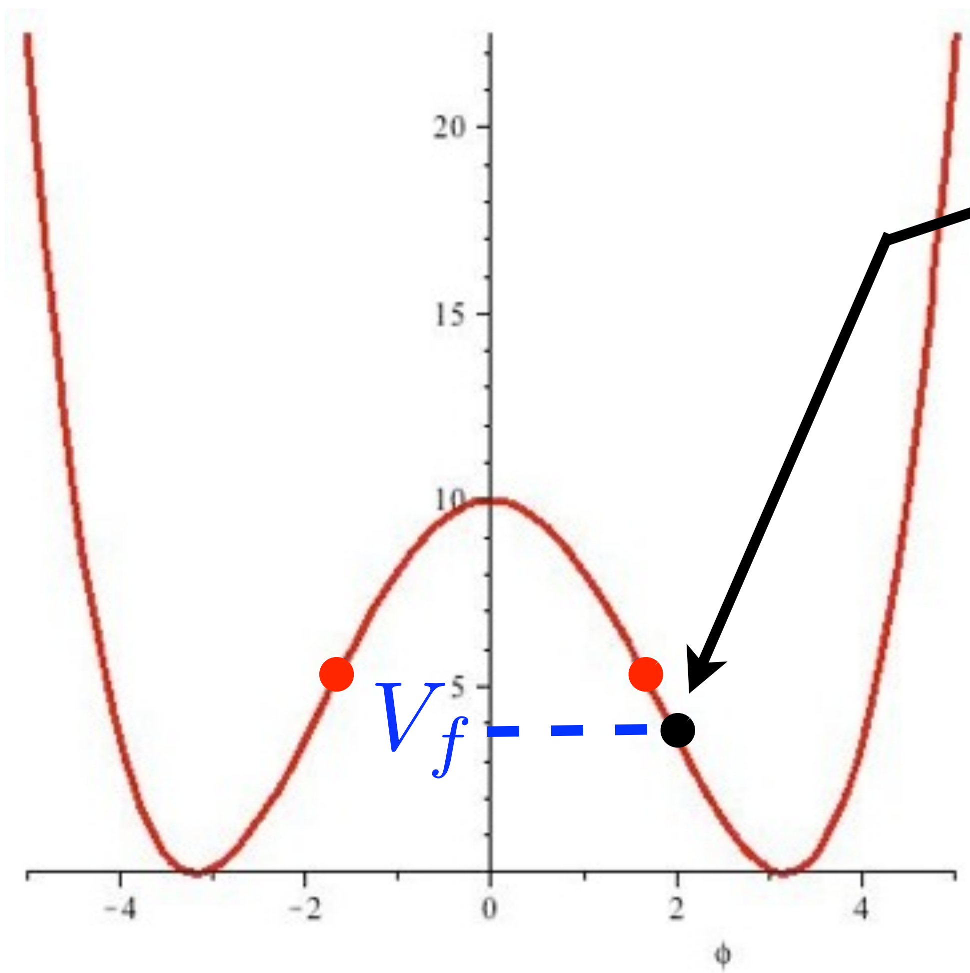
Some reasonable technique to evade these divergences?

"tadpole resummations"

Tadpole resummations in field theory

by Dudas-Nicolosi-Pradisi-Sagnotti (2005)

Possibility to obtain true values of physical quantities in "wrong vacua": vacuum energy, for example

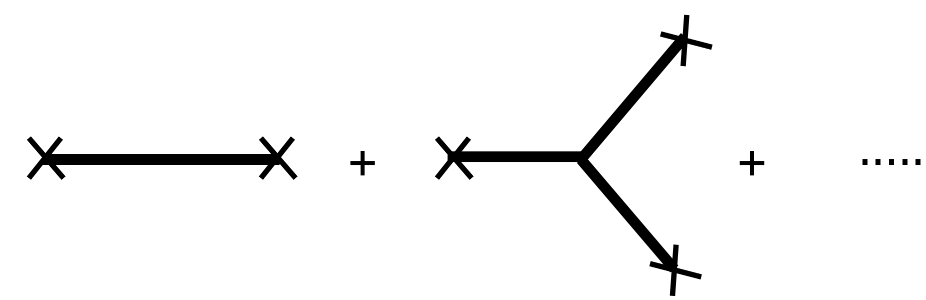


"wrong vacua"

$$V = -4a(v^2 - v_f^2)v_f \phi + \cdots$$

tadpole

$V_{\mathrm{res}} =$



$$V_f + V_{\mathrm{res}} = 0$$

The same is possible in String Theory?

Similar resummations are possible in String Theory using boundary state formalism

N.K. (2008)

This technique gives one positive result:

The vacuum energy of a "Dp-brane" in Bosonic String Theory is canceled by the tree-level contribution from tadpole resummations,

$$\Lambda_p^{\mathrm{cl}} + \Lambda_p^{\mathrm{res}} = 0 \qquad (\Lambda_p^{\mathrm{cl}} = T_p)$$

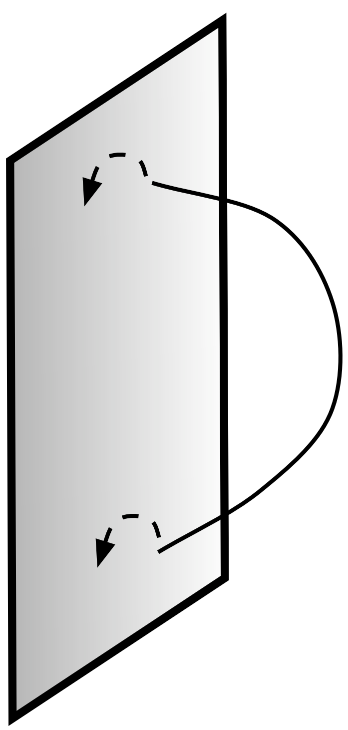
which is consistent with Sen's conjecture of "Dp-brane decay in Bosonic String Theory", or "tachyon condensation".

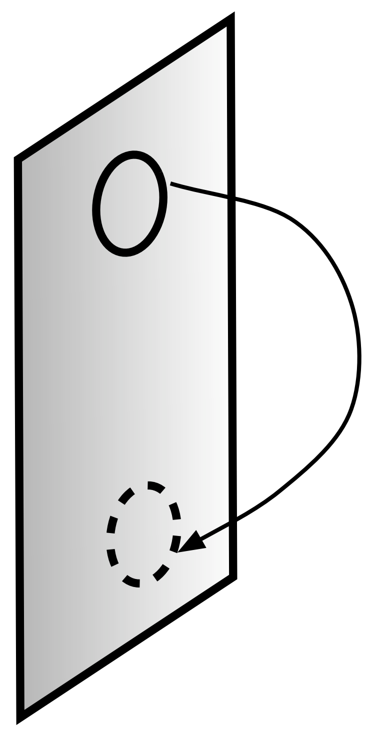
Boundary State Formalism

For simplicity we consider Bosonic String Theory, although the same is applicable to Superstring Theory.

The cylinder amplitude

open string	$\longleftrightarrow$	closed string
one loop		tree





Dp-brane boundary state

$$\frac{1}{2!}\langle B_p|D|B_p\rangle$$

second order of tadpole insertion

propagator operator

The concrete form of boundary states

$$|B_p\rangle = |B_p^X\rangle |B^{\rm gh}\rangle,$$

$$|B_p^X\rangle = N_p \delta^{d_\perp}(\hat{x}) \exp\left(-\sum_{n=1}^{\infty} \frac{1}{n} \alpha^\mu_{-n} S_{\mu\nu} \tilde{\alpha}^\nu_{-n} \right) |0\rangle,$$

$$|B^{\rm gh}\rangle = \exp\left(\sum_{n=1}^{\infty} (c_{-n}\tilde{b}_{-n} - b_{-n}\tilde{c}_{-n}) \right) |0\rangle_{\rm gh}$$

$$S^{\mu\nu} \equiv (\eta^{\alpha\beta}, -\delta^{ij})$$

$$d_\perp \equiv d - (p+1)$$

$$d \equiv 26$$

$$\boxed{N_p \equiv T_p/2}$$

These are solution of boundary conditions:

$\partial_\tau X^\alpha(\sigma, \tau = 0)|B_p\rangle = 0, \quad \alpha = 0, 1, \cdots, p$: Neumann (open string)

$X^i(\sigma, \tau = 0)|B_p\rangle = 0, \quad i = p+1, p+2, \cdots, 25$: Dirichlet (open string)

(conformal transformation from open to closed gives $\tau \longleftrightarrow \sigma$.)

These are something like coherent states.

The Cylinder Amplitude

or one-loop correction to the vacuum energy of Dp-brane

$$A_p = \frac{1}{2!}\langle B_p|D|B_p\rangle = \frac{1}{2!}V_{p+1}N_p^2\Delta_p$$

$$D \equiv \frac{\alpha'}{4\pi}\int_0^\infty dt \int_0^{2\pi} d\varphi \; z^{L_0}\bar{z}^{\tilde{L}_0} \qquad (z = e^{-t}e^{i\varphi})$$

$$\Delta_p \equiv \frac{\pi\alpha'}{2}\int_0^\infty ds \int \frac{d^{d_\perp}p}{(2\pi)^{d_\perp}} e^{-\frac{\pi\alpha'}{2}p_\perp^2 s}\frac{1}{(\eta(is))^{24}}$$

"propagator"

$$= \frac{\pi\alpha'}{2}\int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}}\frac{1}{(\eta(is))^{24}}$$

$$d_\perp \equiv d - (p+1) \;\; \text{with} \;\; d = 26$$

Infrared divergence due to massless tadpoles of dilaton, graviton and tachyon (we will see ot later).

Tadpole couplings from boundary states

$$A^{\mu\nu} \equiv \langle 0; k | a_1^{\mu} \tilde{a}_1^{\nu} | B_p \rangle = -\frac{T_p}{2} V_{p+1} S^{\mu\nu} \qquad S^{\mu\nu} \equiv (\eta^{\alpha\beta}, -\delta^{ij})$$

$$\begin{cases} A_{\text{grav}} = A^{\mu\nu}\epsilon^{(h)}_{\mu\nu} = -V_{p+1} T_p \eta^{\alpha\beta} \epsilon^{(h)}_{\alpha\beta}, \\ A_{\text{dil}} = A^{\mu\nu}\epsilon^{(\phi)}_{\mu\nu} = V_{p+1} T_p a. \end{cases} \qquad \left(a \equiv \frac{d-2p-4}{2\sqrt{d-2}} \right)$$

D-brane effective action in Einstein frame

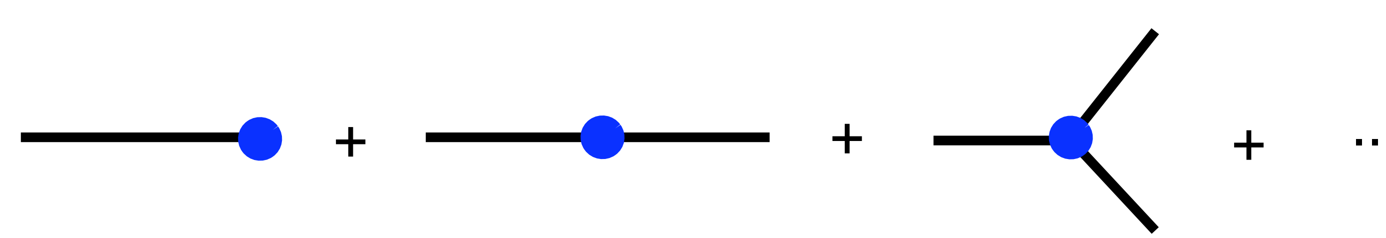
$$S_{Dp} = -T_p \int d^{p+1}\xi e^{-a\phi} \sqrt{-\det g_{\alpha\beta}}$$

ignoring B-field and gauge field

$$\mathcal{L}_{\text{dil}} = -T_p e^{-a\phi} = -T_p + T_p a \phi - \frac{1}{2!} T^p a^2 \phi^2 + \frac{1}{3!} T^p a^3 \phi^3 - \cdots$$

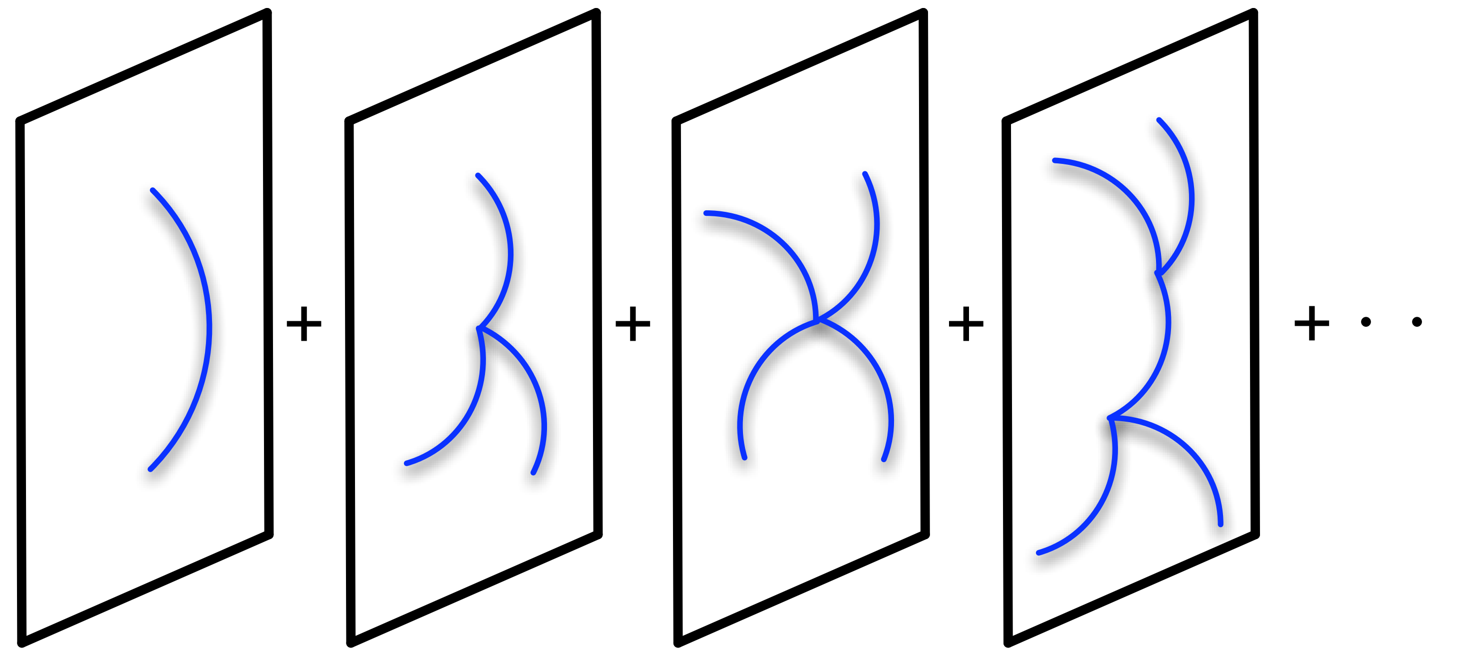
$$\Lambda_p^{\text{cl}} = T_p$$

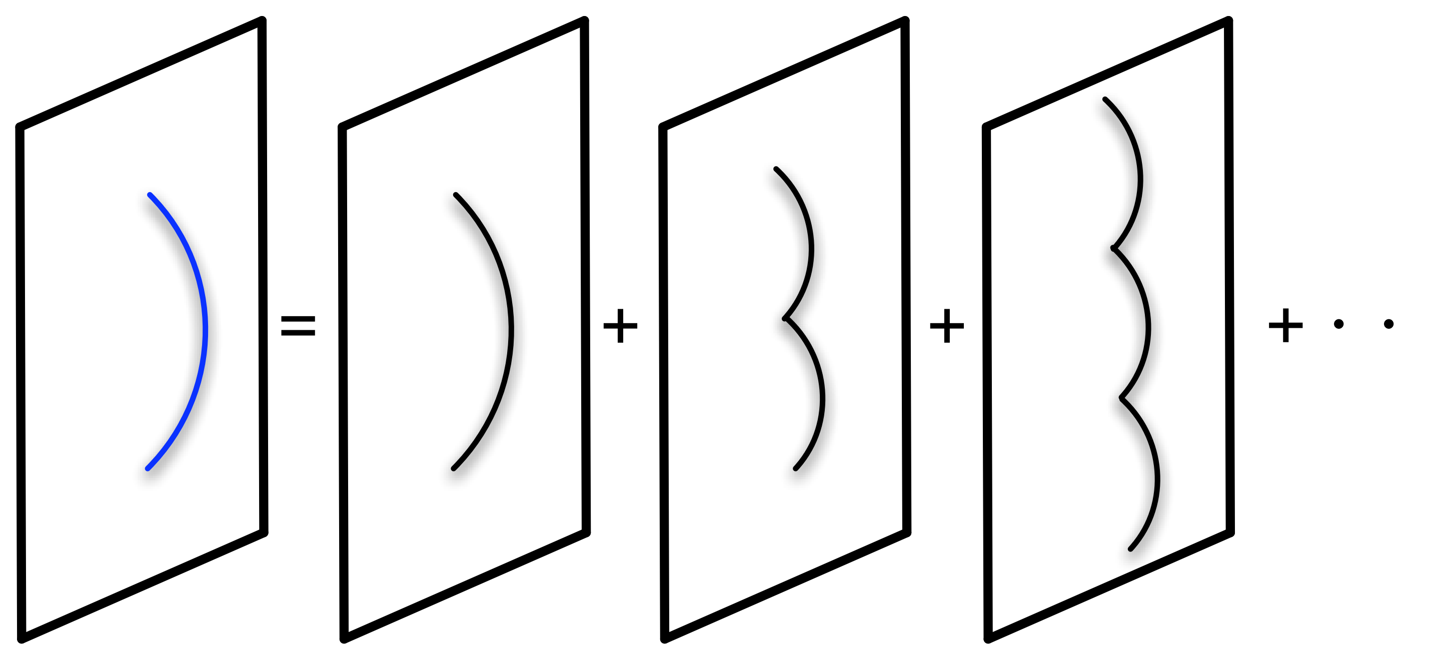
vacuum energy



A strategy for tadpole resummations

$$\Lambda_p = \Lambda_p^{\mathrm{cl}} +$$





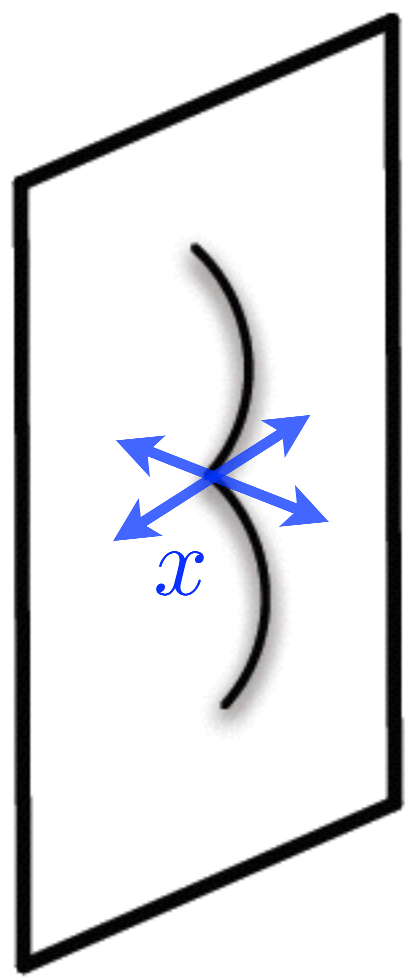
"Closed strings bouncing on a Dp-brane"

Multi-point vertices in boundary state formalism

$-T_p a^2$

$$\hat{M} \equiv \int d^d x \delta^{d_\perp}(x) |\tilde{B}_p(x)\rangle (-T_p) \langle \tilde{B}_p(x)|$$

integrating over Dp-brane world volume



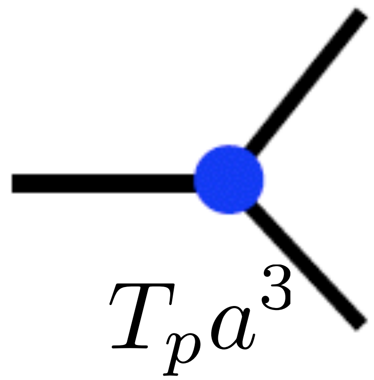
$$|\tilde{B}_p(x)\rangle \equiv \frac{1}{T_p} \delta^{p+1}(\hat{x} - x) |B_p\rangle$$

normalization
for correct coupling

specify one point
on Dp-brane

$$\left(T_p a \times \frac{1}{T_p} \right)^2 \times (-T_p) \times d^d x \delta^{d_\perp}(x) \to -T_p a^2 \times V_{p+1}$$

Similar for three-point vertex and higher



$$\hat{M}^{(3)} \equiv \frac{1}{3!} T_p \left(\frac{N_p}{T_p}\right)^3 \int d^d x \delta^{d_\perp}(x) \times |\tilde{B}_p(x)\rangle |\tilde{B}_p(x)\rangle |\tilde{B}_p(x)\rangle$$

Closer look at "propagator"

$$\Delta_p = \frac{\pi\alpha'}{2} \int_0^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \frac{1}{(\eta(is))^{24}}$$

$$\rightarrow \frac{\pi\alpha'}{2} \int^\infty ds \frac{1}{(2\pi^2\alpha' s)^{d_\perp/2}} \{e^{2\pi s} + 24 + \mathcal{O}(e^{-2\pi s})\}$$

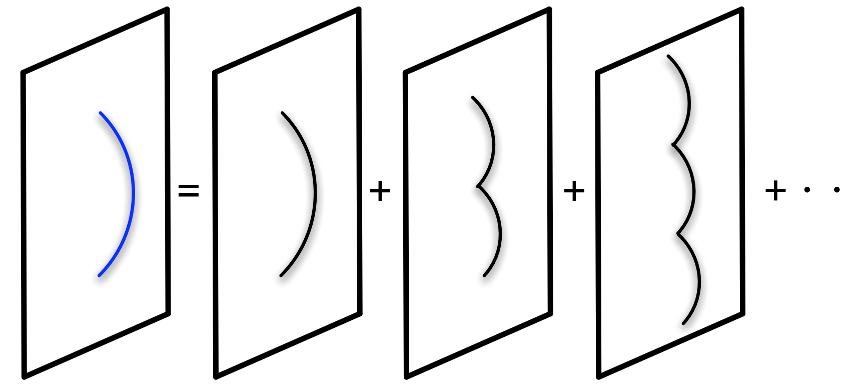
tachyon tadpole

dilaton/graviton tadpoles

Regularize via an "ultraviolet" cutoff on s

Actual calculations for vacuum energies

Full two-point function



"one bounce"

$$\frac{1}{2!}\langle B_p|D\hat{M}D|B_p\rangle = \frac{1}{2!}\int d^dx\,\delta^{d_\perp}(x)\langle B_p|D|\tilde{B}_p(x)\rangle(-T_p)\langle \tilde{B}_p(x)|D|B_p\rangle$$

$$= \frac{1}{2!}V_{p+1}N_p^2\left(\frac{N_p}{T_p}\right)^2(-T_p)(\Delta_p)^2$$

similar to the cylinder

Similar for "two bounces", and more

$$A_p^{(2)} = \frac{1}{2!}\left\{\langle B_p|D|B_p\rangle + \langle B_p|D\hat{M}D|B_p\rangle + \langle B_p|D\hat{M}D\hat{M}D|B_p\rangle + \cdots\right\}$$

geometric series

$$\equiv \frac{1}{2!}\langle B_p|D_M|B_p\rangle$$

$$= \frac{1}{2!}V_{p+1}N_p^2\frac{\Delta_p}{1+T_p(N_p/T_p)^2\Delta_p} \to \frac{1}{2!}V_{p+1}T_p$$

$$\boxed{\Lambda_p^{(2)} = -\frac{1}{2!}T_p}$$

Full three-point function

$$A_p^{(3)} = \frac{1}{3!} T_p \int d^d x \delta^{d_\perp}(x) \left(\langle B_p | D_M | \tilde{B}_p(x) \rangle \right)^3$$

$$= \frac{1}{3!} V_{p+1} T_p \left(\frac{(N_p^2/T_p)\Delta_p}{1 + T_p (N_p/T_p)^2 \Delta_p} \right)^3 \to \frac{1}{3!} V_{p+1} T_p$$

vertex

\# of contractions

$$\frac{1}{3!} = \frac{1}{3!} \times \frac{1}{3!} \times 3!$$

third order of tadpole insertions

$$\boxed{\Lambda_p^{(3)} = -\frac{1}{3!} T_p}$$

Full Vacuum Energy of Dp-brane

$$\Lambda_p^{\text{res}} \equiv -\left(A_p^{(2)} + A_p^{(3)} + \cdots \right) / V_{p+1}$$

$$= -T_p \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \boxed{-T_p}$$

exactly cancels

$$\boxed{\Lambda_p^{\text{cl}} = T_p}$$

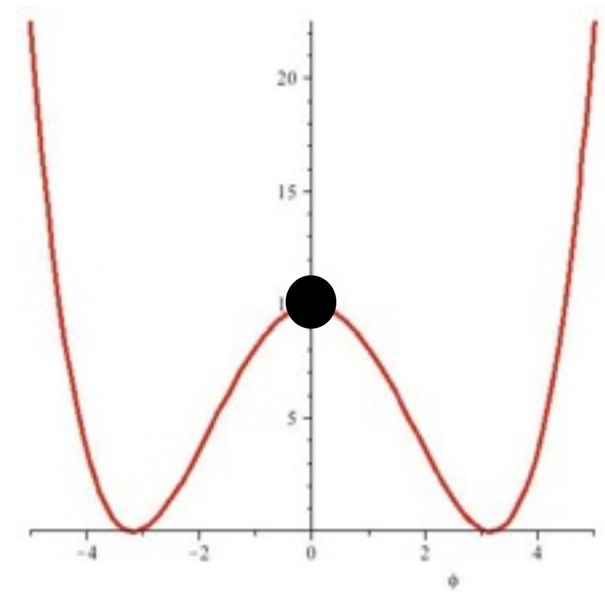
Consistent with Sen's conjecture

Two Consistency Checks

1) "Tadpole resummations" in SO(8192) theory

8192 D25-branes in unoriented Bosonic String Theory
-> (unstable) "solution" of String Theory

- no tadpoles of dilaton and graviton
- tachyon (with tadpole)



"Tadpole resumations" should not give corrections to the vacuum energy, even though tachyon exists, because this is a solution of String Theory.

Two kinds of boundary states and three kinds of amplitudes

$$|B_{25}\rangle \quad \text{and} \quad |C_{25}\rangle$$

for orientifold
fixed plane: O25

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2!}\langle B_{25}|D|B_{25}\rangle \\
\mathcal{M} &= \frac{1}{2!}\left(\langle B_{25}|D|C_{25}\rangle + \langle C_{25}|D|B_{25}\rangle\right) \\
\mathcal{K} &= \frac{1}{2!}\langle C_{25}|D|C_{25}\rangle
\end{aligned}$$

Full cylinder amplitude with bouncing on D25 and O25

$$\begin{aligned}
\mathcal{A}^{(2)} &= \frac{1}{2!} V_{26} N_{25}^2 \frac{\Delta_{25}}{1 + T_{25}(N_{25}/T_{25})^2 \Delta_{25}} \times \frac{1}{1 - T_{25}(N_{25}/T_{25})^2 \Delta_{25}} \\
&\to 0
\end{aligned}$$

(the same for the other two amplitudes)

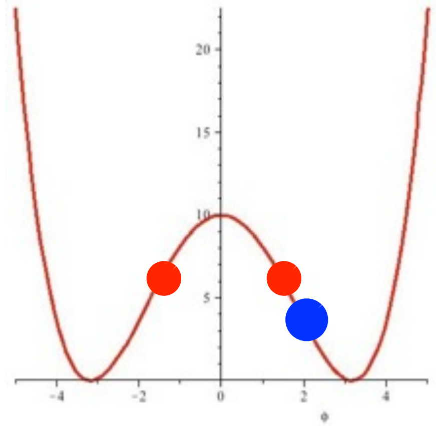
No Correction to Vacuum Energy

(Ignoring tachyon divergence -> $\mathcal{A}^{(2)} + \mathcal{M}^{(2)} + \mathcal{K}^{(2)} = 0$, and the same for multi-point functions. No correction to the vacuum energy.)

2) "Tadpole resummations" in USp(32) type I model

USp(32) type I model is believed to be stable, and anti-D9-brane is not expected to decay.

- NS-NS tadpoles (dilaton and graviton)
- no tachyon



"Tadpole resummations" should not give such a correction to the D9-brane vacuum energy that completely cancels the classical vacuum energy, even though the system is not a "solution".

$$
\begin{aligned}
\mathcal{A}^{(2)} &= \frac{1}{2!} V_{10} N_9^2 \frac{\Delta_{\rm NS}}{1 + T_9 (N_9/T_9)^2 \Delta_{\rm NS}} \times \frac{1}{1 + T_9 (N_9/T_9)^2 \Delta_{\rm NS}} \\
&\to 0
\end{aligned}
$$

No correction to anti-D9-brane vacuum energy

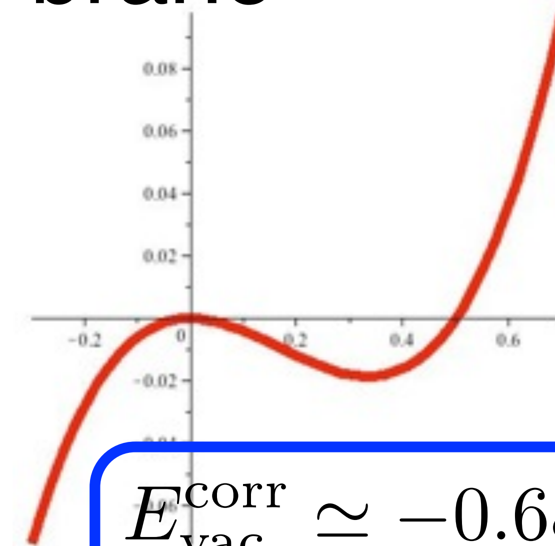
Problems and limitations of this procedure

1) On the relation with open-string field theory analysis on "tachyon condensation" of D25-brane

From Taylor-Zwiebach (hep-th/0311017):

tachyon potential (tachyon mode only)

$$V(\varphi) = -\frac{1}{2}\varphi^2 + \mu\varphi^3$$

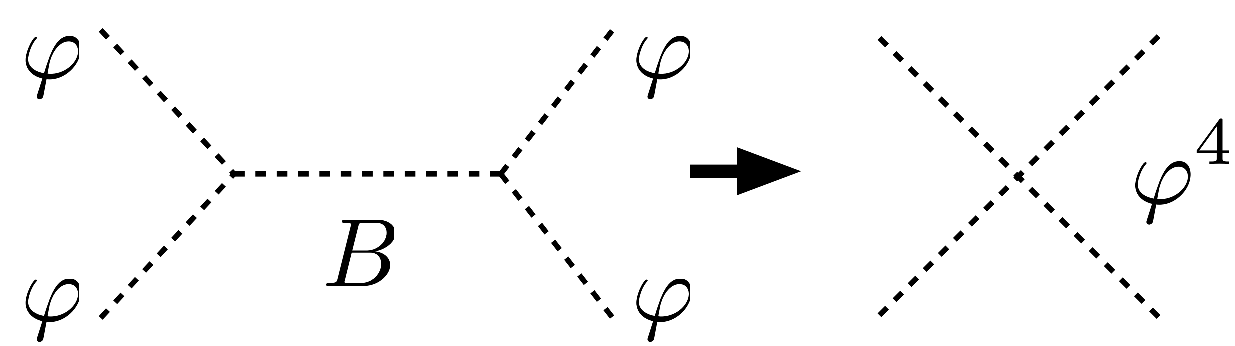


$$\boxed{E^{\rm corr}_{\rm vac} \simeq -0.68T_{25}}$$

a level truncation: $\Psi_s = \varphi|0\rangle + B(\alpha_{-1}\cdot\alpha_{-1})|0\rangle + \beta(b_{-1}c_{-1})|0\rangle + \cdots$

$$\begin{aligned} V &= -\frac{1}{2}\varphi^2 + 26B^2 - \frac{1}{2}\beta^2 \\ &+ \mu\left[\varphi^3 - \frac{130}{9}\varphi^2 B - \frac{11}{9}\varphi^2\beta + \cdots\right] \end{aligned}$$

$$\boxed{E^{\rm corr}_{\rm vac} \simeq -0.95938T_{25}}$$



heavy modes are integrated out

Open string massive modes are important in the analysis of "tachyon condensation" using open-string field theory.
Their role is not evident in the procedure of "tadpole resummations".

One we can say:

Open-closed string duality, which is essential in the procedure of "tadpole resummations", requires infinite number of open string modes.
The effects of open string massive modes are implicitly included.

2) Gravitational back reactions

The procedure of "tadpole resummations" includes "propagations of closed string" in the direction perpendicular to D-branes, assuming flat space-time.

The existence of D-branes should change the background geometry (and background fields).

ex. "spontaneous compactifications" in Sugimoto model by Dudas-Mourad (2000)

↓

It may affect the results of "tadpole resummations", although it is not included also in the analysis with string field theory.

We need to include the back reaction to understand precisely the models without supersymmetry.

The technique of "Tadpole resummations"

"Tadpole resummation in string theory,"
N.K. Phys. Lett. B660 (2008) 415.
"One-loop masses of open-string scalar fields in string theory,"
N.K., JHEP 0809 (2008) 049.

could be one possible direction, and there should be more.

This is a difficult problem,
but there are steady and serious efforts.

"String perturbation theory around dynamically shifted vacuum,"
R.Pius, A.Rudra and A.Sen, JHEP 1410 (2014) 070.

(On the shift of the vacuum by radiative corrections)

Dynamics of gauge symmetry breaking

A typical attitude of gauge symmetry breaking in models of "string phenomenology"

1. give particle contents and gauge symmetry at a high energy.
2. go to the description by quantum field theory assuming decoupling of heavy states.
3. investigate quantum correction to scalar fields and vacuum expectation values of them.

Some times, even a tachyon mode is identified as Higgs!

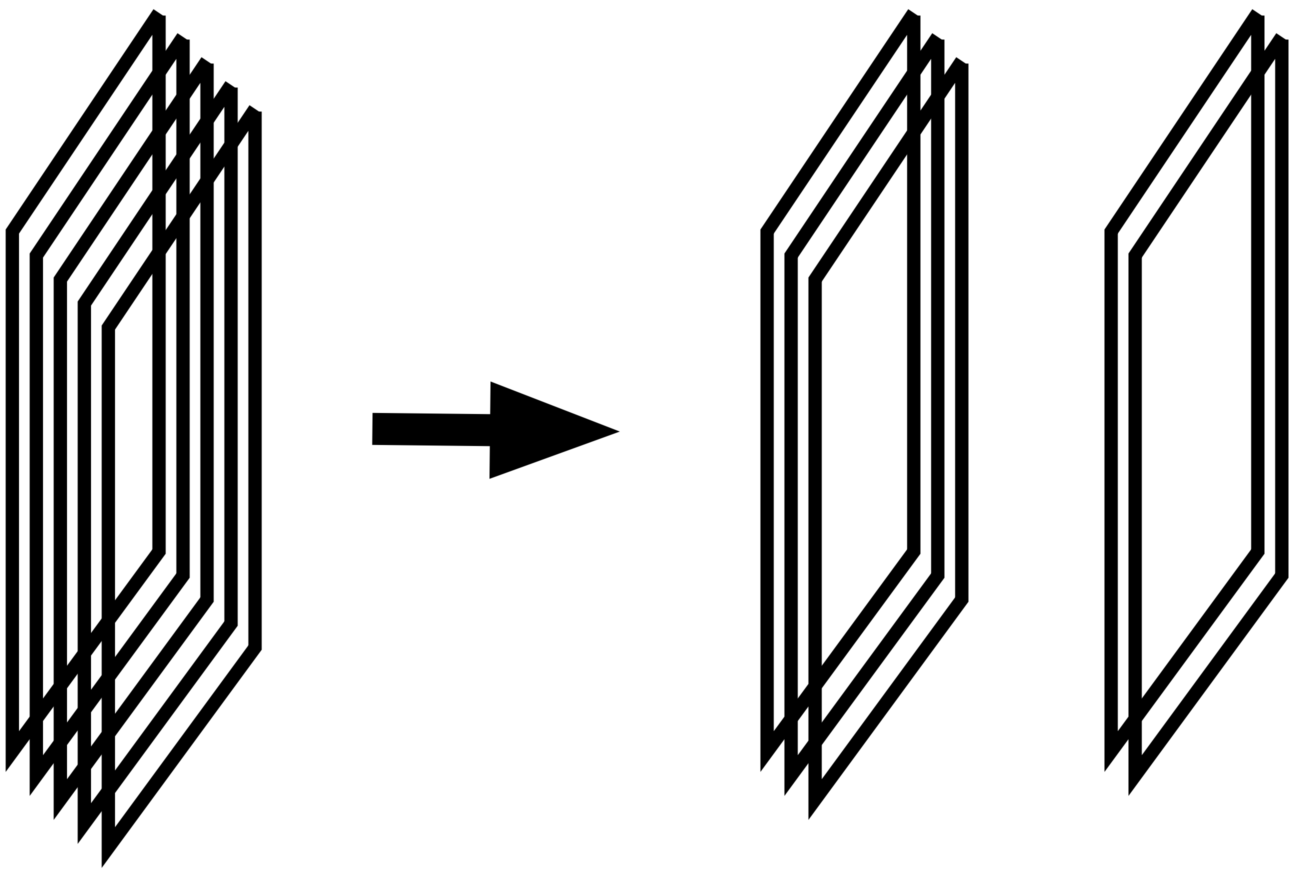
It is mysterious what happens to D-branes.

Remember:
"String Theory" is a tool just like Quantum Field Theory.
Try to use it to describe (understand) particle physics.

Parallel separations of D-branes result
gauge symmetry breaking without rank reduction.

$\mathrm{U}(5)$

rank 5



$\mathrm{U}(3) \times \mathrm{U}(2)$

rank 5

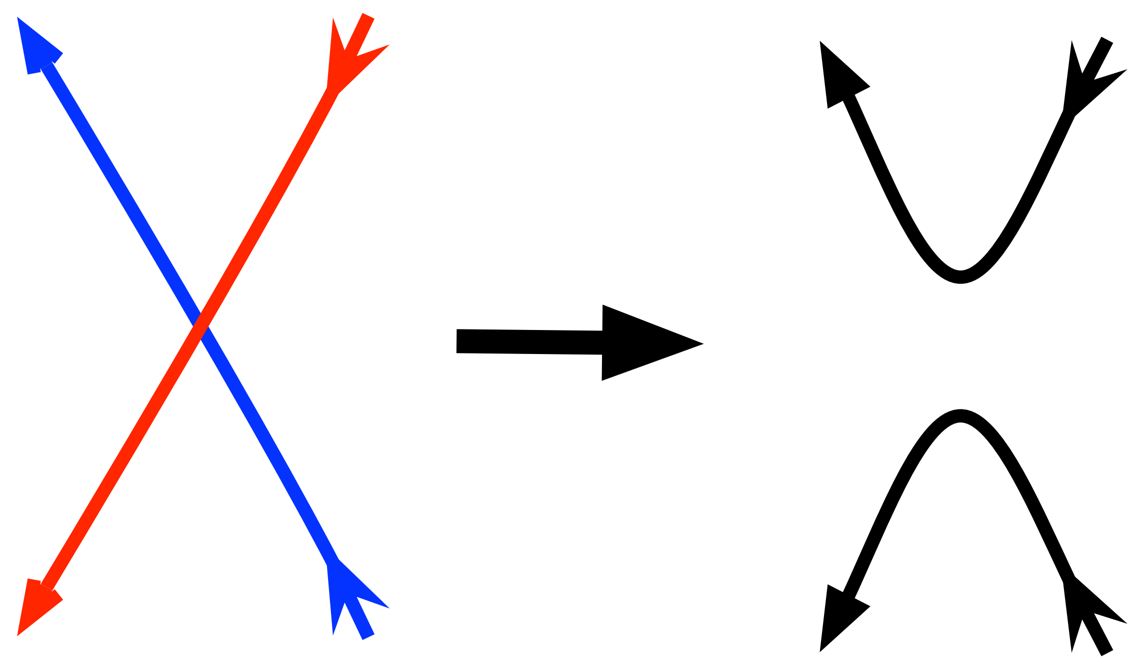
This can happen by radiative corrections in String Theory in models without supersymmetry.
Antoniadis-Benakli-Quiros (2000)

Some D-branes must "disappear" in the process like

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$$

rank 2 rank 1

Expect non-perturbative effects in String Theory?
It could be possible ("D-instanton effects").

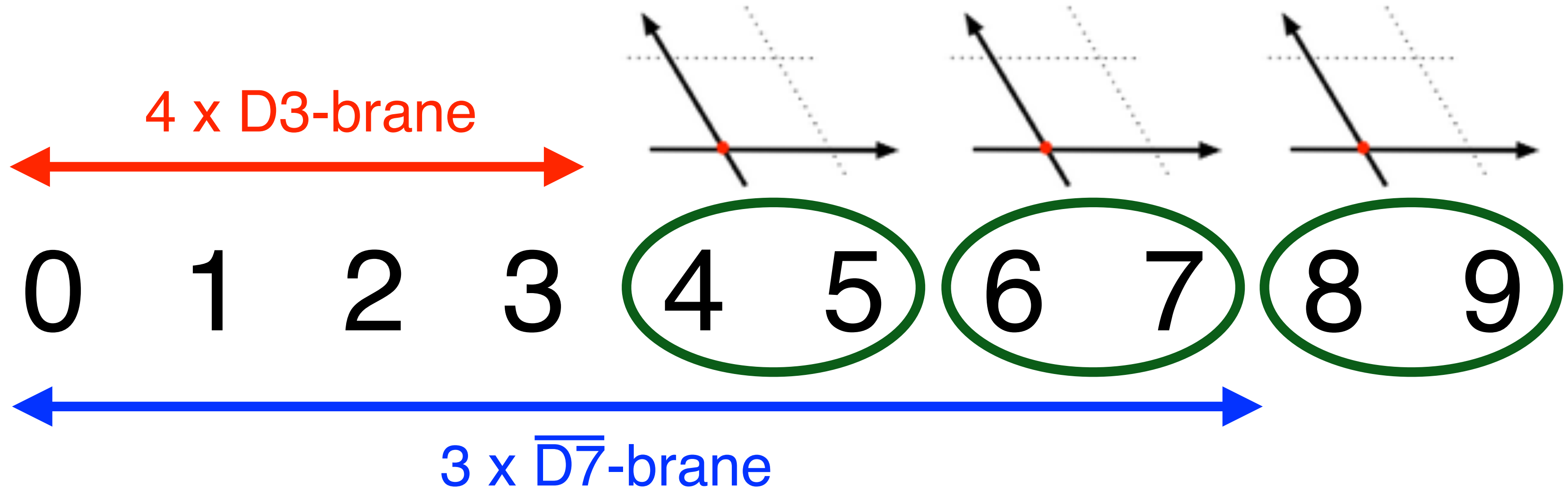


See for example, Camara-Condeescu-Dudas-Lennek (2010).

There are perturbative processes with
geometrical understandings
in the system of "D-branes at singularities".

Consider a system of "brane SUSY breaking"
that we have already seen before.

Four D3-branes and Three anti-D7-branes
at a SUSY $\mathbf{C}^3/\mathbf{Z}_3$ orbifold singularity



the same positions in 8-9 plane

$\mathbf{Z}_3$ operation matrices on each D-brane

$$\gamma_3 = \mathrm{diag}(\mathbf{1}_2, \alpha\mathbf{1}_1, \alpha^2\mathbf{1}_1) \qquad \mathrm{U}(2)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2$$

$$\gamma_{\bar{7}_3} = \mathbf{1}_3 \qquad \mathrm{U}(3)$$

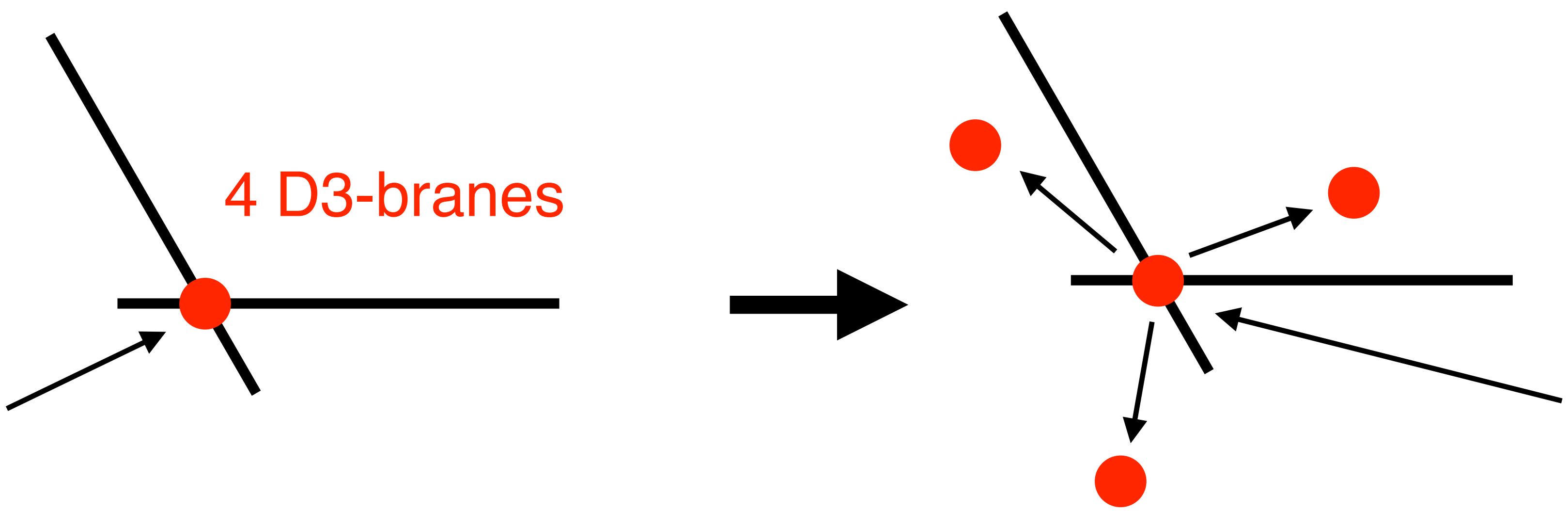
$(\alpha \equiv e^{i2\pi/3})$

$$3\mathrm{Tr}(\gamma_3) - \mathrm{Tr}(\gamma_{\bar{7}_3}) = 0$$

Identifications of D3-branes work as reductions of the number of D3-branes.

In this system with one "fractional D3-brane" and three conventional D3-branes identified by $\mathbf{Z}_3$:

$$\begin{pmatrix} 1_{2\times 2} & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha^2 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\mathrm{U}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$

rank 4

$\mathrm{U}(1) \times \mathrm{U}(1)$

rank 2

a fractional D3-brane
$T_3^{\mathrm{frac}} = T_3/3$

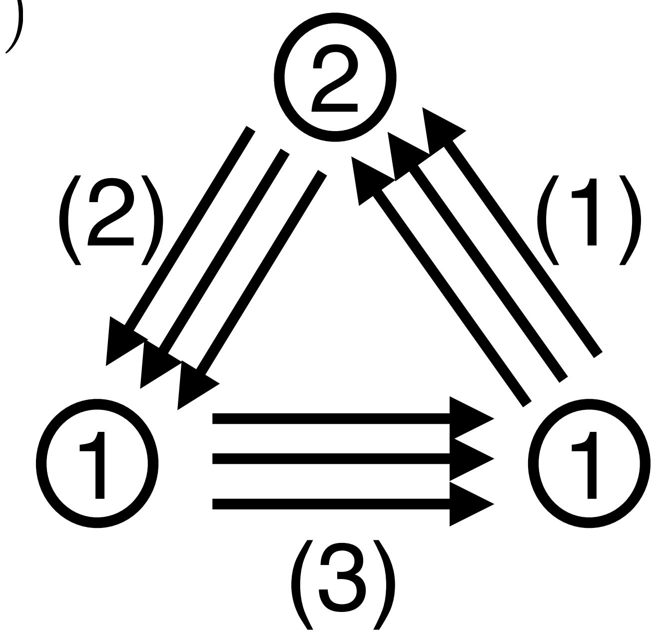
The rank is reduced through the spontaneous gauge symmetry breaking.

Summary of scalar fields in four-dimensional space-time

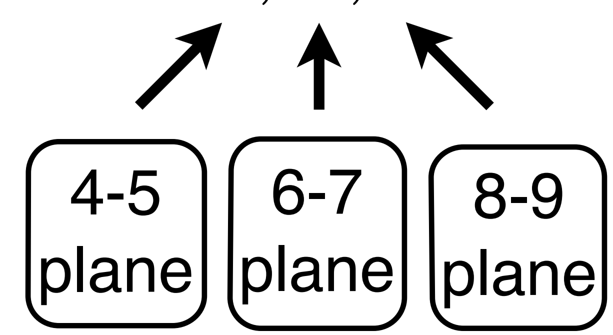
$$\overbrace{\mathrm{U}(2)\times\mathrm{U}(1)_1\times\mathrm{U}(1)_2}^{\text{D3}}\times\overbrace{\mathrm{U}(3)}^{\text{D7}}$$

from D3-D3 open string

$$\begin{cases} A_i^{(1)} \;:\; (2,0,-1,1) \\ A_i^{(2)} \;:\; (\bar{2},+1,0,1) \\ A_i^{(3)} \;:\; (1,-1,+1,1) \end{cases}$$



$i = 1, 2, 3$



from D3-$\overline{\text{D7}}$ open string

$$\begin{cases} \phi_1^{(3\bar{7})} \;:\; (2,0,0,\bar{3}) \\ \phi_2^{(3\bar{7})} \;:\; (\bar{2},0,0,3) \end{cases}$$

We need to identify moduli fields which describe the places of D3-branes (scalar components of $A_i^{(1)}, A_i^{(2)}, A_i^{(3)}$ fields).

Carefully consider the tree-level potential

Kitazawa-S.Kobayashi (2013)

Scalar fields from D3-D3 open string: "D3-D3 sector"

There is 4D N=1 supersymmetry on D3-brane:

$$\boxed{V_{33} = V_F + V_D}$$

$$V_F = |F_i^{(1)}|^2 + |F_i^{(2)}|^2 + |F_i^{(3)}|^2$$

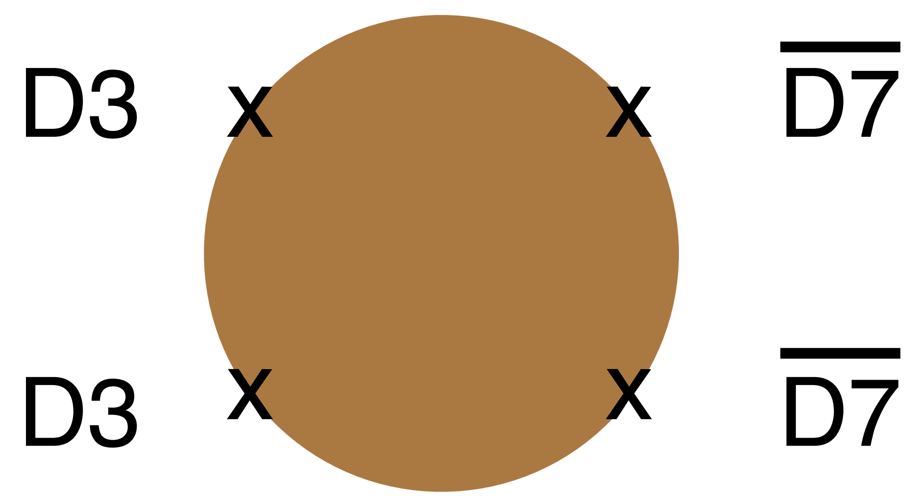
$$F_i^{(1)\dagger\alpha} = g\epsilon_{ijk} A_j^{(3)} A_k^{(2)\alpha} \ , \ F_{i\alpha}^{(2)\dagger} = g\epsilon_{ijk} A_{j\alpha}^{(1)} A_k^{(3)} \ , \ F_i^{(3)\dagger} = g\epsilon_{ijk} A_j^{(2)\alpha} A_{k\alpha}^{(1)} .$$

$$V_D = \frac{1}{2}(D^a_{U(2)})^2 + \frac{1}{2}D_1^2 + \frac{1}{2}D_2^2$$

$$\begin{aligned} D^a_{U(2)} &= -g\left(A_i^{(1)\dagger} T^a A_i^{(1)} - A_i^{(2)\dagger}(T^a)^* A_i^{(2)}\right), \\ D_1 &= -\frac{g}{\sqrt{2}}\left(|A_i^{(2)}|^2 - |A_i^{(3)}|^2\right), \\ D_2 &= -\frac{g}{\sqrt{2}}\left(|A_i^{(3)}|^2 - |A_i^{(1)}|^2\right). \end{aligned}$$

$\boxed{g : \text{gauge coupling}}$

The potential with D3-D3 fields and D3-$\overline{\text{D7}}$ fields can be obtained from string disc amplitudes.



$$
\begin{aligned}
V_{\rm mixed} = & -\frac{g^2}{4} A_1^{(1)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right) A_1^{(1)} + \frac{g^2}{4} A_1^{(2)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right) A_1^{(2)} \\
& + \frac{g^2}{4} A_2^{(1)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right) A_2^{(1)} - \frac{g^2}{4} A_2^{(2)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right) A_2^{(2)} \\
& + \frac{g^2}{4} A_3^{(1)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right) A_3^{(1)} + \frac{g^2}{4} A_3^{(2)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right) A_3^{(2)}
\end{aligned}
$$

No term including U(2) singlet $A_i^{(3)}$

The potential of D3-$\overline{\text{D7}}$ sector only

The general form under gauge symmetry and the trace structure of string disc amplitudes:

$$V_{37} = \kappa_1 g^2 \mathrm{Tr}\left(\phi_1^{(3\bar{7})\dagger}\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger}\phi_1^{(3\bar{7})}\right) + \kappa_2 g^2 \mathrm{Tr}\left(\phi_2^{(3\bar{7})}\phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\phi_2^{(3\bar{7})\dagger}\right)$$
$$+\kappa_3 g^2 \mathrm{Tr}\left(\phi_1^{(3\bar{7})\dagger}\phi_1^{(3\bar{7})}\phi_2^{(3\bar{7})}\phi_2^{(3\bar{7})\dagger}\right) + \kappa_4 g^2 \mathrm{Tr}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)$$

The values of coupling constants strongly depend on the model (namely compactification).

Here, we assume that all of them are positive and that the vacuum expectation values of D3-$\overline{\text{D7}}$ fields are zero.

(Assuming nothing happens on $\overline{\text{D7}}$-brane)

There are flat directions in the potential of D3-D3 scalars.

$$A_i^{(1)} = \begin{pmatrix} a_i \\ 0 \end{pmatrix} \ , \ A_i^{(2)} = \begin{pmatrix} b_i \\ 0 \end{pmatrix} \ , \ A_i^{(3)} = c_i$$

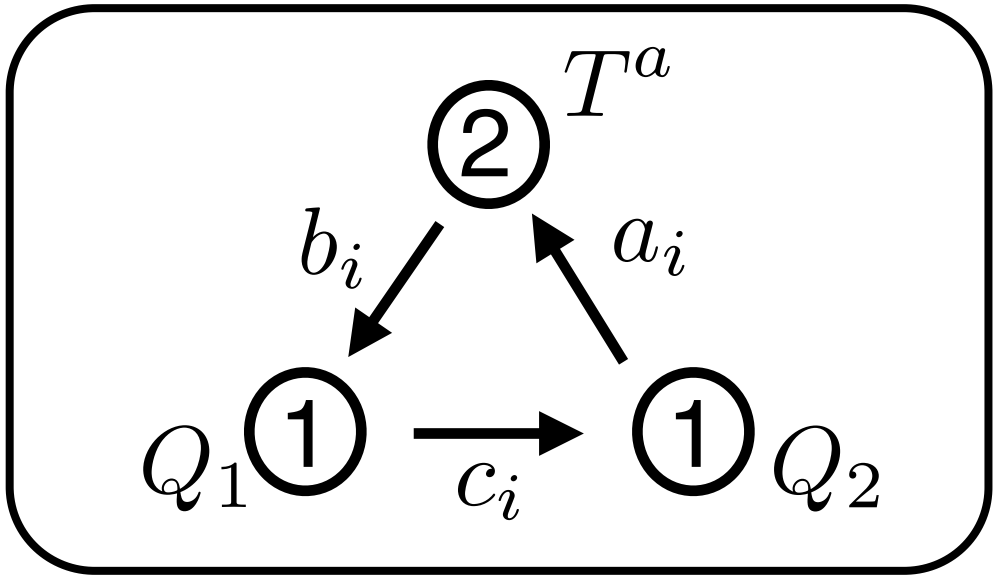
$$a_i = b_i = c_i \equiv v_i \qquad\qquad i = 1, 2, 3$$

Gauge symmetry breaking at the place of non-zero VEV:

$$\begin{array}{ccccc} T^a & Q_1 & Q_2 & (T^0 - T^3) & Q_1 + Q_2 + (T^0 + T^3) \end{array}$$

$$\mathrm{U}(2) \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_2 \longrightarrow \mathrm{U}(1) \times \mathrm{U}(1)'$$

The rank of group decreases.



The gauge boson mass spectrum
(consider i=3 only, for simplicity)

Original gauge bosons

$$\mathrm{U}(2): W^0, W^1, W^2, W^3 \qquad \mathrm{U}(1)_1: B^1 \qquad \mathrm{U}(1)_2: B^2$$

$$\boxed{m^2 = 0}$$

$$A^1 = \frac{1}{\sqrt{2}} W^0 + \frac{1}{\sqrt{2}} W^3$$

$$A^2 = \frac{1}{\sqrt{6}} W^0 - \frac{1}{\sqrt{6}} W^3 + \frac{1}{\sqrt{3}} B^1 + \frac{1}{\sqrt{3}} B^2$$

$$\boxed{m^2 = 3g^2v^2}$$

$$Z^1 = \frac{1}{\sqrt{2}} B^1 - \frac{1}{\sqrt{2}} B^2$$

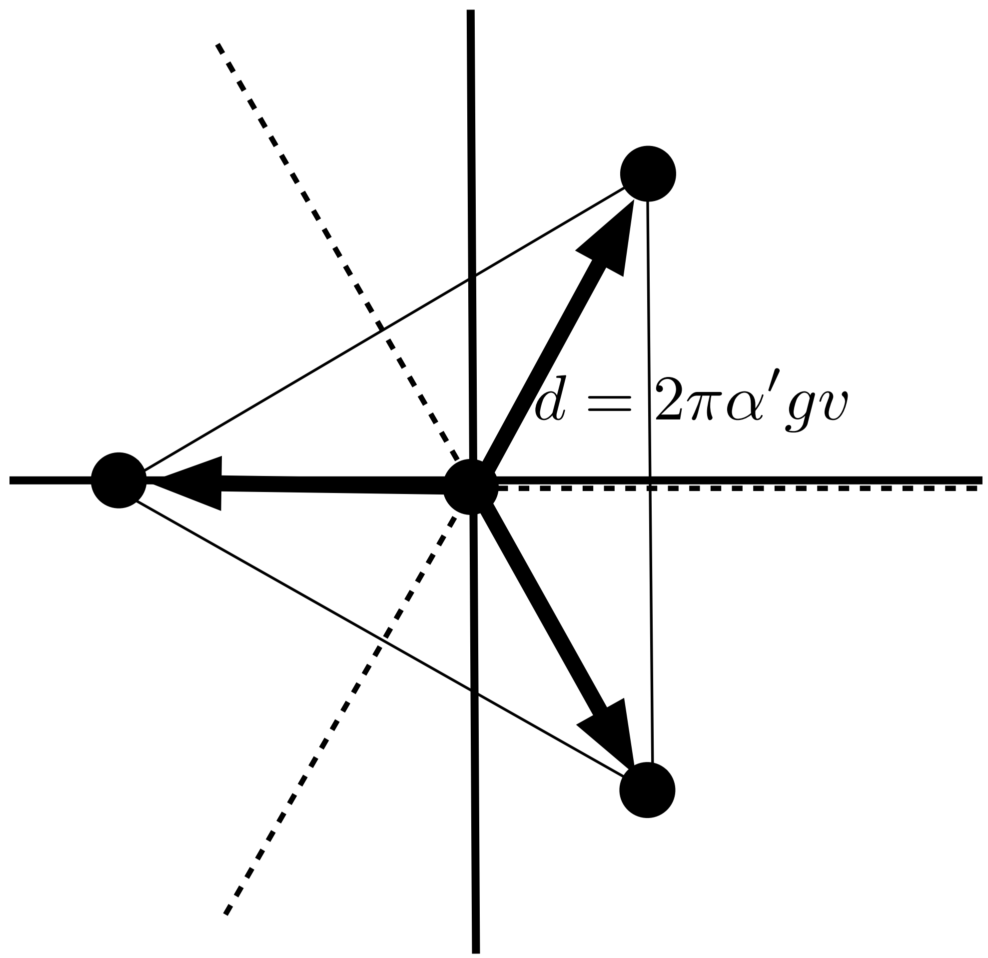
$$Z^2 = \frac{1}{\sqrt{3}} W^0 - \frac{1}{\sqrt{3}} W^3 - \frac{1}{\sqrt{6}} B^1 - \frac{1}{\sqrt{6}} B^2$$

$$\boxed{m^2 = g^2v^2}$$

$$W^{\pm} = \frac{1}{\sqrt{2}} \left(W^1 \mp i W^2 \right)$$

A geometrical interpretation

A separation of three D3-branes in a $\mathbf{Z}_3$ invariant way.



This schematic picture represents D3-brane configuration in 8-9 plane ($i = 3$ case).

The same for 4-5, 6-7 planes for $i = 1, 2$ case, respectively.

$$U \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha^2 \end{pmatrix} U^\dagger = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$(T^0 + T^3)$-Q_1-Q_2 basis

The bases of D3-branes should be changed.

This geometric interpretation is supported by the D3-D3 and D3-$\overline{\text{D7}}$ mixed potential.

$$
\begin{aligned}
V_{\text{mixed}} = & -\frac{g^2}{4} A_1^{(1)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)A_1^{(1)} + \frac{g^2}{4} A_1^{(2)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)A_1^{(2)} \\
& + \frac{g^2}{4} A_2^{(1)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)A_2^{(1)} - \frac{g^2}{4} A_2^{(2)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)A_2^{(2)} \\
& + \frac{g^2}{4} A_3^{(1)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)A_3^{(1)} + \frac{g^2}{4} A_3^{(2)\dagger}\left(\phi_1^{(3\bar{7})}\phi_1^{(3\bar{7})\dagger} + \phi_2^{(3\bar{7})\dagger}\phi_2^{(3\bar{7})}\right)A_3^{(2)}
\end{aligned}
$$

VEV of $A_1^{(1)} = A_1^{(2)}$ and $A_2^{(1)} = A_2^{(2)}$ do not give masses to D3-$\overline{\text{D7}}$ scalar fields, because D3-brane separations in 4-5 and 6-7 planes do not give separations between D3 and $\overline{\text{D7}}$ branes.

VEV of $A_3^{(1)} = A_3^{(2)}$ gives mass to D3-$\overline{\text{D7}}$ scalar fields, because D3 and $\overline{\text{D7}}$ branes are separated in 8-9 plane.

Introduce Fayet-Iliopoulos terms
which are induced by VEVs of twisted moduli fields.

twisted closed string
(closed up to Z_3)

$$D_1 = -\frac{g}{\sqrt{2}}\left(|A_i^{(2)}|^2 - |A_i^{(3)}|^2 + \xi_1\right),$$

$$D_2 = -\frac{g}{\sqrt{2}}\left(|A_i^{(3)}|^2 - |A_i^{(1)}|^2 + \xi_2\right),$$

$$D_{U(2)}^{a=0} = -\frac{g}{2}\left(|A_i^{(1)}|^2 - |A_i^{(2)}|^2 + \xi_3\right).$$

$$\xi_1 = -\frac{1}{2}\phi_R - \frac{\sqrt{3}}{2}\phi_I\ ,\ \xi_2 = -\frac{1}{2}\phi_R + \frac{\sqrt{3}}{2}\phi_I\ ,\ \xi_3 = 2\phi_R.$$

$\phi = \phi_R + i\phi_I$: VEV of a twisted moduli field

(normalized to have dimension 2 by $M_s \equiv 1/\sqrt{\alpha'}$)

-> "resolved singularity"

Douglas-Greene-Morrison (1997)

The flat direction remains (consider i=3 only for simplicity)

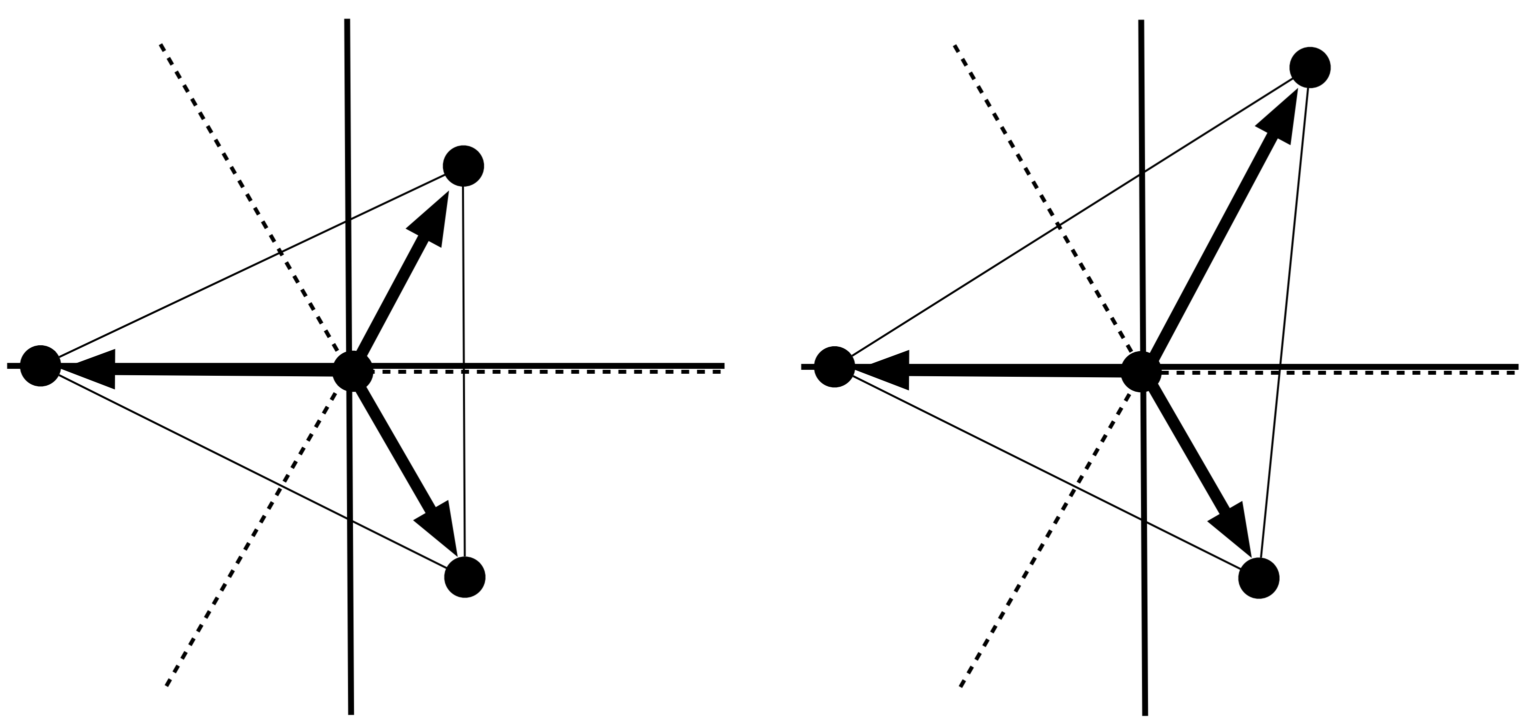
$$A^{(1)} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad A^{(2)} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \quad A^{(3)} = c.$$

$$\begin{aligned} b^2 &= a^2 + \phi_R, \\ c^2 &= a^2 + \frac{1}{2}\phi_R - \frac{\sqrt{3}}{2}\phi_I. \end{aligned}$$

The origin $a = b = c = 0$ is excluded.

$$V_{\text{origin}} = \frac{5g^2}{8}\phi_R^2 + \frac{3g^2}{8}\phi_I^2$$

$$V_{\text{flat}} = \frac{g^2}{4}\phi_R^2$$



$\phi_R = 0,\ \phi_I \neq 0$ $\qquad$ $\phi_R \neq 0,\ \phi_I \neq 0$

We assume the stabilization of twisted moduli.

The singularity is "resolved".

There should be one-loop mass corrections.

$$V_{\mathrm{mass}} = m_1^2|A^{(1)}|^2 + m_2^2|A^{(2)}|^2 + m_3^2|A^{(3)}|^2$$

by
D3-$\overline{\text{D7}}$ string

$$m_1^2 = m_2^2 = m_3^2 \equiv m^2 \simeq \frac{1}{\omega^2}\frac{g^2}{16\pi^2}M_s^2 > 0 \qquad \omega \sim \mathcal{O}(1)$$

(an order estimate)

The flat direction is lifted up, and
there are discrete solutions of stable vacua, for example:

$$\begin{aligned} a^2 &= -\phi_R - \frac{2}{g^2}m^2, \\ b^2 &= 0, \\ c^2 &= -\frac{1}{2}\phi_R - \frac{\sqrt{3}}{2}\phi_I - \frac{2}{g^2}m^2 \end{aligned}$$

The solution with only one SU(2) doublet "Higgs" has VEV.

The relation between gauge symmetry breaking scale and string scale

Reasonably assuming that
the VEV of twisted moduli are less than string scale:

$$v^2 \simeq \frac{4m^2}{g^2} \simeq \frac{M_s^2}{4\pi^2\omega^2} \qquad \left(a = v/\sqrt{2}\right)$$

If $v \simeq 250\mathrm{GeV}$ (electroweak scale), the upper bound:

$$M_s \simeq 1600 \times \omega \ \mathrm{GeV}$$

$M_s > 7\ \mathrm{TeV}$ by CMS, Phys. Rev. Lett. 116 (2016) 071801. $\longrightarrow$ $\omega \gtrsim 4.5$

Anyway:

We have a geometrical understanding of
gauge symmetry breaking in String Theory.

We need some dynamics to stabilize the D-brane moduli
so that the distance should be very small
in comparison with the string scale.

A truly dynamical stabilization:
D3-branes revolving around the origin.

S.Iso-N.K. (2015)

The supersymmetry breaking mass term for moduli
by D3-anti-D7-brane open string acts as an attractive force,
and centrifugal force by revolution acts as a repulsive force.

The problem:
Lorentz symmetry violation in the world volume of D3-brane.

Any other good idea?